

Gauged Flavor Symmetries

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based on

J.H., Werner Rodejohann,
PRD 84 (2011), PRD 85 (2012);
Takeshi Araki, J.H., Jisuke Kubo,
JHEP 07 (2012).

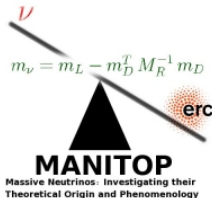
INTERNATIONAL
MAX PLANCK
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FOR PRECISION TESTS
OF FUNDAMENTAL
SYMMETRIES



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Outline

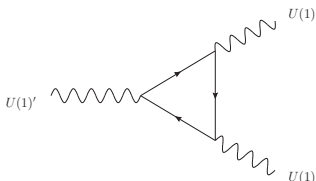
- Motivation for $U(1)$ symmetries in type-I seesaw
- Neutrino hierarchies from $U(1)$
- Texture zeros and vanishing minors
- Summary

Flavor Symmetries

- Most popular way: discrete non-abelian global symmetries (A_4, \dots)
 - Drawbacks: complicated, many parameters and fields, unknown scalar sector at unreachable energies. . .
- Here: continuous abelian local symmetries, i.e. $U(1)$
 - Very simple, i.e. few new particles/parameters
 - Testable outside of neutrino sector, e.g. Z' at LHC
- How to choose the $U(1)$:
 - Same charge for all quarks (FCNC): $Y'(q_L) = Y'(u_R) = Y'(d_R)$.
 - Add three N_i with charges that allow m_D :

$$Y'(\ell_{Li}) = Y'(e_{Ri}) = Y'(N_{Ri}), \quad Y'(\ell_{Li}) \neq Y'(\ell_{Lj}).$$

- Anomaly cancellation:



Maximal Abelian Gauge Group

- Only one constraint on the charges:

$$9 Y'(q_L) + Y'(\ell_{L1}) + Y'(\ell_{L2}) + Y'(\ell_{L3}) = 0.$$

- Two cases:

$$B - \sum_{\alpha} x_{\alpha} L_{\alpha} \text{ with } \sum_{\alpha} x_{\alpha} = 3 \text{ [E. Ma, (1998)],}$$

$$\sum_{\alpha} y_{\alpha} L_{\alpha} \text{ with } \sum_{\alpha} y_{\alpha} = 0.$$

- Famous examples: $B - L$ (GUT?), $L_{\alpha} - L_{\beta}$ (anomaly free in the SM) [R. Foot, *MPL A6 (1991)*]
- More general:

$$SU(3)_{\ell} \times U(1)_{B-L} \supset U(1)_{L_e - L_{\mu}} \times U(1)_{L_{\mu} - L_{\tau}} \times U(1)_{B-L}$$

can be added to G_{SM} without anomalies ($\ell_{Li} \sim \mathbf{3}_{\ell}$, $e_{Ri} \sim \mathbf{3}_{\ell}$, $N_{Ri} \sim \mathbf{3}_{\ell}$).

Now what?

- Three interesting zeroth order approximations:

$$\mathcal{M}_\nu^{L_e} \sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & \times & \times \\ 0 & \times & \times \end{pmatrix}, \quad \mathcal{M}_\nu^{\bar{L}} \sim \begin{pmatrix} 0 & \times & \times \\ \times & 0 & 0 \\ \times & 0 & 0 \end{pmatrix}, \quad \mathcal{M}_\nu^{L_\mu - L_\tau} \sim \begin{pmatrix} \times & 0 & 0 \\ 0 & 0 & \times \\ 0 & \times & 0 \end{pmatrix}.$$

[G. Branco, W. Grimus, L. Lavoura, *NPB (1989)*; S. Choubey, W. Rodejohann, *EPJC 40 (2005)*]

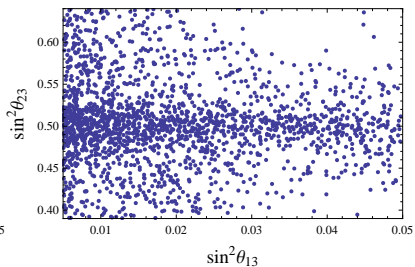
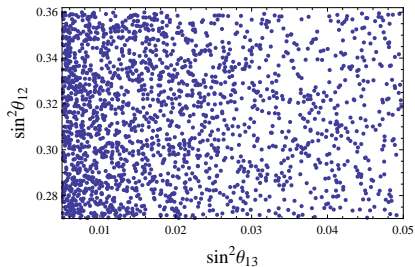
- Conserve L_e , $\bar{L} \equiv L_e - L_\mu - L_\tau$ and $L_\mu - L_\tau$, and lead to NH, IH and QD, respectively.
- \bar{L} often used as global symmetry, gives bimaximal mixing.
- $L_\mu - L_\tau$ is special: $\mathcal{M}_\nu^{L_\mu - L_\tau}$ is invertible \Rightarrow can be obtained from type-I seesaw.

$L_\mu - L_\tau$

- Dirac matrices diagonal due to symmetry, \mathcal{M}_R of the $L_\mu - L_\tau$ symmetric form
- Seesaw gives $L_\mu - L_\tau$ symmetric \mathcal{M}_ν
- Add one or two scalars S that couple to $\overline{N}_i^c N_j$ and get a VEV
- VEV fills zeros in \mathcal{M}_R and \mathcal{M}_ν and gives mass to Z' boson
 $M_{Z'}/g' \sim \langle S \rangle$
- For mixing angles: $\langle S \rangle \sim 10^{-2} |\mathcal{M}_R|$
 \Rightarrow connection of seesaw-scale and $M_{Z'}$
 \Rightarrow LHC can probe low seesaw-scale

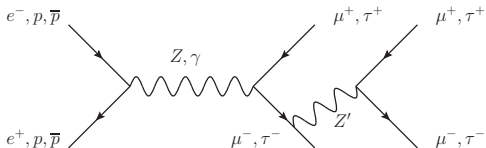
$$L_\mu - L_\tau$$

Two scalars, $\varepsilon = v_S/|\mathcal{M}_R| = 0.02$:

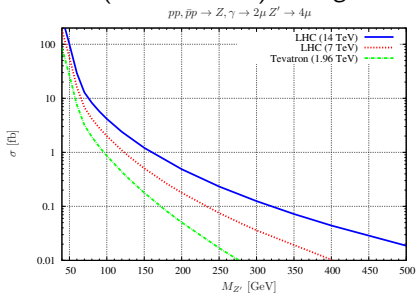


$L_\mu - L_\tau$ at LHC

- Signal at LHC [S. Baek et al, *PRD 64 (2001)*]

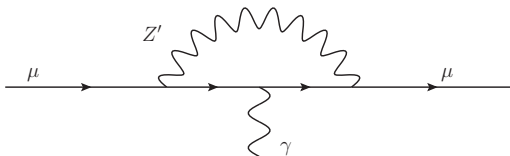


- Full run ($L = 100 \text{ fb}^{-1}$) with $g' = 1$: Up to $M_{Z'} = 350 \text{ GeV}$



$L_\mu - L_\tau$ at low energies

- Magnetic moment of muon:



For $M_{Z'} \gg m_\mu$: $\Delta a_\mu \simeq \frac{1}{12\pi^2} \frac{g'^2}{M_{Z'}^2} m_\mu^2$

- To explain $\sim 3.6\sigma$ deviation: $M_{Z'}/g' \simeq 220 \text{ GeV}$ [E. Ma et al, *PLB* 525 (2002)]

L_e and \bar{L}

- Try $B - 3L_e$ and $B + 3\bar{L}$ as anomaly free symmetries with the right lepton structure.
- Same procedure as for $L_\mu - L_\tau$ does not work, $\mathcal{M}_R^{L_e}$ and $\mathcal{M}_R^{\bar{L}}$ are not invertible.
- Weird coincidence:

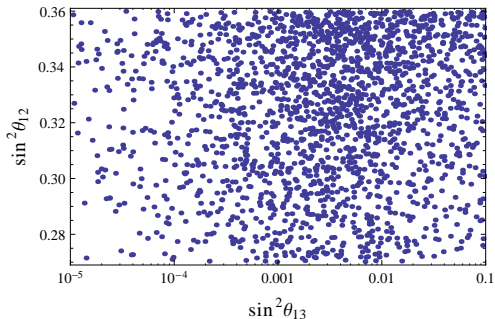
$$\mathcal{M}_R \sim \begin{pmatrix} \varepsilon & 1 & 1 \\ \cdot & \varepsilon & \varepsilon \\ \cdot & \cdot & \varepsilon \end{pmatrix} \quad \Rightarrow \quad \mathcal{M}_\nu \sim \begin{pmatrix} \varepsilon^2 & \varepsilon & \varepsilon \\ \cdot & 1 & 1 \\ \cdot & \cdot & 1 \end{pmatrix}.$$

Low $B + 3\bar{L}$ breaking gives approximate L_e symmetry!

\Rightarrow Broken $B + 3\bar{L}$ (say $\varepsilon = 0.05$) gives NH structure.

Mixing Angles and Collider

- Since μ and τ have same Y' charge, m_D not diagonal $\Rightarrow \theta_{23}$ random (i.e. large).
- One scalar S with charge $Y'(S) = 6$ and VEV $1-10 \text{ TeV} < v_S \sim \varepsilon |\mathcal{M}_R|$ gives:



- LEP-II constraint: $v_S > 2.3 \text{ TeV}$, LHC prospects in [H. S. Lee and E. Ma, *PLB 688 (2010)*].

How to get Inverted Hierarchy

- Problem: anomaly cancellation demands odd number of RHN, but then $\mathcal{M}_R^{\bar{L}}$ is not invertible, which destroys symmetry!
- Solution: Decouple one N_i with a \mathbb{Z}_2 .
- Make N_3 odd:

$$\begin{aligned}\mathcal{L}_{N_3} &= i\bar{N}_3\gamma^\mu(\partial_\mu - i(-3)g'Z'_\mu)N_3 - Y_\chi S\bar{N}_3^c N_3 + \text{h.c.} \\ &= \frac{i}{2}\chi^T C\gamma^\mu\partial_\mu\chi - \frac{3}{2}g'Z'_\mu\chi^T C\gamma^\mu\gamma_5\chi - Y_\chi\frac{v_S}{\sqrt{2}}\chi^T C\chi\left(1 + \frac{s}{v_S}\right).\end{aligned}$$

Majorana fermion χ will be dark matter candidate.

- Neutrino mass:

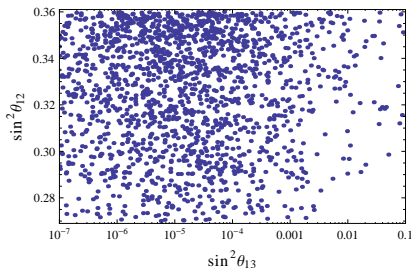
$$\mathcal{M}_\nu \simeq - \begin{pmatrix} a & 0 \\ 0 & b \\ 0 & c \end{pmatrix} \begin{pmatrix} A & X \\ X & B \end{pmatrix}^{-1} \begin{pmatrix} a & 0 & 0 \\ 0 & b & c \end{pmatrix} \sim \begin{pmatrix} a^2 B & -abX & -acX \\ \cdot & b^2 A & bcA \\ \cdot & \cdot & c^2 A \end{pmatrix}.$$

Mixing Angles

- Only two neutrinos massive, \mathcal{M}_ν exhibits “scaling” $\Rightarrow \theta_{13} = 0$.
- Solution: Add five N_i instead of three (still anomaly free) and decouple one of them:

$$\mathcal{M}_\nu \simeq - \begin{pmatrix} a & b & 0 & 0 \\ 0 & 0 & c & d \\ 0 & 0 & e & f \end{pmatrix} \begin{pmatrix} \mathcal{A} & \mathcal{X} \\ \mathcal{X}^T & \mathcal{B} \end{pmatrix}^{-1} \begin{pmatrix} a & 0 & 0 \\ b & 0 & 0 \\ 0 & c & e \\ 0 & d & f \end{pmatrix}.$$

- Need larger breaking than before. $\varepsilon = 0.1$:



Scalar Sector

- Scalar sector identical to $U(1)_{B-L}$ models: [L. Basso et al, *PRD* 80 (2009)]

$$V(H, S) = -\mu_1^2 |H|^2 + \lambda_1 |H|^4 - \mu_2^2 |S|^2 + \lambda_2 |S|^4 + \delta |S|^2 |H|^2,$$

- In unitary gauge, mixing via δ :

$$\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} = \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} h \\ s \end{pmatrix}, \quad \tan 2\alpha = \frac{\delta v v_S}{\lambda_2 v_S^2 - \lambda_1 v^2}.$$

- Masses:

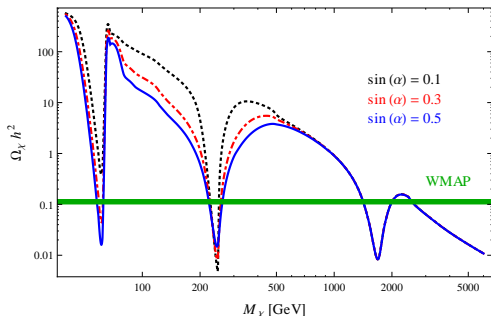
$$M_{Z'} = 6g' v_S, \quad m_2 \simeq m_s \simeq \sqrt{2\lambda_2} v_S, \quad M_\chi = \sqrt{2} Y_\chi v_S.$$

Dark Matter

- Dark matter sector identical to $U(1)_{B-L} \times \mathbb{Z}_2$ sector: [N. Okada and O. Seto, *PRD 82 (2010)*]

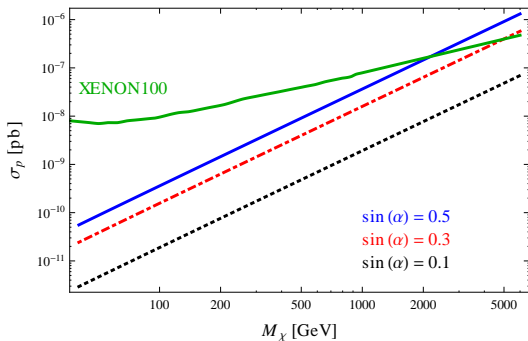
$$\mathcal{L}_\chi = \frac{i}{2} \chi^T C \gamma^\mu \partial_\mu \chi - \frac{3}{2} g' Z'_\mu \chi^T C \gamma^\mu \gamma_5 \chi - Y_\chi \frac{v_S}{\sqrt{2}} \chi^T C \chi \left(1 + \frac{s}{v_S} \right).$$

- Relic density can be obtained around the scalar resonances or the Z' resonance ($m_1 = 125$ GeV, $m_2 = 500$ GeV, $M_{Z'} = 3.5$ TeV):



Direct Detection

- Direct detection cross section via Z' are suppressed by Lorentz structure: $\bar{\chi}\gamma^\mu\gamma_5\chi\bar{f}\gamma_\mu f$.
- XENON1T only sensitive to scalar exchange:



And now for something completely different. . .

Texture Zeros

- $B - \sum_{\alpha} x_{\alpha} L_{\alpha}$ or $\sum_{\alpha} y_{\alpha} L_{\alpha}$ symmetries forbid entries in \mathcal{M}_R
 \Rightarrow texture zeros!

$$Y'(\bar{N}_i^c N_j) = \begin{pmatrix} -2x_e & -x_e - x_{\mu} & x_{\mu} - 3 \\ -x_e - x_{\mu} & -2x_{\mu} & x_e - 3 \\ x_{\mu} - 3 & x_e - 3 & 2x_e + 2x_{\mu} - 6 \end{pmatrix}$$

- Choose x_i so that some entries are allowed, others filled by $\langle S \rangle$, e.g. $B - L_e + L_{\mu} - 3L_{\tau}$ with $Y'(S) = 2$:

$$\begin{aligned} \mathcal{M}_R &= M_0 \begin{pmatrix} 0 & \times & 0 \\ \cdot & 0 & 0 \\ \cdot & \cdot & 0 \end{pmatrix} + \langle S \rangle \begin{pmatrix} \times & 0 & \times \\ \cdot & \times & 0 \\ \cdot & \cdot & 0 \end{pmatrix} \\ &\sim \begin{pmatrix} \times & \times & \times \\ \cdot & \times & 0 \\ \cdot & \cdot & 0 \end{pmatrix} \sim \begin{pmatrix} 0 & 0 & \times \\ \cdot & \times & \times \\ \cdot & \cdot & \times \end{pmatrix}^{-1} \simeq \mathcal{M}_{\nu}^{-1}. \end{aligned}$$

- Dirac matrices diagonal by symmetry, so $\mathcal{M}_R \sim \mathcal{M}_{\nu}^{-1}$.

One Scalar

- With just one scalar S , we can get five of the seven valid patterns:

Symmetry generator Y'	$ Y'(S) $	Texture zeros in \mathcal{M}_R	Texture zeros in \mathcal{M}_ν
$L_\mu - L_\tau$	1	$(\mathcal{M}_R)_{33}, (\mathcal{M}_R)_{22}$ (\mathbf{C}^R)	–
$B - L_e + L_\mu - 3L_\tau$	2	$(\mathcal{M}_R)_{33}, (\mathcal{M}_R)_{13}$ (\mathbf{B}_4^R)	$(\mathcal{M}_\nu)_{12}, (\mathcal{M}_\nu)_{22}$ (\mathbf{B}_3^ν)
$B - L_e - 3L_\mu + L_\tau$	2	$(\mathcal{M}_R)_{22}, (\mathcal{M}_R)_{12}$ (\mathbf{B}_3^R)	$(\mathcal{M}_\nu)_{13}, (\mathcal{M}_\nu)_{33}$ (\mathbf{B}_4^ν)
$B + L_e - L_\mu - 3L_\tau$	2	$(\mathcal{M}_R)_{33}, (\mathcal{M}_R)_{23}$ (\mathbf{D}_2^R)	$(\mathcal{M}_\nu)_{12}, (\mathcal{M}_\nu)_{11}$ (\mathbf{A}_1^ν)
$B + L_e - 3L_\mu - L_\tau$	2	$(\mathcal{M}_R)_{22}, (\mathcal{M}_R)_{23}$ (\mathbf{D}_1^R)	$(\mathcal{M}_\nu)_{13}, (\mathcal{M}_\nu)_{11}$ (\mathbf{A}_2^ν)

- Many patterns are hard to distinguish via neutrino experiments, e.g. \mathbf{D}_1^R and \mathbf{D}_2^R , but the symmetries $B + L_e - 3L_\mu - L_\tau$ and $B + L_e - L_\mu - 3L_\tau$ are very different

⇒ new possibilities to disentangle texture zeros.

- More scalars or discrete \mathbb{Z}_N subgroups can generate other allowed patterns.

Summary

- With three right-handed neutrinos, the SM gauge group can be extended by $U(1)_{B-L} \times U(1)_{L_e-L_\mu} \times U(1)_{L_\mu-L_\tau}$.
- Can use subgroups to influence neutrino mixing by introducing just one or two scalars.
- Applications: Enforcing neutrino hierarchies, two-zero textures, two vanishing minors, maybe other stuff?
- Z' and s can be searched for outside the neutrino sector
⇒ testable models!

Remark

Easy way to see anomaly freedom: Choose different basis in group space:

$$L_e - L_\mu = \text{diag}(1, -1, 0), \quad (L_e - L_\mu) + 2(L_\mu - L_\tau) = \text{diag}(1, 1, -2).$$

Cartan subalgebra of $SU(3)_\ell$ with

$$\ell_{Li} \sim \mathbf{3}_\ell, \quad e_{Ri} \sim \mathbf{3}_\ell, \quad N_{Ri} \sim \mathbf{3}_\ell.$$

Real reducible rep $\Rightarrow [SU(3)_\ell]^3$ anomaly vanishes (like quarks).

Anomalies:

$$SU(3)_\ell - SU(3)_\ell - U(1)_Y : \sum_{\mathbf{3}_\ell} Y = 3 \times (2Y(L_e) - Y(e_R)) = 0,$$

$$SU(3)_\ell - SU(3)_\ell - U(1)_{B-L} : \sum_{\mathbf{3}_\ell} (B - L) = 3 \times (2(-1) + (+1) + (+1)) = 0.$$

$\Rightarrow G_{\text{SM}} \times SU(3)_\ell \times U(1)_{B-L} \supset G_{\text{SM}} \times G_{\text{max}}$ is anomaly free.

One more remark

- For $U(1)$, field strength tensor $F_{\mu\nu}$ is already gauge invariant
⇒ Every gauge group with $U(1)_1 \times U(1)_2$ part allows for “kinetic mixing”:

$$\mathcal{L}_{mix} = c_{mix} F_1^{\mu\nu} F_{2\mu\nu}$$

[B. Holdom, *PLB 166 (1986)*]

- Diagonalize kinetic terms: A_1 couples to $j_2^\mu \Rightarrow$ new effects
- Can be extended to $U(1)_1 \times U(1)_2 \times U(1)_3$ [J.H. and W.R., *PLB 705 (2011)*]

Texture Zeros

- Take \mathcal{M}_ν and set two independent entries to zero \Rightarrow four constraints on the nine low-energy parameters (m_1, m_2, m_3) , $(\theta_{23}, \theta_{12}, \theta_{13})$ and (δ, α, β) (CP violating phases)
- 15 two-zero textures possible, only 7 allowed at 3σ : [H. Fritzsch et al, *JHEP 1109 (2011)*]

$$\mathbf{A}_1^\nu : \begin{pmatrix} 0 & 0 & \times \\ 0 & \times & \times \\ \times & \times & \times \end{pmatrix}, \quad \mathbf{A}_2^\nu : \begin{pmatrix} 0 & \times & 0 \\ \times & \times & \times \\ 0 & \times & \times \end{pmatrix}, \quad \mathbf{B}_1^\nu : \begin{pmatrix} \times & \times & 0 \\ \times & 0 & \times \\ 0 & \times & \times \end{pmatrix}, \quad \mathbf{B}_2^\nu : \begin{pmatrix} \times & 0 & \times \\ 0 & \times & \times \\ \times & \times & 0 \end{pmatrix},$$

$$\mathbf{B}_3^\nu : \begin{pmatrix} \times & 0 & \times \\ 0 & 0 & \times \\ \times & \times & \times \end{pmatrix}, \quad \mathbf{B}_4^\nu : \begin{pmatrix} \times & \times & 0 \\ \times & \times & \times \\ 0 & \times & 0 \end{pmatrix}, \quad \mathbf{C}^\nu : \begin{pmatrix} \times & \times & \times \\ \times & 0 & \times \\ \times & \times & 0 \end{pmatrix}.$$

Vanishing Minors

- Same idea, but with \mathcal{M}_ν^{-1} instead of \mathcal{M}_ν .
- Seven patterns for \mathcal{M}_ν^{-1} allowed:

$$\mathbf{D}_1^R: \begin{pmatrix} \times & \times & \times \\ \cdot & 0 & 0 \\ \cdot & \cdot & \times \end{pmatrix}, \quad \mathbf{D}_2^R: \begin{pmatrix} \times & \times & \times \\ \cdot & \times & 0 \\ \cdot & \cdot & 0 \end{pmatrix}, \quad \mathbf{B}_3^R: \begin{pmatrix} \times & 0 & \times \\ \cdot & 0 & \times \\ \cdot & \cdot & \times \end{pmatrix}, \quad \mathbf{B}_4^R: \begin{pmatrix} \times & \times & 0 \\ \cdot & \times & \times \\ \cdot & \cdot & 0 \end{pmatrix},$$

$$\mathbf{B}_1^R: \begin{pmatrix} \times & \times & 0 \\ \cdot & 0 & \times \\ \cdot & \cdot & \times \end{pmatrix}, \quad \mathbf{B}_2^R: \begin{pmatrix} \times & 0 & \times \\ \cdot & \times & \times \\ \cdot & \cdot & 0 \end{pmatrix}, \quad \mathbf{C}^R: \begin{pmatrix} \times & \times & \times \\ \cdot & 0 & \times \\ \cdot & \cdot & 0 \end{pmatrix}.$$

- Two-zero texture in \mathcal{M}_ν^{-1} corresponds to two vanishing minors in \mathcal{M}_ν . E.g.

$$\mathcal{M}^{-1} = \begin{pmatrix} \times & \times & 0 \\ \times & 0 & \times \\ 0 & \times & \times \end{pmatrix} \Rightarrow \begin{vmatrix} \mathcal{M}_{11} & \mathcal{M}_{13} \\ \mathcal{M}_{31} & \mathcal{M}_{33} \end{vmatrix} = 0 = \begin{vmatrix} \mathcal{M}_{21} & \mathcal{M}_{22} \\ \mathcal{M}_{31} & \mathcal{M}_{32} \end{vmatrix}$$

Two Scalars

\mathcal{M}_R pattern	Symmetry generator Y'	$ Y'(S_i) $
D_1^R	$B - aL_e - 3L_\mu + aL_\tau, a \notin \{-9, -3, 0, 1, 3\}$ $B - 2L_\mu - L_\tau$ $B + \frac{3}{2}L_e - \frac{9}{2}L_\mu$ $B + \frac{6}{7}L_e - \frac{27}{7}L_\mu - \frac{3}{7}L_\tau$ $B + \frac{1}{3}L_e - \frac{7}{3}L_\mu - L_\tau$	$2 a , 3+a $ 1, 2 3, $\frac{3}{2}$ $\frac{18}{7}, \frac{2}{7}$ 2, $\frac{2}{3}$
D_2^R	D_1^R with $L_\mu \leftrightarrow L_\tau$	
B_3^R	D_1^R with $L_e \leftrightarrow L_\tau$	
B_4^R	B_3^R with $L_\mu \leftrightarrow L_\tau$	
B_1^R	$B + 3L_\mu - 6L_\tau$ $B - 2L_\mu - L_\tau$ $B - \frac{9}{2}L_\mu + \frac{3}{2}L_\tau$ $B - 6L_e + 3L_\mu$ $B + \frac{3}{2}L_e - \frac{9}{2}L_\mu$ $B - L_e - 2L_\mu$ $B - L_e + 3L_\mu - 5L_\tau$ $B - 5L_e + 3L_\mu - L_\tau$	3, 12 2, 3 3, $\frac{9}{2}$ 3, 12 3, $\frac{9}{2}$ 2, 3 2, 10 2, 10
B_2^R	B_1^R with $L_\mu \leftrightarrow L_\tau$	
C^R	$B + 3L_e - aL_\mu - (6-a)L_\tau, a \notin \{-3, 0, 1, 3, 5, 6, 9\}$ $B - 6L_\mu + 3L_\tau$ $B - 3L_e \pm 9L_\mu \mp 9L_\tau$	$5, 3-a $ 3, 6 6, 12