Lepton Flavor Structures

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(IHEP, Beijing)

MENU

- ♣ Flavor puzzles
- ♣ Flavor texture
- ♣ Sterile species

@Neutrino Oscillation Workshop, September 9-16, 2012

A Partial History

1897: Discovery of electron (J.J. Thomson)

1928: Prediction of positron (P.A.M. Dirac)

1930: Postulation of neutrino (W. Pauli)

1932: Discovery of positron (C.D. Anderson)

1936: Discovery of muon (J.C. Street *et al*, C.D. Anderson *et al*)

1956: Discovery of electron antineutrino (C.L. Cowan *et al*)

1957: Postulation of neutrino-antineutrino oscillation (B. Pontecorvo)

1962: Discovery of muon neutrino (G. Danby *et al*)

1962: Postulation of neutrino conversion (Z. Maki *et al*)

1968: Discovery of solar neutrino oscillations (R. Davis *et al*)

1975: Discovery of tau (M.L. Perl *et al*)

1987: Discovery of supernova neutrinos (K. Hirata *et al*)

2000: Discovery of tau neutrino (K. Kodama *et al*)

An Exciting Prediction

In 1995 it was an Indian theoretical physicist who first discovered the 39-year gap of charged leptons:

electron (1897) + 39 = muon (1936) + 39 = tau (1975)

2) **NOBEL LEPTONS.**
By K.V.L. Sarma (Tata Inst.),. TIFR-TH-95-56, Dec 1995. 13pp.
Submitted to Curr. Sci.
e-Print Archive: **hep-ph/9512420**

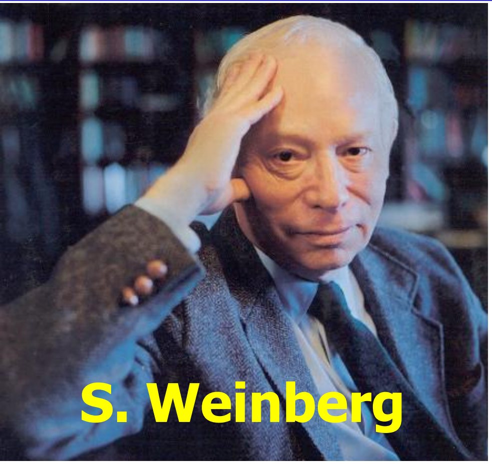
His prediction

A summary of the discoveries made in the world of leptons is given in Table 1. We see that the third generation has started getting Nobel prizes. It is amusing that the charged-leptons crop up with a 39-year gap and may be the 4th one would show up in the year 2114. For the present, the available experimental information implies that there are no charged leptons which are heavier than tau and lighter than 45 GeV.

1975 + 39 = 2014 = 2114 - 100

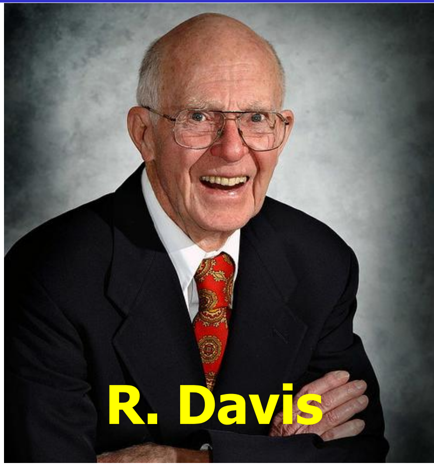
My "contribution": corrected 2114 to 2014, so the discovery would be possible 100 years earlier (2 years later from now on)!

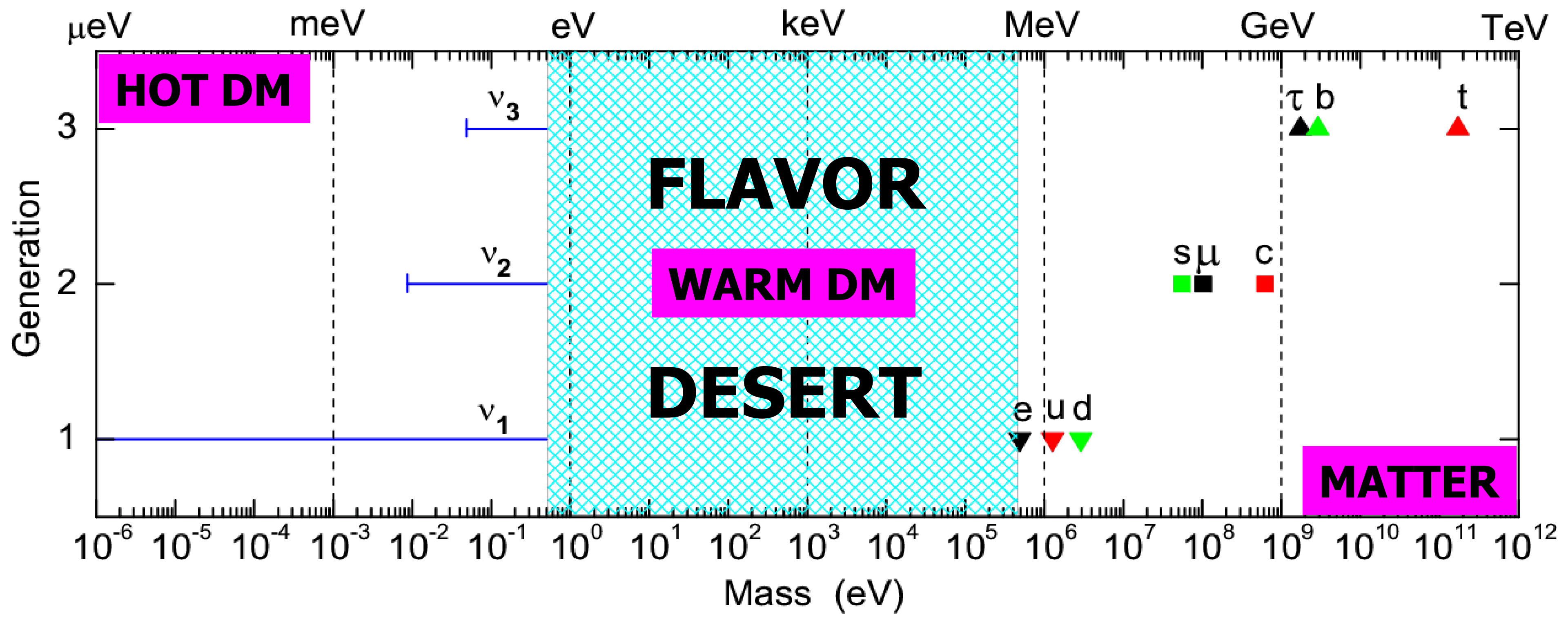
Hierarchy + Desert



Massless neutrinos: a natural assumption when the SM was formulated in 1967, while the solar ν anomaly was observed in 1968.

<u>neutrino oscillation ↔ masses</u>





Part A

Hierarchy or Not?

The CKM quark flavor mixing matrix: $\vartheta_{12} \sim 13^\circ$, $\vartheta_{23} \sim 2^\circ$, $\vartheta_{13} \sim 0.2^\circ$

$$|V| = \begin{pmatrix} 0.97427 \pm 0.00015 & 0.22534 \pm 0.00065 & 0.00351^{+0.00015}_{-0.00014} \\ 0.22520 \pm 0.00065 & 0.97344 \pm 0.00016 & 0.0412^{+0.0011}_{-0.0005} \\ 0.00867^{+0.00029}_{-0.00031} & 0.0404^{+0.0011}_{-0.0005} & 0.999146^{+0.000021}_{-0.000046} \end{pmatrix}$$

It has an amusing structure (hierarchy), and almost symmetric:

$$\underbrace{|V_{tb}| > |V_{ud}| > |V_{cs}|}_{\sim 10^0} \gg \underbrace{|V_{us}| > |V_{cd}|}_{\sim 10^{-1}} \gg \underbrace{|V_{cb}| > |V_{ts}|}_{\sim 10^{-2}} \gg \underbrace{|V_{td}| > |V_{ub}|}_{\sim 10^{-3}} > 0$$

The PMNS lepton flavor mixing matrix: $\theta_{12} \sim 34^\circ$, $\theta_{23} \sim 45^\circ$, $\theta_{13} \sim 9^\circ$

(based on Fogli et al 1205.5254 NH case.)

$$|U| = \begin{pmatrix} 0.7985 \cdots 0.8437 & 0.5162 \cdots 0.5795 & 0.1345 \cdots 0.1764 \\ 0.2765 \cdots 0.5561 & 0.5167 \cdots 0.7412 & 0.5823 \cdots 0.6829 \\ 0.1892 \cdots 0.5067 & 0.3967 \cdots 0.6538 & 0.7132 \cdots 0.7989 \end{pmatrix}$$

Its structure remains unclear, and μ-τ symmetry is essentially broken.

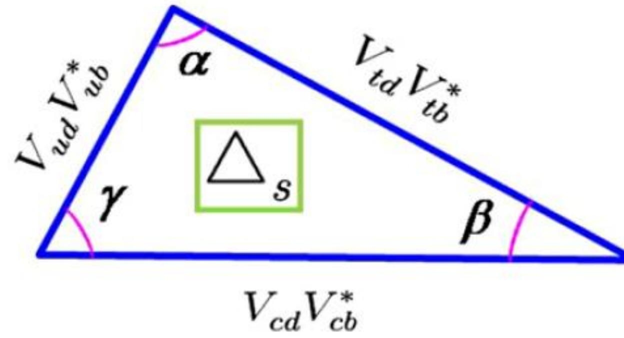
A similarity between CKM and PMNS: (1,3) mixing is the smallest one.

Numerology?

♣ Koide's relation for charged lepton masses:

$$Q_l^{\rm pole} \equiv \frac{m_e + m_\mu + m_\tau}{\left(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau}\right)^2} = \frac{2}{3}$$

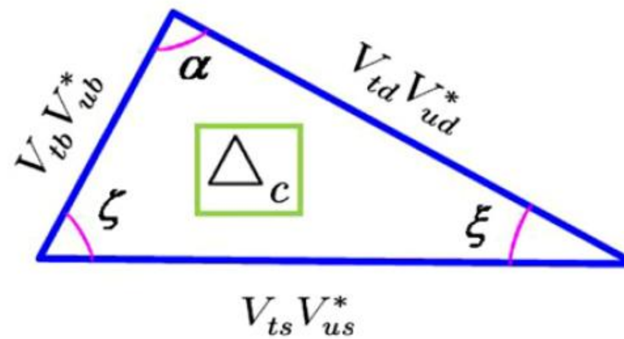
$$-0.00001 \leq Q_l^{\rm pole} - 2/3 \leq +0.00002$$



♣ Right unitarity triangles of the CKM matrix:

$$\alpha = \left(89.0^{+4.4}_{-4.2}\right)^\circ$$

(PDG 2012. Stable against RGE running, Xing 09; Luo, Xing 10)



♣ The quark-lepton complementary relations:

$$\vartheta_{12} + \theta_{12} \simeq 45^\circ, \quad \vartheta_{23} + \theta_{23} \simeq 45^\circ$$

(sensitive to radiative corrections)

$$\left|\frac{U_{e2}}{U_{e1}}\right| \simeq \frac{|V_{ud}| - |V_{us}|}{|V_{ud}| + |V_{us}|}, \qquad \left|\frac{U_{\mu 3}}{U_{\tau 3}}\right| \simeq \frac{|V_{tb}| - |V_{cb}|}{|V_{tb}| + |V_{cb}|}$$

(Li, Lin, He 12)

♣ Correlations of three mixing angles:

$$\theta_{12} + \theta_{13} = \theta_{23}$$

$$\theta_{12} + \theta_{13} + \theta_{23} = \delta = 90^\circ$$

(Also suffer from the RGE running. Luo, Xing 12; Haba et al 12)

Part B

Origin of Flavor Mixing

Weak flavor state:

$$-\mathcal{L}_{\rm mass} = \overline{\begin{pmatrix} e' & \mu' & \tau' \end{pmatrix}_{\rm L}} \, M_l \begin{pmatrix} e' \\ \mu' \\ \tau' \end{pmatrix}_{\rm R} + \frac{1}{2} \overline{\begin{pmatrix} \nu_e & \nu_\mu & \nu_\tau \end{pmatrix}_{\rm L}} \, M_\nu \begin{pmatrix} \nu_e^c \\ \nu_\mu^c \\ \nu_\tau^c \end{pmatrix}_{\rm R} + {\rm h.c.}$$

Mass state:

$$-\mathcal{L}'_{\rm mass} = \overline{\begin{pmatrix} e & \mu & \tau \end{pmatrix}_{\rm L}} \, \widehat{M}_l \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}_{\rm R} + \frac{1}{2} \overline{\begin{pmatrix} \nu_1 & \nu_2 & \nu_3 \end{pmatrix}_{\rm L}} \, \widehat{M}_\nu \begin{pmatrix} \nu_1^c \\ \nu_2^c \\ \nu_3^c \end{pmatrix}_{\rm R} + {\rm h.c.}$$

Weak charged-current interactions:

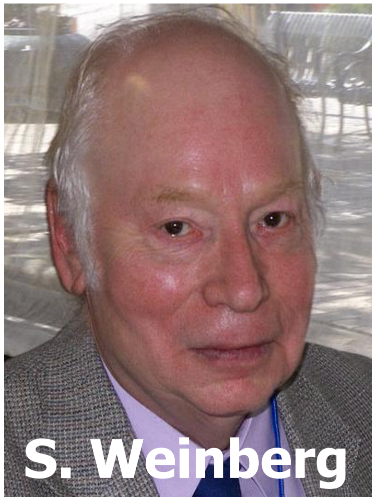
$$-\mathcal{L}_{\rm cc} = \frac{g}{\sqrt{2}} \overline{\begin{pmatrix} e & \mu & \tau \end{pmatrix}_{\rm L}} \, \gamma^\mu \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_{\rm L} W^-_\mu + {\rm h.c.}$$

The Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix: $U = O_l^\dagger O_\nu$

$$O_l^\dagger M_l O_l' = \widehat{M}_l \equiv {\rm Diag}\{m_e, m_\mu, m_\tau\}, \quad O_\nu^\dagger M_\nu O_\nu^* = \widehat{M}_\nu \equiv {\rm Diag}\{m_1, m_2, m_3\}$$

Strategy (A)

The flavor mixing angles are simple functions of 4 lepton mass ratios.



1977

2×2 3×3

Texture zeros

1978

$$\theta_{ij} = f\left(\frac{m_\alpha}{m_\beta}, \frac{m_k}{m_l}, \cdots\right)$$

$$M_{l,\nu} = \begin{pmatrix} 0 & \times & 0 \\ \times & 0 & \times \\ 0 & \times & \times \end{pmatrix}$$

Texture zeros of a fermion mass matrix dynamically mean that some matrix elements are strongly suppressed (in comparison with those weakly suppressed or unsuppressed elements) and may stem from a flavor symmetry (e.g., the Froggatt-Nielsen mechanism 1979)

★ **Texture zeros may lead to some testable relations between fermion mass ratios and flavor mixing angles ---- calculability.**

★ **Texture zeros may not be preserved to all orders or at any scales in an unspecified interaction from which fermion masses are generated.**

Part B

Charged Leptons

In the SM with the Higgs mass ~ 125 GeV, the cutoff scale of vacuum stability is about a few ZeV (Xing, Zhang, Zhou 2012; Lindner's talk).

μ	$m_e(\mu)$ (MeV)	$m_\mu(\mu)$ (MeV)	$m_\tau(\mu)$ (MeV)
$m_c(m_c)$	$0.495473903 \pm 0.000000013$	$104.4617350^{+0.0000059}_{-0.0000060}$	1774.62 ± 0.16
$m_b(m_b)$	$0.493099926 \pm 0.000000013$	$103.9961602^{+0.0000059}_{-0.0000060}$	1767.02 ± 0.16
M_W	$0.486845781^{+0.000000013}_{-0.000000012}$	$102.7721083 \pm 0.0000059$	$1747.05^{+0.15}_{-0.16}$
M_Z	$0.486570154^{+0.000000012}_{-0.000000013}$	$102.7181337^{+0.0000059}_{-0.0000058}$	$1746.17^{+0.15}_{-0.16}$
M_H	$0.485858771^{+0.000000013}_{-0.000000012}$	$102.5788227^{+0.0000058}_{-0.0000059}$	1743.89 ± 0.16
$m_t(m_t)$	$0.485285152^{+0.000000012}_{-0.000000013}$	$102.4664851^{+0.0000059}_{-0.0000058}$	1742.06 ± 0.16
1 TeV	$0.489535765^{+0.000000013}_{-0.000000012}$	$103.3441945 \pm 0.0000059$	1756.81 ± 0.16
$\Lambda_{\rm VS}$	$0.484511554^{+0.000000012}_{-0.000000013}$	$102.2835586^{+0.0000058}_{-0.0000059}$	1738.82 ± 0.16
M_l	$0.510998910 \pm 0.000000013$	105.658367 ± 0.0000040	1776.82 ± 0.16

At the electroweak scale

$$\sqrt{\frac{m_e}{m_\mu}} \simeq 0.069 \Leftrightarrow 4^\circ , \qquad \sqrt{\frac{m_\mu}{m_\tau}} \simeq 0.24 \Leftrightarrow 14^\circ$$

To generate the observed values of three flavor mixing angles, the neutrino sector should contribute in a significant way.

$$\theta_{12} \sim 34^\circ, \; \theta_{23} \sim 45^\circ, \; \theta_{13} \sim 9^\circ$$

(e.g., the Fritzsch texture for both charged leptons and neutrinos)

Part B

Majorana Neutrinos

In the basis of the diagonal charged lepton mass matrix, a symmetric Majorana neutrino mass matrix may have different zero textures.

Definition and counting of texture zeros:

★ **a pair of off-diagonal zeros of M_ν is commonly counted as one zero.**

★ **the number of textures of a 3×3 M_ν with N zeros: 6!/[N!×(6–N)!].**

★ **N=1: totally 6 one-zero textures, and all of them are presently OK.**

★ **N=2: totally 15 two-zero textures, and 7 of them are presently OK.**

★ **N≥3: none of the zero textures compatible with experimental data.**

Pattern A_1	Pattern A_2	Pattern B_1	
$\begin{pmatrix} 0 & 0 & \times \\ 0 & \times & \times \\ \times & \times & \times \end{pmatrix}$	$\begin{pmatrix} 0 & \times & 0 \\ \times & \times & \times \\ 0 & \times & \times \end{pmatrix}$	$\begin{pmatrix} \times & \times & 0 \\ \times & 0 & \times \\ 0 & \times & \times \end{pmatrix}$	
Pattern B_2	**Pattern B_3**	**Pattern B_4**	**Pattern C**
$\begin{pmatrix} \times & 0 & \times \\ 0 & \times & \times \\ \times & \times & 0 \end{pmatrix}$	$\begin{pmatrix} \times & 0 & \times \\ 0 & 0 & \times \\ \times & \times & \times \end{pmatrix}$	$\begin{pmatrix} \times & \times & 0 \\ \times & \times & \times \\ 0 & \times & 0 \end{pmatrix}$	$\begin{pmatrix} \times & \times & \times \\ \times & 0 & \times \\ \times & \times & 0 \end{pmatrix}$

Two-zero textures

WHICH ONE IS UNIQUE?

Part B

Strategy (B)

The PMNS matrix = the constant (leading) term + small perturbations

$$U = \left(U_0 + \Delta U\right) P_\nu = U_0 \left(\mathbf{1} + \Delta U'\right) P_\nu = \left(\mathbf{1} + \Delta U'_{\rm L}\right) U_0 \left(\mathbf{1} + \Delta U'_{\rm R}\right) P_\nu$$

U_0: arising from a certain (discrete or continuous) flavor symmetry;
ΔU: coming from tree-level perturbations or quantum corrections;
P_ν: the diagonal Majorana CP-violating phase matrix.

The first (?) example of this type (Fritzsch, Xing 1996):

$$U \simeq \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{-2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix} + i\sqrt{\frac{m_e}{m_\mu}} \begin{pmatrix} \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{-2}{\sqrt{6}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \\ 0 & 0 & 0 \end{pmatrix} + \frac{m_\mu}{m_\tau} \begin{pmatrix} 0 & 0 & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} \\ \frac{-1}{2\sqrt{3}} & \frac{-1}{2\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix}$$

Flavor democracy of charged leptons
Mass degeneracy of neutrinos

Diagonal but CP-violating perturbation matrices

A common feature of the mass matrix: linear correlations / equalities among the matrix elements, resulting from a certain flavor symmetry.

Part B

Why Zeros or Equalities

Chemical structures determine chemical properties.

The flavor texture determines the flavor mixing properties

Structural Zeros or Equalities

They reduce the number of free parameters, and thus lead to predictions for 3 flavor mixing angles in terms of either the mass ratios or constant numbers.

Typical Example

$$M_{l,\nu} = \begin{pmatrix} 0 & \times & 0 \\ \times & 0 & \times \\ 0 & \times & \times \end{pmatrix}$$

Dependent on mass ratios

Typical Example

$$M_{\nu} = \begin{pmatrix} b+c & -b & -c \\ -b & a+b & -a \\ -c & -a & a+c \end{pmatrix}$$

Independent of mass ratios

→ **PREDICTIONS** ←

Part B

Constant Examples

1st generation:

Cabibbo (78)

$$\begin{pmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & \frac{\omega}{\sqrt{3}} & \frac{\omega^2}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & \frac{\omega^2}{\sqrt{3}} & \frac{\omega}{\sqrt{3}} \end{pmatrix}$$

Wolfenstein (78)

$$\begin{pmatrix} \frac{1}{\sqrt{3}} & \frac{\sqrt{2}}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{3}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

2nd generation:

Democratic (96)

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{\sqrt{2}}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix}$$

Bimaximal (97/98)

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{-1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{-1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

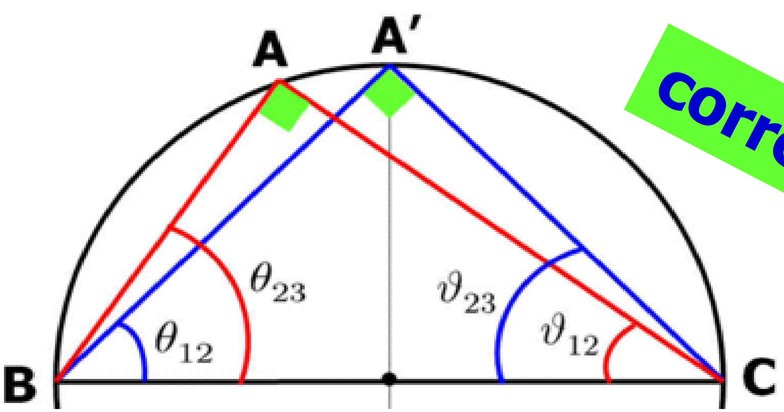
3rd generation:

Tri-bimaximal (02)

$$\begin{pmatrix} \frac{\sqrt{2}}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{-1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Golden-ratio (07)

$$\begin{pmatrix} \frac{\sqrt{2}}{\sqrt{5-\sqrt{5}}} & \frac{\sqrt{2}}{\sqrt{5+\sqrt{5}}} & 0 \\ \frac{-1}{\sqrt{5+\sqrt{5}}} & \frac{1}{\sqrt{5-\sqrt{5}}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{5+\sqrt{5}}} & \frac{-1}{\sqrt{5-\sqrt{5}}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$



correlation

$$\theta_{12} = \pi/4 \qquad \vartheta_{12} = \pi/4 - \theta_*$$
$$\theta_{23} = \pi/4 + \theta_* \qquad \vartheta_{23} = \pi/4$$

Democratic **Tri-bimaximal**

$$\theta_{13} \sim \theta_* \simeq 9.7^\circ$$

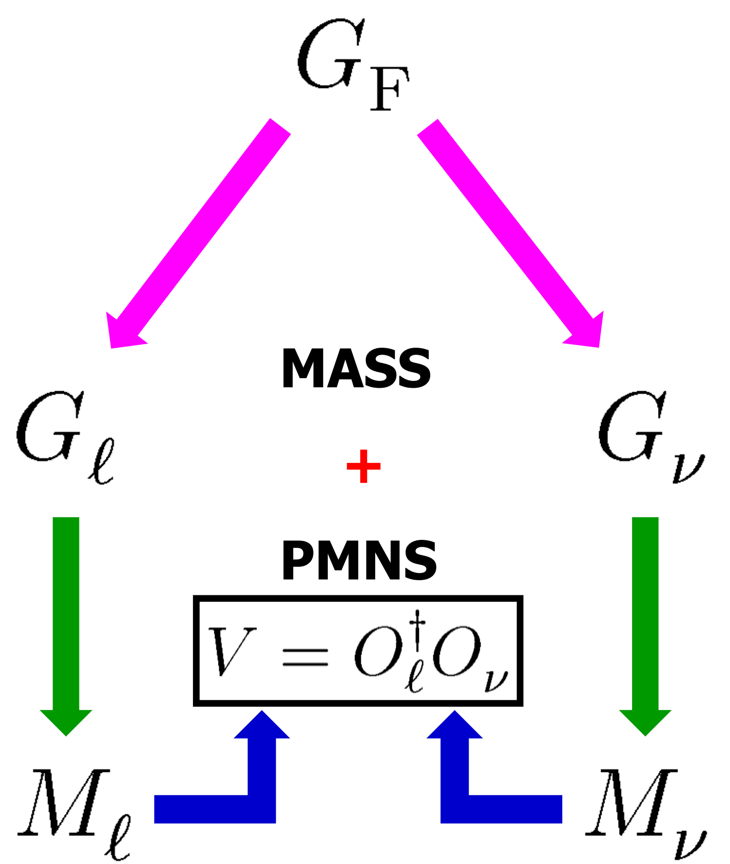
(xing 2011)

Flavor Symmetries

Some small discrete groups for model building (Altarelli, Feruglio 2010).

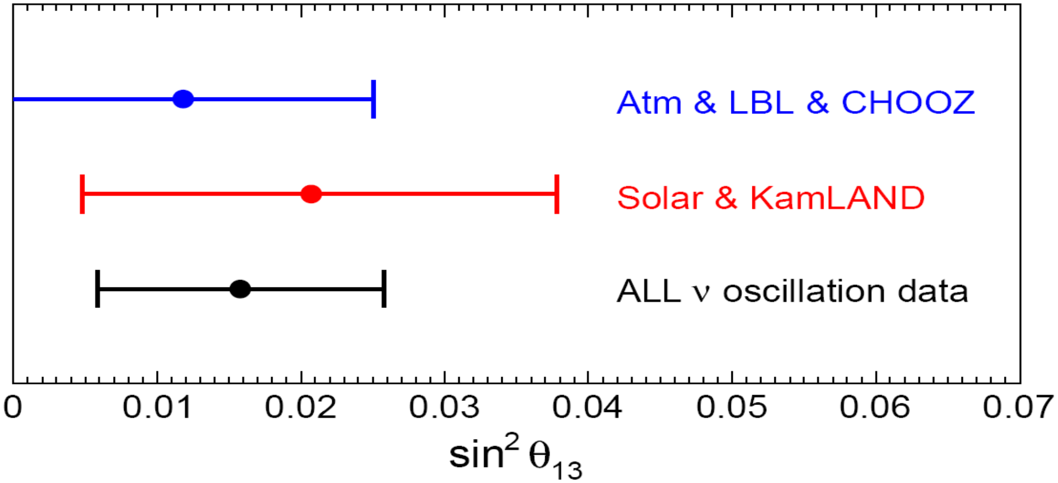
Group	d	Irreducible representation
$D_3 \sim S_3$	6	$1, 1', 2$
D_4	8	$1_1, \ldots, 1_4, 2$
D_7	14	$1, 1', 2, 2', 2''$
A_4	12	$1, 1', 1'', 3$
$A_5 \sim PSL_2(5)$	60	$1, 3, 3', 4, 5$
T'	24	$1, 1', 1'', 2, 2', 2'', 3$
S_4	24	$1, 1', 2, 3, 3'$
$\Delta(27) \sim Z_3 \rtimes Z_3$	27	$1_1, 1_9, 3, \bar{3}$
$PSL_2(7)$	168	$1, 3, \bar{3}, 6, 7, 8$
$T_7 \sim Z_7 \rtimes Z_3$	21	$1, 1', \bar{1}', 3, \bar{3}$

Too many possibilities! Which one stands out?



But θ_{13} Unsuppressed

Preliminary hints from a global fit of neutrino oscillation data in 2008:



Fogli et al., talk in Venice; arXiv:0806.2649

$$\sin^2\theta_{13} = 0.016 \pm 0.010$$

$(1\sigma,\ \text{All oscillation data})$

$$\theta_{13} \simeq 7.3^\circ$$

My plenary talk at ICHEP 2008 in August:

What is the value of θ_{13} ? It must be small. But how small is small?

---- Reactor and accelerator ν-oscillation experiments can answer, but can they answer before a global fit yields a definite prediction?

But I took it seriously 0805.0416

$$U_0 = \begin{pmatrix} \frac{2+\sqrt{2}}{4} & \frac{1}{2} & \frac{2-\sqrt{2}}{4} \\ -\frac{\sqrt{2}}{4} + \frac{i\left(\sqrt{2}-1\right)}{4} & \frac{1}{2} - \frac{i\sqrt{2}}{4} & \frac{\sqrt{2}}{4} + \frac{i\left(\sqrt{2}+1\right)}{4} \\ -\frac{\sqrt{2}}{4} - \frac{i\left(\sqrt{2}-1\right)}{4} & \frac{1}{2} + \frac{i\sqrt{2}}{4} & \frac{\sqrt{2}}{4} - \frac{i\left(\sqrt{2}+1\right)}{4} \end{pmatrix}$$

☺ ☺ ☺

$$\theta_{13} \simeq 8.4^\circ$$

(using 1, 2, *i*)

Perturbations

To illustrate, we typically take $\theta_{12} \simeq 34^\circ,\ \theta_{13} \simeq 9^\circ$ and $\theta_{23} \simeq 45^\circ$

$$U = (U_0 + \Delta U)\, P_\nu = \begin{pmatrix} 0.819 & 0.552 & 0.156e^{-i\delta} \\ -0.395 - 0.092e^{i\delta} & 0.586 - 0.062e^{i\delta} & 0.698 \\ 0.395 - 0.092e^{i\delta} & -0.586 - 0.062e^{i\delta} & 0.698 \end{pmatrix} P_\nu$$

flavor symmetry (↑ U_0)

Democratic

$$\Delta U = \begin{pmatrix} 0.112 & -0.155 & 0.156e^{-i\delta} \\ 0.013 - 0.092e^{i\delta} & 0.178 - 0.062e^{i\delta} & -0.118 \\ -0.182 - 0.092e^{i\delta} & -0.009 - 0.062e^{i\delta} & 0.121 \end{pmatrix}$$

The nine matrix elements are all O(0.1) ---- natural?

Tri-bimaximal

$$\Delta U = \begin{pmatrix} 0.003 & -0.025 & 0.156e^{-i\delta} \\ 0.013 - 0.092e^{i\delta} & 0.009 - 0.062e^{i\delta} & -0.009 \\ -0.013 - 0.092e^{i\delta} & -0.009 - 0.062e^{i\delta} & -0.009 \end{pmatrix}$$

The smallest (largest) angle receives the maximum (minimum) correction ---- unnatural?

Part B

Quantum Corrections

New Physics Scale

A mechanism of neutrino mass generation most likely works at a super-high energy scale. After integrating out heavy degrees of freedom, one is left with the unique Weinberg (79) operator:

$$\frac{\mathcal{L}_{\rm d=5}}{\Lambda} = \frac{1}{2}\kappa_{\alpha\beta}\overline{\ell_{\alpha \rm L}}\tilde{H}\tilde{H}^T\ell^c_{\beta \rm L} + {\rm h.c.}$$

θ_{13}

RGEs = Cable Car

Not impossible to give a favored value of θ_{13}, given new degrees of freedom, but reliable?

Electroweak Scale

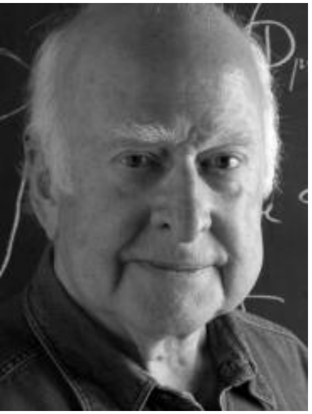
(Antusch et al., 2003, 2005; Mei, 2005; Goswami et al., 2009; Luo, Xing, 2012)

RGE = renormalization-group equation

Part B

The 1st Paper on CPV

Volume 72B, number 3 PHYSICS LETTERS 2 January 1978



TIME REVERSAL VIOLATION IN NEUTRINO OSCILLATION

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Received 11 October 1977

Quarks: $\mathcal{J}_q \simeq 3 \times 10^{-5}$

We discuss the possibility of CP or T violation in neutrino oscillation CP requires $\nu_\mu \longleftrightarrow \nu_e$ and $\bar{\nu}_\mu \longleftrightarrow \bar{\nu}_e$ oscillations to be equal Time reversal invariance requires the oscillation probability to be an even function of time Both conditions can be violated, even drastically, if more than two neutrinos exist

Basis-independent

The Cabibbo Texture: the tri-maximal mixing + maximal CP violation:

$$V_{\rm C} = \begin{pmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & \frac{\omega}{\sqrt{3}} & \frac{\omega^2}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & \frac{\omega^2}{\sqrt{3}} & \frac{\omega}{\sqrt{3}} \end{pmatrix} \Rightarrow V'_{\rm C} = \begin{pmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{-i}{\sqrt{3}} \\ -\frac{1}{2}\left(1+\frac{i}{\sqrt{3}}\right) & \frac{1}{2}\left(1-\frac{i}{\sqrt{3}}\right) & \frac{1}{\sqrt{3}} \\ \frac{1}{2}\left(1-\frac{i}{\sqrt{3}}\right) & -\frac{1}{2}\left(1+\frac{i}{\sqrt{3}}\right) & \frac{1}{\sqrt{3}} \end{pmatrix}$$

The Jarlskog invariant: $\mathcal{J}_{\rm max} = c_{12} s_{12} c_{13}^2 s_{13} c_{23} s_{23} \sin\delta = 1/(6\sqrt{3}) \simeq 9.6 \times 10^{-2}$

Part B

CP / T Violation

Under CPT invariance, the CP and T-violating asymmetries in neutrino oscillations in vacuum are identical (Pascoli's talk):

$$P\left(\nu_\alpha \to \nu_\beta\right) - P\left(\overline{\nu}_\alpha \to \overline{\nu}_\beta\right) = P\left(\nu_\alpha \to \nu_\beta\right) - P\left(\nu_\beta \to \nu_\alpha\right)$$
$$= 16\mathcal{J}\sum_\gamma \epsilon_{\alpha\beta\gamma} \sin\frac{\Delta m^2_{21}L}{4E} \sin\frac{\Delta m^2_{31}L}{4E} \sin\frac{\Delta m^2_{32}L}{4E}$$

Comments:

$\theta_{12} \simeq 34^\circ$

$\theta_{13} \simeq 9^\circ$

$\theta_{23} \simeq 45^\circ$

★ **CP / T violation cannot show up in the disappearance neutrino oscillation experiments ($\alpha = \beta$);**

★ **CP / T violation is a small three-family flavor effect;**

★ **CP / T violation in normal lepton-number-conserving neutrino oscillations depends only on the Dirac phase δ;**

★ **There is no reason for the CP phase δ to be very tiny.**

$$J = \sin\theta_{12}\cos\theta_{12}\sin\theta_{23}\cos\theta_{23}\sin\theta_{13}\cos^2\theta_{13}\sin\delta \simeq 3.6\sin\delta\times 10^{-2}$$

Part B

Recent Global Fits

G.L. Fogli's team
arXiv:1205.5254

parameter	best fit $\pm 1\sigma$	2σ	3σ
Δm^2_{21} $[10^{-5}\text{eV}^2]$	7.62 ± 0.19	7.27–8.01	7.12–8.20
Δm^2_{31} $[10^{-3}\text{eV}^2]$	$2.53^{+0.08}_{-0.10}$ $-(2.40^{+0.10}_{-0.07})$	$2.34-2.69$ $-(2.25-2.59)$	$2.26-2.77$ $-(2.15-2.68)$
$\sin^2\theta_{12}$	$0.320^{+0.015}_{-0.017}$	0.29–0.35	0.27–0.37
$\sin^2\theta_{23}$	$0.49^{+0.08}_{-0.05}$ $0.53^{+0.05}_{-0.07}$	0.41–0.62 0.42–0.62	0.39–0.64
$\sin^2\theta_{13}$	$0.026^{+0.003}_{-0.004}$ $0.027^{+0.003}_{-0.004}$	0.019–0.033 0.020–0.034	0.015–0.036 0.016–0.037
δ	$(0.83^{+0.54}_{-0.64})\,\pi$ 0.07π [a]	$0-2\pi$	$0-2\pi$

J.F. Valle's team
arXiv:1205.4018

Parameter	Best fit	1σ range
$\delta m^2/10^{-5}$ eV^2 (NH or IH)	7.54	7.32 – 7.80
$\sin^2\theta_{12}/10^{-1}$ (NH or IH)	3.07	2.91 – 3.25
$\Delta m^2/10^{-3}$ eV^2 (NH)	2.43	2.33 – 2.49
$\Delta m^2/10^{-3}$ eV^2 (IH)	2.42	2.31 – 2.49
$\sin^2\theta_{13}/10^{-2}$ (NH)	2.41	2.16 – 2.66
$\sin^2\theta_{13}/10^{-2}$ (IH)	2.44	2.19 – 2.67
$\sin^2\theta_{23}/10^{-1}$ (NH)	3.86	3.65 – 4.10
$\sin^2\theta_{23}/10^{-1}$ (IH)	3.92	3.70 – 4.31
δ/π (NH)	1.08	0.77 – 1.36
δ/π (IH)	1.09	0.83 – 1.47

Hints:

μ-τ symmetry breaking

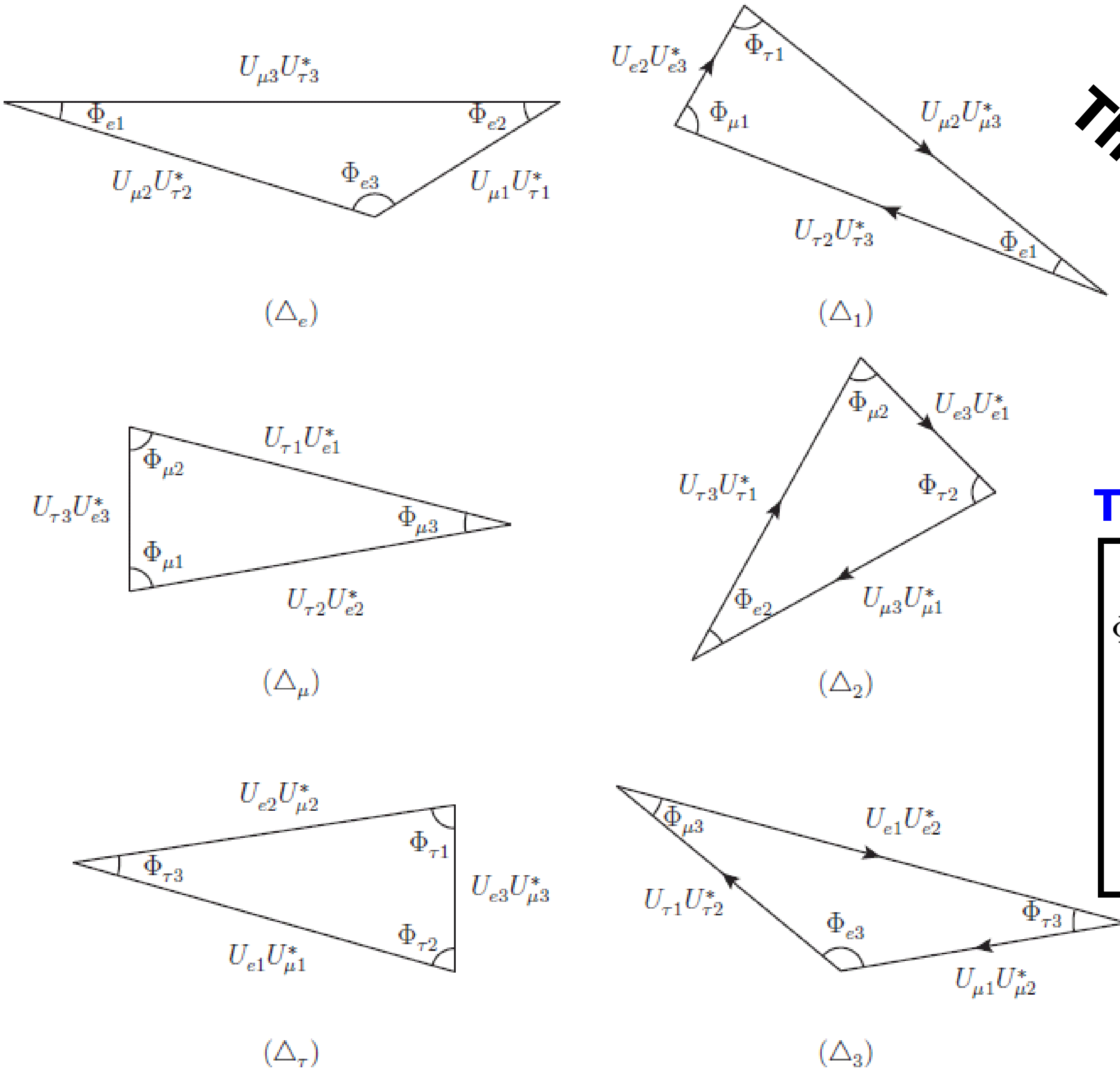
$$|V_{\mu i}| \neq |V_{\tau i}| \quad (i = 1, 2, 3)$$

$$\begin{cases} \theta_{13} = 0 \\ \theta_{23} = \pi/4 \end{cases}$$ ✖

or

$$\begin{cases} \delta = \pm\pi/2 \\ \theta_{23} = \pi/4 \end{cases}$$ ✖

Unitarity Triangles



The same area

$\theta_{12} \simeq 34^\circ$

$\theta_{13} \simeq 9^\circ$

$\theta_{23} \simeq 45^\circ$

$\delta \simeq 90^\circ$

The phase matrix:

$$\Phi \equiv \begin{pmatrix} \Phi_{e1} & \Phi_{e2} & \Phi_{e3} \\ \Phi_{\mu 1} & \Phi_{\mu 2} & \Phi_{\mu 3} \\ \Phi_{\tau 1} & \Phi_{\tau 2} & \Phi_{\tau 3} \end{pmatrix} \simeq \begin{pmatrix} 12.05^\circ & 26.11^\circ & 141.8^\circ \\ 83.98^\circ & 76.94^\circ & 19.08^\circ \\ 83.98^\circ & 76.94^\circ & 19.08^\circ \end{pmatrix}$$

$$\sum_\alpha \Phi_{\alpha i} = \sum_i \Phi_{\alpha i} = 180^\circ$$

Part C

Sterile Neutrinos?

active flavor

sterile flavor

3+3

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ \nu_x \\ \nu_y \\ \nu_z \end{pmatrix} = \mathcal{U} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ \nu_4 \\ \nu_5 \\ \nu_6 \end{pmatrix}$$

Seesaw

LSND

MiniBooNE

RANA

Warm DM

Leptogenesis

......

mass state

e.g. talks by

Rodejohann
Petcov
Rubbia
Lindner
Giunti
Ohlsson .....

Part C

Parametrization

$$\mathcal{U} = \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & U_0 \end{pmatrix} \begin{pmatrix} A & R \\ S & B \end{pmatrix} \begin{pmatrix} V_0 & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$$

sterile part | **interplay** | **active part**

$$\begin{pmatrix} V_0 & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix} = O_{23} O_{13} O_{12} \; ,$$

$$\begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & U_0 \end{pmatrix} = O_{56} O_{46} O_{45} \; ,$$

$$\begin{pmatrix} A & R \\ S & B \end{pmatrix} = O_{36} O_{26} O_{16} O_{35} O_{25} O_{15} O_{34} O_{24} O_{14}$$

Full parametrization:

15 rotation angles

15 phase phases

Xing, arXiv:1110.0083

Approximations

9 active-sterile mixing angles are constrained to be at most of O(0.1).

$$A \simeq \mathbf{1} - \begin{pmatrix} \frac{1}{2}\left(s_{14}^2 + s_{15}^2 + s_{16}^2\right) & 0 & 0 \\ \hat{s}_{14}\hat{s}_{24}^* + \hat{s}_{15}\hat{s}_{25}^* + \hat{s}_{16}\hat{s}_{26}^* & \frac{1}{2}\left(s_{24}^2 + s_{25}^2 + s_{26}^2\right) & 0 \\ \hat{s}_{14}\hat{s}_{34}^* + \hat{s}_{15}\hat{s}_{35}^* + \hat{s}_{16}\hat{s}_{36}^* & \hat{s}_{24}\hat{s}_{34}^* + \hat{s}_{25}\hat{s}_{35}^* + \hat{s}_{26}\hat{s}_{36}^* & \frac{1}{2}\left(s_{34}^2 + s_{35}^2 + s_{36}^2\right) \end{pmatrix}$$

$$B \simeq \mathbf{1} - \begin{pmatrix} \frac{1}{2}\left(s_{14}^2 + s_{24}^2 + s_{34}^2\right) & 0 & 0 \\ \hat{s}_{14}^*\hat{s}_{15} + \hat{s}_{24}^*\hat{s}_{25} + \hat{s}_{34}^*\hat{s}_{35} & \frac{1}{2}\left(s_{15}^2 + s_{25}^2 + s_{35}^2\right) & 0 \\ \hat{s}_{14}^*\hat{s}_{16} + \hat{s}_{24}^*\hat{s}_{26} + \hat{s}_{34}^*\hat{s}_{36} & \hat{s}_{15}^*\hat{s}_{16} + \hat{s}_{25}^*\hat{s}_{26} + \hat{s}_{35}^*\hat{s}_{36} & \frac{1}{2}\left(s_{16}^2 + s_{26}^2 + s_{36}^2\right) \end{pmatrix}$$

$$R \simeq \mathbf{0} + \begin{pmatrix} \hat{s}_{14}^* & \hat{s}_{15}^* & \hat{s}_{16}^* \\ \hat{s}_{24}^* & \hat{s}_{25}^* & \hat{s}_{26}^* \\ \hat{s}_{34}^* & \hat{s}_{35}^* & \hat{s}_{36}^* \end{pmatrix}$$

$$S \simeq \mathbf{0} - \begin{pmatrix} \hat{s}_{14} & \hat{s}_{24} & \hat{s}_{34} \\ \hat{s}_{15} & \hat{s}_{25} & \hat{s}_{35} \\ \hat{s}_{16} & \hat{s}_{26} & \hat{s}_{36} \end{pmatrix}$$

$$\boxed{R \simeq -S^\dagger}$$

Standard weak charged-current interactions:

$$-\mathcal{L}_{\rm cc} = \frac{g}{\sqrt{2}}\, \overline{\left(e \quad \mu \quad \tau\right)_{\rm L}}\, \gamma^\mu \left[V \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_{\rm L} + R \begin{pmatrix} \nu_4 \\ \nu_5 \\ \nu_6 \end{pmatrix}_{\rm L} \right] W_\mu^- + {\rm h.c.}$$

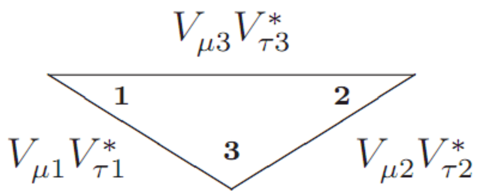
Part C

Unitarity Polygons?

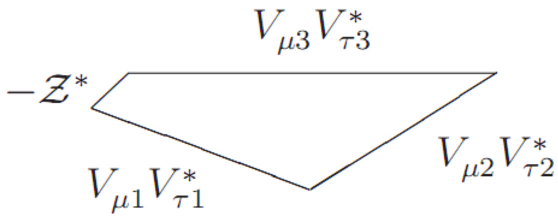
$$\begin{aligned} \triangle_e &: \quad V_{\mu 1}V_{\tau 1}^* + V_{\mu 2}V_{\tau 2}^* + V_{\mu 3}V_{\tau 3}^* = 0 \\ \triangle_\mu &: \quad V_{\tau 1}V_{e 1}^* + V_{\tau 2}V_{e 2}^* + V_{\tau 3}V_{e 3}^* = 0 \\ \triangle_\tau &: \quad V_{e 1}V_{\mu 1}^* + V_{e 2}V_{\mu 2}^* + V_{e 3}V_{\mu 3}^* = 0 \end{aligned}$$

Deformed unitarity triangles

$$\begin{aligned} \triangle_e' &: \quad V_{\mu 1}V_{\tau 1}^* + V_{\mu 2}V_{\tau 2}^* + V_{\mu 3}V_{\tau 3}^* \simeq -\mathcal{Z}^* \\ \triangle_\mu' &: \quad V_{\tau 1}V_{e 1}^* + V_{\tau 2}V_{e 2}^* + V_{\tau 3}V_{e 3}^* \simeq -\mathcal{Y} \\ \triangle_\tau' &: \quad V_{e 1}V_{\mu 1}^* + V_{e 2}V_{\mu 2}^* + V_{e 3}V_{\mu 3}^* \simeq -\mathcal{X}^* \end{aligned}$$



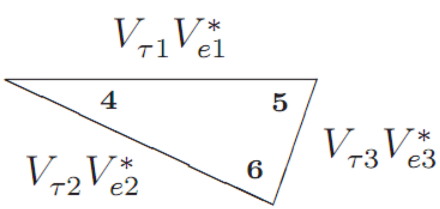
$(\triangle_e)$



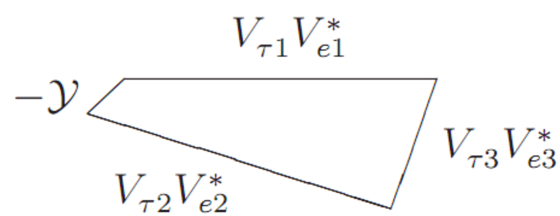
$(\triangle_e')$

$$\begin{aligned} \mathcal{X} &\equiv \hat{s}_{14}\hat{s}_{24}^* + \hat{s}_{15}\hat{s}_{25}^* + \hat{s}_{16}\hat{s}_{26}^* \\ \mathcal{Y} &\equiv \hat{s}_{14}\hat{s}_{34}^* + \hat{s}_{15}\hat{s}_{35}^* + \hat{s}_{16}\hat{s}_{36}^* \\ \mathcal{Z} &\equiv \hat{s}_{24}\hat{s}_{34}^* + \hat{s}_{25}\hat{s}_{35}^* + \hat{s}_{26}\hat{s}_{36}^* \end{aligned}$$

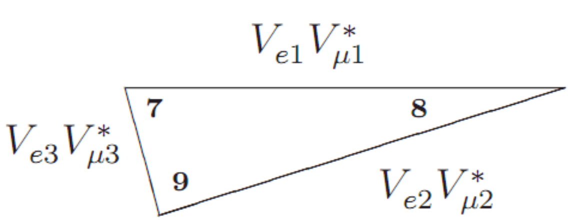
$\leq$ 1%



$(\triangle_\mu)$

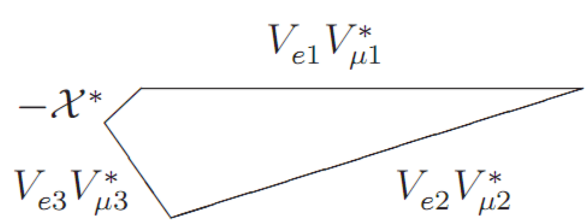


$(\triangle_\mu')$



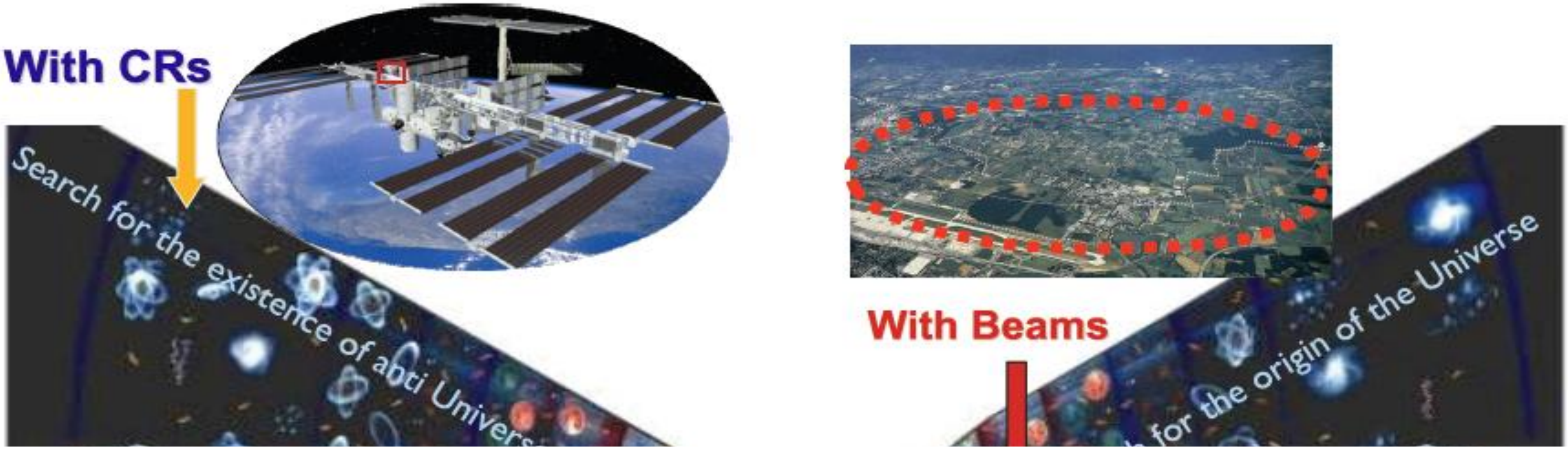
$(\triangle_\tau)$

New effects of CP violation



$(\triangle_\tau')$

Cosmic CP Violation



A link to low-energy CP violation via leptogenesis? **A phase counting: 9 at high scales, 3 at low scales ---- related indirectly via SEESAW.**

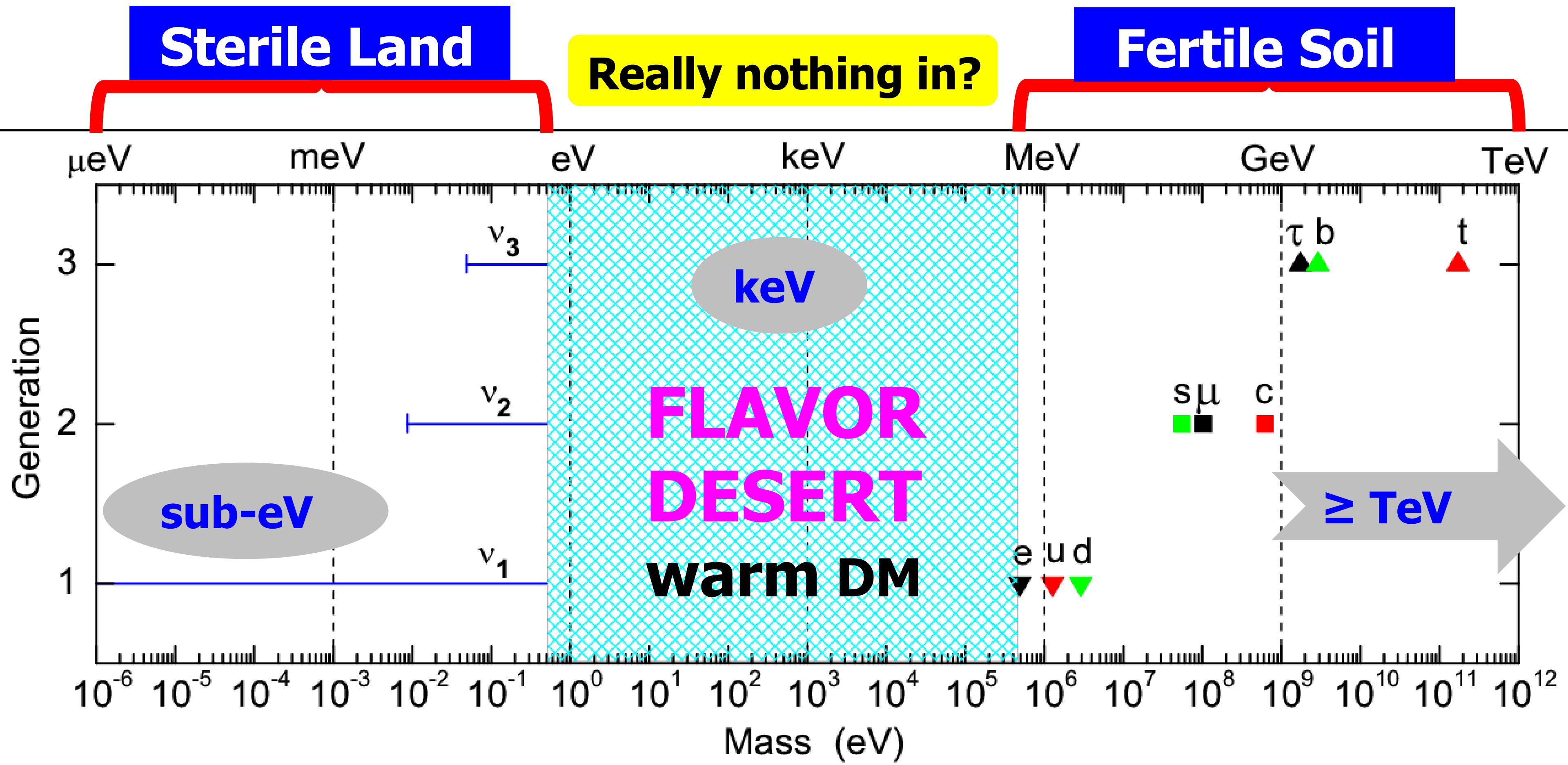
1933

Dirac's conjecture

Part C

Sterile Neutrino Physics?

arXiv:1204.5379



Weinberg's ***3rd law of progress in theoretical physics*** **(83):**

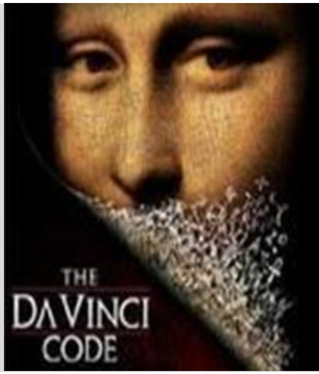
You may use any degrees of freedom you like to describe a physical system, but if you use the wrong ones, you will be sorry .... What could be better?

The Flavor Code?

What distinguishes different families of fermions?

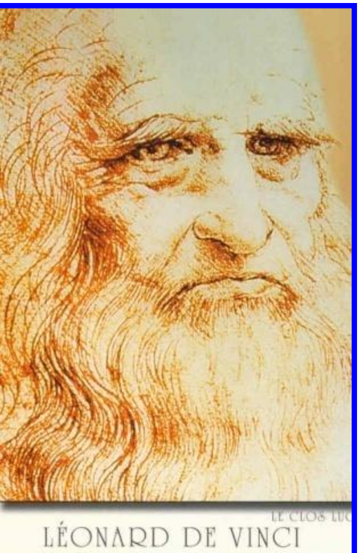
----- they have the same gauge quantum numbers,

yet they are quite different from one another.



We are blind today: no flavor theory of quarks and leptons

The flavor texture determines the flavor mixing properties



Bottom-Up Way

Although nature commences with reason and ends in experience, it is necessary for us to do the opposite, that is, to commence with experience and from this to proceed to investigate the reason

It’s an Animal