

Sterile neutrinos and oscillation coherence *for neutrinos produced in decays*

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Two related coherence issues

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- Neutrino production and detection coherence is related to localization of the neutrino source and absorber. Is e.g. the production coherence condition satisfied in the case of extended sources?

When are ν oscillations observable?

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Observability conditions for ν oscillations:

- **Coherence of ν production and detection:** the produced and detected ν 's are flavor eigenstates = coherent superpositions of different mass eigenstates
- **Coherence of ν propagation:** The produced neutrino state does not (irreversibly) lose coherence due to the wave packet separation in the course of propagation

Both conditions put upper limits on neutrino mass squared differences Δm^2 :

$$(1) \quad \Delta E_{jk} \sim \frac{\Delta m_{jk}^2}{2E} \ll \sigma_E;$$

$$(2) \quad \frac{\Delta m_{jk}^2}{2E^2} L \ll \sigma_x \simeq v_g / \sigma_E$$

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Sterile neutrinos: hints from SBL accelerator experiments (LSND, MiniBooNE), reactor neutrino anomaly, keV sterile neutrinos, pulsar kicks, leptogenesis via ν oscillations, SN r -process nucleosynthesis, unconventional contributions to $2\beta 0\nu$ decay ...

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Production/detection coherence has to be re-checked – important implications for some neutrino experiments!

ν_{ster} (de)coherence: decays of free π 's and μ 's

$\pi \rightarrow \mu \nu$ decay

For free pions: in the rest frame $\sigma_E \simeq \Gamma_0 \simeq 2.5 \times 10^{-8}$ eV.

Neutrino energy: $E_0 \simeq 30$ MeV. For a sterile neutrino with $\Delta m^2 \sim 2$ eV²

$$\frac{\Delta m^2}{2E_0} \simeq 3.3 \times 10^{-8} \text{ eV}; \quad \text{compare with} \quad \Gamma_0 \simeq 2.5 \times 10^{-8} \text{ eV}$$

$\Rightarrow$ the coherence condition $\Delta m^2/2E \ll \sigma_E$ violated!

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$\mu \rightarrow e \nu_\mu \nu_e$ decay:

$\Gamma_0 \simeq 3 \times 10^{-10}$ eV. Neutrino energy: $E_0 \sim 40 - 50$ MeV. For $\Delta m^2 \sim 2$ eV²

$$\frac{\Delta m^2}{2E_0} \simeq 2 \times 10^{-8} \text{ eV}^2; \quad \text{compare with} \quad \Gamma_0 \simeq 3 \times 10^{-10} \text{ eV}$$

$\Rightarrow$ expected violation of the the coherence condition is even much stronger!
Can occur even for smaller values of Δm^2 .

What if prod./det. coherence is violated?

Should LSND & MiniBooNE results be reconsidered?

What about other experiments?

If production/detection coherence is strongly violated, the osc. probabilities $P_{\alpha\beta}$ take their averaged values (even for $L = 0$ – “zero distance effect”).

⇒ No L/E dependence; 2-detector setups are useless!
No energy spectrum distortion!



Careful examination within the QM wave packet formalism
is necessary

W. packets of ν 's produced in π decays

Consider w. packets of neutrinos produced in decays of free pions confined to a decay tunnel of length l_p . Need the pion and muon w. functions!

Pions: produced by $pN \rightarrow \pi X$ processes; $\sigma_{x\pi} \sim \sigma_{xp} \lesssim 10^{-4}$ cm.

Muons: not detected ($\Leftrightarrow$ completely delocalized, $\sigma_{x\mu} \rightarrow \infty$).

$\sigma_{x\pi}$ completely negligible compared to all distances of interest (l_{osc} , L , l_p) $\Rightarrow$ can be set $\rightarrow 0$. The coordinate-state w. functions of the pion and muon:

$$\psi_\pi(x, t) = C_\pi e^{iQx - iE_\pi(Q)t - \Gamma t/2} \delta(x - v_\pi t) \text{box}(x; l_p, 0),$$

$$\psi_\mu(x, t) = C_\mu e^{iKx - iE_\mu(K)t},$$

$$E_\pi(Q) = (Q^2 + m_\pi^2)^{1/2}, \quad E_\mu(K) = (K^2 + m_\mu^2)^{1/2}$$

$$\text{box}(x; l_p, 0) = \begin{cases} 1, & l_p \geq x \geq 0, \\ 0, & \text{otherwise} \end{cases}$$

Pions assumed produced at $t = 0$, $x = 0$.

Neutrino wave packet – contd.

The amplitude of $\pi \rightarrow \mu \nu_\mu$ decay with production of muon with momentum K and mass-eigenstate neutrino ν_j with momentum p :

$$f_j^S(p) = M_P \int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} dx e^{iE_j(p)t - ipx} \psi_\mu(x, t)^* \psi_\pi(x, t).$$

Here: $E_j(p) = (p^2 + m_j^2)^{1/2}$, M_P – mom.-space pion decay amplitude. For fixed $K \Rightarrow$ mom. distribution amplitude (mom.-space w. function) of ν_j .

$$f_j^S(p) = C_j \frac{1 - e^{i[E_j(p) - E_P - v_\pi(p - P) + i\Gamma/2] l_p / v_\pi}}{E_j(p) - E_P - v_\pi(p - P) + i\Gamma/2}.$$

$$P \equiv Q - K, \quad E_P \equiv E_\pi(Q) - E_\mu(K),$$

Coordinate-space w. function of ν_j :

$$\psi_j^S(x, t) = \int \frac{dp}{2\pi} f_j^S(p) e^{-iE_j(p)t + ipx}$$

Coordinate-space ν wave function

The result:

$$\psi_j^S(x, t) = C e^{-iE_j(P_j)t + iP_j x} \left\{ e^{-\frac{\Gamma}{2(v_j - v_\pi)}(v_j t - x)} \left[\theta(v_j t - x) - \theta\left(v_j t - x - \frac{v_j - v_\pi}{v_\pi} l_p\right) \right] \right\}$$

$$P_j \equiv P + \frac{E_P - E_j(P)}{v_j - v_\pi}, \quad \text{where} \quad v_j \equiv \left. \frac{\partial E_j(p)}{\partial p} \right|_{p=P} = \frac{P}{E_j(P)},$$

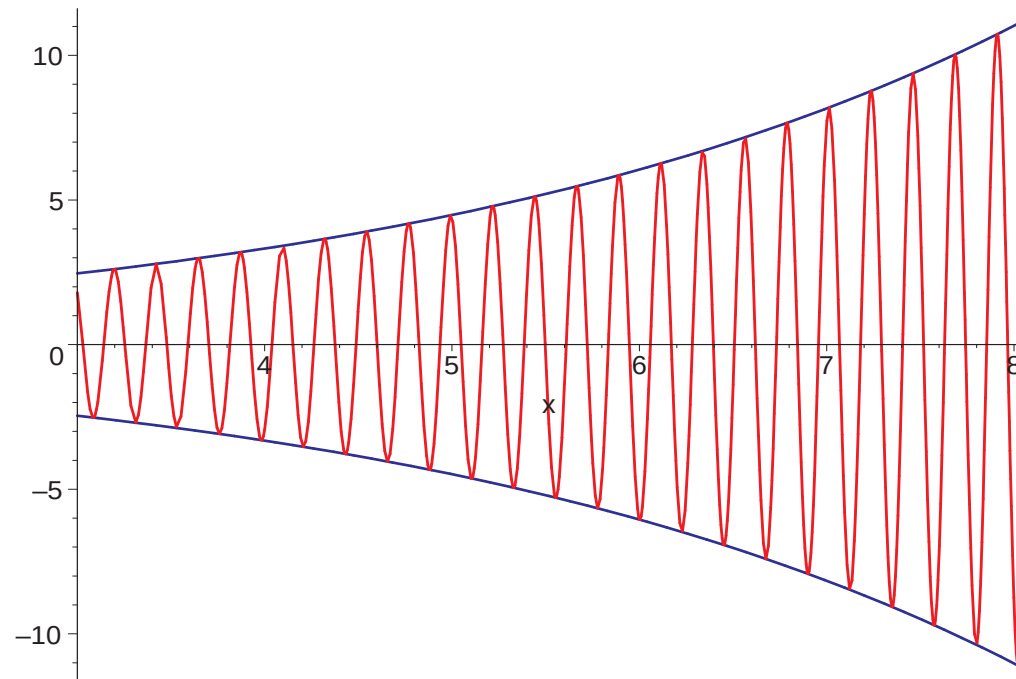
$$E_j(P_j) \simeq E_j(P) + v_j(P_j - P) = E_j(P) + v_j \frac{E_P - E_j(P)}{v_j - v_\pi}.$$

Neutrino wave packet:

- Has sharp edges
- Arrives at point x at $t_1 = x/v_j$; leaves it at $t_2 = x/v_j + (1/v_\pi - 1/v_j)l_p$
- Reaches its maximum at the front edge and exponentially decays towards the rear edge

⇒ An asymmetric wave packet!

Neutrino WF



Trumpet-like wave packet

Neutrino WF – contd.

Limiting cases:

In the limit $\Gamma l_p / v_\pi \gg 1$ (l_p large compared to the pion decay length $l_{\text{decay}} = v_\pi / \Gamma \Rightarrow$ decay of unconfined free pions):

$$\bar{x} \approx v_j t - \frac{v_j - v_\pi}{\Gamma}, \quad \sigma_{xj} \approx \frac{v_j - v_\pi}{\Gamma}$$

In the opposite limit, $\Gamma l_p / v_\pi \ll 1$:

$$\bar{x} \approx v_j t - \frac{v_j - v_\pi}{2v_\pi} l_p, \quad \sigma_{xj} \approx \frac{1}{2\sqrt{3}} \frac{v_j - v_\pi}{v_\pi} l_p.$$

(In this limit only a small fraction of pions decays before being absorbed by the wall at the end of the decay tunnel).

Calculating the oscillation probabilities

The transition amplitude:

$$\mathcal{A}_{\alpha\beta}(L, t) = \sum_j U_{\alpha j}^* U_{\beta j} \mathcal{A}_j(L, t)$$

Contribution of ν_j :

$$\mathcal{A}_j(L, t) \equiv \int dx \psi_j^{D*}(x) \psi_j^S(x, t) .$$

The detected state $\nu_j^D(x)$: a wave packet centered on the point $x = L$.
Assume the detection process is well localized: $\psi_j^D(x, t) = \delta(x - L) \Rightarrow$

$$\mathcal{A}_j(L, t) = \psi_j^S(L, t) .$$

The oscillation probability:

$$P_{\alpha\beta}(L) = \sum_{j,k} U_{\alpha j}^* U_{\beta j} U_{\alpha k} U_{\beta k}^* I_{jk}(L) ,$$

Here:

$$I_{jk}(L) \equiv \int_{-\infty}^{\infty} dt \mathcal{A}_j(L, t) \mathcal{A}_k^*(L, t) = \int_{-\infty}^{\infty} dt \psi_j^S(L, t) \psi_k^{S*}(L, t).$$



$$I_{jk}(L) = \frac{1}{(1 - e^{-\Gamma l_p/v_\pi})} \cdot \frac{i\Gamma}{v_\pi \frac{\Delta m_{jk}^2}{2P} + i\Gamma} \left[e^{-i \frac{\Delta m_{jk}^2}{2P} L} - e^{-\Gamma l_p/v_\pi} e^{-i \frac{\Delta m_{jk}^2}{2P} (L - l_p)} \right]$$

The absolute normalization fixed by imposing the unitarity constraint

$$\sum_{\beta} P_{\alpha\beta}(L) = 1 \quad \Rightarrow \quad I_{jj}(L) = 1.$$

Consider SBL experiments in the 3+1 scheme (only $\Delta m_{41}^2 \equiv \Delta m^2$ can be considered to be nonzero) $\Rightarrow$ an effective two-flavour approximation

Survival probabilities $P_{\alpha\alpha}$: $s = |U_{\alpha 4}|$, $c = (1 - |U_{\alpha 4}|^2)^{1/2}$.

Transition probability $P_{\alpha\beta}$: $\sin^2 2\theta = 4|U_{\alpha 4}|^2 |U_{\beta 4}|^2$.

ν_μ survival probability

$$\diamond \quad P_{\mu\mu} = c^4 + s^4 + \frac{2c^2 s^2}{\xi^2 + 1} \frac{1}{(1 - e^{-\Gamma l_p / v_\pi})} [\cos \phi + \xi \sin \phi - e^{-\Gamma l_p / v_\pi} [\cos(\phi - \phi_p) + \xi \sin(\phi - \phi_p)]]$$

Here:

$$\phi \equiv \frac{\Delta m^2}{2P} L, \quad \phi_p \equiv \frac{\Delta m^2}{2P} l_p, \quad \xi \equiv v_\pi \frac{\Delta m^2}{2P\Gamma},$$

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N.B.: for non-zero ξ and $\Gamma l_p / v_\pi$ the probability $P_{\mu\mu} \neq 1$ even for $L = 0$
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The parameter ξ : essentially the ratio of $\Delta m^2/2P$ and Γ – what we expected to be the (de)coherence parameter. For $\xi \gg 1$ the oscillating term strongly suppressed due to production decoherence (unless $\Gamma l_p/v_p \ll 1$).

A relation between ξ , ϕ_p and $\Gamma l_p/v_\pi$:

$$\xi \cdot \frac{\Gamma l_p}{v_\pi} = \phi_p.$$

In the limit $\Gamma l_p/v_p \ll 1$:

$$P_{\mu\mu} = c^4 + s^4 + \frac{2s^2c^2}{\phi_p} [\sin \phi - \sin(\phi - \phi_p)].$$

The decoherence parameter is ϕ_p . For $\phi_p \gg 1$: $P_{\mu\mu} \simeq \bar{P}_{\mu\mu} = c^4 + s^4$.

For $\phi_p \ll 1$:

$$P_{\mu\mu} = P_{\mu\mu}^{\text{stand}} = c^4 + s^4 + 2s^2c^2 \cos \phi.$$

This result does not depend on whether ξ is small or large!

Two distinct regimes:

- $\Gamma l_p/v_\pi \ll 1$ (pion decay length large compared to l_p , decoherence parameter: ϕ_p)
- $\Gamma l_p/v_\pi \gg 1$ (pion decay length small compared to l_p , decoherence parameter: ξ).

The decoherence parameters

For $\Gamma l_p / v_\pi < 1$ ($l_p < l_{\text{decay}}$) $\Rightarrow l_{\text{prod}} \sim l_p$

For $\Gamma l_p / v_\pi > 1$ ($l_p > l_{\text{decay}}$) $\Rightarrow l_{\text{prod}} \sim l_{\text{decay}} (< l_p)$.

The condition $l_p \ll l_{\text{osc}}/2\pi$ in any case guarantees that the production coherence condition is satisfied.

What was wrong with the argument that the prod. coherence always requires $\xi \ll 1$? $\xi = v_P(\Delta m^2/2P\Gamma)$ becomes > 1 for small enough Γ (long-lived parent particle). But then l_{dec} may exceed l_p , and prod. coherence will be governed by ϕ_p rather than by ξ .

$$\xi > 1 \Rightarrow \frac{\Gamma}{v_\pi} < \frac{\Delta m^2}{2P}; \quad l_p < l_{\text{dec}} \Rightarrow \frac{\Gamma}{v_\pi} < l_p^{-1}.$$

The second cond. follows from the first if $\Delta m^2/2P < l_p^{-1}$, i.e. $\phi_p < 1$.

It is only in the case $\phi_p > 1$ ($\gg 1$) that the production coherence may be governed by ξ .

A completely different approach

Assume each neutrino production event is completely coherent $\Rightarrow$
neutrino flavor transitions are described by the standard oscillation formula.

E.g. for $P_{\mu\mu}$:

$$P_{\mu\mu}(E, L) = P_{\mu\mu}^{\text{stand}}(L, E) = c^4 + s^4 + 2s^2c^2 \cos \phi.$$

Sum the effects of pion decays at different points along the decays tunnel at the probabilities level:

$$F_{\mu}(E, L) = F_{\pi}(E, 0)\Gamma \int_0^{l_p} e^{-\frac{\Gamma}{v_{\pi}}x} P_{\mu\mu}^{\text{stand}}(E, L - x)dx$$

The effective oscillation probability:

$$P_{\mu\mu}^{\text{eff}}(L, E) \equiv F_{\mu}(L, E)/F_{\mu}^0(L, E)$$

(Hernandez & Smirnov, 2011; also performed a simplified calculation at the amplitude level)

Effective probability

The result:

$$\diamond \quad P_{\mu\mu}^{\text{eff}} = c^4 + s^4 + \frac{2c^2 s^2}{\xi^2 + 1} \frac{1}{(1 - e^{-\Gamma l_p / v_\pi})} \left[\cos \phi + \xi \sin \phi \right. \\ \left. - e^{-\Gamma l_p / v_\pi} [\cos(\phi - \phi_p) + \xi \sin(\phi - \phi_p)] \right]$$

Exactly the same expression as that obtained by summation (integration) at the amplitude level!

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The simple integration of the oscillation probability along the decay tunnel (source) – in general with the decay exponential taken into account – automatically takes care of the production coherence condition!

Prod. coherence $\Leftrightarrow$ localization of the ν production process.

Why does this happen?

It is well known that decoherence gives the same result as averaging; but why the exact probabilities (with arbitrary degree of coherence) exactly coincide in the two cases?

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The spatial size of the production region is very small – given by the size of the smallest WP of the particles participating in neutrino production, in our case of the pion. Since we consider pions to be point-like and detection to be fully localized, the *position* of the production point is completely defined. $\Rightarrow$ There is no interference between the amplitudes of neutrino production in different (even closely located) points.

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For non-zero $\sigma_{x\pi}$ one can expect some differences between the results of the two approaches, but for $\sigma_{x\pi} \ll \min\{l_p, l_{\text{dec}}\}$ they should be small (though some enhancement possible).

Finite-width pion WP

Two models of finite-size pion WP, Gaussian and box-type. For $\Gamma l_p / v_\pi \gg 1$:

$$\diamond P_{\mu\mu}^{\text{eff}} = c^4 + s^4 + \frac{2c^2 s^2}{\xi^2 + 1} [(\cos \phi + \xi \sin \phi) - A_\pi \xi (\xi \cos \phi - \sin \phi)]$$

The parameter A_π :

$$A_{\pi\text{box}} = \frac{v_g}{v_\pi} \frac{\Gamma}{v_g - v_\pi} \sigma_{x\pi}, \quad A_{\pi\text{Gauss}} = \frac{2}{\sqrt{2\pi}} \frac{v_g}{v_\pi} \frac{\Gamma}{v_g - v_\pi} \sigma_{x\pi}.$$

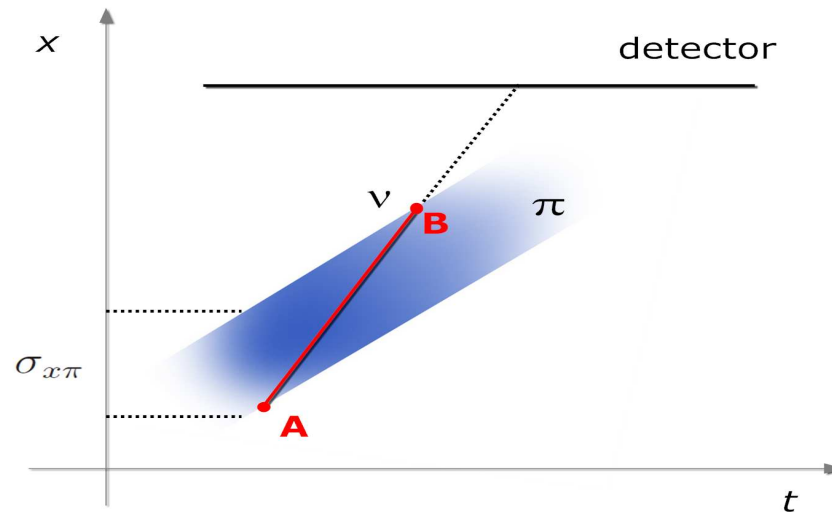
i.e. $A_\pi \sim (v_g/v_\pi) \sigma_{x\pi} / \sigma_{x\nu}$. The correction is of order

$$A_\pi \xi \sim \left[\frac{\Delta m^2}{2P} \sigma_{x\pi} \right] \cdot \frac{v_g}{v_g - v_\pi} = 2\pi \frac{\sigma_{x\pi}}{l_{\text{osc}}} \cdot \frac{v_g}{v_g - v_\pi}$$

– small since $\sigma_{x\pi} \lll l_{\text{osc}}$ (unless $v_\pi \simeq v_g$ to a very high accuracy).

An interesting point: summation at the probabilities level for finite-thickness ($= d$) proton target and point-like neutrino production gives similar expression, but with $A_\pi \xi = (\Delta m^2 / 2P) d$ (no factor $[v_g / (v_g - v_\pi)]$).

Finite-width pion WP – contd.



Additional phase for the segment AB :

$$\Delta\phi = -[E_j(P_j) - E_k(P_k)]\Delta t + (P_j - P_k)\Delta x.$$

Δt and Δx : projections of AB on the t and x axes. $\Rightarrow$

$$\Delta t = \frac{\sigma_{x\pi}}{v_g - v_\pi}, \quad \Delta x = \sigma_{x\pi} \frac{v_g}{v_g - v_\pi}.$$

$$\Delta\phi \simeq -\frac{v_g}{v_g - v_\pi} \cdot \frac{\Delta m_{jk}^2}{2P} \sigma_{x\pi}$$

Finite-width pion WP – contd.

Are deviations between the results of the coherent amplitude summation and incoherent probability summation approaches experimentally observable?

Requires extremely high energies of the parent pion:

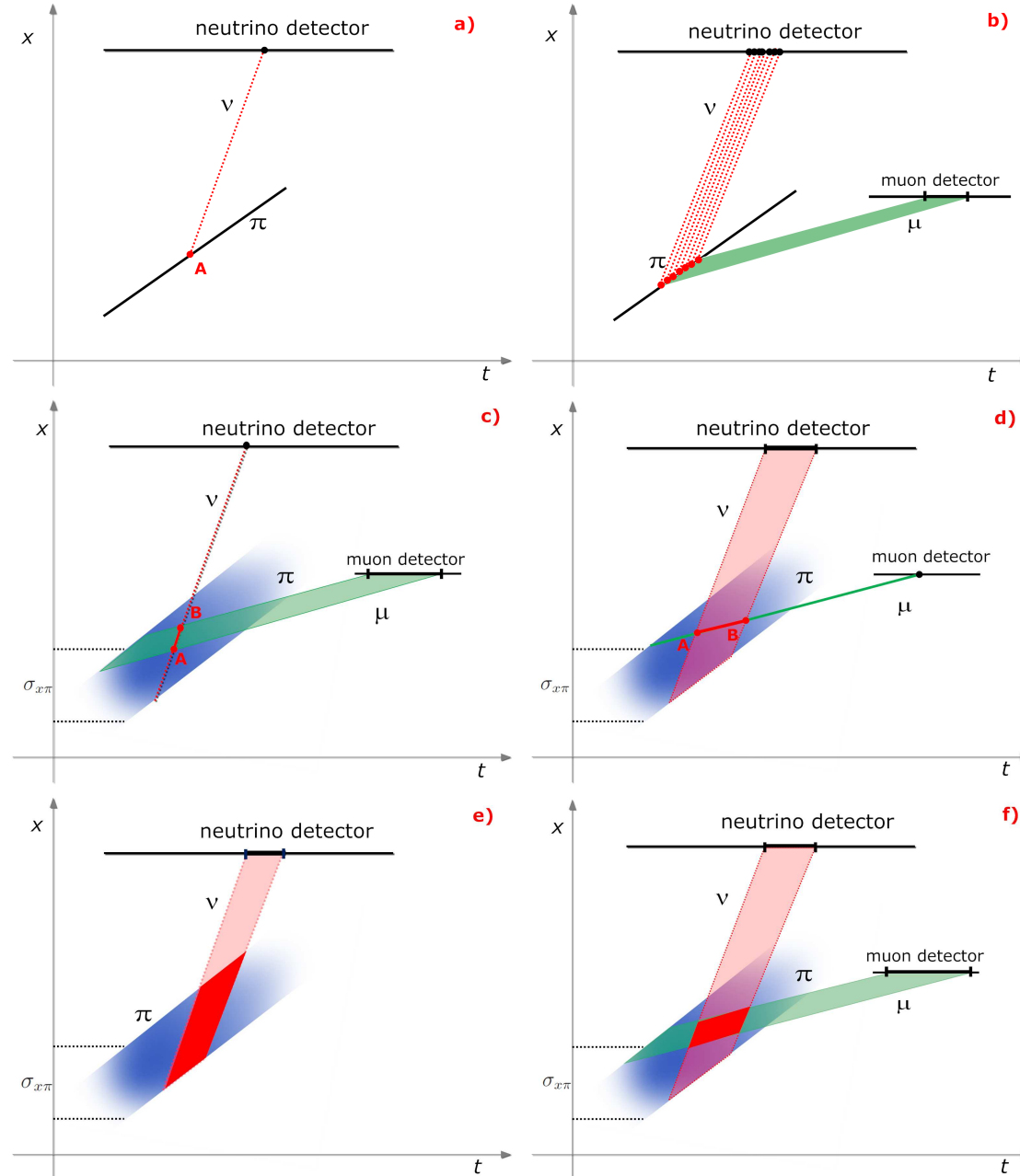
$$2 (E_{\pi} \sigma_{x\pi}) \frac{\Delta m^2}{m_{\pi}^2} \gtrsim 1.$$

E.g. for $\sigma_{x\pi} \sim 10^{-4}$ cm and $\Delta m^2 \sim 1$ eV² A_{π} would be ~ 1 for pion energies $E_{\pi} \gtrsim 10^3$ TeV – not feasible,

Another possibility: increase significantly the spatial width of w. packets of ancestor protons, which would increase the values of $\sigma_{x\pi}$. But: not clear how this could be achieved.

Other possibilities...

Graphical interpretation



Production coherence for some experiments

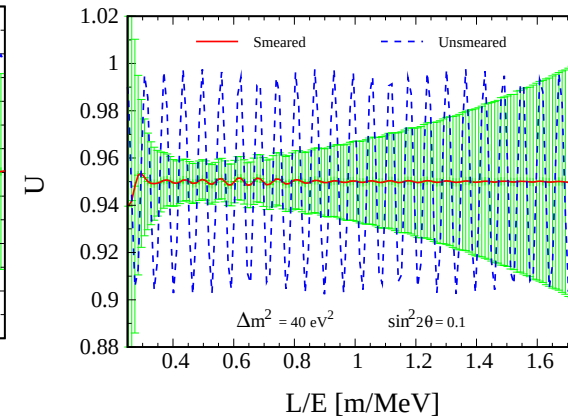
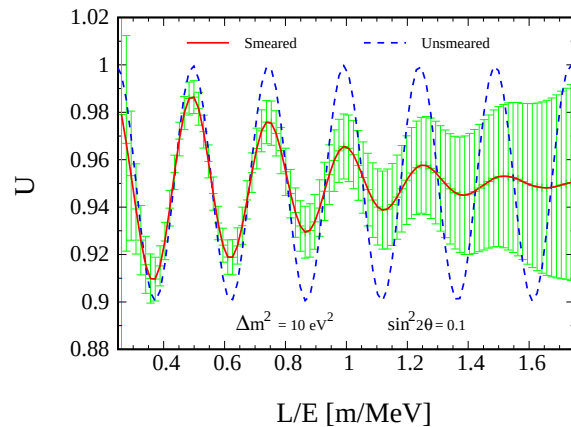
Unless otherwise specified, $\Delta m^2 = 2 \text{ eV}^2$. For β -beams $E_0 = 2 \text{ MeV}$, $\tau_0 = 1 \text{ s}$, $\gamma = 100$.

Experiment	$\langle E_\nu \rangle (\text{MeV})$	$L(\text{m})$	$l_p(\text{m})$	$l_{\text{dec}}(\text{m})$	$l_{\text{osc}}(\text{m})$	ϕ	$\Gamma l_p / v_P$	ϕ_p	ξ
LSND	~ 40	30	0	0	50	3.8	-	0	0
KARMEN	~ 40	17.7	0	0	50	2.24	-	0	0
MiniBooNE	~ 800	541	50	89	992	3.43	0.56	0.32	0.56
NOMAD	$2.7 \cdot 10^3$	770	290	3009	33480	0.145	0.1	0.054	0.56
(20 eV^2)					3348	1.45	0.1	0.54	5.64
CCFR(10^2 eV^2)	$5 \cdot 10^4$	891	352	5570	1240	4.51	0.06	1.78	28.2
CDHS	3000	130	52	334	3720	0.22	0.155	0.088	0.56
(20 eV^2)					372	2.2	0.155	0.878	5.64
K2K	1500	300	200	167	1861	1.01	1.2	0.68	0.56
T2K	600	280	96	66.4	744	2.36	1.45	0.81	0.56
Minos	3300	1040	675	368	4092	1.6	1.84	1.04	0.56
NO ν A	2000	1040	675	223	2480	2.64	3.03	1.71	0.56
β -beams	400	$1.3 \cdot 10^5$	2500	$3 \cdot 10^{10}$	496	1647	$8.3 \cdot 10^{-8}$	31.7	$3.8 \cdot 10^8$

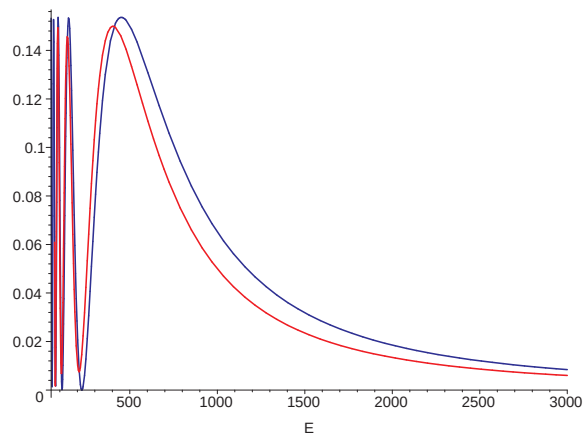
Noticeable effects for MiniBooNE, NOMAD (20 eV^2), CCFR (100 eV^2), CDHS (20 eV^2), K2K, T2K, MINOS, NO ν A, very large effects for β -beams

Examples of prod. coherence violation

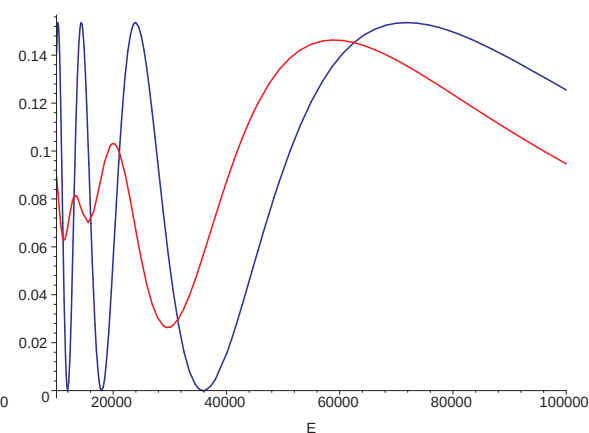
$\nu_e \rightarrow \nu_s$ oscillations in β -beam expts. (Agarwalla, Huber & Link, arXiv:0907.3145).
Ratio of oscillated and unoscillated fluxes ($\gamma = 30$, $l_p = 10\text{m}$, $L = 50\text{ m}$):



T2K



CCFR



Summary

- For sterile neutrinos with $\Delta m^2 \gtrsim 1 \text{ eV}^2$ production/detection coherence is no longer automatic and has to be carefully examined.

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Summary

- For sterile neutrinos with $\Delta m^2 \gtrsim 1 \text{ eV}^2$ production/detection coherence is no longer automatic and has to be carefully examined.
- QM wave packet formalism provides a consistent framework for that and allows to obtain the result through the summation of amplitudes corresponding to decays of parent particles in different points at the amplitude level.
- The production coherence depends on the oscillation phase acquired over the neutrino production region and is satisfied when this phase is small. The parameters governing the coherence are ϕ_p for $l_{\text{dec}} > l_p$ and ξ for $l_{\text{dec}} < l_p$.

Summary – contd.

- The obtained probability coincides exactly with the one obtained by incoherent summation of oscill. probabilities over the neutrino production points (with the proper decay exponential included). This coincidence is a consequence of vanishing spatial size of the wave packets of parent particles σ_{xP} and perfect space-time localization of the detection process. Approximately holds even if $\sigma_{xP} \neq 0$ provided that it is small compared to l_p and l_{osc} .

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- Detection of particles accompanying neutrino production leads to neutrino tagging. It can increase the localization of the production region and so improve production coherence.
- When averaging over the neutrino source and neutrino detector is properly done, possible production/detection decoherence effects are automatically taken into account.

Backup slides

How can this be understood?

The prod. coherence condition (ensures that different neutrino mass eigenstates forming a flavor neutrino state are emitted coherently):

$$\Delta E \ll \sigma_E ,$$

For a decay of a free particle in a box (e.g. decay tunnel):

$$\sigma_E \sim \max\{\Gamma, v_\pi/l_p\} \cdot \frac{v_g}{v_g - v_\pi} .$$

σ_E also determines the spatial width of the neutrino wave packet: $\sigma_x \simeq v_g/\sigma_E$

$\Rightarrow$ for $\Gamma l_p/v_\pi \gg 1$ the width of the wave packet $\sigma_x \sim (v_g - v_\pi)/\Gamma$;

for $\Gamma l_p/v_\pi \ll 1$ $\sigma_x \sim [(v_g - v_\pi)/v_\pi] l_p$.

$$\Delta E \equiv |E_j(P_j) - E_k(P_k)| \simeq \frac{\Delta m^2}{2P} \frac{v_\pi v_g}{(v_g - v_\pi)} \quad \Rightarrow$$

The decoherence parameters

Two limiting regimes:

- $\Gamma l_p / v_\pi \ll 1$ (pion decay length $l_{\text{decay}} = v_\pi / \Gamma \gg l_p$)

$$\frac{\Delta E}{\sigma_E} \simeq \frac{\Delta m^2}{2P} l_p = \phi_p.$$

- $\Gamma l_p / v_\pi \gg 1$ (pion decay length $l_{\text{decay}} = v_\pi / \Gamma \ll l_p$)

$$\frac{\Delta E}{\sigma_E} \simeq \frac{\Delta m^2}{2P} \cdot \frac{v_\pi}{\Gamma} = \xi.$$

When expressed through $l_{\text{osc}} = 4\pi E / \Delta m^2$:

$$\phi_p = 2\pi \frac{l_p}{l_{\text{osc}}}, \quad \xi = 2\pi \frac{l_{\text{decay}}}{l_{\text{osc}}}.$$

The meaning of the production coherence condition:

The osc. phase acquired over the neutrino production region must be small $\Leftrightarrow$
the production region must be small compared to $l_{\text{osc}} / 2\pi$.

The decoherence parameters – contd.

For $\Gamma l_p / v_\pi < 1$ ($l_p < l_{\text{decay}}$) $\Rightarrow l_{\text{prod}} \sim l_p$

For $\Gamma l_p / v_\pi > 1$ ($l_p > l_{\text{decay}}$) $\Rightarrow l_{\text{prod}} \sim l_{\text{decay}} (< l_p)$.

The condition $l_p \ll l_{\text{osc}}/2\pi$ in any case guarantees that the production coherence condition is satisfied.

What was wrong with the argument that the prod. coherence always requires $\xi \ll 1$? $\xi = v_P(\Delta m^2/2P\Gamma)$ becomes > 1 for small enough Γ (long-lived parent particle). But then l_{dec} may exceed l_p , and prod. coherence will be governed by ϕ_p rather than by ξ .

$$\xi > 1 \Rightarrow \frac{\Gamma}{v_\pi} < \frac{\Delta m^2}{2P}; \quad l_p < l_{\text{dec}} \Rightarrow \frac{\Gamma}{v_\pi} < l_p^{-1}.$$

The second cond. follows from the first if $\Delta m^2/2P < l_p^{-1}$, i.e. $\phi_p < 1$.

It is only in the case $\phi_p > 1$ ($\gg 1$) that the production coherence may be governed by ξ .

Effects of muon detection (for pointlike pion)

If muons is detected: plane wave $\rightarrow$ wave packet

$$\psi_\mu(x, t) = e^{iKx - iE_\mu(K)t} g_\mu[(x - x_S) - v_\mu(t - t_S)] .$$

Shape factor $g_\mu[(x - x_S) - v_\mu(t - t_S)]$ determined by the muon detection process. The argument of g_μ : initial condition that at time $t = t_S$ the peak of the w. packet is at $x = x_S$. Choose x_S as the coordinate of the center of the muon w. packet at the neutrino production time. For pointlike pions x_S should lie on the pion's trajectory $\Rightarrow x_S = v_\pi t_S$.

$$I_{jk}(L) = C_0 \int_0^{l_p} dx \left| g_\mu \left((x - x_S) \frac{v_\pi - v_\mu}{v_\pi} \right) \right|^2 e^{-i \frac{\Delta m_{jk}^2}{2P} (L-x) - \Gamma \frac{x}{v_\pi}} .$$

When the muon is undetected: $g_\mu \rightarrow 1$. Eff. width of the muon w. packet:

$$\tilde{\sigma}_{x\mu} \equiv \sigma_{x\mu} \frac{v_\pi}{v_\pi - v_\mu} .$$

The results of amplitude summation and probability summation approaches again coincide.

Limiting cases

(1) $\tilde{\sigma}_{x\mu} \rightarrow \infty$: plane wave limit. $g_\mu \rightarrow \text{const}$ – previous results recovered.

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$$I_{jk}(L) = \text{const.} e^{-\Gamma \frac{x_S}{v_\pi}} e^{-i \frac{\Delta m_{jk}^2}{2P} (L - x_S)} \Rightarrow P_{\alpha\beta}^{\text{stand}}(L - x_S).$$

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For $\tilde{\sigma}_{x\mu} \not\rightarrow \infty$ – oscillations of a “tagged” neutrino, i.e. of a neutrino produced together with the muon which was detected and whose production coordinate was found to be x_S with the accuracy $\tilde{\sigma}_{x\mu}$.

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$$P_{\mu\mu} = c^4 + s^4 + 2s^2 c^2 e^{-\frac{1}{2} \left(\frac{\Delta m^2}{2P} \right)^2 \tilde{\sigma}_{x\mu}^2} \cos \left(\frac{\Delta m^2}{2P} (L - x_S) \right).$$

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$\Rightarrow$ the decoherence parameter is $\frac{\Delta m^2}{2P} \tilde{\sigma}_{x\mu}$. For $\tilde{\sigma}_{x\mu} \ll l_{\text{osc}}/2\pi$ the stand. probability is recovered.

The case of muon interacting with medium

The case when the muon interacts with the medium but there are no muon detectors (the muon's position not measured). Neutrinos are not tagged $\Rightarrow$ one has to integrate

$$I_{jk}(L) = C_0 \int_0^{l_p} dx \left| g_\mu \left((x - x_S) \frac{v_\pi - v_\mu}{v_\pi} \right) \right|^2 e^{-i \frac{\Delta m_{jk}^2}{2E} (L-x) - \Gamma \frac{x}{v_\pi}} .$$

over x_S .

Integration of $|g_\mu|^2$ gives the normalization constant of this function which does not influence the oscillation probabilities. The results obtained in the case when the muon is not detected are recovered.

Pion interactions

Interaction of the pions in the bunch btw themselves or with other particles may identify the individual pion whose decay produces a given neutrino. E.g. pion decay may lead to some recoil of the neighbouring particles which may be detected. $\Rightarrow$

Would localize the neutrino production point up to an uncertainty of order of the inter-pionic distance (or the distance between the pion and the other particles in the source) $r_0 \Rightarrow$ neutrino tagging.

Production decoherence parameter: $(\Delta m^2/2P)r_0$.

If the information about the interaction between the decaying pion and the surrounding particles is not recorded and not used for neutrino tagging, the oscillations occur in exactly the same way as if pions did not interact with each other or with other particles.

Coherence production conditions

Coherence production conditions:

$$|\Delta E| \ll \sigma_E, \quad |\Delta p| \ll \sigma_p.$$

On the other hand:

$$\Delta E \simeq v_g \Delta p + \frac{\Delta m^2}{2E}.$$

Constraint $|\Delta E| \ll \sigma_E \Rightarrow$

$$\left| \frac{v_g \Delta p}{\sigma_E} + \frac{\Delta m^2}{2E \sigma_E} \right| \ll 1. \quad (*)$$

(a) The two terms in ΔE do not approximately cancel each other. $\Rightarrow$

$v_g |\Delta p| \ll \sigma_E \leq \sigma_p$, i.e. for relativistic neutrinos $|\Delta p| \ll \sigma_p$ follows from $|\Delta E| \ll \sigma_E$.

(b1) There is a strong cancellation, but both terms on the l.h.s. of (*) are small – see case (a).

(b2) Strong cancellation, but both terms on the l.h.s. of (*) are $\gtrsim 1$: momentum condition is independent. But: the only known case – Mössbauer neutrinos.

When are neutrino oscillations observable?

How are the oscillations destroyed? Suppose by measuring momenta and energies of particles at neutrino production (or detection) we can determine its energy E and momentum p with uncertainties σ_E and σ_p . From

$$E_i^2 = p_i^2 + m_i^2:$$

$$\sigma_{m^2} = \left[(2E\sigma_E)^2 + (2p\sigma_p)^2 \right]^{1/2}$$

When are neutrino oscillations observable?

If $\sigma_{m^2} < \Delta m^2 = |m_i^2 - m_k^2|$ – one can tell which mass eigenstate is emitted.

$\sigma_{m^2} < \Delta m^2$ implies $2p\sigma_p < \Delta m^2$, or $\sigma_p < \Delta m^2/2p \simeq l_{\text{osc}}^{-1}$.

But: To measure p with the accuracy σ_p one needs to measure the momenta of particles at production with (at least) the same accuracy $\Rightarrow$ uncertainty of their coordinates (and the coordinate of ν production point) will be

$$\sigma_{x, \text{prod}} \gtrsim \sigma_p^{-1} > l_{\text{osc}}$$

$\Rightarrow$ Oscillations washed out. Similarly for neutrino detection.

Natural necessary condition for coherence (observability of oscillations):

$$L_{\text{source}} \ll l_{\text{osc}}, \quad L_{\text{det}} \ll l_{\text{osc}}$$

No averaging of oscillations in the source and detector

Satisfied with very large margins in most cases of practical interest

Wave packet separation

Wave packets representing different mass eigenstate components have different group velocities $v_{gi} \Rightarrow$ after time t_{coh} (coherence time) they separate $\Rightarrow$ Neutrinos stop oscillating! (Only averaged effect observable).

Coherence time and length:

$$\Delta v \cdot t_{\text{coh}} \simeq \sigma_x; \quad l_{\text{coh}} \simeq v t_{\text{coh}}$$

$$\Delta v = \frac{p_i}{E_i} - \frac{p_k}{E_k} \simeq \frac{\Delta m^2}{2E^2}$$

$$l_{\text{coh}} \simeq \frac{v}{\Delta v} \sigma_x = \frac{2E^2}{\Delta m^2} v \sigma_x$$

The standard formula for P_{osc} is obtained when the decoherence effects are negligible.

Lorentz invariance of oscillation probability

1. “Paradox” of neutrino w. packet length

For neutrino production in decays of unstable particles at rest (e.g. $\pi \rightarrow \mu \nu_\mu$):

$$\sigma_E \simeq \tau^{-1} = \Gamma_\pi, \quad \sigma_x \simeq \frac{v_g}{\sigma_E} \simeq \frac{v_g}{\Gamma_\pi} (= v_g \tau)$$

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For decay in flight: $\Gamma'_\pi = (m_\pi/E_\pi)\Gamma_\pi$. One might expect

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The solution: pion decay takes finite time. During the decay time the pion moves over distance $l = u\tau'$ (“chases” the neutrino if $u > 0$).

$$\sigma'_x \simeq v'_g/\Gamma' - l = v'_g\tau' - u\tau' = (v'_g - u)\gamma_u\tau = \frac{v_g\tau}{\gamma_u(1 + v_g u)},$$

[the relativ. law of addition of velocities: $v'_g = (v_g + u)/(1 + v_g u)$].

Lorentz invariance issues – contd.

That is

$$\sigma'_x = \frac{\sigma_x}{\gamma_u(1 + v_g u)}$$

For relativistic neutrinos $v_g \approx v'_g \approx 1 \Rightarrow$

$$\sigma'_x = \sigma_x \sqrt{\frac{1 - u}{1 + u}}$$

$\Rightarrow$ when the pion is boosted in the direction of neutrino emission ($u > 0$) the neutrino wave packet gets contracted; when it is boosted in the opposite direction ($u < 0$) – the wave packet gets dilated.

Oscillations and QM uncertainty relations

Neutrino oscillations – a QM interference phenomenon, owe their existence to QM uncertainty relations

Neutrino energy and momentum are characterized by uncertainties σ_E and σ_p related to the spatial localization and time scale of the production and detection processes. These uncertainties

- allow the emitted/absorbed neutrino state to be a coherent superposition of different mass eigenstates
- determine the size of the neutrino wave packets $\Rightarrow$ govern decoherence due to wave packet separation

σ_E – the effective energy uncertainty, dominated by the smaller one between the energy uncertainties at production and detection. Similarly for σ_p .

The paradox of σ_E and σ_p

QM uncertainty relations: σ_p is related to the spatial localization of the production (detection) process, while σ_E to its time scale $\Rightarrow$ independent quantities.

On the other hand: Neutrinos propagating macroscopic distances are on the mass shell. For on-shell mass eigenstates $E^2 = p^2 + m_i^2$ means

$$E\sigma_E = p\sigma_p$$

How can this be understood?

The solution: At production, neutrinos are *not* on the mass shell. They go on shell only after they propagate $x \sim (\text{a few}) \times$ De Broglie wavelengths. After that their energy and momentum get related by $E^2 = p^2 + m_i^2 \Rightarrow$ the larger uncertainty shrinks towards the smaller one to satisfy $E\sigma_E = p\sigma_p$.

On-shell relation between E and p allows to determine the less certain of the two through the more certain one, reducing the error of the former.

What determines the length of ν w. packets?

The length of ν w. packets: $\sigma_x \sim 1/\sigma_p$. For propagating on-shell neutrinos:

$$\sigma_p \simeq \min\{\sigma_p^{\text{prod}}, (E/p)\sigma_E^{\text{prod}}\} = \min\{\sigma_p^{\text{prod}}, (1/v_g)\sigma_E^{\text{prod}}\}$$

Which uncertainty is smaller at production, σ_p^{prod} or σ_E^{prod} ?

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● If $T_S < \tau$ (τ – lifetime of the parent unstable particle) $\Rightarrow$
 $\sigma_E \simeq T_S^{-1}$ (collisional broadening). Mom. uncertainty: $\sigma_p \simeq L_S^{-1}$.

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● If $T_S > \tau$ (quasi-free parent particle) $\Rightarrow \sigma_E \simeq \tau^{-1} = \Gamma$.

$\sigma_p \simeq [(p/E)\tau]^{-1} \simeq [(p/E)\sigma_E]^{-1}$, i.e. $\sigma_E \simeq (p/E)\sigma_p < \sigma_p$.

The length of ν w. packets – contd.

In both cases $\sigma_E^{\text{prod}} < \sigma_p^{\text{prod}}$ $\Leftarrow$ also when $\nu's$ are produced in collisions.

$$\Rightarrow \sigma_{p \text{ eff}} \simeq \frac{\sigma_E}{v_g},$$

$$\sigma_x \simeq \frac{v_g}{\sigma_E}$$

In the stationary limit ($\sigma_E \rightarrow 0$) one has $\sigma_{p \text{ eff}} \rightarrow 0$ even though σ_p is finite!
Therefore $\sigma_x \rightarrow \infty$ and so the coherence length $l_{\text{coh}} \rightarrow \infty$
– a well known result.

Size of the pion w. packet

$$\frac{1}{\sigma_{xP}^2} \equiv \frac{1}{\sigma_{xN}^2} + \frac{1}{\sigma_{xp}^2}.$$

$$v_P \equiv \sigma_{xP}^2 \left(\frac{v_p}{\sigma_{xp}^2} + \frac{v_N}{\sigma_{xN}^2} \right).$$

$$\sigma_{x\pi} \approx [\sigma_{xP}^2 + (v_P - v_\pi)^2 / \sigma_{e\pi}^2]^{1/2},$$

$$\sigma_{e\pi} \approx \frac{v_p - v_N}{(\sigma_{xp}^2 + \sigma_{xN}^2)^{1/2}} \approx \frac{v_p}{\sigma_{xp}}.$$

We shall assume that the target nuclei are at rest, $v_N = 0$. $\Rightarrow$

$$v_P \approx v_p (\sigma_{xN} / \sigma_{xp})^2 \ll v_p.$$

$$\sigma_{x\pi} \approx [\sigma_{xN}^2 + (v_\pi / v_p)^2 \sigma_{xp}^2]^{1/2}.$$

By integrating first over time t (in the expression for $I_{jk}(L)$) and momenta (in Fourier transformations $f_{j,k}^S(p) \rightarrow \psi_{j,k}^S(x, t)$):

$$I_{jk}(L) = \frac{|M_P|^2}{v_g} e^{-i \frac{\Delta m_{jk}^2}{2P} L} \int dx_1 dt_1 \int dx_2 dt_2 g_\pi(x_1, t_1) g_\pi^*(x_2, t_2) e^{i \frac{\Delta m_{jk}^2}{2P} x_2 - \frac{\Gamma}{2} (t_1 + t_2)} \\ \times e^{i[E_j(P) - E_P](t_1 - t_2)} \delta[(x_1 - x_2) - v_g(t_1 - t_2)].$$

For pointlike pions: $g_\pi(x_1, t_1) \propto \delta(x_1 - v_\pi t_1)$, $g_\pi(x_2, t_2) \propto \delta(x_2 - v_\pi t_2)$ $\Rightarrow$

$$\delta[(x_1 - x_2) - v_g(t_1 - t_2)] = \delta[(v_g - v_\pi)(t_1 - t_2)].$$

$$\Rightarrow t_1 = t_2 \Rightarrow x_1 = x_2, \text{ i.e.}$$

$$g_\pi(x_1, t_1) g_\pi^*(x_2, t_2) \rightarrow |g_\pi(x_1, t_1)|^2$$

No interf. terms in the expression for $|\mathcal{A}|^2$, summation at the probabilities level.

$$B_j \equiv \frac{E_P - E_j(P)}{v_j - v_\pi}.$$

Note a useful relation

$$E_j(P_j) - E_k(P_k) = v_\pi(B_j - B_k).$$

From the definition of B_j :

$$B_j - B_k \simeq \frac{E_k(P) - E_j(P)}{v_g - v_\pi} + \left[E_P - \frac{E_j(P) + E_k(P)}{2} \right] \frac{v_k - v_j}{(v_g - v_\pi)^2},$$

where v_g is the average group velocity of the neutrino mass eigenstates ν_j and ν_k , and terms $\sim (\Delta m_{jk}^2)^2$ have been discarded. The second term here has the smallness $\sim \Gamma/P$ compared to the first term and so can be safely neglected.

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⇒ Does not always mean that prod. coherence is violated! For $l_p < l_{\text{dec}}$ prod. coherence is governed by ϕ_p rather than by ξ .

The analysis also applies to neutrinos produced in any other decay process (K -decay, μ -decay, β -decay, ...), provided that particles accompanying ν production are undetected. If they are detected, production coherence depends on the degree of their localization (coherence typically improved).