(not only) Neutrinos and GUT's

A tribute to Guido Altarelli

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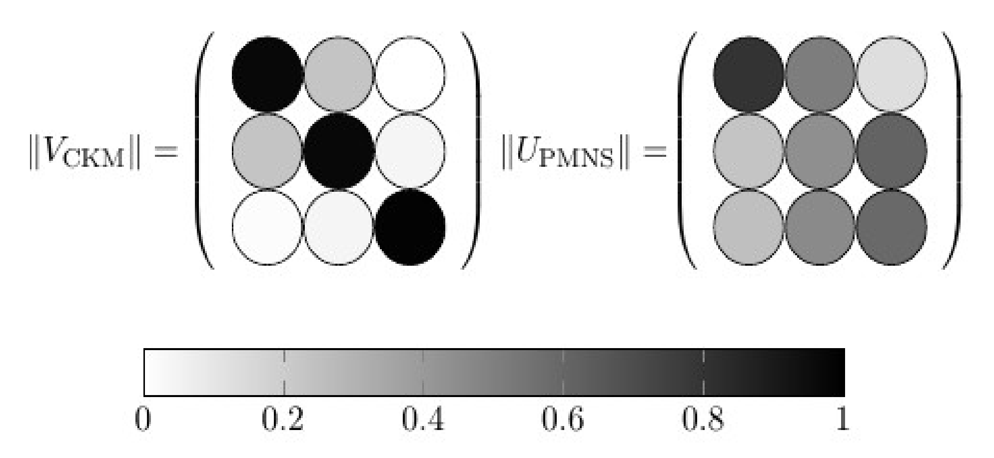
Standard oscillations

- Mixing matrix has the same structure in both contexts

$$U_{CKM\,,\,PMNS}=\begin{pmatrix}1&0&0\\0&c_{23}&s_{23}\\0&-s_{23}&c_{23}\end{pmatrix}\times\begin{pmatrix}c_{13}&0&s_{13}e^{i\delta}\\0&1&0\\-s_{13}e^{-i\delta}&0&c_{13}\end{pmatrix}\times\begin{pmatrix}c_{12}&s_{12}&0\\-s_{12}&c_{12}&0\\0&0&1\end{pmatrix}$$

thanks to Andrea Di Iura



PMNS	CKM
all (but 1-3) matrix elements are of O(1)	almost diagonal
one small and two large mixing angles	the three mixings are all small

(PMNS vs CKM)

in the Standard Model they do not talk to each other although the mechanism producing them is essentially the same

Mixing matrices

- U_{PMNS} and V_{CKM} have contributions from two different sectors

<u>leptons</u>

$$U_{PMNS} = U^{+l}_{j\alpha} U^{\nu}_{\alpha i}$$

from the diagonalisation of the charged lepton mass matrix

from the diagonalisation of the neutrino mass matrix

<u>quarks</u>

$$V_{CKM} = U^{+d}_{j\alpha} U^{u}_{\alpha i}$$

How to relate these two sectors ?

The need of New Physics

How to relate these two sectors ?

- Invoking GUT theories (gauge groups larger than the Standard Model):

  leptons and quarks sit in the same irreducible representations of the group

↓

Mass matrices are related

ex: SU(5)

$$\bar{5} = \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ e^- \\ -\nu_e \end{pmatrix}_L \qquad m_d = m_e^T$$

→ Not enough for producing the correct mixings

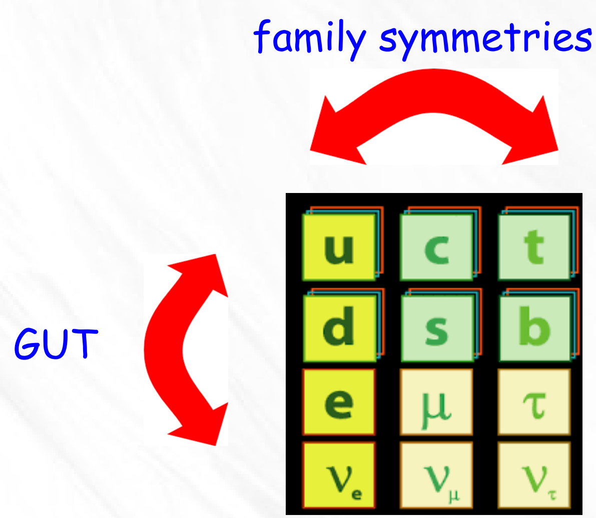
The need of New Physics

- to improve predictability: Invoke family symmetries:

different families sit in the same irreducible representations of the group

↓

Matrix elements of mass matrices are related



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Being less ambitious...QLC

- Numerically, one sees that: $\theta_{12} + \theta_C \sim \pi/4$ → quark-lepton complementarity (QLC)

$\theta_{12} + O(\theta_C) \sim \pi/4$ is called *weak complementarity*

- Numerically, one also sees that: $\theta_{13} \sim \theta_C/\mathrm{sqrt}[2]$

this suggests that the Cabibbo angle is a key-role parameter

Where θ_c enters in the lepton sector?

Nature seems to help us !

- $m_\mu/m_\tau \sim \theta^2_C$
- $m_e/m_\mu \sim \theta_c^{3\text{-}4}$

we have to deal with **mass matrices !**

Introducing θ_c into the charged lepton masses

- for large fermion masses, we can use renormalizable operators (d=4):

$$\overline{\psi_L} H \psi_R$$

- to generate hierarchies, we can use non-renormalizable operators (d >= 5):

$$\overline{\psi_L} H \psi_R \left(\frac{\varphi}{\Lambda}\right)^n$$

φ: new scalar fields, with vev = $\langle\phi\rangle$ transforming non-trivially under some flavor symmetry

Λ: cut-off of the theory (and $\langle\phi\rangle/\Lambda$ is smaller than 1)

After the breaking of the flavor symmetry

$$m_\mu/m_\tau \sim (d=6) / (d=4) \sim (\langle\phi\rangle/\Lambda)^2$$

$$\Downarrow$$

$$\frac{\langle\varphi\rangle}{\Lambda} \sim \theta_C$$

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Getting the QLC relation

- The strategy:

Start with LO prediction in the neutrino sector as $\theta_{12} = \pi/4$ ← family symmetries

↓

Corrections from charged leptons of $O(\theta_c)$: weak QLC

↓

Connecting quarks and leptons: obtaining $V_{us} \sim \theta_c$

GUT

Getting the QLC

Start with a model whose LO prediction in the neutrino sector is $\theta_{12} = \pi/4$

An easy task with family symmetries

Plethora of models in the literature

Frampton, Petcov and Rodejohann,
Nucl. Phys. B687 (2004) 31
T.Ohlsson,
Phys.Lett.B622, 159 (2005)
Altarelli, Feruglio and Merlo,
JHEP0905, 020 (2009)
D.Meloni,
JHEP1110, 010 (2011)
Altarelli, Machado and Meloni,
arXiv:1504.05514 [hep-ph]

$$M_\nu = \begin{pmatrix} x & y & y \\ y & z & x-z \\ y & x-z & z \end{pmatrix}$$

diagonalization $\Rightarrow$

$$U_{BM} = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Bi-Maximal mixing

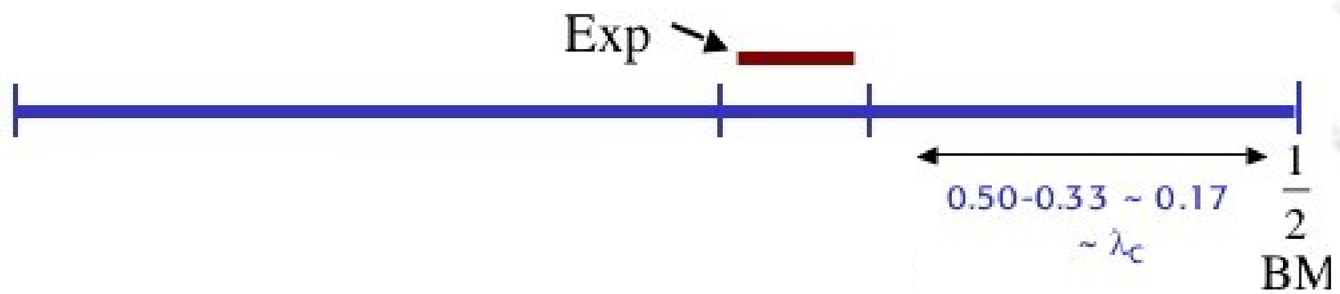
$$\sin^2\theta_{12} = \frac{1}{2} \quad \sin^2\theta_{23} = \frac{1}{2} \quad \sin^2\theta_{13} = 0$$

Corrections

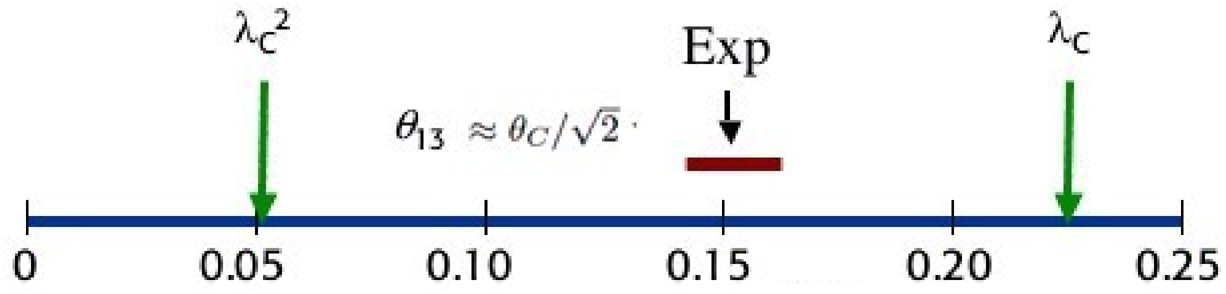
- The strategy:

Now needs corrections to fall on the experimental value $\theta_{12} \sim 33°$

$\sin^2\theta_{12}$



$\sin\theta_{13}$



Corrections provided by the diagonalization of the charged leptons

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An SU(5) example

- Example from SU(5) x S_4 → group of permutation of 4 objects

Altarelli, Machado, Meloni
arXiv:1504.05514

$$m_e \sim \begin{bmatrix} a_{11}\lambda_C^5 & a_{12}\lambda_C^4 & a_{13}\lambda_C^4 \\ a_{21}\lambda_C^4 & -c\lambda_C^3 & 0 \\ a_{13}\lambda_C^4 & c\lambda_C^3 & a_{33}\lambda_C \end{bmatrix} \Rightarrow U_l \sim \begin{bmatrix} 1 & u_{12}\lambda_C & u_{13}\lambda_C \\ -u_{12}^*\lambda_C & 1 & 0 \\ -u_{13}^*\lambda_C & -u_{12}^* u_{13}^*\lambda_C^2 & 1 \end{bmatrix}$$

- a_{ij}, u_{ij} are O(1) coefficients
- u_{ij} is a linear combination of a_{ij}

$$U_{PMNS} = U_l^+ U_{BM}$$

this gives $\sin^2\theta_{12} = \frac{1}{2} - u_{12}\lambda_C$ which is perfectly OK

- this relation is of the *weak complementarity* form
- we also ask the model to generate Vus ~ $O(\lambda_c)$

⇒ link with GUT

The Vus matrix element

- the down sector

$$m_d = m_e^T \Rightarrow U_d \sim \begin{bmatrix} 1 & d_{12}\lambda_C & d_{13}\lambda_C^3 \\ -d_{12}^*\lambda_C & 1 & d_{23}^*\lambda_C^2 \\ (d_{12}^* d_{23}^* - d_{13}^*)\lambda_C^3 & -d_{23}^*\lambda_C^2 & 1 \end{bmatrix}$$

d_{ij} are a different combination of a_{ij}

so mixings are different but the off-diagonal (1-2) element is again of $O(\lambda_c)$

(obviously we have to be sure that the up-quark sector does not destroy the scheme)

Since Vus is not u12*λ_c, we did not realize the "true" QLC

What about BM and SO(10) ?

$$L = 16\left(f \cdot 126_H + h \cdot 10_H\right)16$$

all fermions are here,
including nu-right

- no SO(10) singlets for right-handed neutrinos → more difficult explanation of the difference between charged fermions and neutrinos
- see-saw type II is an useful ansatz to separate the neutrino masses from the dominant contribution to the charged fermions (given by the Yukawa h)

$$M_{\nu R} \sim f \langle 126_H \rangle_3 + \not{type} - I$$

vev of the triplet component

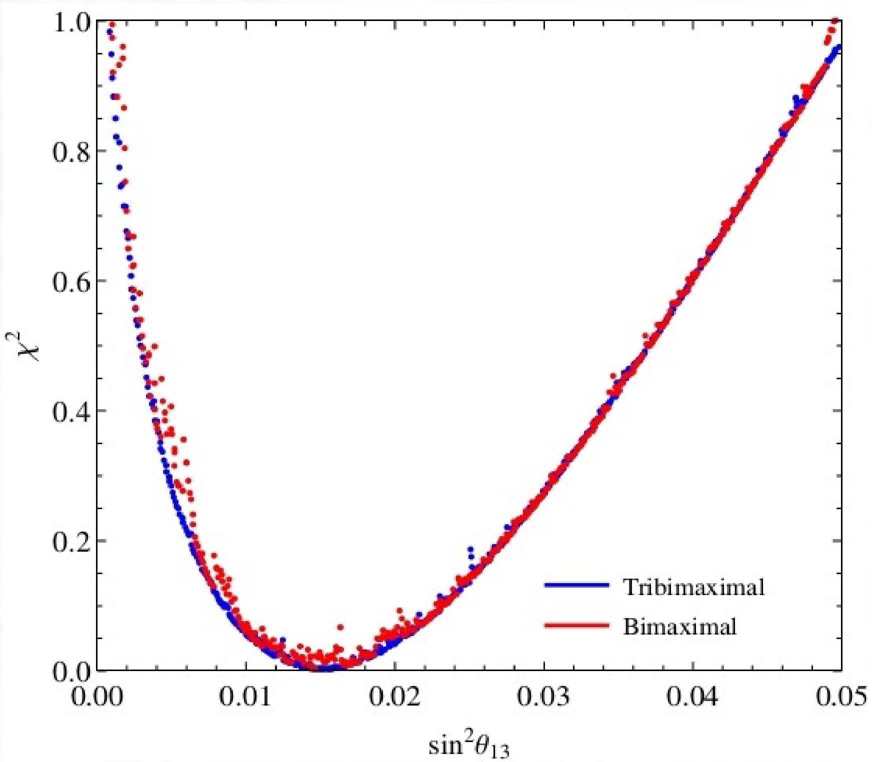
Better BM or TBM in SO(10) ?

- Are the data compatible with BM?

The answer is YES but not very conclusive

in fact, we could have started from f of the TBM form and still obtain a good description of the data, i.e., of θ_{13}

the set of parameters used in one fit are functions of the parameters of the other fit, so the χ^2 in the two cases are simply related to each other



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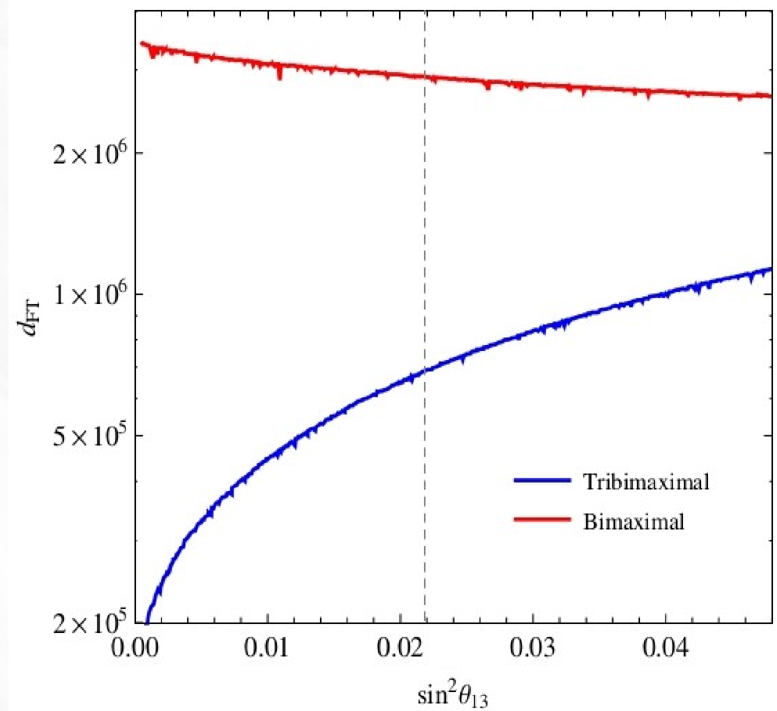
Better BM or TBM in SO(10) ?

- we have to use some estimator: the *fine-tuning parameter*

$$d_{FT} = \Sigma_i \left| \frac{par_i}{err_i} \right|$$

shift of the best-fit parameter that changes the χ^2 by 1 unit



the TBM fit to the data is slightly less fine-tuned than BM

Conclusions

- Weak form of complementarity can be easily implemented in GUT context
- BM is a good starting point in a SU(5) + family symmetry framework
- No clear preference in the description of the data emerged from SO(10), weak QLC a bit more fine-tuned than a fit from QLC