Nonunitary mixing: current constraints and new ambiguity

Omar Miranda

Cinvestav

September 7, 2016

Outline

Seesaw schemes

$$\begin{bmatrix} M_L & D \\ D^T & M_R \end{bmatrix}$$

$$\begin{bmatrix} 0 & D & 0 \\ D^T & 0 & M \\ 0 & M^T & \mu \end{bmatrix}$$

$\frac{n(n-1)}{2}$ mixing angles

$\frac{(n-1)(n-2)}{2}$ phases

Minkowski 1977, Gell-Mann Ramond Slanski 1979, Yanagida 1979, Mohapatra Senjanovic 80, Schechter Valle 1980.

Mixing matrix

$$U^{n\times n} = \omega_{n-1\,n}\,\omega_{n-2\,n}\,\ldots\,\omega_{1\,n}\,\omega_{n-2\,n-1}\,\omega_{n-3\,n-1}\,\ldots\,\omega_{1\,n-1}\,\ldots\,\omega_{2\,3}\,\omega_{1\,3}\,\omega_{1\,2}\,,$$

$$\omega_{ij} = \begin{pmatrix} 1 & 0 & & \cdots & 0 & \cdots & & & 0 \\ 0 & 1 & & & & & & & \vdots \\ \vdots & & c_{ij} & \cdots & 0 & \cdots & \eta_{ij} & & \\ & & \vdots & \ddots & & & \vdots & & \\ & & 0 & & 1 & & 0 & & \\ & & \vdots & & & \ddots & \vdots & & \\ & & \bar{\eta}_{ij} & \cdots & 0 & \cdots & c_{ij} & & \vdots \\ \vdots & & & & & & & 1 & 0 \\ 0 & & & \cdots & 0 & \cdots & & 0 & 1 \end{pmatrix}.$$

Mixing matrix

$$U^{NP} = \omega_{n-1\,n}\omega_{n-2\,n}\dots\omega_{2\,n}\omega_{1\,n}\omega_{n-2\,n-1}\dots\omega_{2\,n-1}\omega_{1\,n-1}\dots\omega_{3\,4}\,\omega_{2\,4}\omega_{1\,4}\,,$$

$$U^{3\times 3} = \omega_{2\,3}\,\omega_{1\,3}\,\omega_{1\,2}\,.$$

$$\omega_{13} = \begin{pmatrix} c_{13} & 0 & e^{-i\phi_{13}}s_{13} & \\ 0 & 1 & 0 & \vdots \\ -e^{i\phi_{13}}s_{13} & 0 & c_{13} & \\ & \dots & & 1 \end{pmatrix}$$

with $s_{ij} = \sin\theta_{ij}$, $c_{ij} = \cos\theta_{ij}$, $\eta_{ij} = e^{-i\phi_{ij}}\,\sin\theta_{ij}$, and $\bar{\eta}_{ij} = -e^{i\phi_{ij}}\,\sin\theta_{ij}$

Mixing matrix

$$U_{\alpha i}^{n\times n} = \begin{pmatrix} N & S \\ V & T \end{pmatrix}$$

$$NN^\dagger + SS^\dagger = I,$$

$$N^\dagger N + V^\dagger V = I.$$

Mixing matrix

$$N = N^{NP}\, U^{3\times 3} = \begin{pmatrix} \alpha_{11} & 0 & 0 \\ \alpha_{21} & \alpha_{22} & 0 \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{pmatrix} U^{3\times 3}$$

$$\begin{aligned} \alpha_{11} &= c_{1\,n}\, c_{1 n-1} c_{1\,n-2} \cdots c_{14}, \\ \alpha_{22} &= c_{2\,n}\, c_{2 n-1} c_{2\,n-2} \cdots c_{24}, \\ \alpha_{33} &= c_{3\,n}\, c_{3 n-1} c_{3\,n-2} \cdots c_{34}, \end{aligned}$$

Escrihuela, Forero, OGM, Tortola, Valle **PRD 93** 053009 (2015)

Mixing matrix

$$\alpha_{21} = c_{2\,n}\, c_{2n-1} \ldots c_{2\,5}\, \eta_{24}\bar{\eta}_{14} + c_{2\,n} \ldots c_{2\,6}\, \eta_{25}\bar{\eta}_{15}\, c_{14} + \ldots + \eta_{2n}\bar{\eta}_{1n}\, c_{1n-1}\, c_{1n-2} \ldots c_{14}$$

$$\alpha_{32} = c_{3\,n}\, c_{3n-1} \ldots c_{3\,5}\, \eta_{34}\bar{\eta}_{24} + c_{3\,n} \ldots c_{3\,6}\, \eta_{35}\bar{\eta}_{25}\, c_{24} + \ldots + \eta_{3n}\bar{\eta}_{2n}\, c_{2n-1}\, c_{2n-2} \ldots c_{24}$$

$\eta_{ij} = e^{-i\phi_{ij}} \sin\theta_{ij}$
$\bar{\eta}_{ij} = -e^{i\phi_{ij}} \sin\theta_{ij}$

Mixing matrix

$$NN^\dagger = \begin{pmatrix} \alpha_{11}^2 & \alpha_{11}\alpha_{21}^* & \alpha_{11}\alpha_{31}^* \\ \alpha_{11}\alpha_{21} & \alpha_{22}^2 + |\alpha_{21}|^2 & \alpha_{22}\alpha_{32}^* + \alpha_{21}\alpha_{31}^* \\ \alpha_{11}\alpha_{31} & \alpha_{22}\alpha_{32} + \alpha_{31}\alpha_{21}^* & \alpha_{33}^2 + |\alpha_{31}|^2 + |\alpha_{32}|^2 \end{pmatrix}$$

Mixing matrix

$$N = N^{NP}\, U^{3\times 3} = \begin{pmatrix} \alpha_{11} & 0 & 0 \\ \alpha_{21} & \alpha_{22} & 0 \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{pmatrix} U^{3\times 3}$$

$$\begin{aligned} \alpha_{11} &= c_{1\,n}\, c_{1 n-1} c_{1\,n-2} \cdots c_{14}, \\ \alpha_{22} &= c_{2\,n}\, c_{2 n-1} c_{2\,n-2} \cdots c_{24}, \\ \alpha_{33} &= c_{3\,n}\, c_{3 n-1} c_{3\,n-2} \cdots c_{34}, \end{aligned}$$

Escrihuela, Forero, OGM, Tortola, Valle **PRD 93** 053009 (2015)

Current constraints

beta decay

$$\propto \left[\bar{e}_L \gamma_\mu \sum N_{1i} \nu_{iL}\right] \quad (1)$$

$$G_\beta = G_F \sqrt{(NN^\dagger)_{11}} = G_F \sqrt{\alpha_{11}^2}.$$

Current constraints

muon decay

$$\propto \left[\sum N^*_{2j}\bar{\nu}_{jL}\gamma^\mu \mu_L\right]\ \left[\bar{e}_L\gamma_\mu \sum N_{1i}\nu_{iL}\right] \tag{2}$$

$$G_\mu = G_F\sqrt{(NN^\dagger)_{11}(NN^\dagger)_{22}} = G_F\sqrt{\alpha_{11}^2(\alpha_{22}^2 + |\alpha_{21}|^2)},$$

Current constraints

$$\sum_{i=1}^{3} |V_{ui}|^2 = \left(\frac{G_\beta}{G_\mu}\right)^2 = \left(\frac{G_F\sqrt{(NN^\dagger)_{11}}}{G_F\sqrt{(NN^\dagger)_{11}(NN^\dagger)_{22}}}\right)^2 = \frac{1}{(NN^\dagger)_{22}},$$

$$\sum_{i=1}^{3} |V_{ui}|^2 = \frac{1}{\alpha_{22}^2 + |\alpha_{21}|^2} = 0.9999 \pm 0.0006,$$

PDG Chin.Phys. **C38** (2014) 090001

Current constraints

$$R_\pi = \frac{\Gamma(\pi^+ \to e^+\nu)}{\Gamma(\pi^+ \to \mu^+\nu)}.$$

$$r_\pi = \frac{R_\pi}{R_\pi^{SM}} = \frac{(NN^\dagger)_{11}}{(NN^\dagger)_{22}} = \frac{\alpha_{11}^2}{\alpha_{22}^2 + |\alpha_{21}|^2}$$

$$r_\pi = 0.9956 \pm 0.0040$$

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Oscillation probabilities

$$P_{\mu e} = \sum_{i,j}^{3} N^*_{\mu i} N_{ei} N_{\mu j} N^*_{ej} \quad - \quad 4 \sum_{j>i}^{3} Re\left[N^*_{\mu j} N_{ej} N_{\mu i} N^*_{ei}\right] \sin^2\left(\frac{\Delta m^2_{ji} L}{4E}\right)$$
$$+ \quad 2 \sum_{j>i}^{3} Im\left[N^*_{\mu j} N_{ej} N_{\mu i} N^*_{ei}\right] \sin\left(\frac{\Delta m^2_{ji} L}{2E}\right).$$

Oscillation probabilities

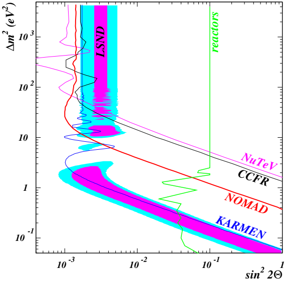
$$P_{\mu e} = (\alpha_{11}\alpha_{22})^2 P_{\mu e}^{3\times 3} + \alpha_{11}^2\alpha_{22}|\alpha_{21}|P_{\mu e}^I + \alpha_{11}^2|\alpha_{21}|^2,$$

$$\begin{aligned} P_{\mu e}^I &= -2\left[\sin(2\theta_{13})\sin\theta_{23}\sin\left(\frac{\Delta m_{31}^2 L}{4E_\nu}\right)\sin\left(\frac{\Delta m_{31}^2 L}{4E_\nu}+\phi+\delta_{CP}\right)\right] \\ &- \cos\theta_{13}\cos\theta_{23}\sin(2\theta_{12})\sin\left(\frac{\Delta m_{21}^2 L}{2E_\nu}\right)\sin(\phi), \end{aligned}$$

with $-\delta_{CP} = \phi_{12} - \phi_{13} + \phi_{23}$ and $\phi = \mathrm{I}_{\mathrm{NP}} = \phi_{12} - \mathrm{Arg}(\alpha_{21})$.

Oscillation probabilities

$$P_{\mu e} = \alpha_{11}^2 |\alpha_{21}|^2 = \frac{1}{2} \left[\sin^2 \left(2\theta_{\mu e} \right) \right]_{\text{eff}}$$



NOMAD Coll. PLB **570** (2003) 19

Current constraints

P. Astier et al. Search for nu(mu) $\rightarrow$ nu(e) oscillations in the NOMAD experiment. *Phys. Lett.*, B570:19–31, 2003.

$$|\alpha_{21}|^2 \leq 0.0007.$$

Current constraints

K.A. Olive et al. Review of Particle Physics. *Chin.Phys.*, C38:090001, 2014.
A. Abada, A.M. Teixeira, A. Vicente, and C. Weiland. *JHEP*, 1402:091, 2014.
G. Czapek et al. *Phys. Rev. Lett.*, 70:17–20, 1993.
P. Astier et al. Search for nu(mu) $\to$ nu(e) oscillations in the NOMAD experiment. *Phys. Lett.*, B570:19–31, 2003.

$$\alpha_{11}^2 \geq 0.989, \qquad \alpha_{22}^2 \geq 0.999, \qquad |\alpha_{21}|^2 \leq 0.0007.$$

Limits at 90 % CL

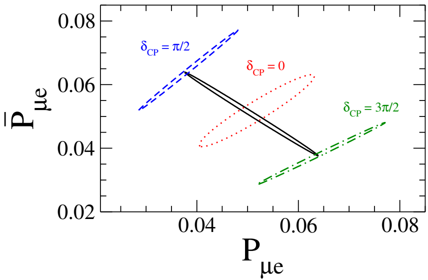
Oscillation probabilities

$$P_{\mu e} = (\alpha_{11}\alpha_{22})^2 P_{\mu e}^{3\times 3} + \alpha_{11}^2\alpha_{22}|\alpha_{21}|P_{\mu e}^I + \alpha_{11}^2|\alpha_{21}|^2,$$

$$\begin{aligned} P_{\mu e}^I &= -2\left[\sin(2\theta_{13})\sin\theta_{23}\sin\left(\frac{\Delta m_{31}^2 L}{4E_\nu}\right)\sin\left(\frac{\Delta m_{31}^2 L}{4E_\nu} + \phi + \delta_{CP}\right)\right] \\ &- \cos\theta_{13}\cos\theta_{23}\sin(2\theta_{12})\sin\left(\frac{\Delta m_{21}^2 L}{2E_\nu}\right)\sin(\phi), \end{aligned}$$

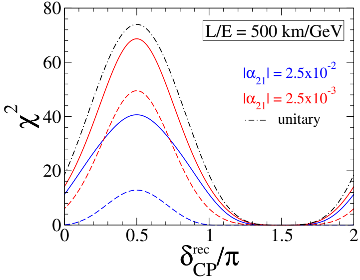
with $-\delta_{CP} = \phi_{12} - \phi_{13} + \phi_{23}$ and $\phi = \mathrm{I}_{\mathrm{NP}} = \phi_{12} - \mathrm{Arg}(\alpha_{21})$.

CP-phase ambiguity



OGM, Tortola, Valle, PRL 117 (2016) 061804

CP-phase ambiguity



OGM, Tortola, Valle, PRL 117 (2016) 061804

On the positive side

- matter effects may also contribute to the signal
- Non-standard interactions may also contribute to the matter potential making the phenomenology more interesting
  Forero, Huber PRL 117 (2016) 031801
  Forero, Huang 1608.04719

The drawbacks

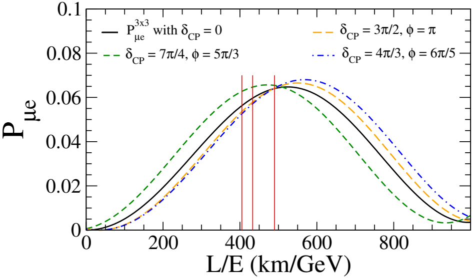
- Any improvement in the restriction of $|\alpha_{21}|$ leads to a diminish in the effect of the new phase (at least in vacuum).
- If we consider specific models for extra heavy neutral isosinglets, such as the seesaw, $|\alpha_{21}|$ gets more restricted.

Conclusions

- We have shown a parametrization that is useful from the phenomenological point of view and it is general for any number of extra neutral heavy leptons.
- The parametrization incorporates naturally the right number of parameters for a non unitary mixing matrix.
- Non unitarity will introduce new phases and their effect in the conversion probability have been shown.
- In the case of big values of the non diagonal α parameters a signal might be hinted if both neutrino chanels are measured.
- Otherwise, LBLN experiments could give complementary constraints on these parameters in future.

Thanks

CP-phase ambiguity



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