CP Violation Predictions from Flavour Symmetries

Arsenii V. Titov

in collaboration with Ivan Girardi and Serguey T. Petcov

SISSA and INFN, Trieste, Italy

Neutrino Oscillation Workshop 2016

September 6, 2016, Otranto, Lecce, Italy

Outline

- 3-Neutrino Mixing
- Discrete Flavour Symmetry Approach
- General Set-up
- Dirac Phase
    - Sum Rules
    - Predictions
    - Statistical Analysis
- Majorana Phases
    - Sum Rules
    - Predictions
- Generalised CP Symmetry
- Neutrinoless Double Beta Decay
- Conclusions

3-Neutrino Mixing

$$\nu_{lL} = \sum_{i=1}^{3} U_{li}\nu_{iL}\,, \quad l = e, \mu, \tau$$

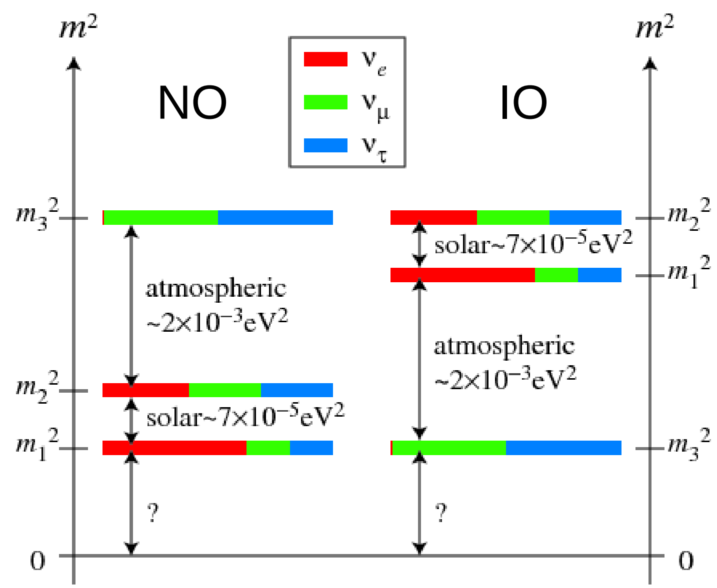
U is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\frac{\alpha_{21}}{2}} & 0 \\ 0 & 0 & e^{i\frac{\alpha_{31}}{2}} \end{pmatrix}$$

Parameter	Best fit	3σ range
$\sin^2\theta_{12}$	0.297	0.250 – 0.354
$\sin^2\theta_{23}$ (NO)	0.437	0.379 – 0.616
$\sin^2\theta_{23}$ (IO)	0.569	0.383 – 0.637
$\sin^2\theta_{13}$ (NO)	0.0214	0.0185 – 0.0246
$\sin^2\theta_{13}$ (IO)	0.0218	0.0186 – 0.0248
δ/π (NO)	1.35	0 – 2
δ/π (IO)	1.32	0 – 2
$\Delta m^2_{21}/10^{-5}$ eV²	7.37	6.93 – 7.97
$\Delta m^2_{31}/10^{-3}$ eV² (NO)	2.54	2.40 – 2.67
$\Delta m^2_{23}/10^{-3}$ eV² (IO)	2.50	2.36 – 2.64

Capozzi *et. al.*, NPB **908** (2016) 218

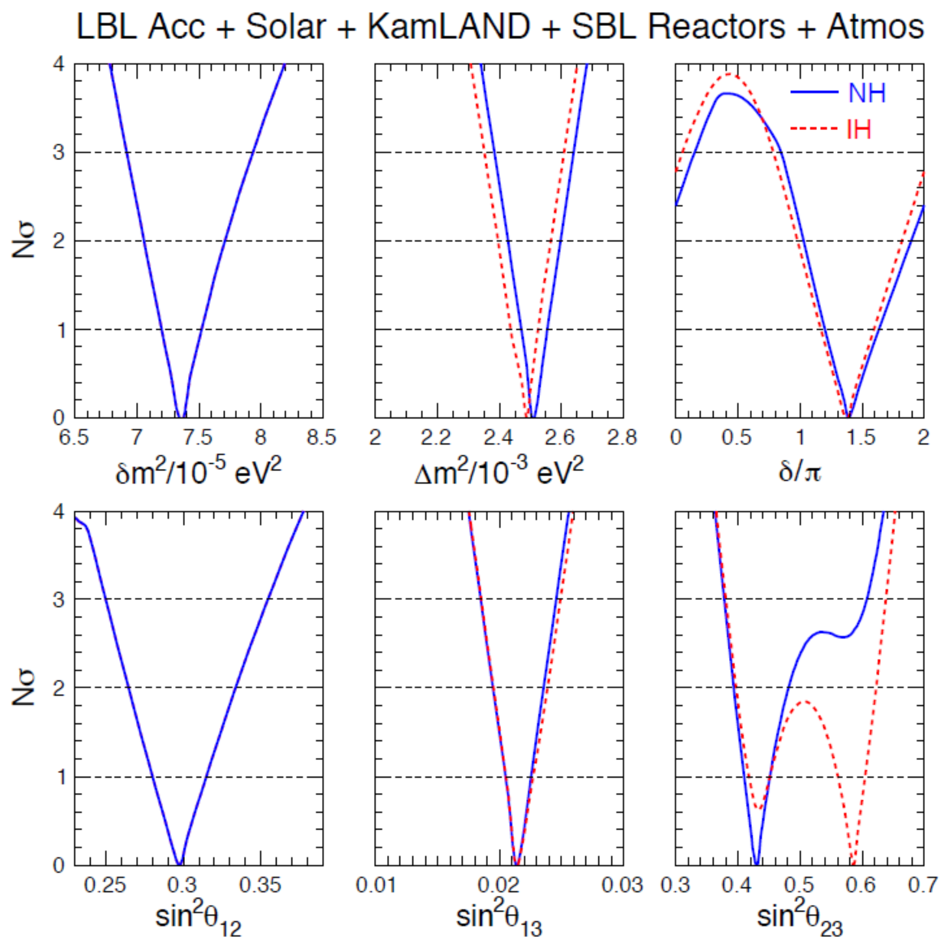
Symmetry behind this?



King and Luhn, RPP **76** (2013) 056201

3-Neutrino Mixing

Bounds on single oscillation parameters
(preliminary update)



CP phase trend:

- $\delta \sim 1.4\,\pi$ at best fit
- CP-conserving cases ($\delta = 0, \pi$) disfavored at $\sim 2\sigma$ level or more
- Significant fraction of the $[0,\pi]$ range disfavored at $>3\sigma$

θ_{23} trend:

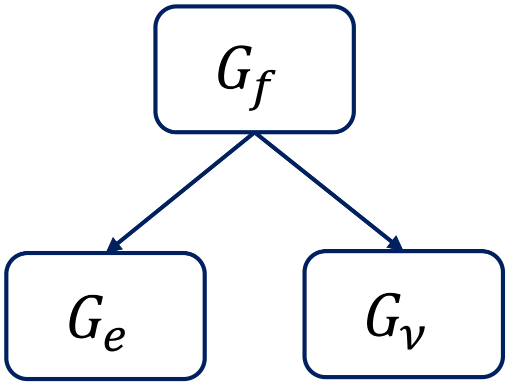
- maximal mixing disfavored at about $\sim 2\sigma$ level
- best-fit octant flips with mass ordering

$$\Delta\chi^2_{\mathrm{IO-NO}} = 3.1$$

inverted ordering slightly disfavored

Talk by Marrone @ Neutrino 2016, London, July 9, 2016

Discrete Flavour Symmetry Approach



Flavour symmetry group (non-Abelian discrete)

Residual symmetries (Abelian) of the charged lepton and neutrino mass matrices M_e and M_ν

$$-\mathcal{L} \supset \overline{l_L}\, M_e\, l_R + \overline{\nu_L^c}\, M_\nu\, \nu_L + h.c.$$

$$\rho(g_e)^\dagger\, M_e\, M_e^\dagger\, \rho(g_e) = M_e\, M_e^\dagger\,, \quad g_e \in G_e \qquad\qquad \rho(g_\nu)^T\, M_\nu\, \rho(g_\nu) = M_\nu\,, \quad g_\nu \in G_\nu$$

ρ is a unitary representation of G_f under which LH fields are transformed

$$U_e^\dagger M_e M_e^\dagger U_e = \mathrm{diag}\left(m_e^2, m_\mu^2, m_\tau^2\right) \qquad\qquad U_\nu^T M_\nu U_\nu = \mathrm{diag}\left(m_1, m_2, m_3\right)$$

$$U_e^\dagger\, \rho(g_e)\, U_e = \rho(g_e)^{\mathrm{diag}} \qquad\qquad U_\nu^\dagger\, \rho(g_\nu)\, U_\nu = \rho(g_\nu)^{\mathrm{diag}}$$

If $G_e = Z_k,\ k > 2$ or $Z_m \times Z_n,\ m,n \geq 2$ and $G_\nu = Z_2 \times Z_2$, the matrices U_e and U_ν are fixed (up to permutations of columns and right multiplication by diagonal phase matrices) $\Rightarrow$ $U = U_e^\dagger U_\nu$ is fixed

Discrete Flavour Symmetry Approach

$G_f = A_4/T', S_4, A_5$ possess a 3-dimensional ρ (unification of 3 flavours at high energies, where G_f is unbroken)

Examples: Bimaximal mixing (S_4) Tri-bimaximal mixing ($A_4/T', S_4$)

$$U_{\rm BM} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix} \qquad U_{\rm TBM} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix}$$

These mixing forms *per se* are excluded by the data ($\theta_{13} = 0$)
However, perturbative corrections are sufficient to reconstitute compatibility of, e.g., tri-bimaximal mixing with the data

If $G_e = 1$ (G_f is fully broken in the charged lepton sector), then U_e is not fixed, and it provides the requisite corrections (charged lepton corrections)

For different breaking patterns see Girardi, Petcov, Stuart, Titov, NPB **902** (2016) 1

Discrete Flavour Symmetry Approach

$G_\nu = Z_2 \times Z_2 \Rightarrow U_\nu$ is fixed (up to permutations of columns and right multiplication by a diagonal phase matrix):

$$U_\nu = \tilde{U}_\nu \, Q_0 \,, \quad Q_0 = \mathrm{diag}\left(1, e^{i\frac{\xi_{21}}{2}}, e^{i\frac{\xi_{31}}{2}}\right)$$

Symmetry Forms of $\tilde{U}_\nu$

$$\tilde{U}_\nu = R_{23}(\theta^\nu_{23})\, R_{12}(\theta^\nu_{12})$$

R_{ij} is a rotation matrix in the i-j plane

Symmetry form	Group	θ^ν_{12}	θ^ν_{23}	θ^ν_{13}
Tri-bimaximal (TBM)	A_4/T'	$\sin^{-1}(1/\sqrt{3}) \approx 35°$		
Bi-maximal (BM)	S_4	$\pi/4 = 45°$		
Golden ratio A (GRA)	A_5	$\sin^{-1}(1/\sqrt{2+r}) \approx 31°$	$-\pi/4 = -45°$	0
Golden ratio B (GRB)	D_{10}	$\sin^{-1}(\sqrt{3-r}/2) = 36°$		
Hexagonal (HG)	D_{12}	$\pi/6 = 30°$		

r is the golden ratio: $r = (1+\sqrt{5})/2$

General Set-up

$$U = U_e^\dagger U_\nu = \tilde{U}_e^\dagger \Psi \tilde{U}_\nu Q_0$$

$$\Psi = \mathrm{diag}\left(1, e^{-i\psi}, e^{-i\omega}\right), \quad Q_0 = \mathrm{diag}\left(1, e^{i\frac{\xi_{21}}{2}}, e^{i\frac{\xi_{31}}{2}}\right)$$

In general, $\tilde{U}_e$ and $\tilde{U}_\nu$ are CKM-like matrices

Frampton, Petcov, Rodejohann, NPB **687** (2004) 31

Considered Cases

Case	$\tilde{U}_e^\dagger$	$\tilde{U}_\nu$
A1	$R_{12}(\theta_{12}^e)$	$R_{23}(\theta_{23}^\nu)\, R_{12}(\theta_{12}^\nu)$
A2	$R_{13}(\theta_{13}^e)$	
B1	$R_{12}(\theta_{12}^e) R_{23}(\theta_{23}^e)$	
B2	$R_{13}(\theta_{13}^e) R_{23}(\theta_{23}^e)$	
C1	$R_{12}(\theta_{12}^e)$	$R_{23}(\theta_{23}^\nu)\, R_{13}(\theta_{13}^\nu)\, R_{12}(\theta_{12}^\nu)$
C2	$R_{13}(\theta_{13}^e)$	

$\tilde{U}_e^\dagger = R_{23}(\theta_{23}^e)$ leads to

- $\theta_{13} = 0$ for $\tilde{U}_\nu$ containing 2 rotations
- $\theta_{13} = \theta_{13}^\nu$ for $\tilde{U}_\nu$ containing 3 rotations

In the case of $\tilde{U}_e^\dagger = R_{12}(\theta_{12}^e) R_{13}(\theta_{13}^e)$ and $\tilde{U}_\nu$ containing 2 rotations, a free phase parameter ω enters resulting sum rules for the CP-violating phases

Dirac Phase: Sum Rules

Case	s^2_{23}	$\cos\delta$
A1	$\dfrac{s^{\nu 2}_{23} - s^2_{13}}{1 - s^2_{13}}$	$\dfrac{(c^2_{13} - c^{\nu 2}_{23})^{\frac{1}{2}}}{\sin 2\theta_{12}\, s_{13}\, \lvert c^{\nu}_{23}\rvert} \left[\cos 2\theta^{\nu}_{12} + (s^2_{12} - c^{\nu 2}_{12})\, \dfrac{s^{\nu 2}_{23} - (1 + c^{\nu 2}_{23})\, s^2_{13}}{c^2_{13} - c^{\nu 2}_{23}}\right]$
A2	$\dfrac{s^{\nu 2}_{23}}{1 - s^2_{13}}$	$-\dfrac{(c^2_{13} - s^{\nu 2}_{23})^{\frac{1}{2}}}{\sin 2\theta_{12}\, s_{13}\, \lvert s^{\nu}_{23}\rvert} \left[\cos 2\theta^{\nu}_{12} + (s^2_{12} - c^{\nu 2}_{12})\, \dfrac{c^{\nu 2}_{23} - (1 + s^{\nu 2}_{23})\, s^2_{13}}{c^2_{13} - s^{\nu 2}_{23}}\right]$
B1	Not fixed	$\dfrac{\tan\theta_{23}}{\sin 2\theta_{12}\, s_{13}} \left[\cos 2\theta^{\nu}_{12} + (s^2_{12} - c^{\nu 2}_{12})\, (1 - \cot^2\theta_{23}\, s^2_{13})\right]$
B2	Not fixed	$-\dfrac{\cot\theta_{23}}{\sin 2\theta_{12}\, s_{13}} \left[\cos 2\theta^{\nu}_{12} + (s^2_{12} - c^{\nu 2}_{12})\, (1 - \tan^2\theta_{23}\, s^2_{13})\right]$
C1	$\dfrac{c^2_{13} - c^{\nu 2}_{23}\, c^{\nu 2}_{13}}{1 - s^2_{13}}$	$\dfrac{(c^2_{13} - c^{\nu 2}_{13}\, c^{\nu 2}_{23})\, s^2_{12} + c^2_{12}\, s^2_{13}\, c^{\nu 2}_{13}\, c^{\nu 2}_{23} - c^2_{13}\, (c^{\nu}_{12}\, s^{\nu}_{13}\, c^{\nu}_{23} - s^{\nu}_{12}\, s^{\nu}_{23})^2}{\sin 2\theta_{12}\, s_{13}\, \lvert c^{\nu}_{13}\, c^{\nu}_{23}\rvert\, (c^2_{13} - c^{\nu 2}_{13}\, c^{\nu 2}_{23})^{\frac{1}{2}}}$
C2	$\dfrac{s^{\nu 2}_{23}\, c^{\nu 2}_{13}}{1 - s^2_{13}}$	$-\dfrac{(c^2_{13} - c^{\nu 2}_{13}\, s^{\nu 2}_{23})\, s^2_{12} + c^2_{12}\, s^2_{13}\, c^{\nu 2}_{13}\, s^{\nu 2}_{23} - c^2_{13}\, (c^{\nu}_{12}\, s^{\nu}_{13}\, s^{\nu}_{23} + s^{\nu}_{12}\, c^{\nu}_{23})^2}{\sin 2\theta_{12}\, s_{13}\, \lvert c^{\nu}_{13}\, s^{\nu}_{23}\rvert\, (c^2_{13} - c^{\nu 2}_{13}\, s^{\nu 2}_{23})^{\frac{1}{2}}}$

Petcov, NPB **892** (2015) 400; Girardi, Petcov, Titov, EPJC **75** (2015) 345

In cases A1 and A2 for $\theta^{\nu}_{23} = -\pi/4$, $s^2_{23} \approx 1/2\,(1 \mp s^2_{13})$, i.e., $\theta_{23} \approx \pi/4$

In cases B1 and B2 the best fit values of all the three mixing angles can be reproduced

Dirac Phase: Predictions

δ [°], using the best fit values of the neutrino mixing angles for NO

Case	TBM	GRA	GRB	HG	BM
A1	$102 \vee 258$	$77 \vee 283$	$107 \vee 253$	$65 \vee 295$	—
A2	$78 \vee 282$	$103 \vee 257$	$73 \vee 287$	$115 \vee 245$	—
B1	$100 \vee 260$	$78 \vee 282$	$105 \vee 255$	$67 \vee 293$	—
B2	$75 \vee 285$	$104 \vee 256$	$69 \vee 291$	$118 \vee 242$	—
	$[\pi/20, -\pi/4]$	$[\pi/10, -\pi/4]$	$[a, -\pi/4]$	$[\pi/20, b]$	$[\pi/20, \pi/6]$
C1	$109 \vee 251$	$45 \vee 315$	$30 \vee 330$	$155 \vee 205$	$133 \vee 227$
	$[\pi/20, c]$	$[\pi/20, \pi/4]$	$[\pi/10, \pi/4]$	$[a, \pi/4]$	$[\pi/20, d]$
C2	$146 \vee 214$	$71 \vee 289$	$135 \vee 225$	$150 \vee 210$	$139 \vee 221$

$\theta^{\nu}_{23} = -\pi/4$

The values in square brackets are those of $[\theta^{\nu}_{13}, \theta^{\nu}_{12}]$

$$a = \sin^{-1}(1/3), \quad b = \sin^{-1}\left(1/\sqrt{2+r}\right), \quad c = \sin^{-1}\left(1/\sqrt{3}\right), \quad d = \sin^{-1}\left(\sqrt{3-r}/2\right)$$

Non-zero values of θ^{ν}_{13}:

Bazzocchi, arXiv:1108.2497;
Toorop, Feruglio, Hagedorn, PLB **703** (2011) 447;
Rodejohann and Zhang, PLB **732** (2014) 174

Dirac Phase: Statistical Analysis

Likelihood: $$L(\cos\delta) = \exp\left(-\frac{\chi^2(\cos\delta)}{2}\right), \quad \chi^2(\cos\delta) = \min\left[\chi^2(\vec{x})|_{\cos\delta=\text{const}}\right]$$

Present: $$\chi^2(\vec{x}) = \sum_{i=1}^{4} \chi_i^2(x_i)\,, \quad \vec{x} = (s_{12}^2, s_{13}^2, s_{23}^2, \delta)$$

χ_i^2 are the 1-dimensional projections from the global analysis performed in Capozzi *et. al.*, PRD **89** (2014) 093018

Future: $$\chi^2(\vec{x}) = \sum_{i=1}^{3} \frac{(x_i - \overline{x_i})^2}{\sigma_{x_i}^2}\,, \quad \vec{x} = (s_{12}^2, s_{13}^2, s_{23}^2)$$

$\overline{x_i}$ are the current best fit values of $\sin^2\theta_{12}$, $\sin^2\theta_{13}$ and $\sin^2\theta_{23}$

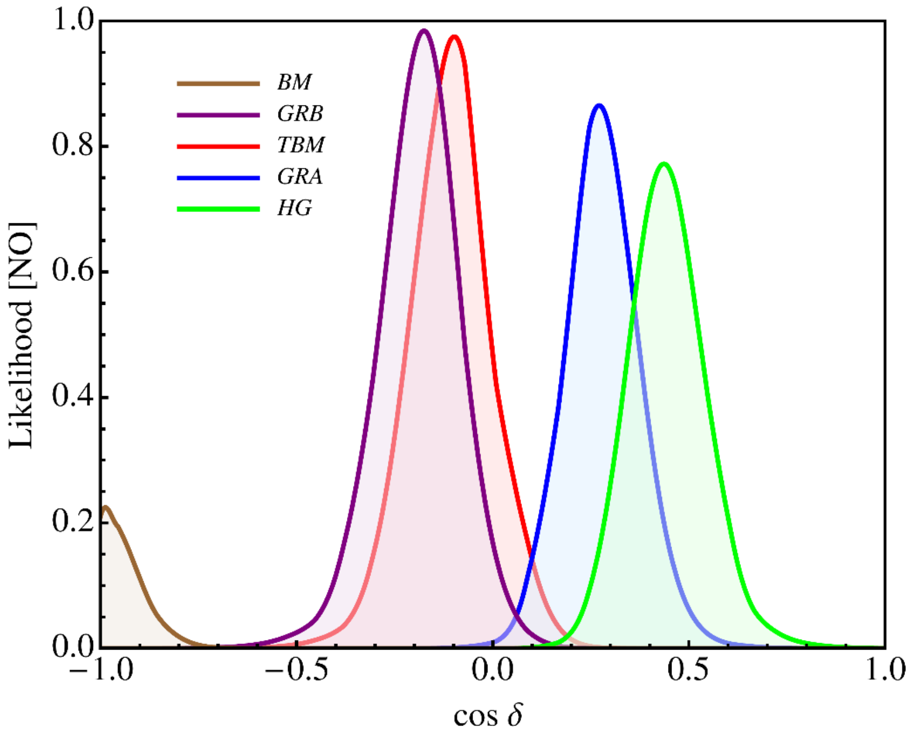
σ_{x_i} are the prospective 1σ uncertainties:

- 0.7% for $\sin^2\theta_{12}$ (JUNO)
- 3% for $\sin^2\theta_{13}$ (Daya Bay)
- 5% for $\sin^2\theta_{23}$ (NOvA and T2K)

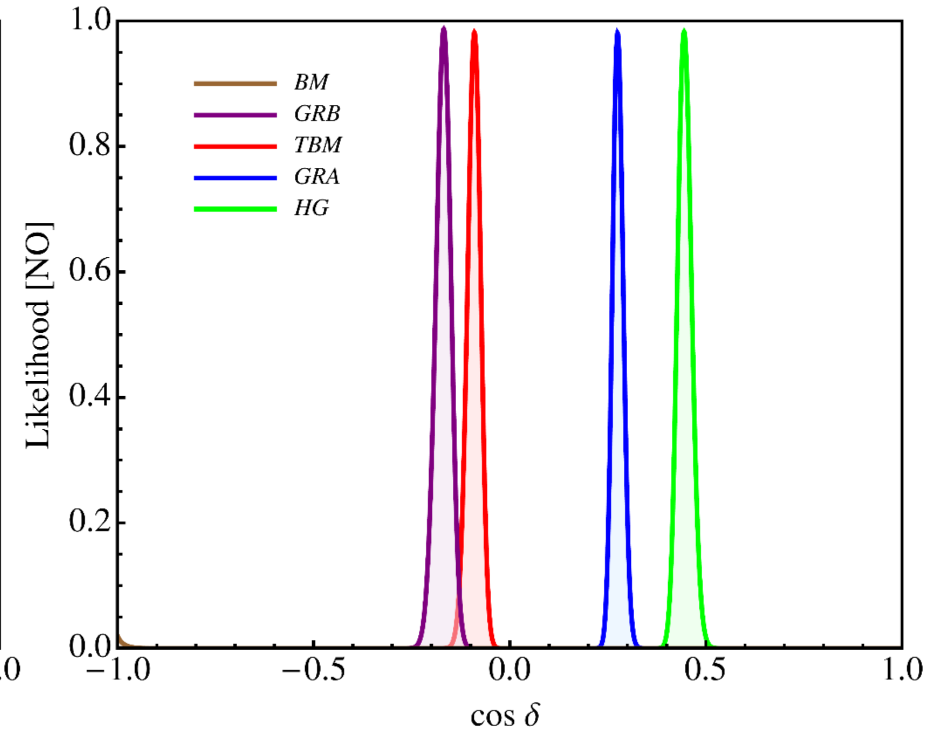
Dirac Phase: Statistical Analysis

Case **B1**: $\widetilde{U}_e^{\dagger} = R_{12}(\theta_{12}^e)R_{23}(\theta_{23}^e)$

Present



Future

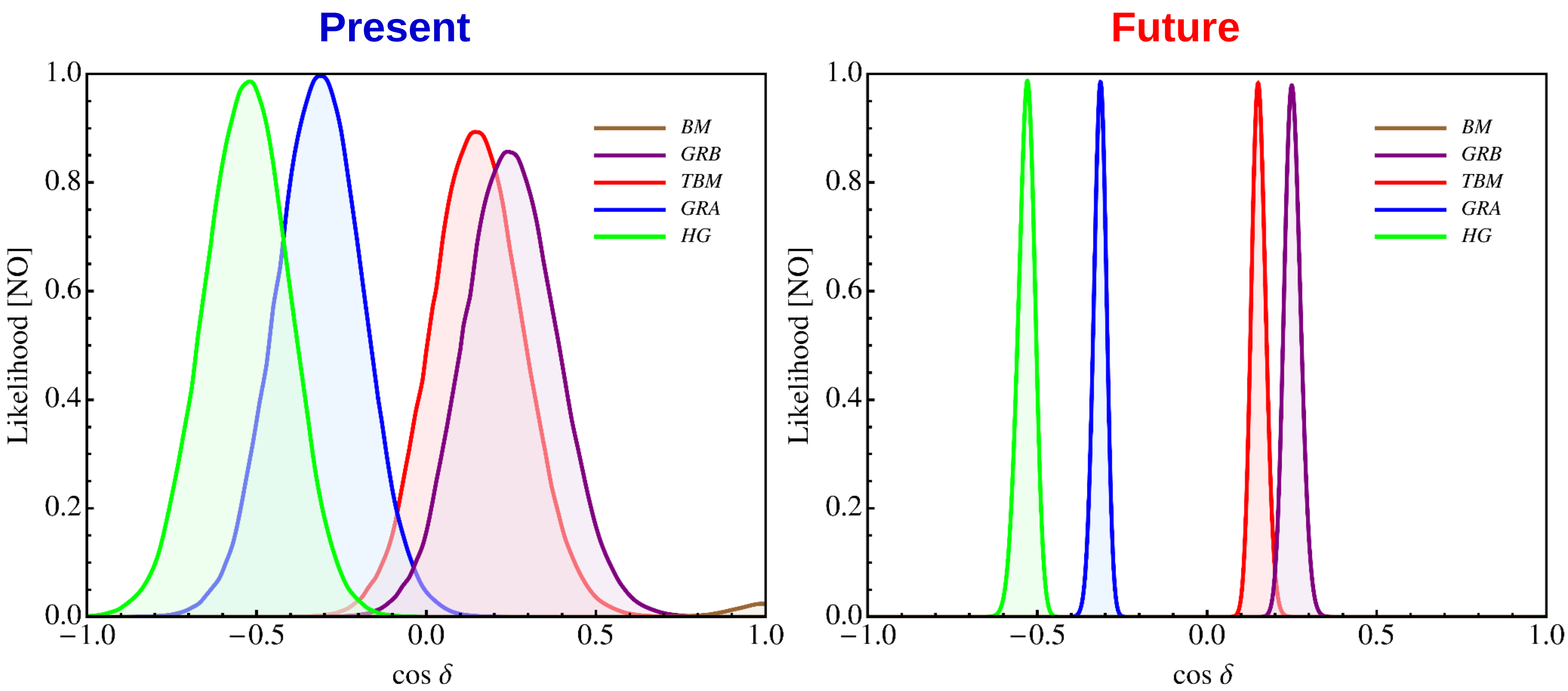


Girardi, Petcov, Titov, NPB **894** (2015) 733

RG corrections to sum rule predictions are negligible within the SM extended by the Weinberg (dimension 5) operator, see Gehrlein, Petcov, Spinrath, Titov, arXiv:1608.08409

Dirac Phase: Statistical Analysis

Case **B2**: $\tilde{U}_e^\dagger = R_{13}(\theta_{13}^e) R_{23}(\theta_{23}^e)$



Girardi, Petcov, Titov, EPJC **75** (2015) 345

Rephasing Invariant J_{CP}: Statistical Analysis

$$J_{\mathrm{CP}} = \mathrm{Im}\left\{U_{e1}^{*} U_{\mu 3}^{*} U_{e3} U_{\mu 1}\right\} = \frac{1}{8} \sin\delta \sin 2\theta_{13} \sin 2\theta_{23} \sin 2\theta_{12} \cos\theta_{13}$$

J_{CP} determines the magnitude of CP-violating effects in neutrino oscillations

Krastev and Petcov, PLB **205** (1988) 84

$$N_\sigma = \sqrt{\chi^2}$$

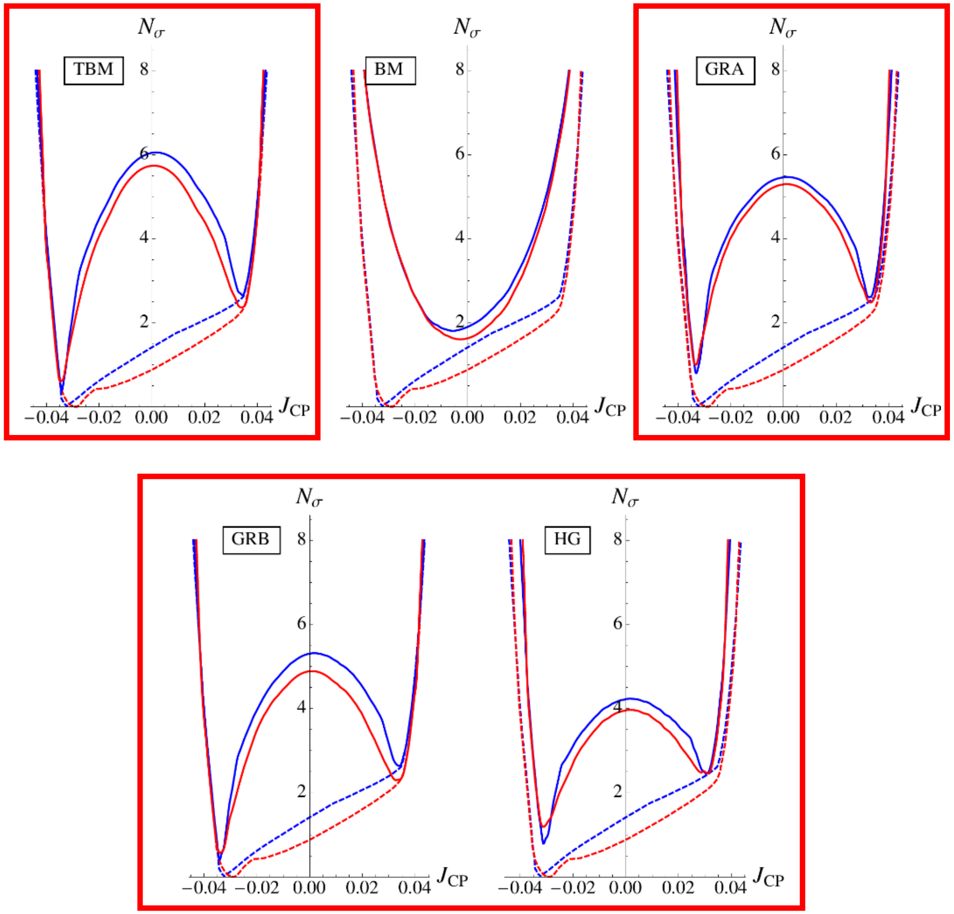
NO case B1

IO case B1

NO global fit

IO global fit

Case **B1**: $\tilde{U}_e^{\dagger} = R_{12}(\theta_{12}^e) R_{23}(\theta_{23}^e)$



Relatively large CP-violating effects in neutrino oscillations in the cases of TBM, GRA, GRB, HG:
$J_{\mathrm{CP}} \approx -0.03$, $|J_{\mathrm{CP}}| \geq 0.02$ @ 3σ
and suppressed effects in the case of BM:
$J_{\mathrm{CP}} \approx 0$

Girardi, Petcov, Titov, NPB **894** (2015) 733

Majorana Phases: Sum Rules

Cases	$\alpha_{21}/2$	$\alpha_{31}/2$
A1, B1, C1	$\arg\left(U_{\tau 1}U^*_{\tau 2}e^{i\frac{\alpha_{21}}{2}}\right) + \varkappa_{21} + \xi_{21}/2$	$\arg\left(U_{\tau 1}\right) + \varkappa_{31} + \xi_{31}/2$
A2, B2, C2	$\arg\left(U_{\mu 1}U^*_{\mu 2}e^{i\frac{\alpha_{21}}{2}}\right) + \varkappa_{21} + \xi_{21}/2$	$\arg\left(U_{\mu 1}\right) + \varkappa_{31} + \xi_{31}/2$

In these expressions U is in the standard parametrisation, and the corresponding sum rules for $\sin^2\theta_{23}$ and δ (slide 9) should be used

The phases κ_{21} and κ_{31} are 0 or π and known when the angles θ_{ij}^{ν} are fixed for all the cases, but B1 and B2, for which $\kappa_{31} = 0\ (\pi) + \beta$, where β is a free phase parameter

Case	$\varkappa_{21}$	$\varkappa_{31}$
A1	$\arg\left(-s^{\nu}_{12}c^{\nu}_{12}\right)$	$\arg\left(s^{\nu}_{12}s^{\nu}_{23}c^{\nu}_{23}\right)$
A2	$\arg\left(-s^{\nu}_{12}c^{\nu}_{12}\right)$	$\arg\left(-s^{\nu}_{12}s^{\nu}_{23}c^{\nu}_{23}\right)$
B1	$\arg\left(-s^{\nu}_{12}c^{\nu}_{12}\right)$	$\arg\left(s^{\nu}_{12}\right) + \beta$
B2	$\arg\left(-s^{\nu}_{12}c^{\nu}_{12}\right)$	$\arg\left(-s^{\nu}_{12}\right) + \beta$
C1	$\arg\left[-\left(c^{\nu}_{12}s^{\nu}_{23} + s^{\nu}_{12}c^{\nu}_{23}s^{\nu}_{13}\right)\left(s^{\nu}_{12}s^{\nu}_{23} - c^{\nu}_{12}c^{\nu}_{23}s^{\nu}_{13}\right)\right]$	$\arg\left[c^{\nu}_{23}c^{\nu}_{13}\left(s^{\nu}_{12}s^{\nu}_{23} - c^{\nu}_{12}c^{\nu}_{23}s^{\nu}_{13}\right)\right]$
C2	$\arg\left[-\left(c^{\nu}_{12}c^{\nu}_{23} - s^{\nu}_{12}s^{\nu}_{23}s^{\nu}_{13}\right)\left(s^{\nu}_{12}c^{\nu}_{23} + c^{\nu}_{12}s^{\nu}_{23}s^{\nu}_{13}\right)\right]$	$\arg\left[-s^{\nu}_{23}c^{\nu}_{13}\left(s^{\nu}_{12}c^{\nu}_{23} + c^{\nu}_{12}s^{\nu}_{23}s^{\nu}_{13}\right)\right]$

Girardi, Petcov, Titov, arXiv:1605.04172

Majorana Phases: Predictions

$\alpha_{21}/2 - \xi_{21}/2$ [°], using the best fit values of the neutrino mixing angles for NO

Case	TBM	GRA	GRB	HG	BM
A1	$342 \vee 18$	$341 \vee 19$	$343 \vee 17$	$342 \vee 18$	—
A2	$18 \vee 342$	$19 \vee 341$	$17 \vee 343$	$18 \vee 342$	—
B1	$340 \vee 20$	$339 \vee 21$	$341 \vee 19$	$340 \vee 20$	—
B2	$15 \vee 345$	$16 \vee 344$	$14 \vee 346$	$15 \vee 345$	—
	$[\pi/20, -\pi/4]$	$[\pi/10, -\pi/4]$	$[a, -\pi/4]$	$[\pi/20, b]$	$[\pi/20, \pi/6]$
C1	$163 \vee 197$	$167 \vee 193$	$171 \vee 189$	$353 \vee 7$	$348 \vee 12$
	$[\pi/20, c]$	$[\pi/20, \pi/4]$	$[\pi/10, \pi/4]$	$[a, \pi/4]$	$[\pi/20, d]$
C2	$12 \vee 348$	$17 \vee 343$	$13 \vee 347$	$9 \vee 351$	$14 \vee 346$

First number corresponds to $\delta = \cos^{-1}(\cos\delta)$, second is for $\delta = 2\pi - \cos^{-1}(\cos\delta)$

$\theta^{\nu}_{23} = -\pi/4$

The values in square brackets are those of $[\theta^{\nu}_{13}, \theta^{\nu}_{12}]$

$a = \sin^{-1}(1/3)$, $b = \sin^{-1}\left(1/\sqrt{2+r}\right)$, $c = \sin^{-1}\left(1/\sqrt{3}\right)$, $d = \sin^{-1}\left(\sqrt{3-r}/2\right)$

Majorana Phases: Predictions

$\alpha_{31}/2 - \xi_{31}/1$ [°] ($\alpha_{31}/2 - \xi_{31}/1 - \beta$ [°] in cases B1 and B2), using the best fit values of the neutrino mixing angles for NO

Case	TBM	GRA	GRB	HG	BM
A1	$168 \vee 192$	$167 \vee 193$	$168 \vee 192$	$167 \vee 193$	—
A2	$192 \vee 168$	$193 \vee 167$	$192 \vee 168$	$193 \vee 167$	—
B1	$346 \vee 14$	$345 \vee 15$	$347 \vee 13$	$345 \vee 15$	—
B2	$10 \vee 350$	$11 \vee 349$	$10 \vee 350$	$11 \vee 349$	—
	$[\pi/20, -\pi/4]$	$[\pi/10, -\pi/4]$	$[a, -\pi/4]$	$[\pi/20, b]$	$[\pi/20, \pi/6]$
C1	$349 \vee 11$	$350 \vee 10$	$353 \vee 7$	$175 \vee 185$	$172 \vee 188$
	$[\pi/20, c]$	$[\pi/20, \pi/4]$	$[\pi/10, \pi/4]$	$[a, \pi/4]$	$[\pi/20, d]$
C2	$189 \vee 171$	$191 \vee 169$	$190 \vee 170$	$187 \vee 173$	$190 \vee 170$

First number corresponds to $\delta = \cos^{-1}(\cos\delta)$, second is for $\delta = 2\pi - \cos^{-1}(\cos\delta)$

$\theta^{\nu}_{23} = -\pi/4$

The values in square brackets are those of $[\theta^{\nu}_{13}, \theta^{\nu}_{12}]$

$a = \sin^{-1}(1/3)$, $b = \sin^{-1}\left(1/\sqrt{2+r}\right)$, $c = \sin^{-1}\left(1/\sqrt{3}\right)$, $d = \sin^{-1}\left(\sqrt{3-r}/2\right)$

Generalised CP Symmetry

$$X^T M_\nu X = M_\nu^*$$

X are generalised CP transformations

Generalised CP symmetry should be consistent with (residual) flavour symmetry:

$$X\,\rho^*(g_\nu)\,X^{-1} = \rho(g'_\nu)\,, \quad g_\nu, g'_\nu \in G_\nu$$

It can be shown that

$$\tilde{U}_\nu^\dagger\, X\, \tilde{U}_\nu^* = \mathrm{diag}\left(\pm e^{i\xi_1}, \pm e^{i\xi_2}, \pm e^{i\xi_3}\right)$$

$$\xi_{21} = \xi_2 - \xi_1, \quad \xi_{31} = \xi_3 - \xi_1$$

Thus, the phases ξ_i are known once $\tilde{U}_\nu$ is fixed by G_ν, and X consistent with G_ν are identified

Generalised CP Symmetry

Example: $G_f = A_4$

$$S^2 = T^3 = (ST)^3 = 1$$

$G_\nu = Z_2^S \times Z_2^{acc}$ (Z_2^{acc} is a $\mu - \tau$ symmetry which arises accidentally) leads to tri-bimaximal mixing in the neutrino sector

The generalised CP transformations consistent with the preserved S generator are $X = \rho(1)$ and $X = \rho(S)$. Then

$$U_{\rm TBM}^\dagger\, \rho(1)\, U_{\rm TBM}^* = {\rm diag}(1,1,1)$$
$$U_{\rm TBM}^\dagger\, \rho(S)\, U_{\rm TBM}^* = {\rm diag}(-1,1,-1)$$

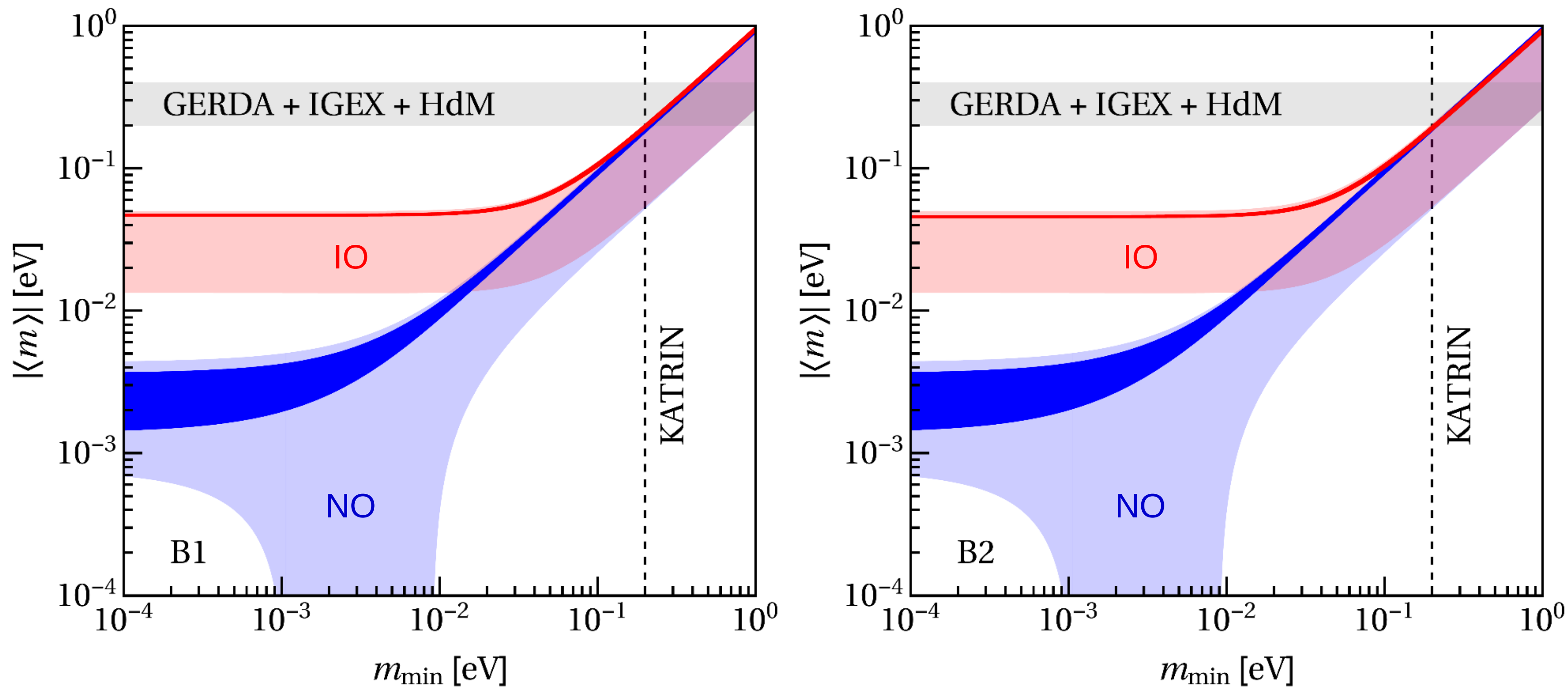
Thus, the phases ξ_i, and hence ξ_{21} and ξ_{31}, can be either 0 or π

A similar situation takes place for $G_f = S_4$ and A_5
(BM and GRA mixing forms, respectively)

Neutrinoless Double Beta Decay

Effective Majorana mass: $\langle m \rangle = \sum_{i=1}^{3} m_i U_{ei}^2 = m_1 c_{12}^2 c_{13}^2 + m_2 s_{12}^2 c_{13}^2 e^{i\alpha_{21}} + m_3 s_{13}^2 e^{i(\alpha_{31} - 2\delta)}$

Using the best fit values of θ_{12}, θ_{13}, Δm_{21}^2, $\Delta m_{31(23)}^2$ and the predicted values of the Dirac phase and Majorana phases for $(\xi_{21}, \xi_{31}) = (0, 0)$



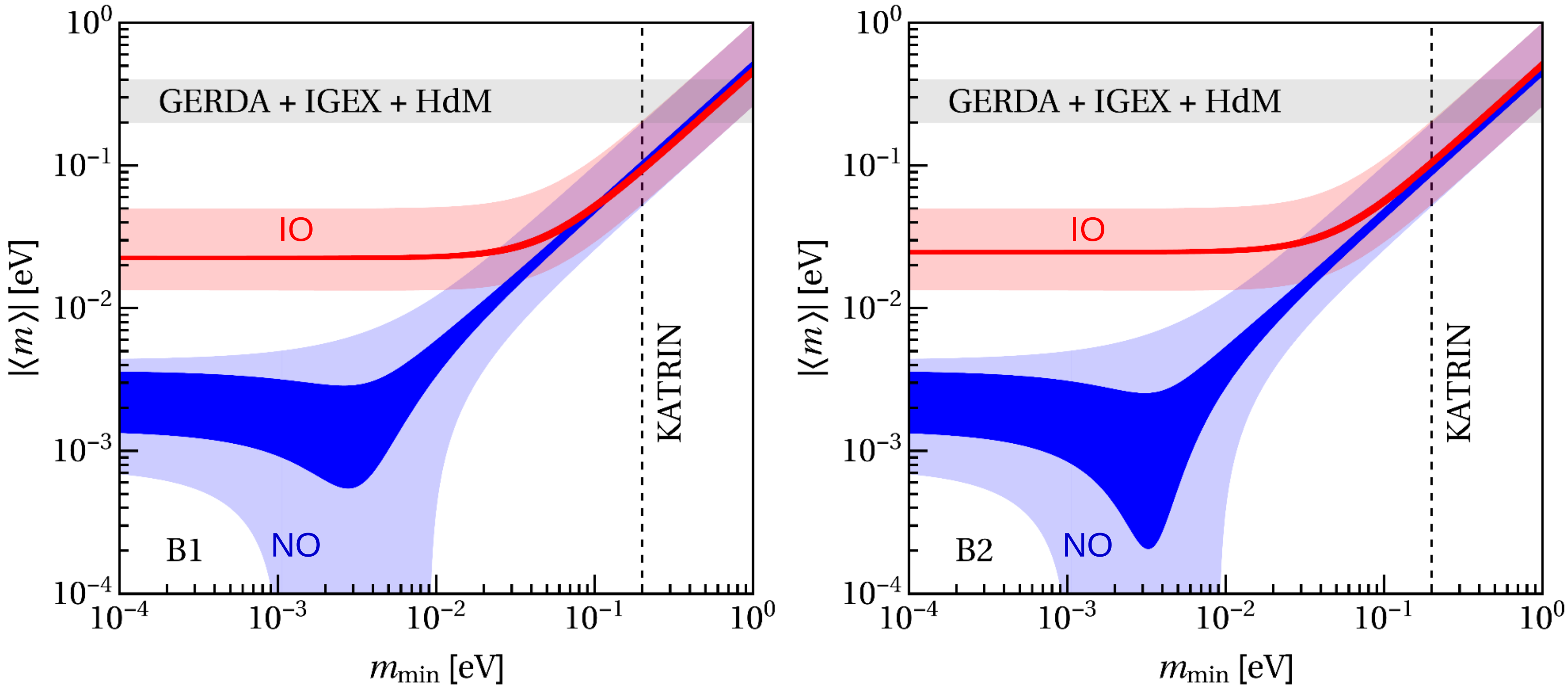
TBM, GRA, GRB, HG $\beta \in [0, \pi]$

Girardi, Petcov, Titov, arXiv:1605.04172

Neutrinoless Double Beta Decay

Effective Majorana mass: $\langle m \rangle = \sum_{i=1}^{3} m_i U_{ei}^2 = m_1 c_{12}^2 c_{13}^2 + m_2 s_{12}^2 c_{13}^2 e^{i\alpha_{21}} + m_3 s_{13}^2 e^{i(\alpha_{31} - 2\delta)}$

Using the best fit values of θ_{12}, θ_{13}, Δm_{21}^2, $\Delta m_{31(23)}^2$ and the predicted values of the Dirac phase and Majorana phases for $(\xi_{21}, \xi_{31}) = (\pi, \pi)$



TBM, GRA, GRB, HG $\beta \in [0, \pi]$ Girardi, Petcov, Titov, arXiv:1605.04172

Conclusions

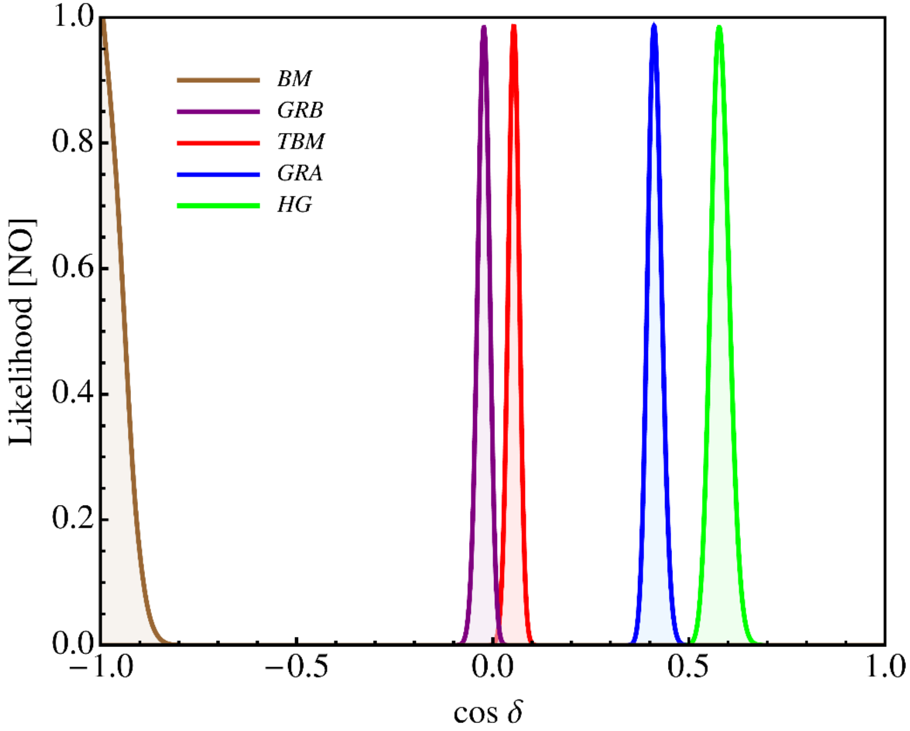
- Exact (within the schemes considered) sum rules for the cosine of the Dirac phase and the Majorana phases were derived and numerical predictions were obtained
- Sufficiently precise measurements of the Dirac phase and the mixing angles are the key to the possible discrete symmetry origin of the observed pattern of neutrino mixing
- Relatively large CP-violating effects in neutrino oscillations in the cases of TBM, GRA, GRB, HG and suppressed effects in the case of BM were found
- Constrained parameter space in neutrinoless double beta decay is predicted

Backup

Dirac Phase: Statistical Analysis

Case **B1**: Dependence on the best fit values

Larger b.f.v. of $\sin^2\theta_{12}$

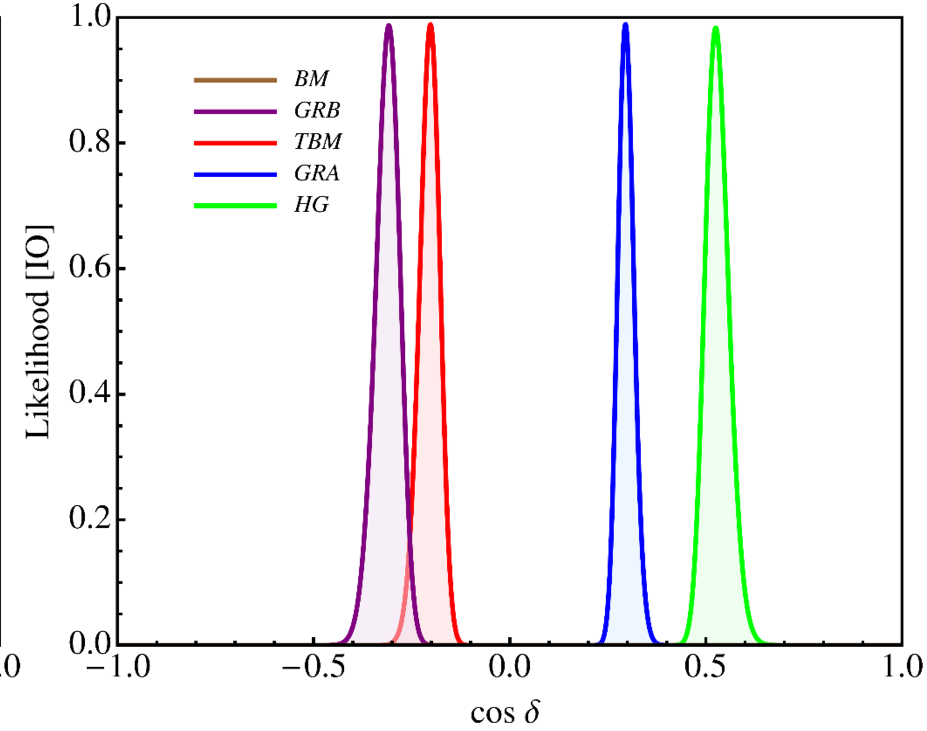


$(s^2_{12})_{\rm bf} = 0.332$

$(s^2_{23})_{\rm bf} = 0.437$

$(s^2_{13})_{\rm pbf} = 0.0234$

Larger b.f.v. of $\sin^2\theta_{23}$



$(s^2_{12})_{\rm bf} = 0.304$

$(s^2_{23})_{\rm bf} = 0.579$

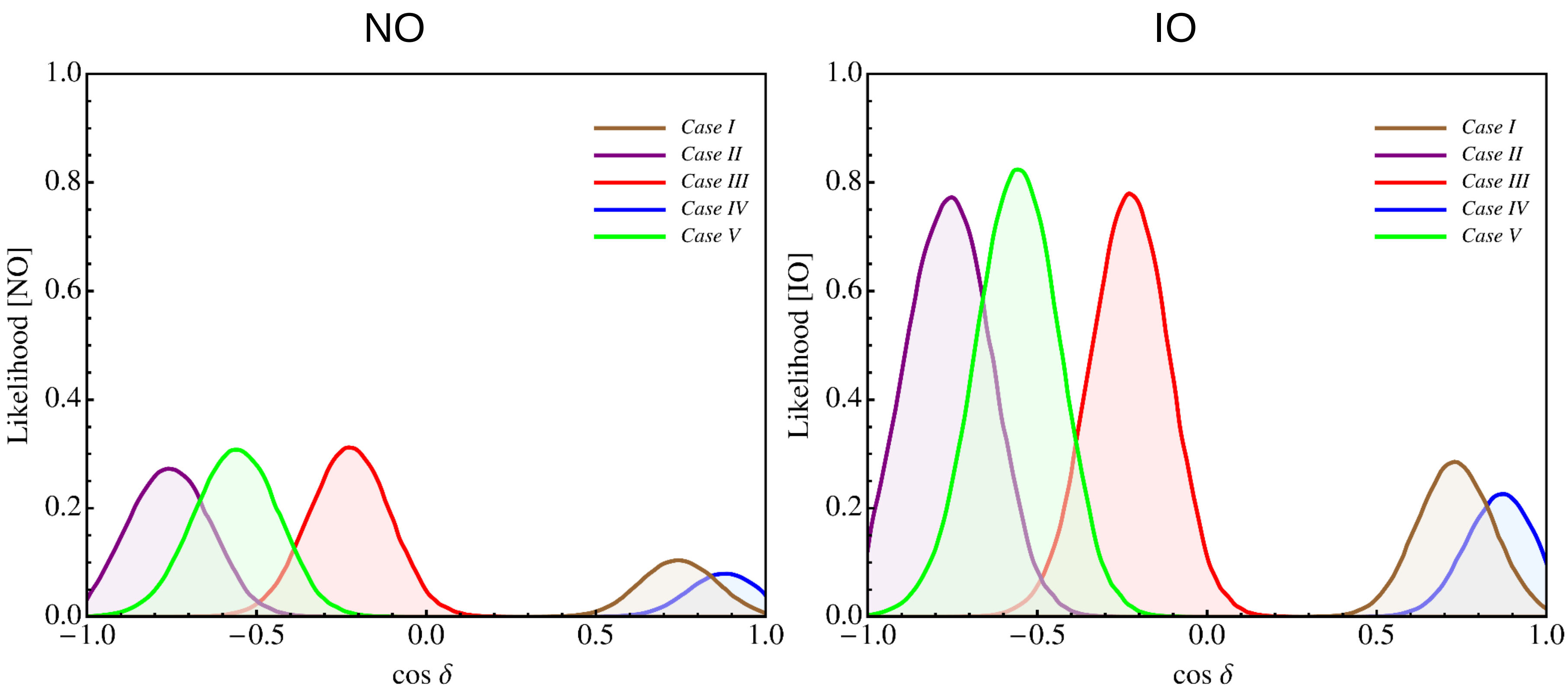
$(s^2_{13})_{\rm pbf} = 0.0219$

IO neutrino mass spectrum

Gonzalez-Garcia *et. al.*, JHEP **1411** (2014) 052

Dirac Phase: Statistical Analysis

Case **C1**: Present



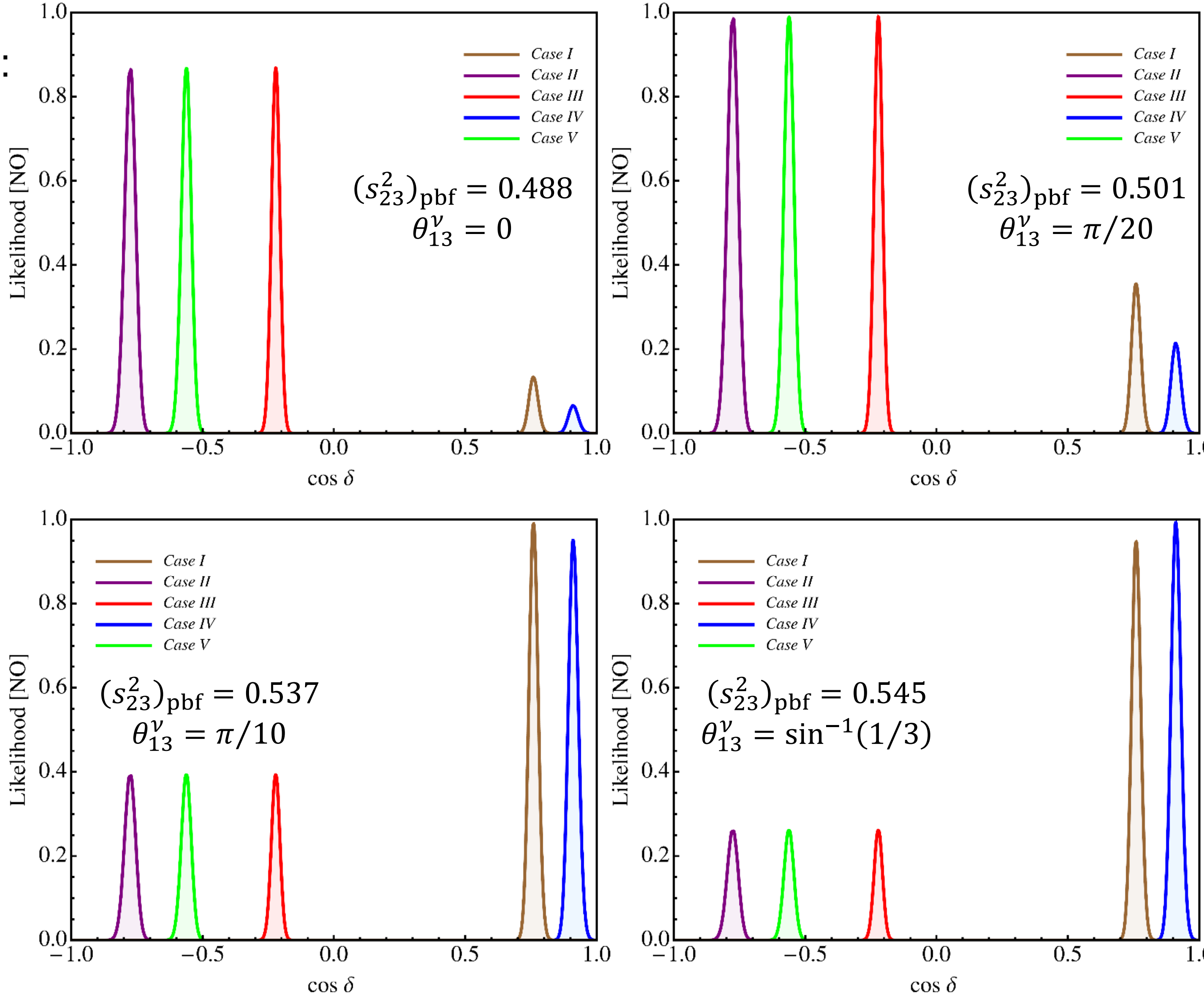
$[\theta_{13}^{\nu}, \theta_{12}^{\nu}]$: $Case\ I = [\pi/20, -\pi/4]$ $Case\ II = [\pi/10, -\pi/4]$ $Case\ III = [\sin^{-1}(1/3), -\pi/4]$

$Case\ IV = [\pi/20, \sin^{-1}(1/\sqrt{2+r})]$ $Case\ V = [\pi/20, \pi/6]$

Girardi, Petcov, Titov, EPJC **75** (2015) 345

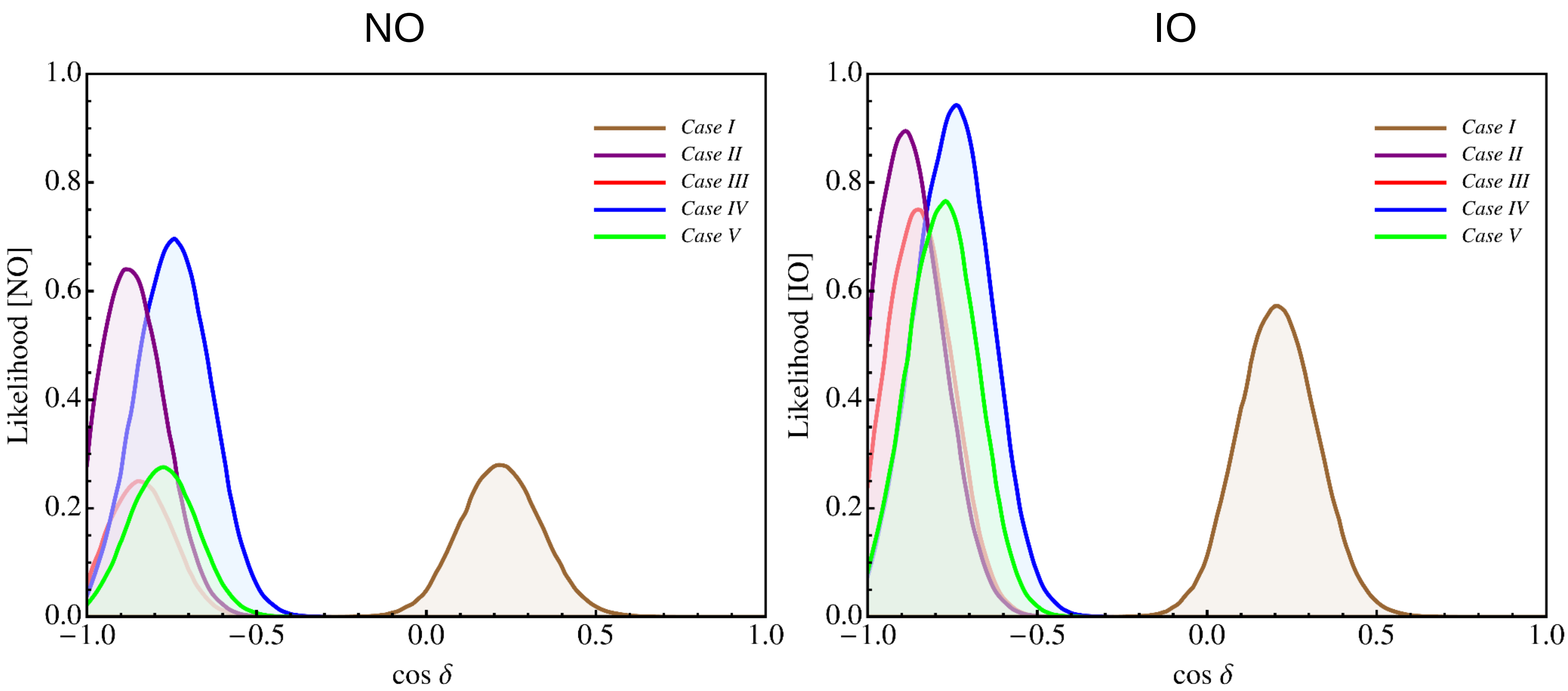
Dirac Phase: Statistical Analysis

Case **C1**:
Future



Dirac Phase: Statistical Analysis

Case **C2**: Present



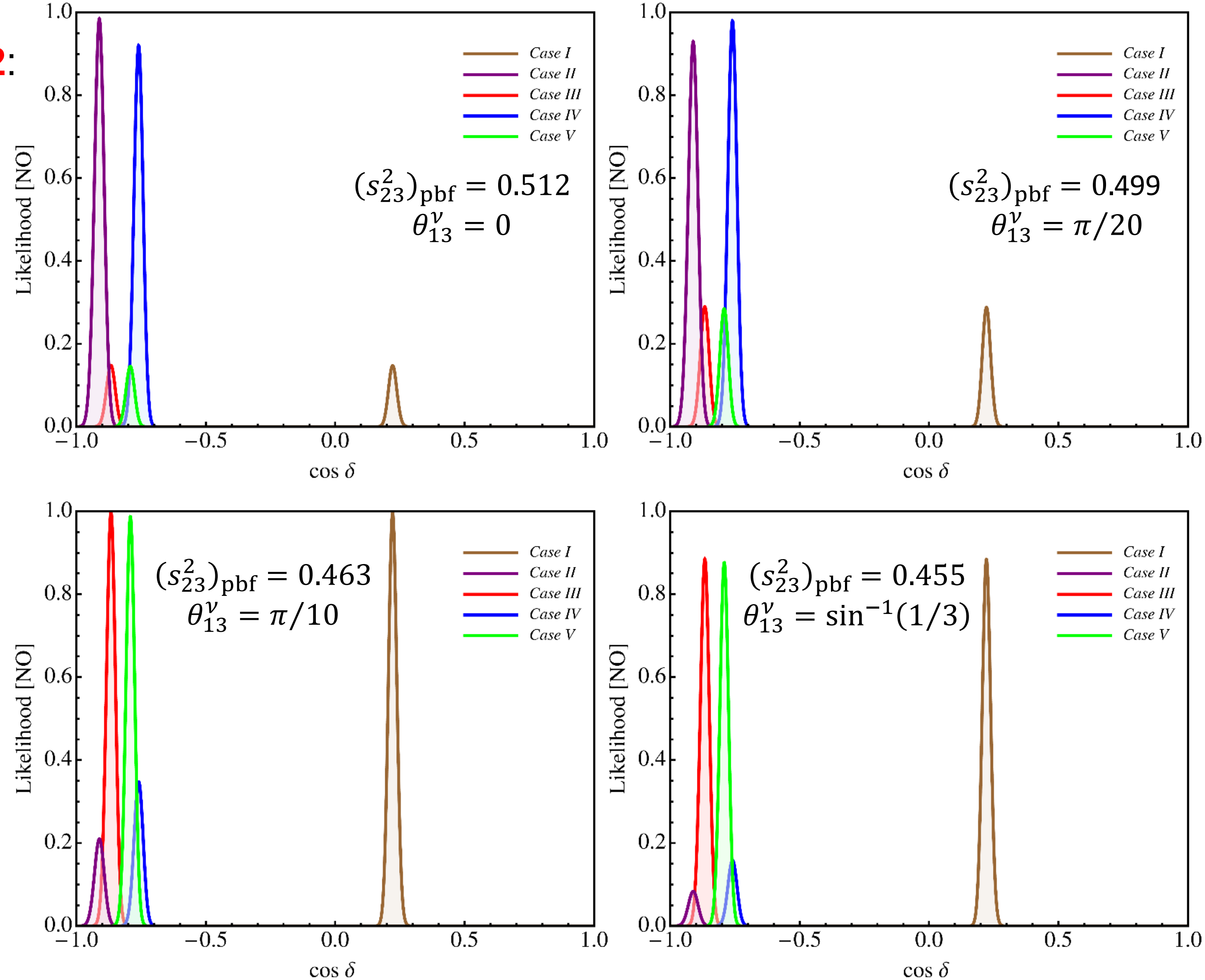
$[\theta_{13}^{\nu}, \theta_{12}^{\nu}]$: $Case\ I = \left[\pi/20, \sin^{-1}(1/\sqrt{3})\right]$ $Case\ II = [\pi/20, \pi/4]$ $Case\ III = [\pi/10, \pi/4]$

$Case\ IV = [\sin^{-1}(1/3), \pi/4]$ $Case\ V = \left[\pi/20, \sin^{-1}(\sqrt{3-r}/2)\right]$

Girardi, Petcov, Titov, EPJC **75** (2015) 345

Dirac Phase: Statistical Analysis

Case **C2**:
Future



$\sin^2\theta_{23}$: Statistical Analysis

Case **B1**

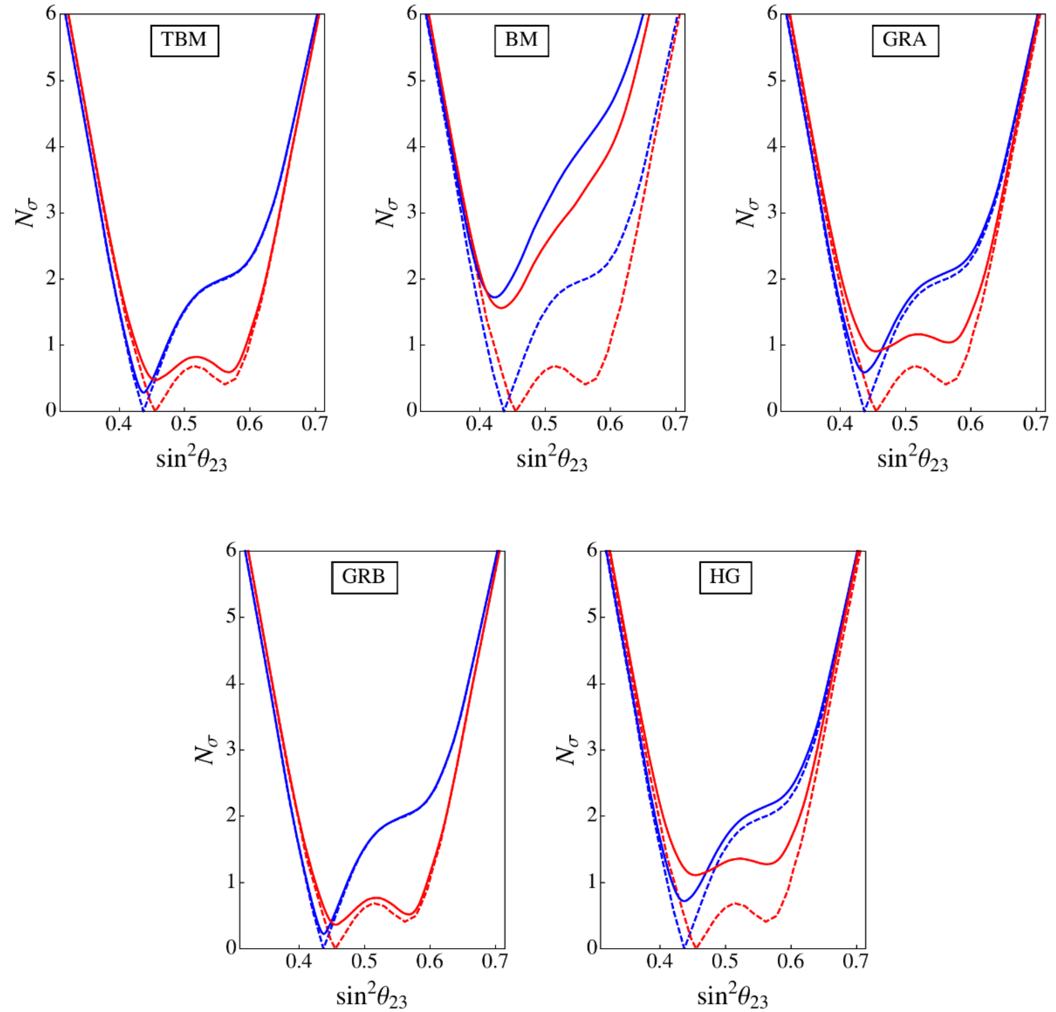
$$N_\sigma = \sqrt{\chi^2}$$

NO case B1

IO case B1

NO global fit

IO global fit



Girardi, Petcov, Titov, NPB **894** (2015) 733

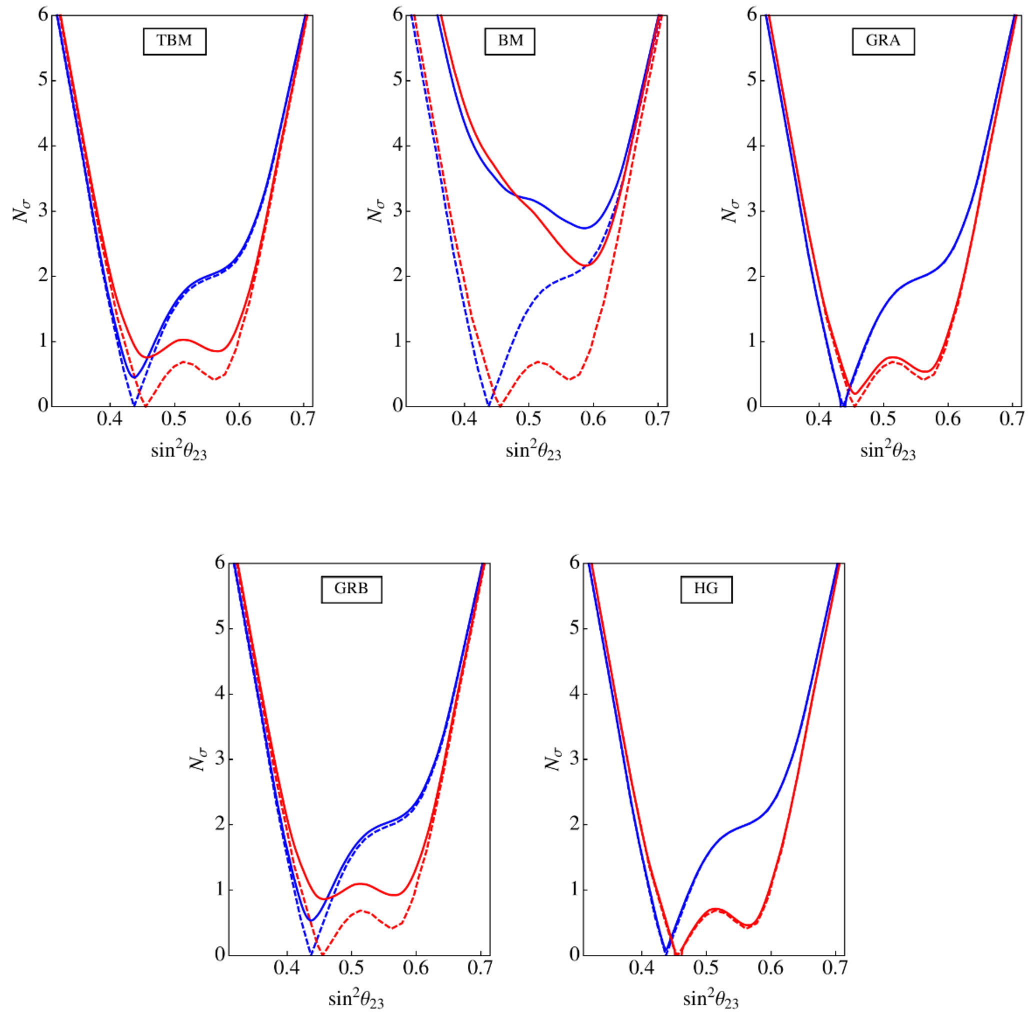
$\sin^2 \theta_{23}$: Statistical Analysis

Case **B2**

$$N_\sigma = \sqrt{\chi^2}$$

NO case B2
IO case B2

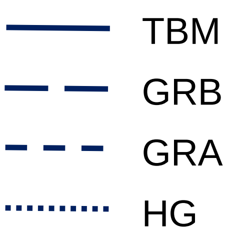
NO global fit
IO global fit

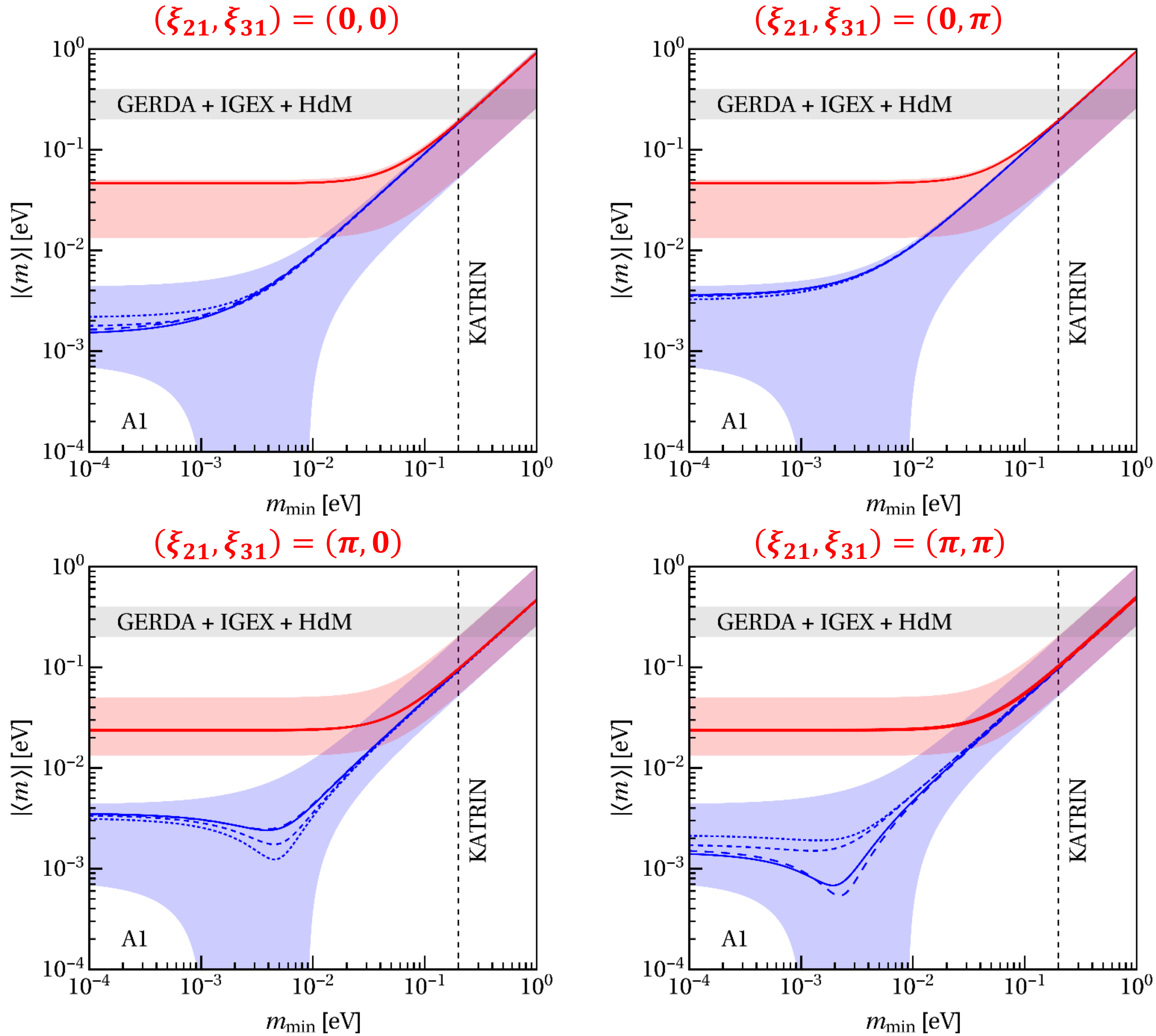


Girardi, Petcov, Titov, EPJC **75** (2015) 345

Neutrinoless Double Beta Decay

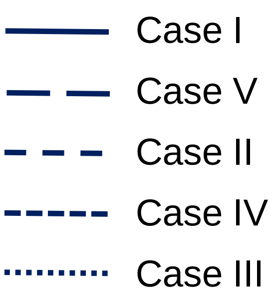
Case **A1**
(“= **A2**”
in terms of
predictions
for $|\langle m \rangle|$)

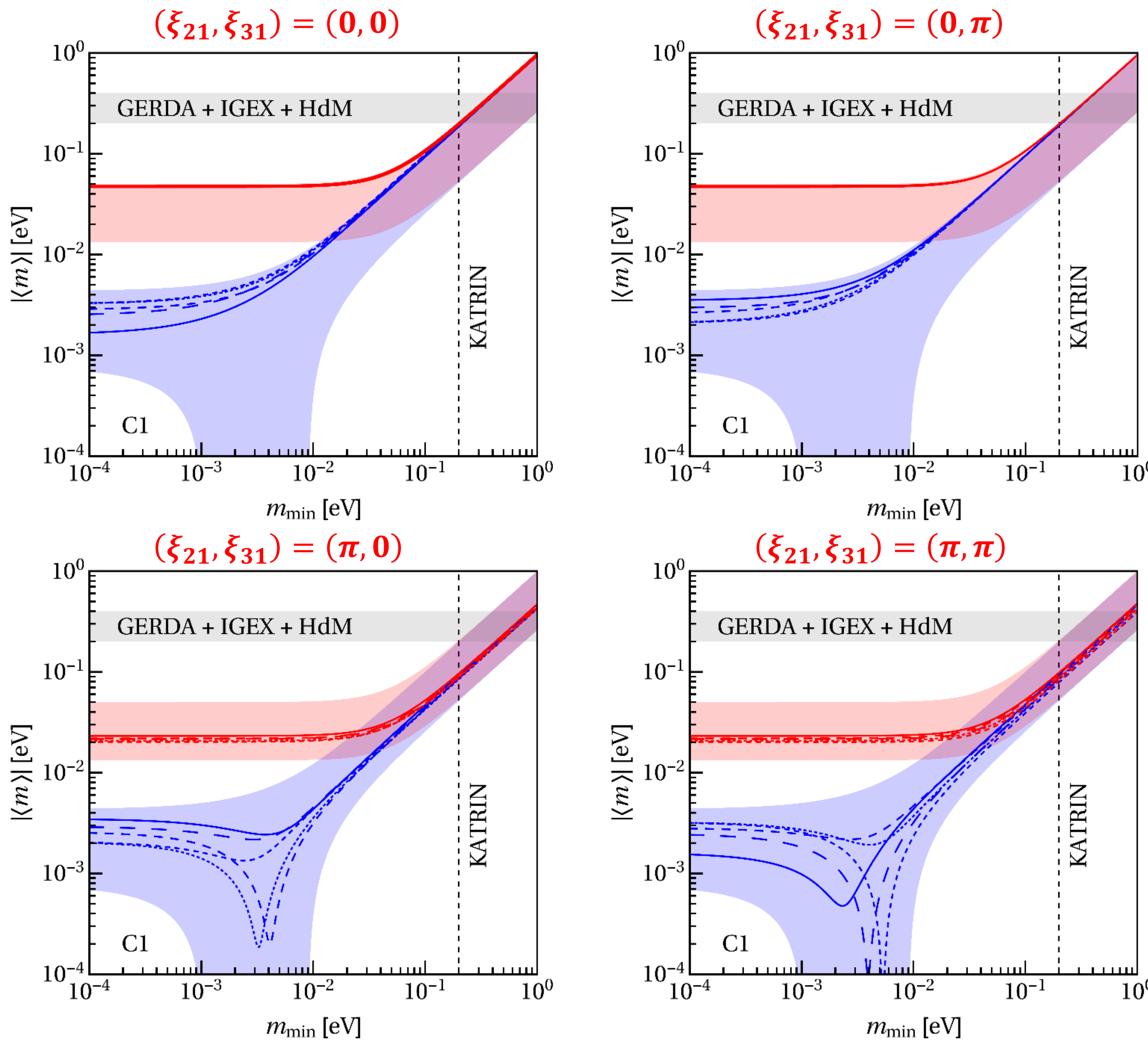




Neutrinoless Double Beta Decay

Case **C1**





Neutrinoless Double Beta Decay

Case **C2**