MONOPOLE CONDENSATION IN FULL QCD

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OUTLINE

- MINI-REVIEW ON DUAL SUPERCONDUCTIVITY IN QCD
- MINI-REVIEW ON THE FORMULATION AND SIMULATION OF FULL QCD ON THE LATTICE
- TECHNICAL DETAILS AND TROUBLES ABOUT STUDYING DUAL SUPERCONDUCTIVITY IN FULL QCD
- PRELIMINARY PHYSICAL RESULTS AND CONCLUSIONS

MINI-REVIEW ON DUAL SUPERCONDUCTIVITY IN QCD

COLOUR CONFINEMENT IN YANG-MILLS THEORIES IS NOW WELL UNDERSTOOD AS A ORDER-DISORDER DUALITY PHENOMENON: CONDENSATION OF MAGNETIC MONOPOLES MAKES THE QCD VACUUM A DUAL SUPERCONDUCTOR AND CHROMOELECTRIC FIELDS ARE CONFINED IN FLUX TUBES $\Longrightarrow$ LINEARLY RISING POTENTIAL BETWEEN ELECTRIC COLOUR CHARGES

ONE WAY TO DETECT DUAL SUPERCONDUCTIVITY IS TO MEASURE THE V.E.V. OF AN OPERATOR μ CARRYING THE QUANTUM NUMBERS OF THE TOPOLOGICAL EXCITATIONS ASSOCIATED TO ORDER-DISORDER DUALITY: IN (3+1) THESE ARE MAGNETIC MONOPOLES

μ IS AN OPERATOR WHICH CREATES A MONOPOLE BY TRANSLATING THE QUANTUM FIELDS BY THE CORRESPONDING CLASSICAL MONOPOLE FIELDS

$$\mu(\vec{y},t) = \exp\left[i \sum_k \int d^3x \, \Pi_k(\vec{x},t) \, \Phi_k(\vec{x},\vec{y}) \right]$$

$\Pi_i(\vec{x},t) \Rightarrow$ CONJUGATE MOMENTUM TO THE FIELD $\varphi_i(\vec{x},t)$

$\Phi_i(\vec{x},\vec{y}) \Rightarrow$ CLASSICAL FIELD CONFIGURATION FOR A MONOPOLE LOCATED AT $\vec{y}$

$$\mu(\vec{y},t)\,|\varphi_i(\vec{x},t)\rangle = |\varphi_i(\vec{x},t) + \Phi_i(\vec{x},\vec{y})\rangle$$

EXAMPLE OF COMPACT U(1) GAUGE THEORY

THE MOMENTUM CONJUGATE TO THE VECTOR POTENTIAL $\vec{A}(\vec{x},t)$ IS THE ELECTRIC FIELD, $\vec{E}(\vec{x},t)$

$$\mu(\vec{y},t) = \exp\left[\frac{i}{e}\int d^3x\, \vec{E}(\vec{x},t)\, \vec{b}(\vec{x}-\vec{y})\right]$$

$\vec{b}(\vec{x}-\vec{y})$ IS THE VECTOR POTENTIAL OF A MONOPOLE LOCATED AT $\vec{y}$

ON THE LATTICE:

$$a^2 E_i \simeq \frac{1}{e}\,\mathrm{Im}\,\Pi^{0i} + O(a^4)$$

Π^{0i} IS THE TEMPORAL PLAQUETTE IN THE i-SPATIAL DIRECTION

$$\left(\Pi_{\mu\nu}(n) \equiv \exp\left(i\theta_{\mu\nu}(n)\right);\ \theta_{\mu\nu} = \Delta_\mu\theta_\nu - \Delta_\nu\theta_\mu \underset{a\to 0}{\simeq} a^2 e F_{\mu\nu};\ U_\mu(n) = e^{i\theta_\mu(n)} \right)$$

A DISCRETIZATION OF μ WHICH AUTOMATICALLY RESPECTS THE COMPACTNESS OF THE THEORY IS:

$$\mu(\vec{y},n_0) \equiv \exp\left\{\beta\sum_{\vec{n}}\left[S\left(\theta^{0i}(\vec{n},n_0) + b_i(\vec{n}-\vec{y})\right) - S\left(\theta^{0i}(\vec{n},n_0)\right)\right]\right\}$$

WHERE $S(\theta_{\mu\nu}(n))$ IS THE ACTION ASSOCIATED TO THE PLAQUETTE $\theta_{\mu\nu}(n)$, FOR INSTANCE THE WILSON ACTION

$$S(\theta_{\mu\nu}) = \cos(\theta_{\mu\nu})$$

EXTENSION TO NON-ABELIAN GAUGE THEORIES

SUPERCONDUCTIVITY IS A PHENOMENON ASSOCIATED TO THE SPONTANEOUS BREAKING OF AN ABELIAN SYMMETRY

IN NON-ABELIAN GAUGE SYMMETRY IT IS POSSIBLE TO ASSOCIATE AN ABELIAN CONSERVED MONOPOLE CHARGE TO EACH OPERATOR IN THE ADJOINT REPRESENTATION BY THE SO CALLED ABELIAN PROJECTION

FOR INSTANCE, IN SU(3), LET US CONSIDER A LOCAL OPERATOR IN THE ADJOINT REPR.

$$\phi(x) = \sum_{i=1}^{8} \phi^i(x) \cdot F^i \quad ; \quad F^i = \frac{\lambda_i}{2}$$

WE CAN FIX A GAUGE IN WHICH $\phi(x)$ IS DIAGONAL, BY CHOOSING A GAUGE TRANSFORMATION $\Omega(x)$ SUCH THAT $\Omega(x)\,\phi(x)\,\Omega^+(x) = \phi_{DIAG}(x)$

THIS LEAVES A RESIDUAL $U(1) \otimes U(1)$ LOCAL GAUGE FREEDOM.
THE MONOPOLE CHARGE CAN BE ASSOCIATED TO A $U(1)$ SUBGROUP OF THIS RESIDUAL GROUP, FIXED BY A LINEAR COMBINATION F_D OF THE DIAGONAL GENERATORS F^3, F^8

THE 'T HOOFT FIELD TENSOR

$$F_{\mu\nu} = \frac{1}{2} tr\left(\varphi \cdot G_{\mu\nu} - \frac{i}{g} \varphi \cdot \frac{4}{3} [D_\mu \varphi, D_\nu \varphi] \right) \; ; \quad \varphi(x) = (\Omega(x))^+ F_D \, \Omega(x)$$

BECOMES ABELIAN IN THE GAUGE WHERE ϕ IS DIAGONAL

THE MONOPOLE CREATION OPERATOR IS IN THIS CASE

$$\mu(\vec{y}, t_0) \equiv \exp\left\{\frac{\beta}{3}\sum_{\vec{n},i} \mathrm{Tr}\left[\Pi_{i0}(\vec{n}, t_0) - \Pi'_{i0}(\vec{n}, t_0)\right]\right\} \equiv \exp(-\beta \Delta S)$$

WHERE Π'_{i0} IS OBTAINED FROM THE USUAL PLAQUETTE

$$\Pi_{i0}(\vec{n}, t_0) = U_i(\vec{n}, t_0)\, U_0(\vec{n}+i, t_0)\left(U_i(\vec{n}, t_0+1)\right)^{\dagger}\left(U_0(\vec{n}, t_0)\right)^{+}$$

BY $\quad U_i(\vec{n}, t_0) \to U_i(\vec{n}, t_0)\, e^{i A^M_{\perp i}(\vec{n}, \vec{y})}\, f(\vec{n}+i, t)$

WHERE $A^M_{\perp i}(\vec{n}, \vec{y})$ IS THE VECTOR POTENTIAL OF A MONOPOLE, ASSUMED HERE COMPLETELY TRANSVERSE

THE V.E.V. OF μ IS DEFINED AS

$$\langle \mu \rangle \equiv \frac{\int [dU]\, e^{-\beta S}\, e^{-\beta \Delta S}}{\int [dU]\, e^{-\beta S}} = \frac{\int [dU]\, e^{-\beta S'}}{\int [dU]\, e^{-\beta S}}$$

$S' = S + \Delta S$ AND S IS THE USUAL PURE GAUGE ACTION

A DIRECT LATTICE DETERMINATION OF $\langle \mu \rangle$ IS VERY DIFFICULT: IT IS THE EXPONENTIAL OF A QUANTITY WHICH FLUCTUATES PROPORTIONALLY TO THE SQUARE ROOT OF THE PHYSICAL VOLUME

IT IS MUCH MORE CONVENIENT TO DEFINE

$$\rho \equiv \frac{d}{d\beta} \log \langle \mu \rangle = \langle S \rangle_S - \langle S + \Delta S \rangle_{S+\Delta S}$$

WHICH IS MUCH MORE EASIER TO DETERMINE AND GIVES ALL THE RELEVANT INFORMATION ON $\langle \mu \rangle$

IT IS POSSIBLE TO SHOW THAT ADDING $\mu(\vec{y}, t_0)$ TO THE FUNCTIONAL INTEGRAL ACTUALLY CORRESPONDS TO ADDING, AT ALL TIMES $t > t_0$, A MONOPOLE MAGNETIC FIELD TO THE ORIGINAL CONFIGURATION.

THE CORRELATION FUNCTION $\langle \mu(\vec{y}, 0)\, \bar{\mu}(\vec{y}, x_0) \rangle$

THEN DESCRIBES A MONOPOLE CREATED AT TIME $t = 0$ AND ANNIHILATED AT TIME $t = x_0$

THE ASYMPTOTIC BEHAVIOUR AT LARGE x_0,

$$\langle \mu(\vec{y}, 0)\, \bar{\mu}(\vec{y}, x_0) \rangle \simeq C \exp(-x_0 M) + \langle \mu \rangle^2$$

CAN THEN SHOW MONOPOLE CONDENSATION, i.e. $\langle \mu \rangle \neq 0$

AT FINITE T, HOWEVER, IT IS NOT POSSIBLE TO DEFINE TIME CORRELATION FUNCTIONS, SO $\langle \mu \rangle$ HAS TO BE MEASURED DIRECTLY.

PERIODIC B.C. IN TIME CANNOT BE USED IN THIS CASE!

IF A MONOPOLE IS CREATED AT $t = 1$, THE MAGNETIC CHARGE AT $t = N_t$ WOULD BE DIFFERENT FROM THAT AT $t = 0$ IN CONTRAST WITH PERIODICITY.

C^* BOUNDARY CONDITIONS, $U_\mu(\vec{n}, N_t) \equiv U^*_\mu(\vec{n}, 0)$

SOLVE THE PROBLEM $\Rightarrow$ THE MAGNETIC CHARGE IS DESTROYED AT THE TIME BOUNDARY

WHAT BEHAVIOUR IS EXPECTED FOR ρ ?

ON AN ASYMMETRIC LATTICE, THE TEMPERATURE IS GIVEN BY

$$T(\beta) = \frac{1}{N_t\, a(\beta)}$$

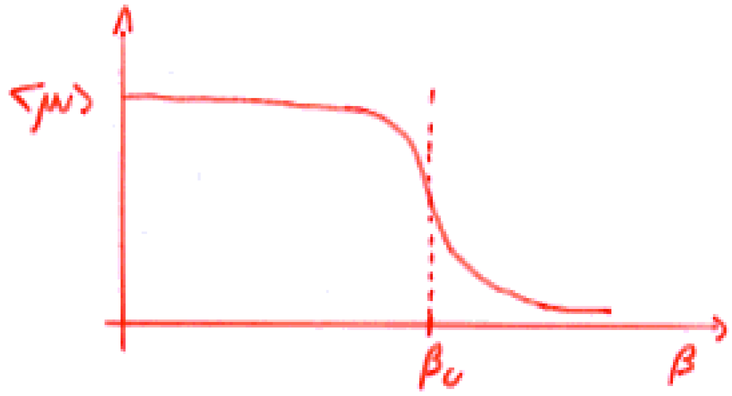
THEREFORE T IS AN INCREASING FUNCTION OF β

IF CONFINEMENT IS RELATED TO MONOPOLE CONDENSATION, WE EXPECT, IN THE THERMODYNAMIC LIMIT

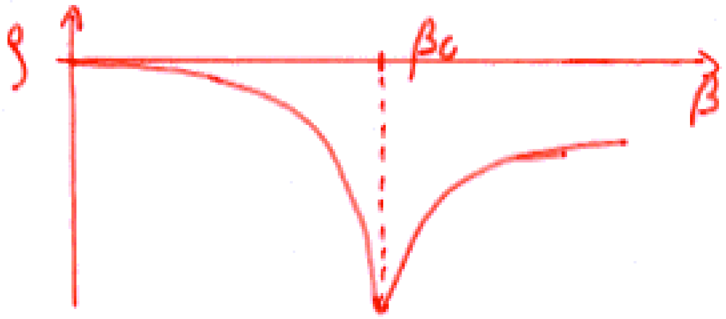
$$\langle \mu \rangle \neq 0 \qquad \beta < \beta_c \quad ; \quad T(\beta_c) = T_c$$

$$\langle \mu \rangle = 0 \qquad \beta \geq \beta_c$$

ON A FINITE SPATIAL LATTICE $\langle \mu \rangle$ CANNOT BE EXACTLY ZERO FOR $\beta \geq \beta_c$ WITHOUT BEING SO EVERYWHERE, SO WE EXPECT A QUALITATIVE BEHAVIOUR:



WHICH CORRESPONDS TO THE FOLLOWING BEHAVIOUR FOR $\rho = \frac{d}{d\beta} \ln \langle \mu \rangle$



ALL THE RELEVANT PHYSICAL INFORMATION CAN THEN BE EXTRACTED FROM A FINITE SIZE SCALING ANALYSIS OF ρ

RESULTS OBTAINED IN PURE GAUGE THEORIES CAN BE SUMMARIZED AS FOLLOWS

- THE QCD VACUUM IS A DUAL SUPERCONDUCTOR IN THE CONFINED PHASE
  DUAL SUPERCONDUCTIVITY DISAPPEARS AT THE DECONFINEMENT PHASE TRANSITION

- IN AGREEMENT WITH A THEORETICAL GUESS BY 'T HOOFT, DIFFERENT ABELIAN PROJECTIONS, CORRESPONDING TO DIFFERENT OPERATORS $\phi(x)$, ARE EQUIVALENT TO EACH OTHER

WE WANT TO EXTEND THIS STUDY TO FULL QCD, i.e. INCLUDING DYNAMICAL FERMIONS.

THIS IS OF GREAT IMPORTANCE, SINCE:

1) TRADITIONAL PARAMETERS WHICH CHARACTERIZE THE DECONFINEMENT PHASE TRANSITION IN PURE GAUGE THEORY, FAIL IN FULL QCD

   STRING TENSION σ: THE STRING BREAKS IN FULL QCD DUE TO $\bar{q}q$ PAIR PRODUCTION

   POLYAKOV LINE P: IN PURE GAUGE THEORY IT IS AN ORDER PARAMETER WHICH SIGNALS THE SPONTANEOUS BREAKING OF CENTRE Z_3 SYMMETRY IN THE DECONFINED PHASE: BUT Z_3 IS NOT MORE A SYMMETRY IN PRESENCE OF QUARKS.

$\langle\mu\rangle$ IS INSTEAD WELL DEFINED ALSO IN PRESENCE OF DYNAMICAL QUARKS, SO IT IS A NATURAL CANDIDATE FOR THE CHARACTERIZATION OF DECONFINEMENT PHASE TRANSITION IN THE COMPLETE THEORY

2) THIS IS ALSO A NATURAL EXPECTATION IF THE IDEA THAT THE $N_c \to \infty$ LIMIT IS A GOOD PHYSICAL APPROXIMATION OF $N_c = 3$ IS CORRECT.
$O(1/N_c)$ CORRECTIONS, AND SO ALSO THE INTRODUCTION OF DYNAMICAL QUARKS, ARE MERELY PERTURBATIONS WHICH DO NOT ALTER THE MAIN PHYSICAL FEATURES OF THE THEORY, IN PARTICULAR CONFINEMENT, WHICH CAN SO BE ASCRIBED EXCLUSIVELY TO THE GAUGE DEGREES OF FREEDOM.

OUR AIM IS TO TEST THIS CONJECTURE ON THE LATTICE

AS A PILOT STUDY, WE ARE INVESTIGATING THE CASE OF FULL QCD WITH 4 FLAVOURS OF STAGGERED FERMIONS, WITH A DEGENERATE FERMION MASS $a \cdot m = 0.05$

FOR $N_f = 4$, A FIRST ORDER PHASE TRANSITION HAS BEEN OBSERVED AT $\beta_c \simeq 5.04$
(BROWN ET AL., PHYS. LETT. B 251 (1990) 181)

MINI-REVIEW ON THE FORMULATION AND SIMULATION OF FULL QCD

THE LATTICE QCD ACTION CAN BE WRITTEN AS

$$S = S_F + S_G$$

$S_G \rightarrow$ PURE GAUGE TERM

$S_F \rightarrow$ FERMIONIC TERM

FOR STAGGERED FERMIONS

$$S_F = \sum_{i,j} \bar{\chi}_i M_{ij} \chi_j$$

χ_i IS A COLOUR TRIPLET OF GRASSMANN FIELDS AND

$$M_{ij} = m\,\delta_{ij} + \frac{1}{2}\sum_{\mu} \eta_{\mu}(i)\left(\delta_{i+\hat{\mu},j} U_{\mu}(i) - \delta_{i,j+\hat{\mu}} \left(U_{\mu}(j)\right)^{+}\right)$$

$$\eta_{\mu}(i) = (-1)^{i_1 + \dots + i_{\mu-1}}$$

FROM THE 16 STAGGERED FIELDS ON EACH HYPERCUBE IT IS POSSIBLE TO RECONSTRUCT 4 DIRAC FIELDS: THE THEORY DESCRIBES 4 DEGEN. FLAVOURS

IN THE FUNCTIONAL INTEGRAL, PERFORMING THE INTEGRATION ON GRASSMANN VARIABLES, WE HAVE

$$Z = \int [dU\, d\chi\, d\bar{\chi}]\, e^{-S_G - S_F} = \int [dU] \det(M)\, e^{-S_G}$$

THE FERMION DETERMINANT CAN BE REWRITTEN IN TERMS OF A BOSONIC FIELD, φ, CALLED THE PSEUDO FERMIONIC FIELD

$$Z = \int [dU\, d\varphi\, d\varphi^{+}]\, e^{-\left[S_G + \phi^{+}(M^{+}M)^{-1}\phi\right]}$$

IN THIS FORM THE PARTITION FUNCTION IS SUITABLE FOR A SIMULATION WITH THE HYBRID MONTE CARLO ALGORITHM (HMC)

ACTUALLY $\det(M^+M) = (\det M)^2 \Rightarrow$ FLAVOUR NUMBER DOUBLING

BUT

$$M^+M = \begin{pmatrix} (M^+M)_{ee} & 0 \\ 0 & (M^+M)_{oo} \end{pmatrix} \quad ; \quad \det(M^+M)_{ee} = \det(M^+M)_{oo}$$

SO DOUBLING IS AVOIDED BY DEFINING φ ON EVEN SITES ONLY

DESCRIPTION OF THE HMC ALGORITHM

FICTITIOUS MOMENTA $H_{J,\mu}$, CONJUGATE TO THE LINKS, ARE INTRODUCED

$$Z = \int [dU\, d\varphi\, d\varphi^+ dH]\, e^{-\mathcal{H}}$$

$$\mathcal{H} = \frac{1}{2} \sum_{J,\mu} \mathrm{tr}\, H^2_{J,\mu} + S_G + \varphi^+ (M^+M)^{-1} \varphi$$

THE FIELDS EVOLVE WITH A MIXED DYNAMICS WHICH ALTERNATES DETERMINISTIC AND STOCHASTIC STEPS:

DETERMINISTIC PART: FIELDS ARE EVOLVED IN A FICTITIOUS TIME τ, ACCORDING TO THE HAMILTONIAN $\mathcal{H}$ OF THE (4+1)-DIM. SYSTEM

$$\dot{U}_{J,\mu} = i\, H_{J,\mu} U_{J,\mu}$$

AND $H_{J,\mu}$ FIELDS EVOLVE SO THAT THE HAMILTONIAN STAY CONSTANT

THE INTEGRATION IS PERFORMED NUMERICALLY WITH $\delta\tau \sim O(10^{-2}-10^{-3})$ FOR A TOTAL TRAJECTORY LENGTH $\tau \sim O(1)$

STOCHASTIC PART: AT THE BEGINNING OF EACH TRAJECTORY, NEW φ AND H FIELDS ARE GENERATED ACCORDING TO THEIR PROB. DISTRIBUTION

$\delta\tau \to 0$ LIMIT : EXACT ALGORITHM, U FIELDS CORRECTLY SAMPLED

FINITE $\delta\tau$: A METROPOLIS ACCEPT-REJECT STEP AT THE END OF EACH TRAJECTORY MAKES THE ALGORITHM EXACT

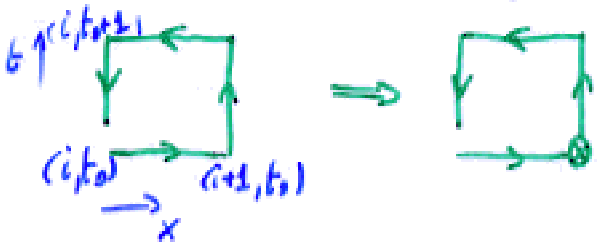
DETECTING DUAL SUPERCONDUCTIVITY WITH DYNAMICAL FERMIONS

WE WANT TO MEASURE $\langle \mu \rangle$, WHERE μ CARRIES A NON-TRIVIAL MAGNETIC CHARGE

μ CAN BE CONSTRUCTED EXACTLY AS IN THE PURE GAUGE CASE, IN TERMS OF GAUGE DEGREES OF FREEDOM ONLY

$$\langle \mu \rangle = \frac{\int [dU\, d\chi\, d\bar{\chi}]\, e^{-S'}}{\int [dU][d\chi\, d\bar{\chi}]\, e^{-S}}$$

WHERE $S' = S'_G + S_F$ IS A MODIFIED ACTION IN WHICH ONLY THE PURE GAUGE TERM HAS BEEN CHANGED BY CHANGING THE DEFINITION OF TEMPORAL PLAQUETTES



$$U_i(\vec{m}, t_0) \to U_i(\vec{m}, t_0)\, e^{i A^M_{\perp i}(\vec{m}, \vec{y})\, \ell(\vec{m}+i, t_0)}$$

$\ell(\vec{m}, t_0) = \Omega(\vec{m}, t_0)\, F_0\, (\Omega(\vec{m}, t_0))^{+}$; $\Omega(\vec{m}, t)$ DIAGONALIZES A LOCAL OPERATOR $\phi(\vec{m}, t)$ IN THE ADJ. REPR.

WE HAVE CHOSEN THE POLYAKOV LINE AS $\phi(\vec{m}, t)$

WE THEN MEASURE $\rho = \frac{d}{d\beta} \ln \langle \mu \rangle = \langle S_G \rangle_{S_G + S_F} - \langle S'_G \rangle_{S'_G + S_F}$

ALGORITHM IMPLEMENTATION

THE EQUATIONS OF MOTION OF THE CONJUGATE MOMENTA MUST BE CHANGED IN ORDER TO MAINTAIN CONSTANT THE MODIFIED HAMILTONIAN

$$\mathcal{H} = \frac{1}{2} \sum_{J,\mu} \mathrm{tr}\, H^2_{J,\mu} + S'_G + \varphi^{+} (M^{+} M)^{-1} \varphi$$

THE EVOLUTION OF MOMENTA $H_{J,0}$ CHANGES IN A SUBTLE WAY

$U_{J,0}$ CHANGES $\Rightarrow$ POLYAKOV LINE CHANGES $\Rightarrow$ $\Omega(\vec{m}, t_0)$ CHANGES $\Rightarrow$ MONOPOLE FIELD CHANGES $\Rightarrow$ S'_G CHANGES

TROUBLES

1st TROUBLE: C^* BOUNDARY CONDITIONS

WE HAVE SEEN THAT AT FINITE T WE CAN ONLY MEASURE $\langle \mu \rangle$ AND THIS REQUIRES C^* B.C. FOR THE GAUGE FIELDS

$$U_\mu(\vec{n}, N_t) = U^*_\mu(\vec{n}, 0)$$

THE SAME B.C. HAS TO BE ADOPTED IN THE DEFINITION OF THE FERMION MATRIX

BUT IF WE NAIVELY IMPLEMENT C^* B.C. ON GAUGE FIELDS ONLY, THE RESULTING FERMIONIC DETERMINANT LOOSES GAUGE INVARIANCE

C^* B.C. MUST BE IMPOSED ON FERMIONIC VARIABLES TOO!

FOR STAGGERED FERMIONS THIS MEANS

$$\chi(\vec{x}, t+N_t) = (-1)^{x+y+z+t}\, \chi^*(\vec{x}, t)$$
$$\bar{\chi}(\vec{x}, t+N_t) = -(-1)^{x+y+z+t}\, \chi^T(\vec{x}, t)$$

(WIESE, 1992)

IN ORDER TO IMPLEMENT THESE B.C. ONE MUST FIRST REWRITE S_F

$$S_F = \bar{\chi} M \chi = \frac{1}{2}\left[(\bar{\chi} M \chi) + (\bar{\chi} M \chi)^T\right] =$$

$$= \frac{1}{2} (\chi^T \ \bar{\chi})\, A \begin{pmatrix} \chi \\ \chi^* \end{pmatrix} \quad ; \quad A = \begin{pmatrix} 0 & -M^T \\ M & 0 \end{pmatrix}$$

WRITING $\psi \equiv (\chi \ \ \chi^*)$ WE HAVE

$$\int [d\chi\, d\bar{\chi}]\, e^{-S_F} = \int [d\psi]\, e^{-\frac{1}{2}\psi^T A \psi} = \mathrm{Pf}(A) = \det(M)$$

$$\left((\mathrm{Pf}(A))^2 = \det(A) \right)$$

C^* B.C. ARE THEN IMPLEMENTED BY INSERTING NON-ZERO BOUNDARY TERMS IN THE DIAGONAL BLOCKS OF A

THIS IMPLIES RELEVANT CHANGES IN THE ALGORITHM IMPLEMENTATION, SEE FOR DETAILS (J. CARMONA, M. D'E., A. DI GIACOMO, B. LUCINI, HEP-LAT/0003002)

ONE IMPORTANT CHANGE IS THAT, SINCE WE CAN ONLY SIMULATE $\det(A)$ AND NOT $\mathrm{Pf}(A)$, THERE IS AN UNRECOVERABLE FLAVOUR DOUBLING
IN ORDER TO STAY WITH 4 FLAVOURS WE MUST SWITCH TO A NON EXACT VERSION OF THE ALGORITHM.

2nd TROUBLE: THE MODIFIED ACTION IS NON-ANALYTIC IN THE GAUGE FIELDS, THIS MAKES THE MOLECULAR DYNAMICS INTEGRATION OF THE EQUATION OF MOTION DIFFICULT

IN THE ABELIAN PROJECTION ONE HAS TO FIX ALSO THE ORDER OF THE EIGENVALUES OF THE DIAGONALIZED OPERATOR $\phi_D(x)$, FOR EXAMPLE ACCORDING TO THEIR IMAGINARY PART

$$\mathrm{Im}\,\lambda_1 > \mathrm{Im}\,\lambda_2 > \mathrm{Im}\,\lambda_3$$

AFTER ABELIAN PROJECTION WE WANT A PURE $U(1)\otimes U(1)$ REMNANT GAUGE FREEDOM, WITHOUT ORDERING THERE IS AN ADDITIONAL PERMUTATION SYMMETRY

DURING MOLECULAR DYNAMICS: TWO EIGENVALUES CAN CROSS EACH OTHER $\Longrightarrow$ THE ORDERING CHANGES $\Longrightarrow$ THE DIAGONALIZING MATRIX $\Omega(x)$ CHANGES $\Longrightarrow$ THE MONOPOLE FIELD CHANGES $\Longrightarrow$ THE ACTION CHANGES DISCONTINOUSLY !

THIS IS NOT AVOIDABLE BY CHANGING ORDERING CRITERION

A POSSIBLE WAY OUT IS TO CHANGE THE DEFINITION OF μ
(LATER WE WILL LOOK AT POSSIBLE SOLUTIONS WHICH DO NOT CHANGE THE DEFINITION OF μ)

1) RELAX THE ORDERING CONDITION : EIGENVALUES ARE ORDERED AT THE BEGINNING OF EACH TRAJECTORY AND THEN ARE LET FREE DURING THE MOLECULAR DYNAMICS

2) FIX $\Omega(x) = \mathbb{1}$ ALWAYS, i.e. THE MONOPOLE FIELD IS DIAGONAL IN THE GAUGE INDICATED BY THE FIELD CONFIGURATION ITSELF (RANDOM GAUGE)

THE APPROACH SEEMS REASONABLE IN VIEW OF THE INDICATION FROM PURE GAUGE THEORY THAT ALL ABELIAN PROJECTIONS SEEM PHYSICALLY EQUIVALENT

CONCLUSIONS

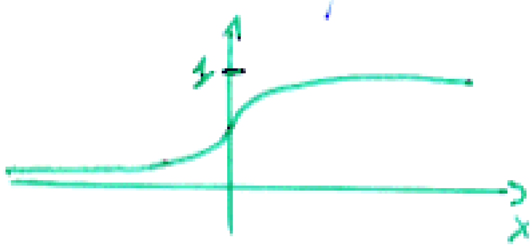
THE PRELIMINARY RESULTS INDICATE THAT DUAL SUPERCONDUCTIVITY IS THE MECHANISM OF CONFINEMENT ALSO IN FULL QCD. THE PRESENCE OF DYNAMICAL QUARKS ACTS MERELY AS A PERTURBATION TO GLUODYNAMICS (IN AGREEMENT WITH $1/N_c$)

A FINITE SIZE SCALING ANALYSIS IS HOWEVER NECESSARY TO MAKE THE STATEMENT MORE RIGOROUS.
THIS IS CURRENTLY BEING DONE. HOWEVER GOING TO LARGER SPATIAL LATTICES IN FULL QCD REQUIRES A LOT OF COMPUTER POWER. BUT WE WILL SOON BENEFIT FROM THE APEMILLE GIGAFLOPS.

IT IS NECESSARY TO OVERCOME THE EIGENVALUE ORDERING PROBLEM WITHOUT GIVING UP THE ORIGINAL DEFINITION OF μ
TWO POSSIBILITIES ARE VIABLE :

1) THE ORDERING CAN BE "REGULARIZED" :
WHEN DEFINING THE MODIFIED ACTION, THETA FUNCTIONS LIKE $\theta(\mathrm{Im}\lambda_1 - \mathrm{Im}\lambda_2)$, DEFINING THE ORDERING, CAN BE SUBSTITUTED BY A CONTINUOS FUNCTION, FOR INSTANCE

$$\theta(\lambda) \rightarrow \frac{1}{1+e^{-\lambda/a}}$$

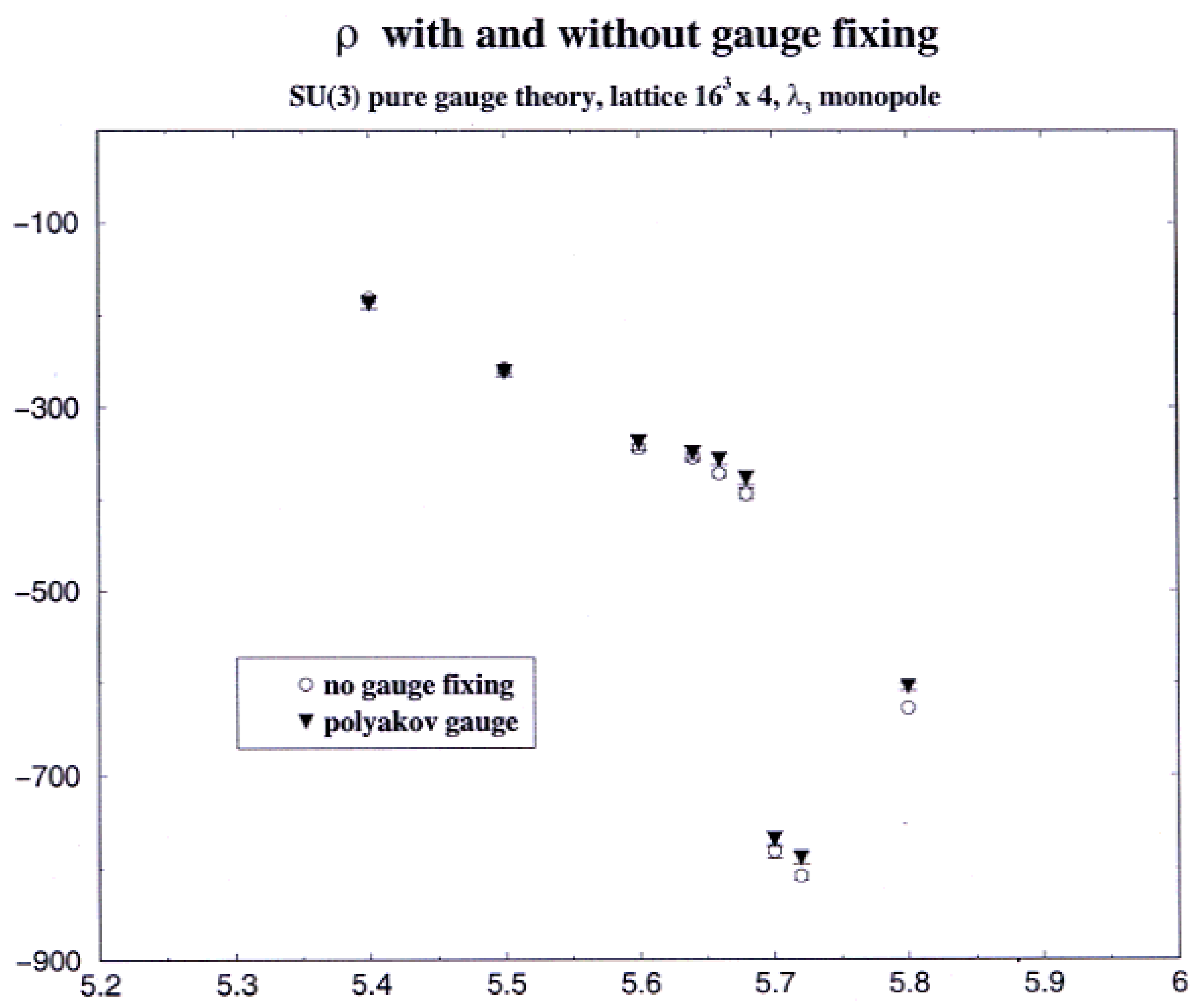


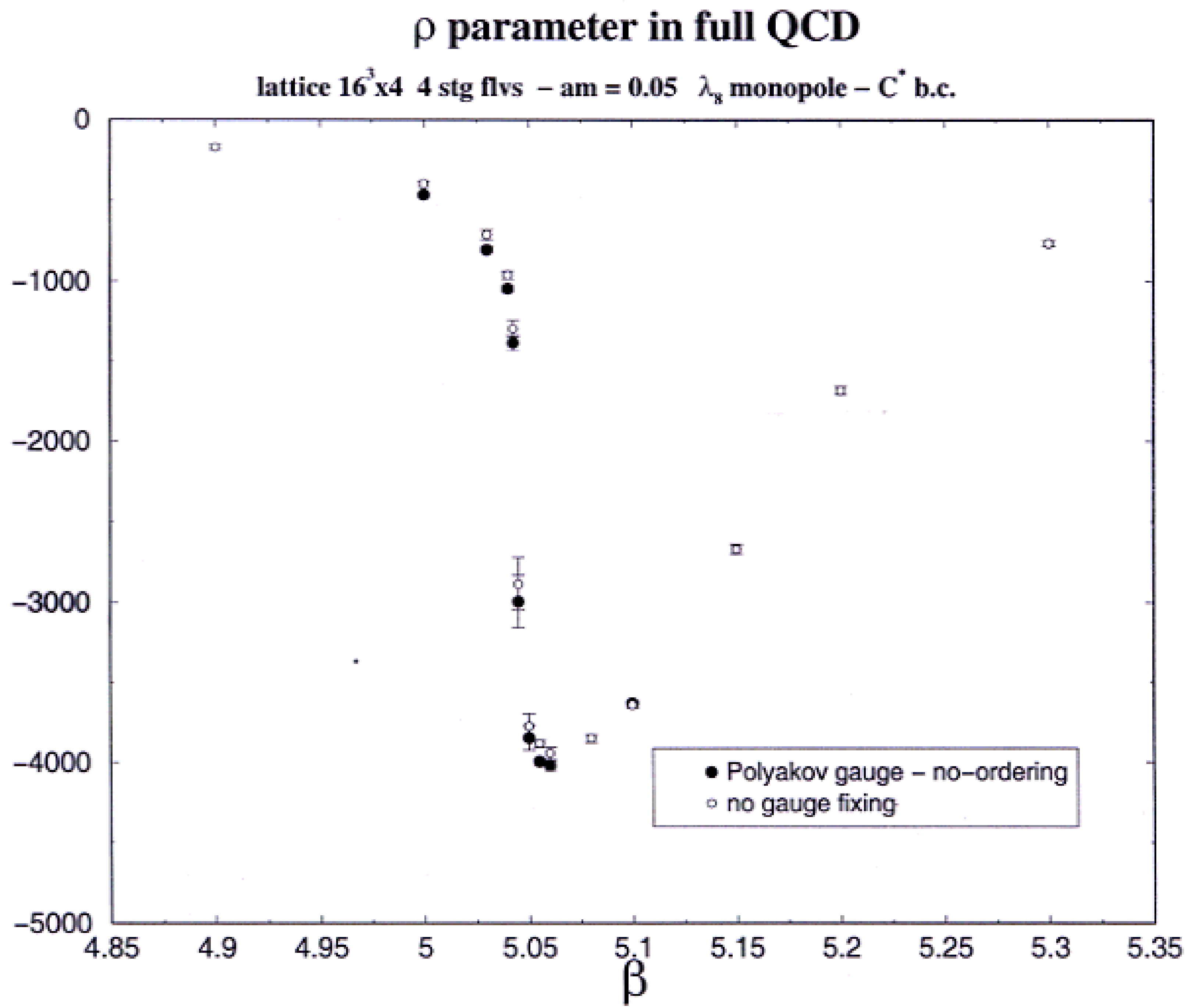
THE REGULARIZED THETA FUNCTIONS CAN THEN BE INTEGRATED IN THE EQUATIONS OF MOTION

2) CHANGE SIMULATION ALGORITHM

THE PROBLEM WITH HMC IS THAT IT USES MOLECULAR DYNAMICS. A DIFFERENT ALGORITHM COULD AVOID THE PROBLEM.
A POSSIBILITY IS THE LÜSCHER MULTIBOSON ALGORITHM, WHERE THE UPDATING IS PERFORMED LOCALLY

BOTH POSSIBILITIES ARE CURRENTLY UNDER STUDY


ρ with and without gauge fixing
SU(3) pure gauge theory, lattice 16³ x 4, λ₃ monopole
−100
−300
−500
−700
−900
5.2
5.3
5.4
5.5
5.6
5.7
5.8
5.9
6
no gauge fixing
polyakov gauge


ρ parameter in full QCD
lattice 16³x4 4 stg flvs – am = 0.05 λ8 monopole – C* b.c.
0
-1000
-2000
-3000
-4000
-5000
4.85
4.9
4.95
5
5.05
5.1
5.15
5.2
5.25
5.3
5.35
β
Polyakov gauge – no-ordering
no gauge fixing