BARI 00

CONFINEMENT OF COLOUR

A REVIEW

Adriano DI GIACOMO

CONFINEMENT OF COLOUR

1 – QUARKS AND GLUONS HAVE NEVER BEEN OBSERVED : ASYMPTOTIC STATES ARE COLOUR SINGLETS – (CONFINEMENT)

IN NATURE $\frac{n_q}{n_p} \lesssim 10^{-27}$

MILLIKAN LIKE EXPERIMENTS $\sim 1g$ MATTER

EXPECTATION IN ABSENCE OF CONFINEMENT $\frac{n_q}{n_p} \simeq 10^{-12}$ (OKUN)

IF quark gluon plasma exists

$q\bar{q} \to$ hadrons

$qq \to \bar{q}$ + hadrons

$\sigma_0 = \lim_{v \to 0} v\sigma \simeq m_\pi^{-2}$

$T \sim m_q$

$\sigma_0 n_q = G_N^{1/2} T^2$ $\qquad n_\gamma = T^3$

$$\frac{n_q}{n_\gamma} = \frac{G_N^{1/2}}{T^2 \sigma_0} \gtrsim \frac{10^{-19} m_\pi^2}{m_p T} \simeq 10^{-28} \Rightarrow \frac{n_q}{n_p} \gtrsim 10^{-12}$$

$T \sim 10$ GeV

FACTOR 10^{-15} VERY SMALL.

ONLY NATURAL EXPLANATION CAN BE IN TERMS OF SYMMETRY [LIKE σ IN A SUPERCONDUCTOR] – NOT IN TERMS OF A TUNABLE SMALL PARAMETER.

L. Del Debbio, A. Di G., B. Lucini hep-lat 0006012

A. Di Giacomo, B. Lucini, L. Montesi, G. Paffuti PRD 61 (00) 034503/034504

IN QCD.

2. QCD AT FINITE TEMPERATURE. DECONFINING TRANSITION

- $Z_T = \int [dA] \exp[-\beta S_T]$ $\qquad \beta = \frac{2N_c}{g^2}$

$$S_T = \int d^3x \int_0^{1/T} \mathcal{L}\, dt$$

p.b.c bosons
a.b.c fermions

ON A LATTICE $N_t \times N_s^3$

$N_s \to \infty \qquad (N_t/N_s \to 0)$

$$\frac{1}{T} = N_t \cdot a \qquad T = \frac{1}{N_t \cdot a}$$

$$a(\beta) \propto \frac{1}{\Lambda} \exp[-b_0 \beta]$$ (ASYMPTOTIC FREEDOM)

$$T = \frac{\Lambda}{N_T} \exp[b_0 \beta]$$

- HIGH T — WEAK COUPLING
- LOW T — STRONG COUPLING

USUALLY WEAK COUPLING IS LOW T, STRONG COUPLING HIGH T
$g^2 \simeq T$

QUENCHED QCD

T_c

SU(2) 2nd order Phase Transition (I_3)
SU(3) weak first order
$T_c \sim 150$ MeV

$T < T_c$ $\begin{cases} \sigma \neq 0 \\ \langle |L| \rangle = 0 \end{cases}$

$T > T_c$ $\begin{cases} \sigma = 0 \\ \langle |L| \rangle \neq 0 \end{cases}$

σ STRING TENSION — DISORDER PARAMETER

$\langle |L| \rangle$ POLYAKOV LOOP $\equiv \exp(-\mu_q)$ — ORDER PARAMETER

μ_q CHEMICAL POTENTIAL OF A QUARK

FULL QCD (UNQUENCHED)

σ NOT DEFINED

Z_N SYMMETRY BROKEN $\langle L \rangle$ NOT AN ORDER PARAMETER

CHIRAL PHASE TRANSITION

$\langle \bar{\psi} \psi \rangle \neq 0 \quad T < T_\chi \qquad \langle \bar{\psi} \psi \rangle = 0 \quad T > T_\chi$

$T_\chi \sim 140$ MEV

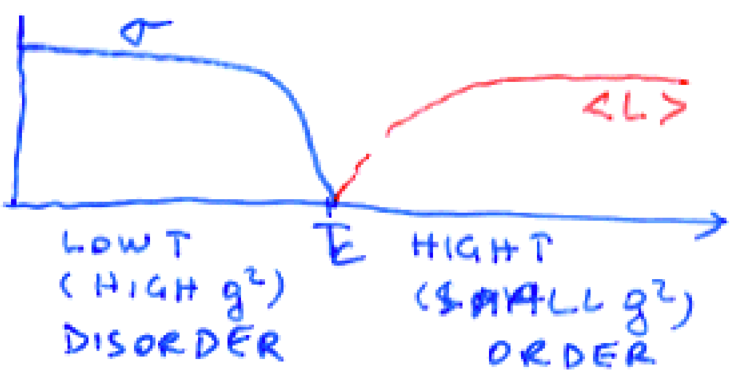
- ORDER OF THE TRANSITION & UNIVERSALITY CLASS UNCLEAR.
- RELATION TO CONFINEMENT UNCLEAR

- PROBLEM 1
  - DEFINE AN ORDER PARAMETER FOR CONFINEMENT-DECONFINEMENT, INDEPENDENT OF THE PRESENCE OF QUARKS

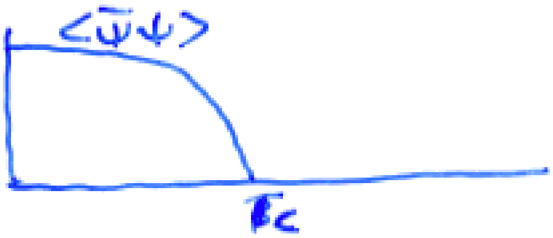
- PROBLEM 2.

  UNDERSTAND THE SYMMETRY OF THE CONFINED PHASE

QUENCHED



UNQUENCHED



?

– DUALITY

WEAK COUPLING REGIME $\Rightarrow$ SYMMETRY DESCRIBED BY ORDER PARAMETERS

$\langle \Phi \rangle$ Φ fundamental fields

EX GINZBURG–LANDAU Φ IN SUPERCONDUCTORS

$\langle$HIGGS FIELD$\rangle$ IN STANDARD MODEL

$\langle \vec{m} \rangle$ MAGNETIZATION IN SPIN MODELS

THERE EXIST SYSTEMS WITH TOPOLOGICALLY NON TRIVIAL NON LOCAL EXCITATIONS μ, UNDERGOING PHASE TRANSITIONS TO DISORDER, WHICH ADMIT A DUAL DESCRIPTION, IN WHICH μ'S BECOME LOCAL

WEAK COUPLING		STRONG COUPLING
Local Φ NON LOCAL μ	$\longrightarrow$	Local μ Non local Φ
ORDERED PHASE	$\longleftarrow$	DISORDERED PHASE

$$\beta \rightleftarrows \beta^* \simeq \frac{1}{\beta}$$

KRAMERS–WANNIER DUALITY

- ISING MODEL (KADANOFF–CEVA 73)
- LIQUID He_4 (3d X-Y MODEL) [Di Cecio et al 97]
- HEISENBERG MAGNET 3d [DG et al 98]
- NON COMPACT 4d U(1) [FROLICH 9[illegible], Del Debbio et al 94]
- SUSY QCD (WITTEN – SEIBERG)

GUIDING PRINCIPLE # 1

$\Rightarrow$ CONFINEMENT IN QCD PRODUCED BY CONDENSATION OF TOPOLOGICAL EXCITATIONS μ IN THE DISORDERED PHASE ['t Hooft 78], I.E. BY DUAL SYMMETRY BREAKING

$\langle \mu \rangle$ DISORDER PARAMETER

GUIDING PRINCIPLE #2

- $1/N_c$ LIMIT $\lambda = g^2 N_c$ fixed $N_c \to \infty$

[SU(N_c) GAUGE GROUP] PHYSICS IS DETERMINED AT $N_c = \infty$; $\frac{1}{N_c}$ CORRECTIONS ARE SMALL PERTURBATIONS [Witten, t'Hooft, Veneziano]

CONFINEMENT EXISTS AT $N_c = \infty$. THE DISORDER PARAMETER SHOULD BE N_c INDEPENDENT

σ $\langle |L| \rangle$ NOT DEFINED IN THE PRESENCE OF DYNAMICAL QUARKS

$$\simeq N_f g^n \simeq \frac{\lambda^{n/2} N_f}{N_c^{n/2}}$$

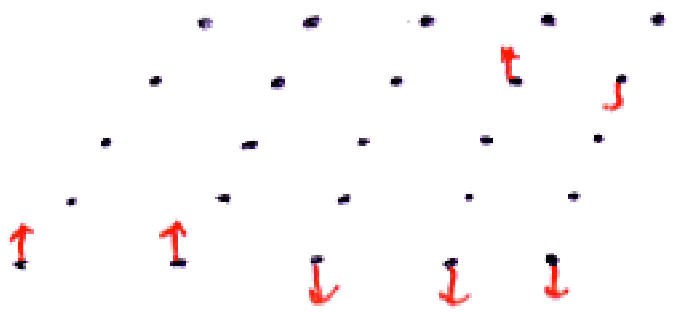
NEGLIGIBLE FOR $N > 2$

$\frac{N_f}{N_c} \sim O(1)$ CORRECTS β FUNCTION

- QUENCHED APPROXIMATION ON LATTICE WORKS AT 10% ACCURACY

- $U_A(1)$ PROBLEM SOLVED BY $1/N_c$ ARGUMENT

2d ISING – A TOY MODEL



$\sigma(\vec{n}) = \pm 1$

$$S[\sigma] = J \sum_{\vec{n},\vec{\mu}} \sigma(\vec{n})\sigma(\vec{n}+\vec{\mu}) \qquad Z = \exp\left[-\frac{1}{T} S[\sigma]\right]$$

- EXACTLY SOLVABLE (ONSAGER)

$$T_c = \frac{2J}{\ln(\sqrt{2}+1)} \qquad \langle \sigma \rangle = \lim_{V \to \infty} \frac{1}{V} \sum_{\vec{n}} \langle \sigma(\vec{n}) \rangle$$

$T < T_c$ $\quad \langle \sigma \rangle \neq 0$ $\quad$ SYMMETRY $\sigma \to -\sigma$ broken
ORDER PARAMETER

$T > T_c$ $\quad \langle \sigma \rangle = 0$ $\quad$ (DISORDERED PHASE)

$$\langle \sigma \rangle \underset{T \to T_c}{\sim} (T_c - T)^{\beta} \qquad \beta = \frac{1}{8}$$

$$\langle \sigma(\vec{i})\,\sigma(\vec{j}) \rangle \sim \exp\left[-\frac{d}{\xi(T)}\right] + \langle \sigma \rangle^2$$ CLUSTER PROPERTY

$$\xi(T) \underset{T \to T_c}{\sim} a\,(T - T_c)^{\nu} \qquad \nu = 1$$

A (1+1) d Field Theory [J. Cannon, ADG, B. Lucini]

$$S = \frac{J}{2} \sum_{\mu\,\vec{n}} [\Delta_\mu \sigma(\vec{n})]^2 \qquad \Delta_{\vec{\mu}} \sigma = \sigma(n+\vec{\mu}) - \sigma(\vec{n})$$

Eq. motion $\Delta_\mu \Delta_\mu \sigma = 0$

$$j_\mu = \frac{1}{2} \varepsilon_{\mu\nu} \Delta_\nu \sigma \qquad \Delta_\mu j_\mu = 0$$

$$Q = \int dx\, j_0(t,x) = \frac{1}{2}[\sigma(x=+\infty) - \sigma(x=-\infty)]$$

Q = # KINKS – # ANTIKINKS

KINKS ARE THE TOPOLOGICAL EXCITATIONS

DUALITY : (KADANOFF-CEVA 73)

DEFINE ON THE DUAL LATTICE A VARIABLE σ^*

$$\langle \sigma^*(i)\, \sigma^*(j) \rangle = \frac{\tilde{Z}}{Z}$$

$\tilde{Z}^*$ OBTAINED FROM Z BY CHANGE OF SIGN OF THE LINKS $J\sigma(n)\sigma(n+\mu)$ ALONG AN ARBITRARY PATH FROM i TO J

THEN IN THE THERMODYNAMICAL LIMIT $L \to \infty$

1) $\sigma^* = \pm 1$

2) $\tilde{Z}^*$ INDEPENDENT OF THE PATH CHOSEN

3) $Z[\sigma, T] = Z[\sigma^*, T^*]$

$$\sinh \frac{2}{T} = \frac{1}{\sinh 2/T^*} \qquad T \to \frac{1}{T}$$

$\langle \sigma^* \rangle \neq 0 \quad T > T_c$

$\langle \sigma \rangle \langle \sigma^* \rangle = 0$

4) $\sigma^* \pm 1$ IS THE CREATION OPERATOR OF A KINK (ANTIKINK)

($\leftrightarrow$)

spin — kink

$$\langle \sigma \rangle \rightleftarrows \langle \sigma^* \rangle$$

ORDER PARAMETER — DISORDER PARAMETER

– THE DISORDER PARAMETER

BASIC IDEA

$$e^{ipa}\,|x\rangle = |x+a\rangle$$

[L. Del Debbio, A. Di G., G. Paffuti 90]

FIELD CONFIGURATION IN THE SCHRÖDINGER REPRESENTATION

$$\mu(y,t) = e^{i\int d^3x\, \Pi_i(\vec{x},t)\, \phi_i^{cl}(\vec{x}-\vec{y})}\,|\phi(\vec{x},t)\rangle = |\phi(\vec{x},t) + \phi^{cl}(\vec{x}-\vec{y})\rangle$$

$\phi_i^{cl}(\vec{x}-\vec{y})$, A TOPOLOGICAL EXCITATION, IS ADDED TO THE FIELD CON-FIGURATION

ADAPT TO COMPACT FIELDS
- ISING 2d
- 3d X-Y
- 3d Heisenberg
- 4d U(1)

$\langle 0|\mu|0\rangle \neq 0$ SIGNALS BREAKING OF THE TOPOLOGICAL SYMMETRY

EXAMPLE

ISING MODEL (2D)

$$Z = \exp\left[-\frac{J}{T}\sum_{\vec{n}}\sum_{\mu=0}^{1} \sigma(\vec{n})\,\sigma(\vec{n}+\hat{\mu})\right]$$

$$\mu(n_0, n_1) = \exp\left[\frac{2J}{T}\sum_{n\le n_1} \sigma(n_0,n)\,\sigma(n_0+1,n)\right]$$

$$\langle \mu(n,n_1)\rangle = \frac{\tilde{Z}}{Z} \qquad \tilde{Z} = \exp\left[-\frac{1}{T}\tilde{S}\right]$$

$\tilde{S}$ OBTAINED FROM S BY CHANGING SIGN TO ALL TEMPORAL LINKS AT TIME n_0 AND $n \le n_1$

COMPUTE NUMERICALLY

$$\langle \mu(n_0, n_1)\, \mu(n_0+t, n_1) \rangle \underset{t\to\infty}{\simeq} c\, \exp(-Mt) + \langle \mu \rangle^2$$

$$\langle \mu \rangle = \left\langle \exp \sum_{n<n_1} \frac{2J}{T}\, \sigma(n_0, n_1)\, \sigma(n_0+1, n_1) \right\rangle$$

$$\sim \exp V \implies \text{WILD FLUCTUATIONS}$$

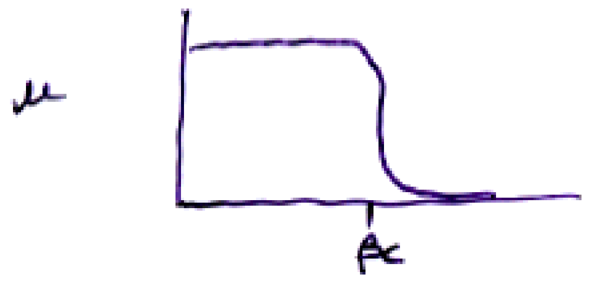
$\Downarrow$

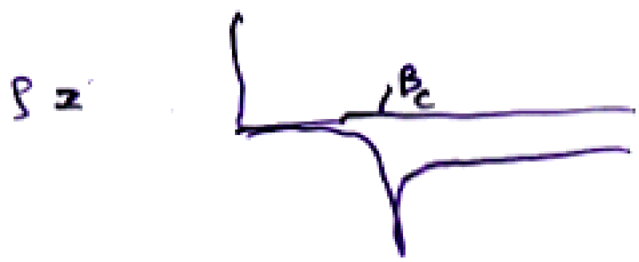
COMPUTE INSTEAD

$$\left.\begin{aligned} \rho &= \frac{d}{d\beta} \ln \langle \mu \rangle \\ &= \frac{d}{d\beta} \ln \frac{\tilde{Z}}{Z} \end{aligned}\right\} = \langle S \rangle_S - \langle \tilde{S} \rangle_{\tilde{S}}$$

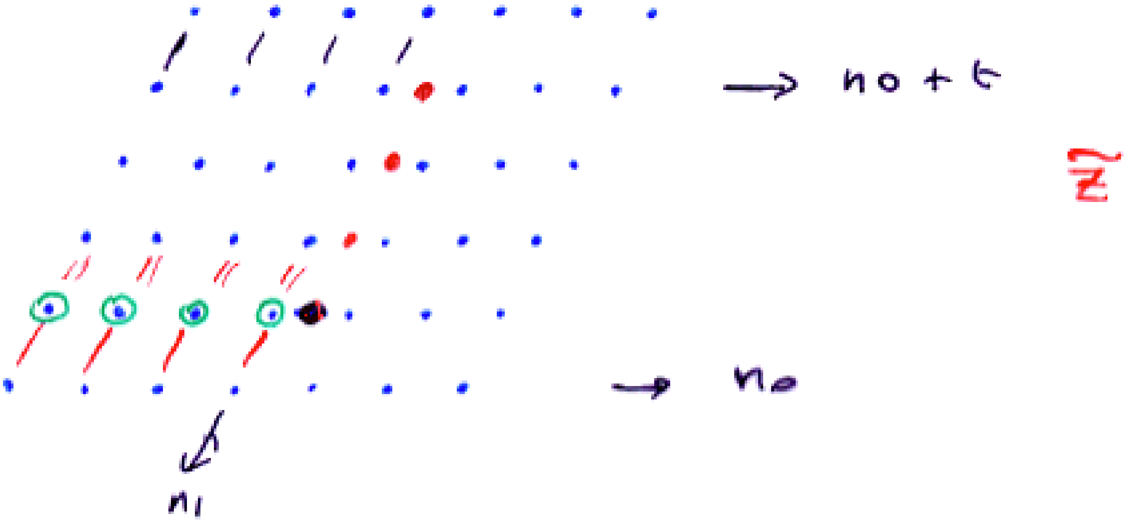
$$\langle \mu \rangle = \exp\left(\int_0^\beta dx\, \rho(x) \right)$$

EXPECT





GO TO THERMODYNAMICAL LIMIT BY FINITE SIZE SCALING ANALYSIS



THEOREM μ COINCIDES WITH KADANOFF'S DUAL VARIABLE σ^* [G. Carmona, A.DG, B.Lucini 2000]

CONSIDER $\tilde{Z} = \langle \mu(n_0\, n_1)\ \mu(n_0+t, n_1) \rangle$

PROOF: CHANGE VARIABLES IN COMPUTING $\tilde{Z}$ FROM $\sigma(n_0, n;\ n \le n_1)$ TO $-\sigma(n_0+1, n : n \le n_1)$ (see fig) THEN

(i) THE TEMPORAL LINKS $n \le n_1$ n_0 ARE RESET TO THE ORIGINAL SIGN

(ii) THE SPATIAL LINK n_0+1 $n_1 \to n_1+1$ CHANGES SIGN

(iii) THE TEMPORAL LINKS n_0+1 $n \le n_1$ CHANGE SIGN

....

REPEAT THE PROCEDURE AT n_0+1 . ...

IF ANOTHER KINK IS PUT AT n_0+t, n_1

$\tilde{Z}$ IS OBTAINED FROM Z BY CHANGING SIGN TO THE SPACE LINKS $n_0)$ $n_i \to n_{i+1}$ to n_0+t, $n_i\ n_{i+1}$ WHICH IS KADANOFF DEFINITION

$T < T_c \qquad \langle\mu\rangle \xrightarrow[L\to\infty]{} \text{finite} \neq 0 \qquad \rho \to$ finite bounded

$T > T_c \qquad \langle\mu\rangle \xrightarrow[L\to\infty]{} \exp\left[-\frac{a}{T} L\right] \qquad \mu = \exp \int\int_0^{1/T} \rho(x) dx^0$

$T \sim T_c \qquad \langle\mu\rangle \xrightarrow[T\to T_c]{} \left(\frac{T_c - T}{T_c}\right)^\delta \equiv t^\delta \qquad t \equiv \left(1 - \frac{T}{T_c}\right)$

scaling $\qquad \langle\mu\rangle \simeq t^\delta f\left(\frac{a}{\xi}, \frac{L}{\xi}\right) \simeq t^\delta f(L/\xi)$

$$\langle\mu\rangle = t^\delta f(L^{1/\nu} t)$$

$\xi \propto t^{-\nu} \qquad\qquad s \equiv L^{1/\nu} t$

$$\frac{\rho}{L^{1/\nu}} = \frac{\delta}{s} - \frac{1}{f}\frac{df}{ds}$$

$\rho / L^{1/\nu} = \phi(L^{1/\nu} t)$

fig 1,2,3

FIT
- $\nu = 1$
- $\delta = .120(5)$
- β_c consistent with expectation.

SIMILAR CHECK DONE FOR

3d XY	(Liquid He_4)	vortices
3d Heisenberg		NONABELIAN VORTICES
4d U(1)		MONOPOLES

- SU(2) – SU(3) MONOPOLES OF DIFFERENT ABELIAN PROJECTIONS [FIG]

[A.DG. B.LUCINI, L.MONTESI & PAFFUTI PRD 2000]

MAGNETIC CHARGES IN "ALL" ABELIAN PROJECTIONS CONDENSE: DUAL S.C. ~~EVERYWHERE~~ IN THE CONFINING PHASE. IT DISAPPEARS ABOVE T_c.

A DISORDER PARAMETER FOUND WORKING ALSO ~~INSENSIBLE~~ IN THE PRESENCE OF QUARKS ??

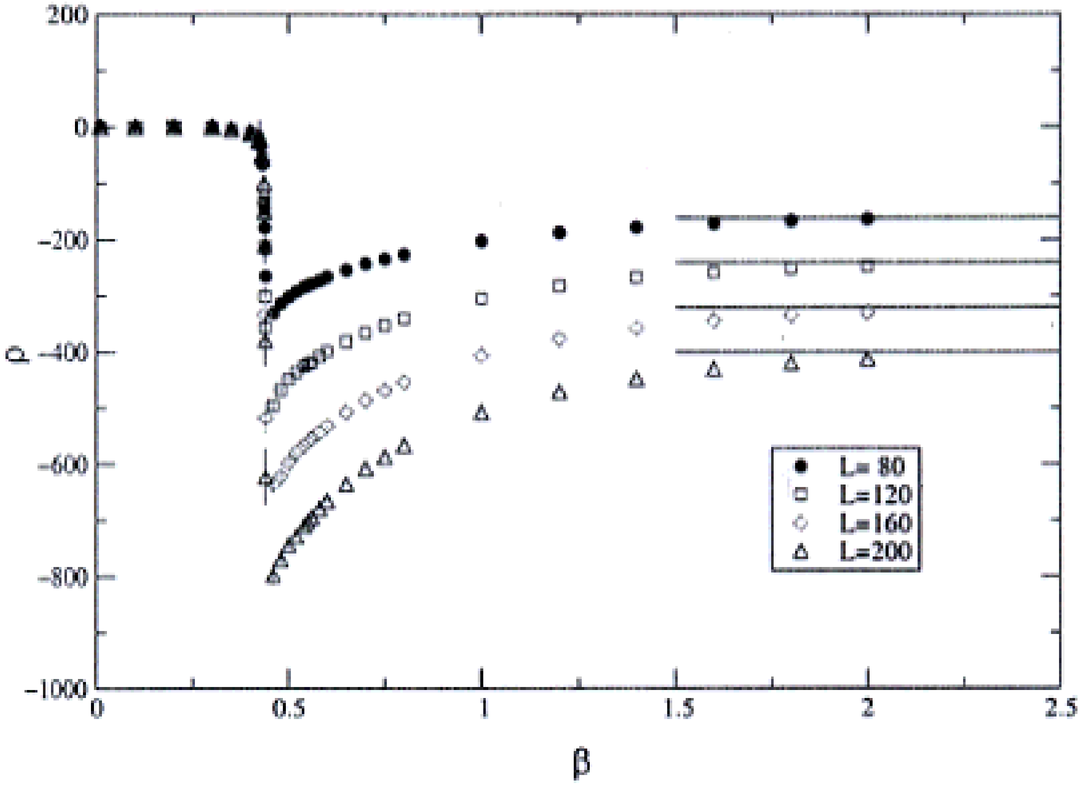


Fig. 1. ρ as a function of β for different lattice sizes. Continuous lines refer to low temperature calculations.

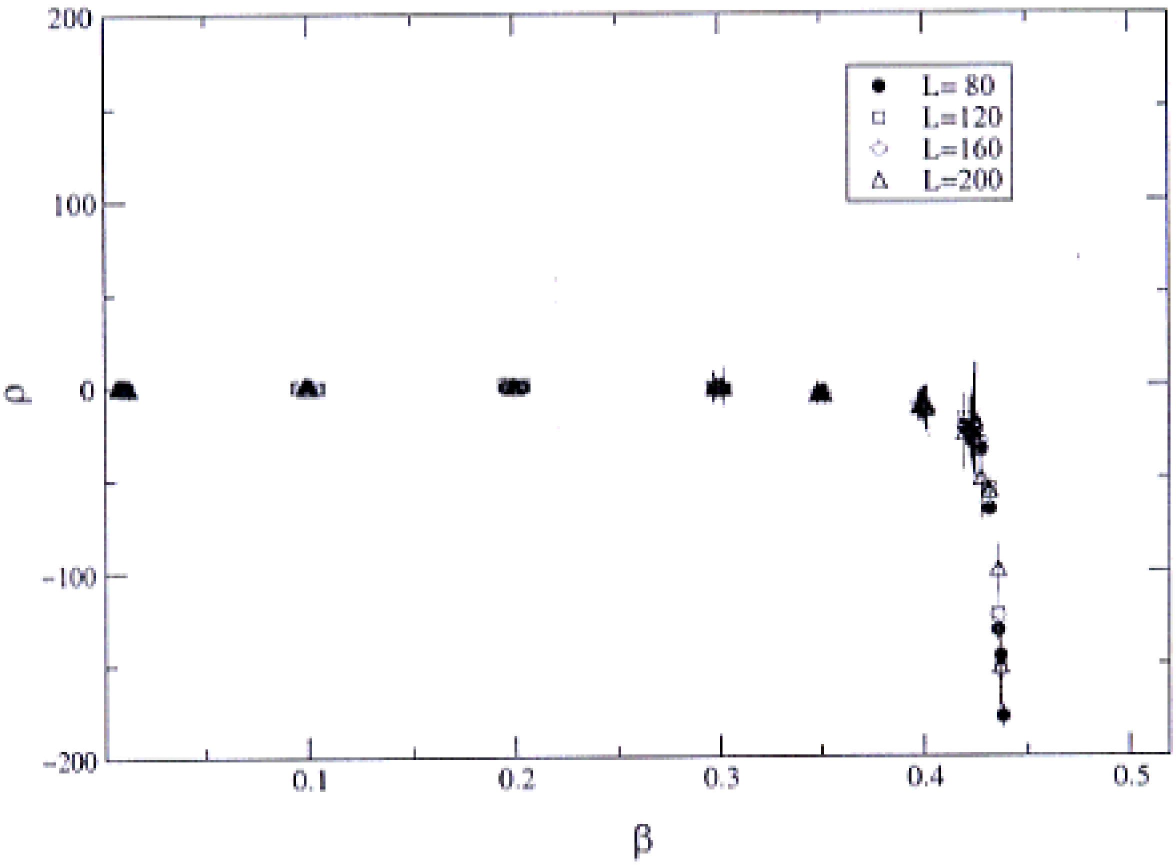


Fig. 2. Low β data for ρ.

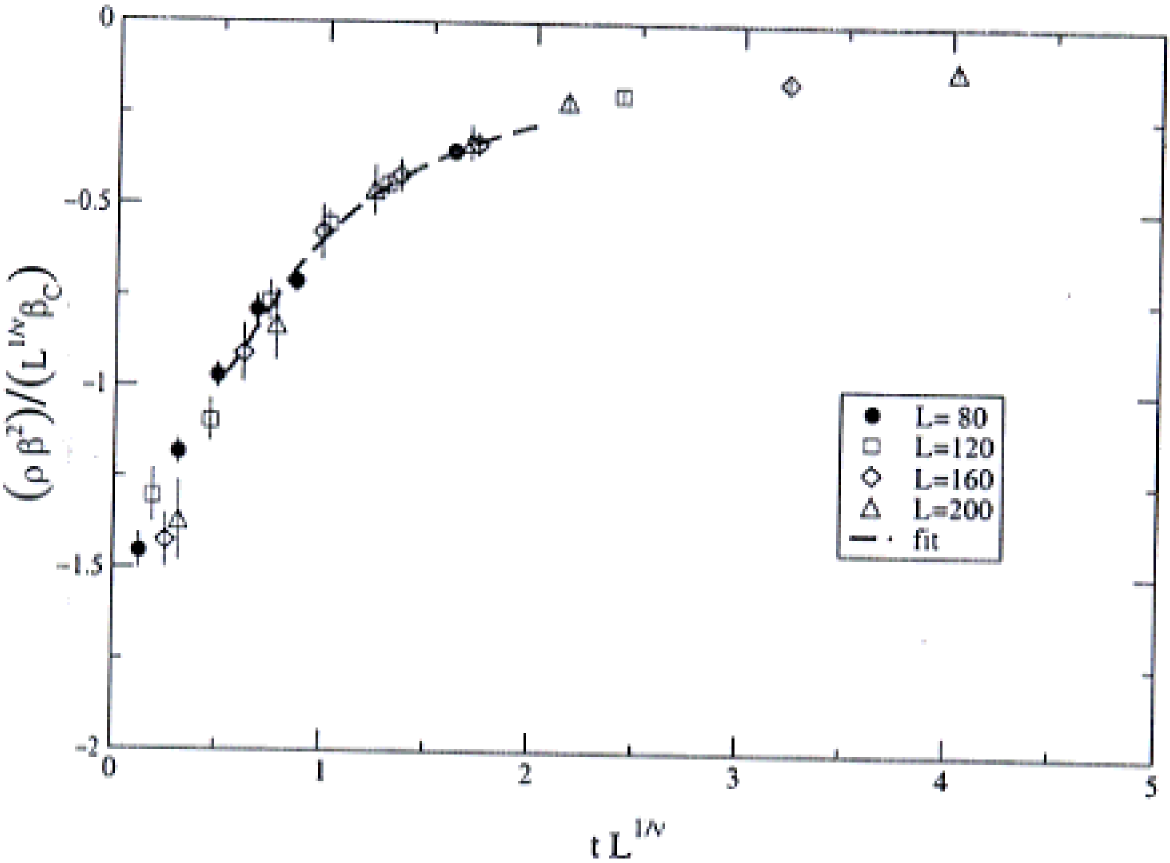


Fig. 3. Rescaled plot of ρ data.

- POSSIBLE PATTERNS OF DUAL SYMMETRY IN QCD

⇒ t'Hooft 1981

- TOPOLOGICAL EXCITATIONS ARE MAGNETIC CHARGES (MONOPOLES) ⟹ DUAL SUPERCONDUCTIVITY : CONFINEMENT VIA DUAL MEISSNER EFFECT [t'Hooft Mandelstam]

NATURAL TOPOLOGY IN 3d

$$\pi_2(SU(2)) = Z_n = \pi_1(U(1))$$

MONOPOLES

{ t HOOFT, POLYAKOV, COLEMAN } 74.

STATUS : MAGNETIC CHARGES CONDENSE, FOR SU(2) SU(3) WITH AND WITHOUT QUARKS [$N_c \to \infty$] duality.

HOWEVER : SYMMETRY PATTERN NOT FULLY UNDERSTOOD

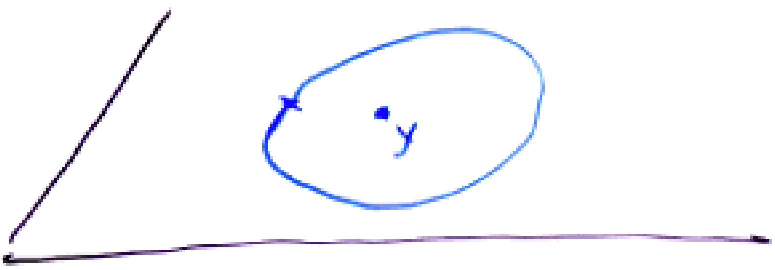
⇐ t'Hooft 1978

TOPOLOGICAL EXCITATIONS : Z_N VORTICES

2+1 d

GEORGI-GLASHOW MODEL : NO QUARKS

SO(3) SYMMETRY, HIGGS FIELD H(x) IN THE ADJOINT REPR



SU(2)

IF $H = \vec{H}\cdot\vec{\sigma}$ HAS NO ZEROS THE GAUGE TRANSFORMATION $\Omega(x)$ WHICH DYAGONALIZES IT

$$\Omega(x)\,\vec{H}(x)\vec{\sigma}\,\Omega^\dagger(x) = H(x)\,\sigma_3$$

IS NON SINGULAR AND

$$A_\mu = i\partial_\mu\Omega\,\Omega^\dagger \qquad \left[\Omega_x(x) = P\exp i\int_{x_0}^{x} A_\mu dx_\mu\right.$$

IS A PURE GAUGE, AND THE INTEGRAL PATH INDEPENDENT

IF A SINGULARITY EXISTS, THEN

$$\Omega(x+2\pi) \neq \Omega(x)$$

[COMP AB Prob]

AND THE PATH CANNOT BE SHRINKED TO ZERO.

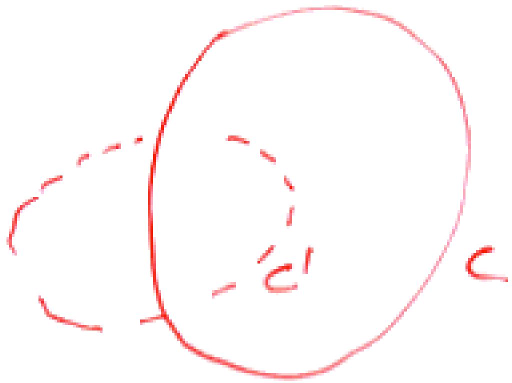
IF SINGLE VALUEDNESS OF THE FIELDS IS REQUIRED, THEN (IN THE ABSENCE OF QUARKS) THE ONLY AMBIGUITY LEFT IS THE CENTRE Z_N

$$\Omega(x+2\pi) = \Omega(x)\, Z_N$$

- THE # OF VORTICES IS A CONSERVED QUANTITY. $\langle \phi(y) \rangle$ IS A DISORDER PARAMETER.

BUT: IN (2+1)d THERE IS NO DECONFINING TRANSITION

<u>3+1 d</u> ONLY WAY TO HAVE A NON TRIVIAL CONNECTION IS BY HAVING A LINE OF VORTICES



$$B(C)\, W(C') = W(C')\, B(C)\, Z_N$$

t'HOOFT: $\langle B(C) \rangle$ area law $\longrightarrow$ $\langle W(C') \rangle$ perimeter law

$\langle B(C) \rangle$ perimeter law $\longleftarrow$ $\langle W(C') \rangle$ area law.

WHO IS THE DISORDER PARAMETER?
WHAT IS THE SYMMETRY + THE CONSERVED QUANTITY?

TOPOLOGICAL EXCITATIONS IN QCD — MONOPOLES

3d MAPPING OF S_2 ON A GROUP

$$\pi_2(SU(N)) = Z_N = \pi_1(U(1))$$ MONOPOLES [t'Hooft 81, t'Hooft 74, Polyakov 74]

AN APPEALING POSSIBILITY: CONDENSATION OF MONOPOLES GIVES DUAL SUPERCONDUCTIVITY, AND CONFINEMENT BY DUAL MEISSNER EFFECT [t'Hooft; Mandelstam 75]

$E \simeq \sigma R$

SU(2) $\quad \phi \equiv \vec{\phi}(x)\cdot\vec{\sigma}$ AN OPERATOR IN THE ADJOINT REPRESENTATION

Def $\quad \hat{\phi}(x) = \dfrac{\vec{\phi}(x)}{|\vec{\phi}(x)|}$ (EXCEPT AT $\vec{\phi}=0$)

Def $\quad F_{\mu\nu} = \hat{\phi}\cdot\vec{G}_{\mu\nu} - \frac{1}{g}(D_\mu\hat{\phi}\wedge D_\nu\hat{\phi})\cdot\hat{\phi}$

$D_\mu = \partial_\mu - g\vec{A}_\mu\wedge$ (covariant derivative)

$\vec{G}_{\mu\nu} = \partial_\mu\vec{A}_\nu - \partial_\nu\vec{A}_\mu + g\vec{A}_\mu\wedge\vec{A}_\nu$ (field strength tensor)

$$F_{\mu\nu} = \hat{\phi}(\partial_\mu\vec{A}_\nu - \partial_\nu\vec{A}_\mu) - \frac{1}{g}(\partial_\mu\hat{\phi}\wedge\partial_\nu\hat{\phi})\cdot\hat{\phi}$$

Def $\quad F^*_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\rho\sigma}F_{\rho\sigma} \qquad \partial_\mu F^*_{\mu\nu} = j^M_\nu$

$\partial_\nu j^M_\nu = 0$ [MAGNETIC U(1) SYMMETRY]

GAUGE ROTATE $U\vec{\phi} = (0,0,1)$ (ABELIAN PROJECTION)

$F_{\mu\nu} = \hat{\phi}(\partial_\mu\vec{A}_\nu - \partial_\nu A_\mu)$

- A CONSERVED MAGNETIC CHARGE IS ASSOCIATED TO ANY $\phi(x)$ IN THE ADJOINT REPRESENTATION $\Rightarrow$ (SU(3))
- ALL OF THEM ARE IDENTICALLY CONSERVED
- CONDENSATION WILL PRODUCE DUAL SUPERCONDUCTIVITY VIA HIGGS MECHANISM

~~STATUS~~ HISTORY OF THE INVESTIGATIONS ON MONOPOLES ['t Hooft]

- DESY GROUP (87) DEFINITION OF MONOPOLES ON LATTICE: [DE GRAND-TOUSSAINT] CONSTRUCTION OF ABELIAN PROJECTION ON LATTICE

  COUNTING OF MONOPOLES - MAX ABELIAN

- KANAZAWA ABELIAN DOMINANCE

- KANAZAWA + ILLINOIS MONOPOLE DOMINANCE
  ⇓
  MAX ABELIAN

- KANAZAWA - ITEP SIMULATING DUAL THEORY MAX ABELIAN

- LOUISIANA STATE LONDON CURRENT

- PISA (94) SYMMETRY: CONSTRUCTION OF A DISORDER PARAMETER

  LOOK FOR SYMMETRY, IRRESPECTIVE OF THE ABELIAN PROJECTION

⇓

|| MONOPOLE CONDENSATION IS AT WORK, IN ALL ABELIAN PROJECTIONS - DUAL VARIABLES STILL TO IDENTIFY

~~STATUS~~ HISTORY OF THE INVESTIGATIONS ON VORTICES ['t Hooft 78]

- TOMBOULIS (80) Mack, Petkova — FEYNMAN INTEGRAL IN TERMS OF Z_2 $SU(N)/Z_N$

- COPENHAGEN-VIENNA (94) — COUNTING VORTICES MAX DIAGONAL

- DIFFERENT GROUPS [00] — CREATING VORTICES

MONOPOLES IN QCD - THE DISORDER PARAMETER

[Del Debbio, ADG, G Paffuti, ADG, Lucini, Montesi, Paffuti PRD 2000)

SU(2)

$$U_\mu(\vec{n}) = e^{i\alpha\sigma_3} e^{i\beta\sigma_2} e^{i\gamma\sigma_3}$$

$$= e^{i\vec{\varepsilon}_T \vec{\sigma}} e^{i(\alpha+\gamma)\sigma_3} \qquad \vec{\varepsilon}_T \cdot \vec{\sigma} = e^{i\alpha\sigma_3} \beta\sigma_2 e^{-i\alpha\sigma_3}$$

$$U_\mu(n) = e^{iagA_\mu^T} e^{iagA_3\sigma_3}$$

(ABELIAN PROJECTION)

$$\Pi_{\mu\nu}(n) = \Pi_{\mu\nu}^T \Pi_{\mu\nu}^3 \qquad \Pi_{\mu\nu}^3 = e^{iagF_{\mu\nu}^3} \qquad O(a^2)$$

$$F_{\mu\nu}^3 = \Delta_\mu A_\nu^3 - \Delta_\nu A_\mu^3$$

$$\Pi_{\mu\nu}^{*3} = \frac{1}{2}\varepsilon_{\mu\nu\rho\sigma}\Pi_{\rho\sigma}^3 \simeq iagF_{\mu\nu}^{3*}$$

$$j_\nu^{Magn} = \Delta_\mu \Pi_{\mu\nu}^{*3} \qquad \Delta_\mu j_\nu^{Mag} = 0$$

MAGNETIC U(1) SYMMETRY

- WHO IS σ_3 DEPENDS ON THE GAUGE

Creating a monopole (in a given gauge)

$\langle\mu(\vec{y}, n_0)\rangle \qquad S_W \to \tilde{S}_W$

$$E_i \qquad \Pi_{0i}(n_0, \vec{n}) = U_i(n_0, n)\, U_0(n_0, \vec{n}+\hat{\imath})\, U_{(i)}^+(n_0+1, \vec{n})\, U_0^+(n_0, n)$$

$$\to \tilde{\Pi}_{0i}$$

$$U_i(n_0, n) \to U_i(n_0, \vec{n})\, e^{i\sigma_3 A_i(\vec{n}, \vec{y})}$$

$$E_i^3 \to (E_i^3 + A_i)$$

MEASURE $\langle\mu(\vec{y}, n_0), \mu(\vec{y}, n_0+t)\rangle \simeq C e^{-Mt} + \langle\mu\rangle^2$

$\langle\mu\rangle \neq 0$ SIGNALS BREAKING OF MAGNETIC U(1) SYMMETRY $\to$ DUAL SUPERCONDUCTIVITY

- GAUGE CAN BE FIXED BY DIAGONALIZING ANY OPERATOR Φ IN THE ADJOINT REPRESENTATION

ALTERNATIVE: DO NOT DIAGONALIZE AND WORK WITH THE CONVENTIONAL CHOICE OF σ_3 [see COSMAI]

AGAIN MEASURE

$$\rho = \frac{d}{d\beta} \ln \langle \mu \rangle$$

1) SU(2) PURE GAUGE
- DIFFERENT N_T
- DIFFERENT ABELIAN PROJECTIONS → ALL EQUIVALENT
- DIFFERENT LATTICE SIZES (FINITE SIZE SCALING)

CRITICAL INDICES AND β_c MEASURED : AGREEMENT WITH OTHER METHODS.
IInd ORDER TRANSITION : CRITICAL INDEX

$\nu = .62 \pm .01$ (3d ISING) $\delta =$

2) SU(3) PURE GAUGE
- DIFFERENT N_T
- DIFFERENT SPECIES OF MONOPOLES (λ_8 $\lambda_3 \pm \frac{1}{\sqrt{3}} \lambda_8$)
- DIFFERENT ABELIAN PROJECTIONS ALL EQUIVALENT
- NO GAUGE FIXING (SEE CEA COSMAI)
- FINITE SIZE ANALYSIS

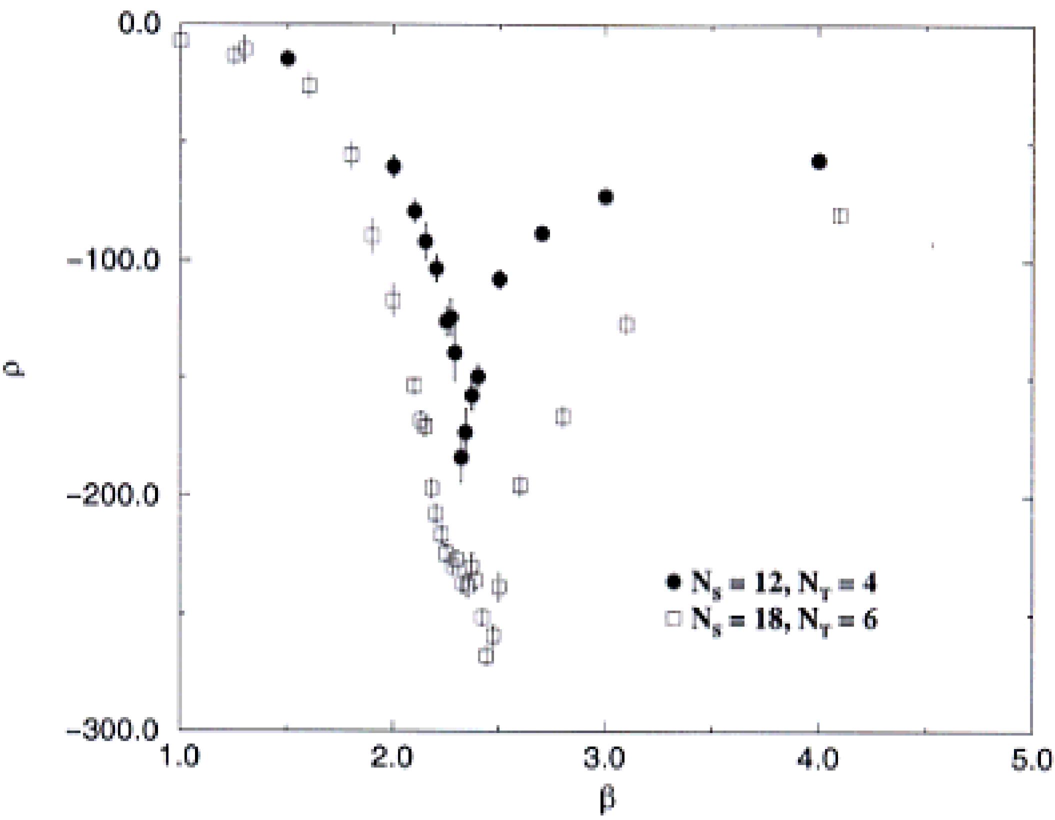
TRANSITION 1ST ORDER $\nu = 1/3$

$\delta =$

3) SU(3) DYNAMICAL QUARKS
- THE DISORDER PARAMETER CAN BE DEFINED CONSISTENTLY ($N_C \to \infty$)
- SIMILAR BEHAVIOUR – FINITE SIZE SCALING UNDER ANALYSIS
- PROBLEMS WITH HYBRID MONTECARLO WHEN GAUGE IS FIXED BY KEEPING THE ORDER OF EIGENVALUES ⇒ LUESHER ALGORITHM

IF THE GAUGE IS NOT FIXED BY ABELIAN PROJECTION NO PROBLEM)

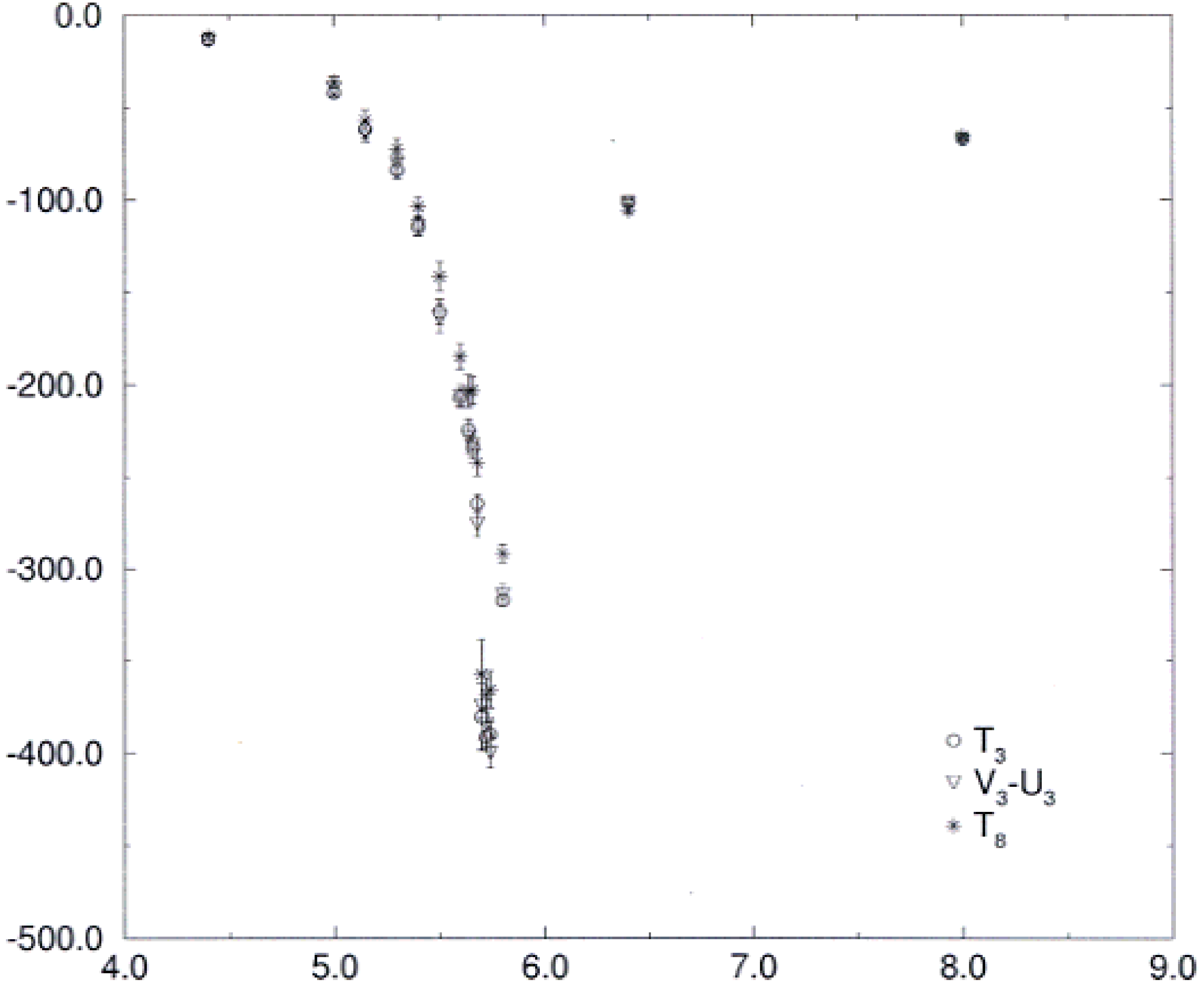
Different time extensions
SU(2) LGT, Polyakov projection

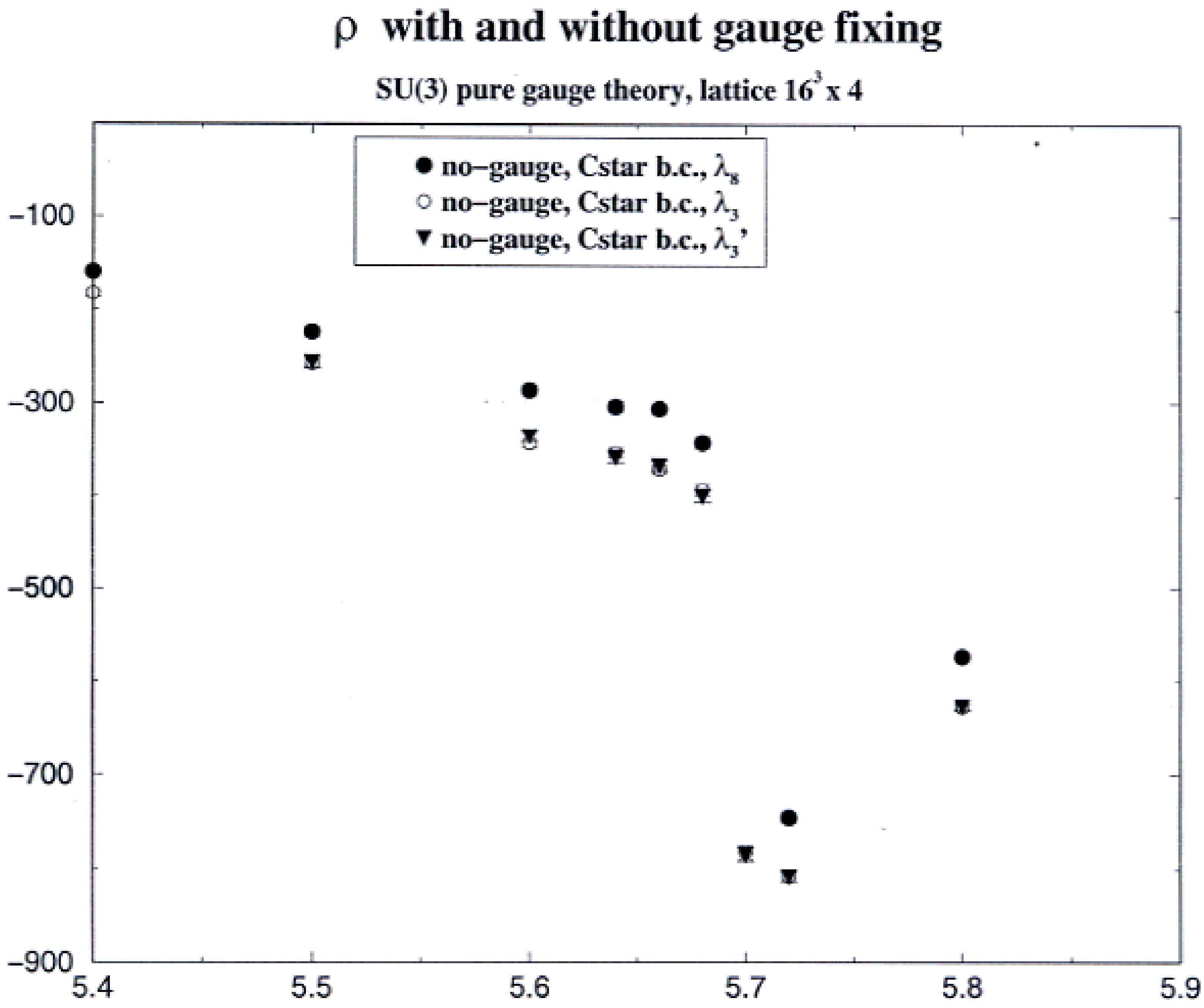


$a(\beta)\, N_T = \frac{1}{T}$

$N_T' T' = \Lambda_L \exp(\beta/b_0)$

SU(3) - Polyakov Gauge
Lattice $12^3 4$



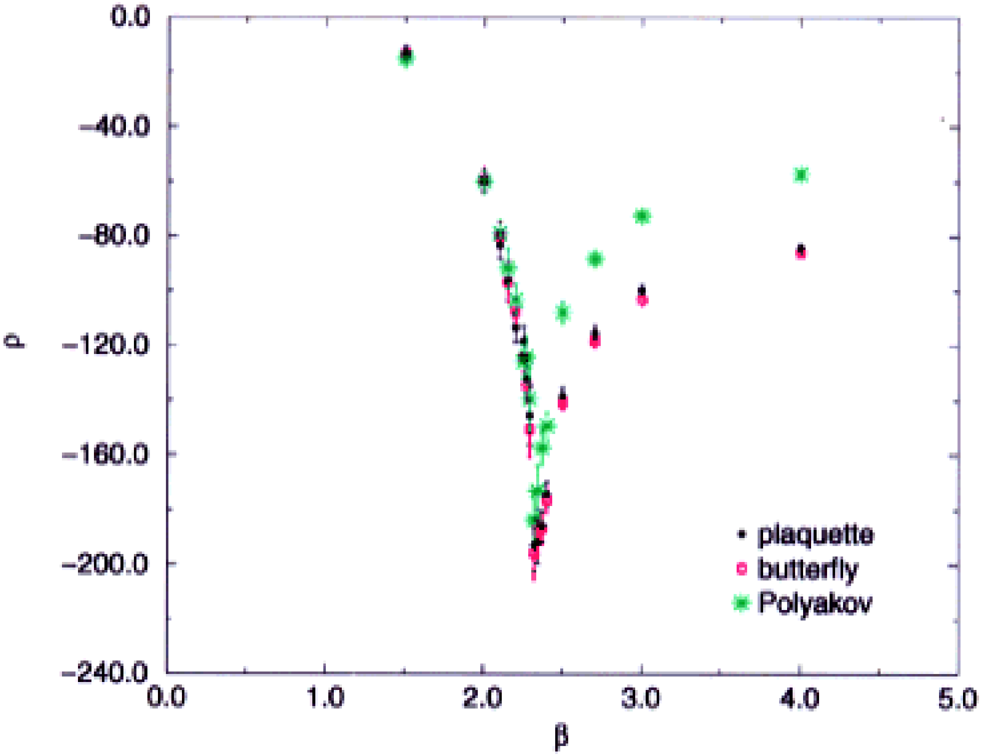


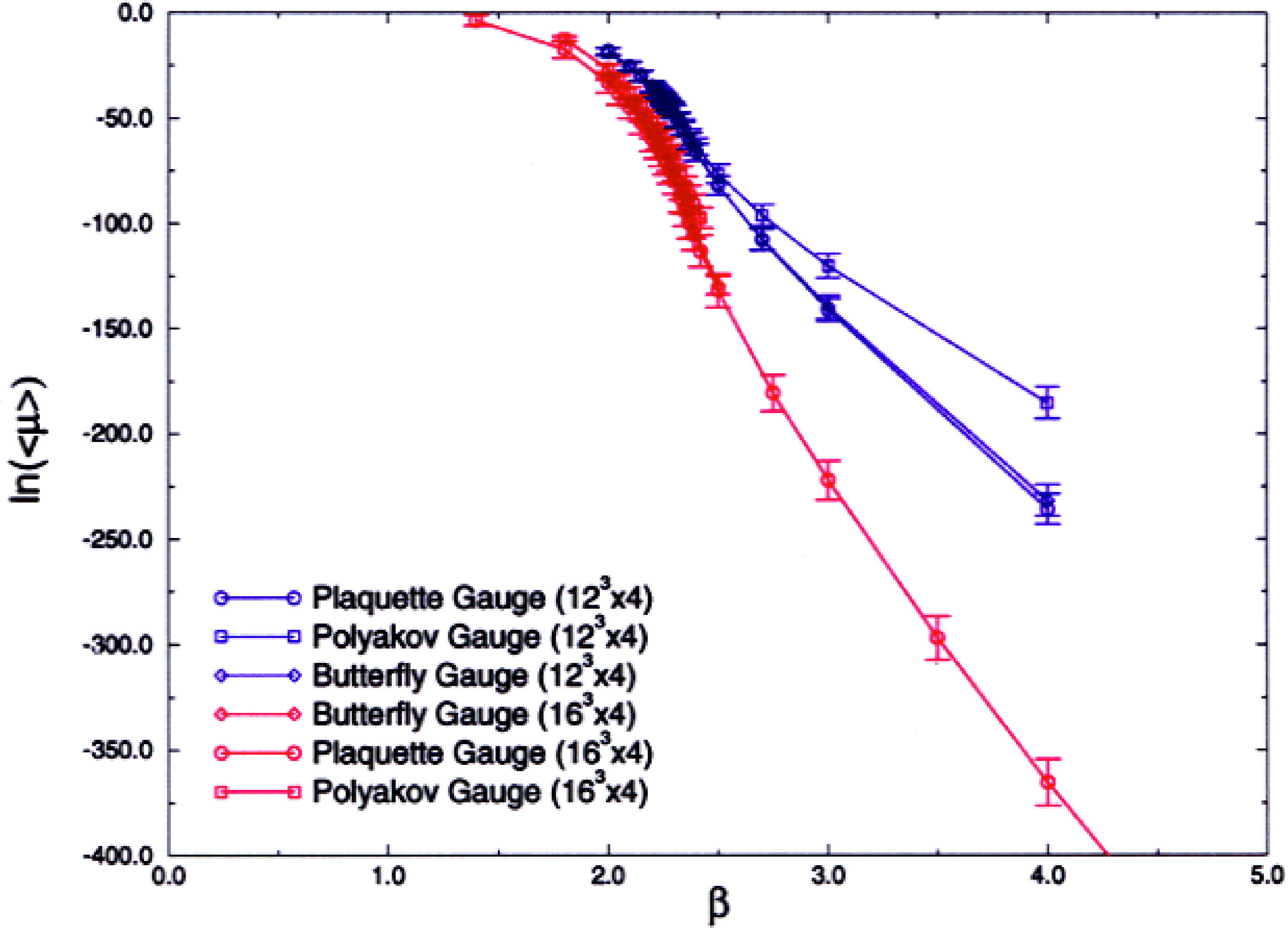
ρ with and without gauge fixing
SU(3) pure gauge theory, lattice 16^3 x 4
no–gauge, Cstar b.c., λ_8
no–gauge, Cstar b.c., λ_3
no–gauge, Cstar b.c., λ_3'
−100
−300
−500
−700
−900
5.4
5.5
5.6
5.7
5.8
5.9

21

Different abelian projections

SU(2) LGT, Lattice $12^3 \times 4$



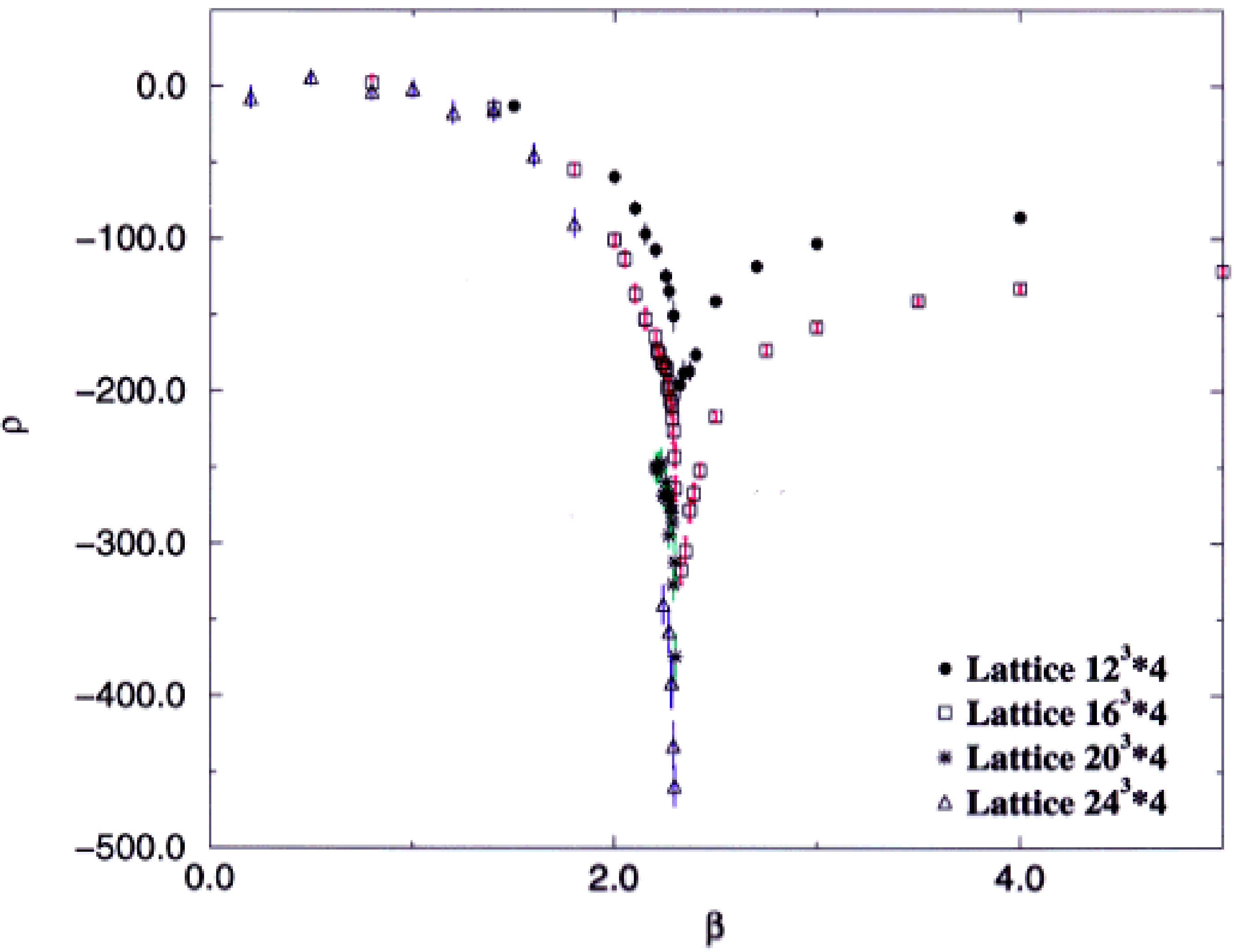

ln(<μ>)
β
0.0
-50.0
-100.0
-150.0
-200.0
-250.0
-300.0
-350.0
-400.0
1.0
2.0
3.0
4.0
5.0
Plaquette Gauge (12^3x4)
Polyakov Gauge (12^3x4)
Butterfly Gauge (12^3x4)
Butterfly Gauge (16^3x4)
Plaquette Gauge (16^3x4)
Polyakov Gauge (16^3x4)

$\beta > \beta_c$

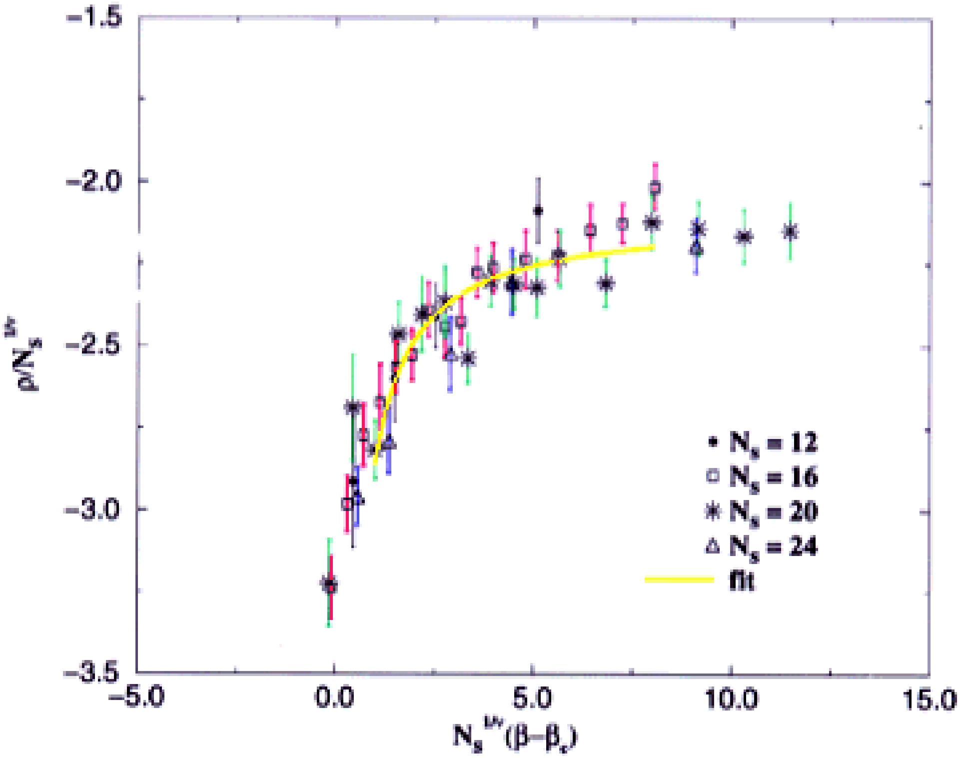
$\rho \sim \exp(-c N_s)$

SU(2) Lattice Gauge Theroy

Plaquette Gauge



Critical region
SU(2) LGT, Plaquette projection



Fit result: $\delta = 0.7(1)$

$\rho/N_S^{1/\nu} = f[N_S^{1/\nu}(\beta_c - \beta)]$

$\langle\mu\rangle = \Phi(\frac{a}{\lambda}, \frac{\lambda}{L})$

$\simeq \Phi(\frac{\lambda}{L})$ $\lambda \simeq (\beta_c - \beta)^{-\nu}$

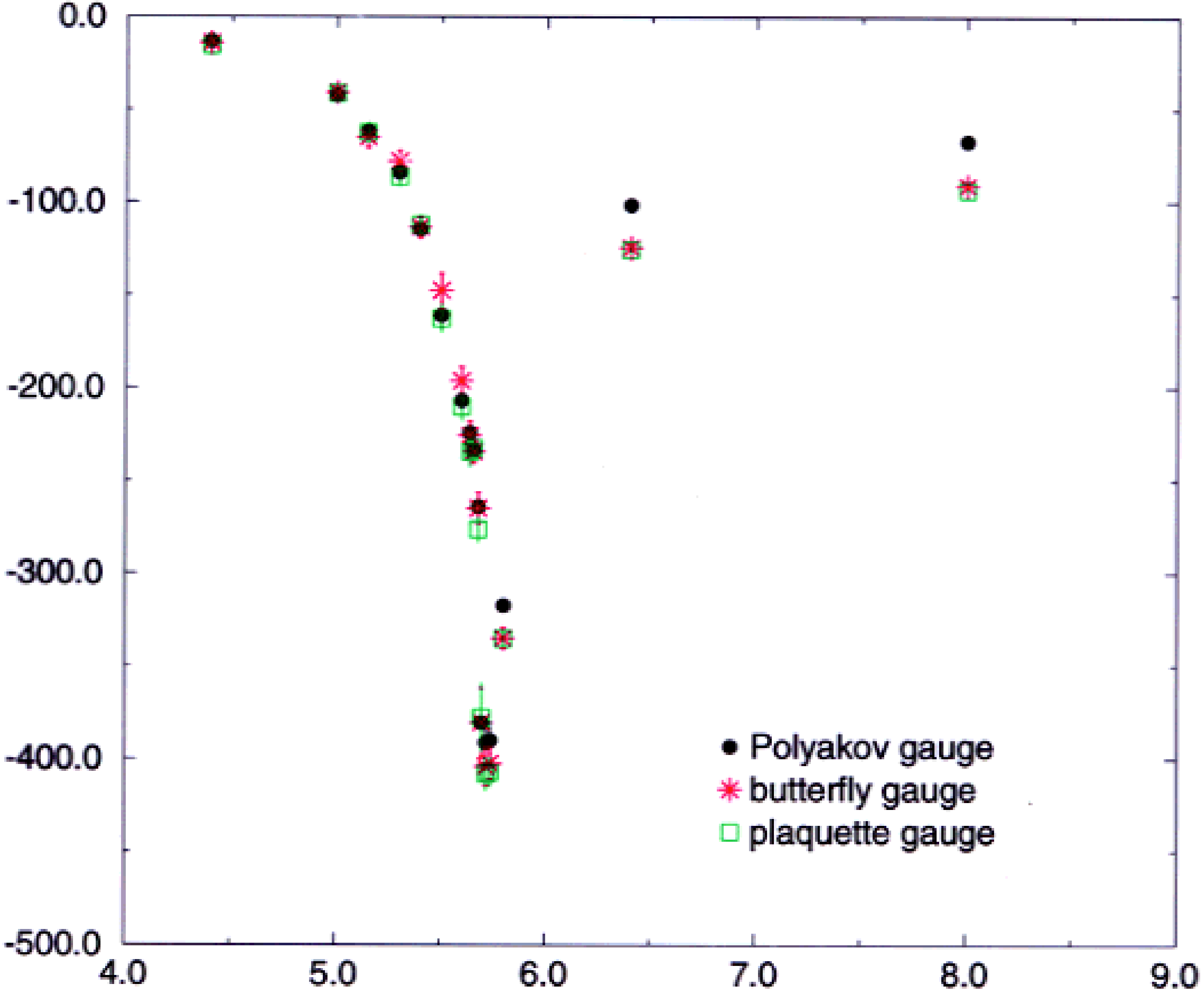
$\langle\mu\rangle = \Phi(0, L^{1/\nu}(\beta_c - \beta))$

$\nu = .62(4)$

$\mu \propto (\beta_c - \beta)^{\delta}$

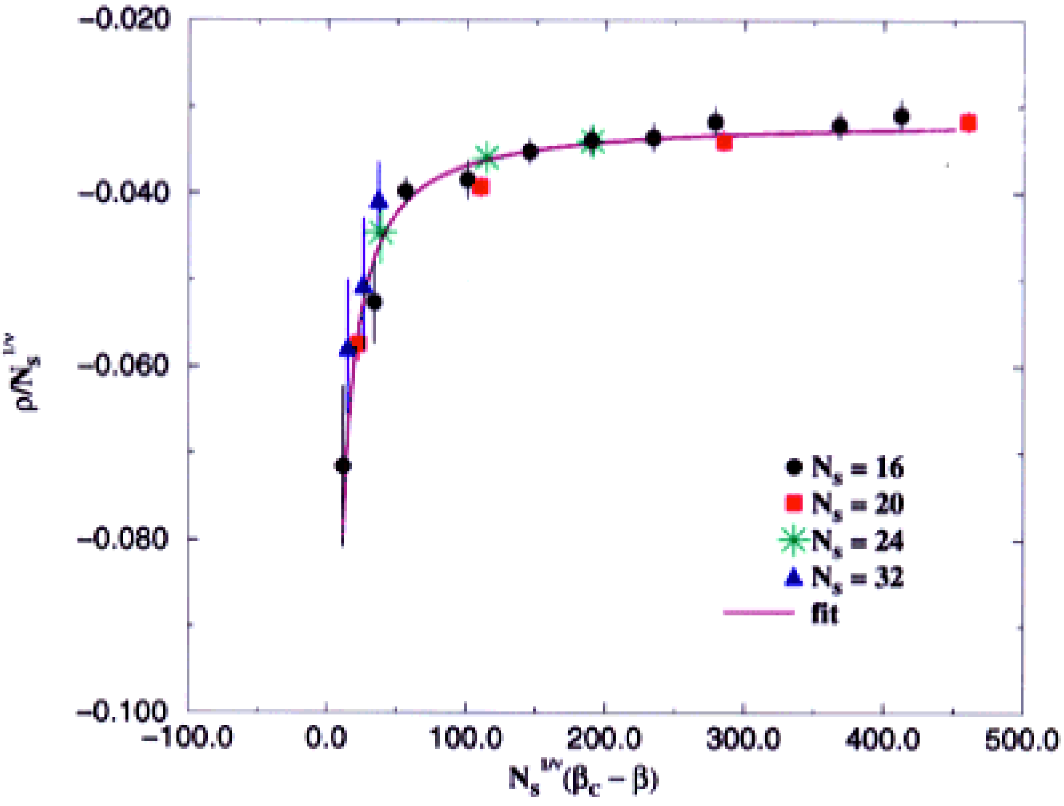
$\rho = \frac{d}{d\beta} \ln\langle\mu\rangle$

SU(3)



0.0
-100.0
-200.0
-300.0
-400.0
-500.0
4.0
5.0
6.0
7.0
8.0
9.0
Polyakov gauge
butterfly gauge
plaquette gauge

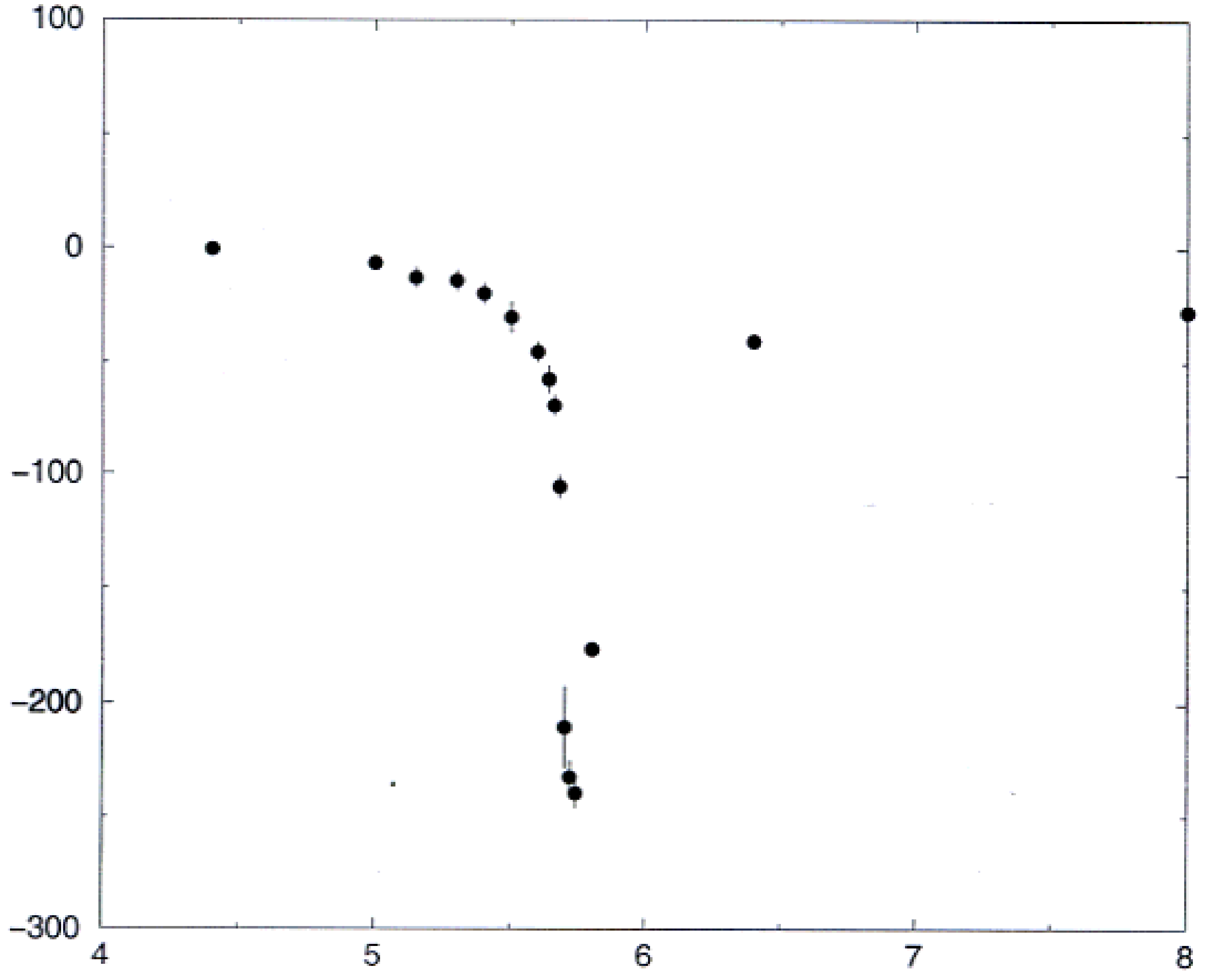
Critical region
SU(3) LGT, Polyakov projection



Fit result: $\delta = 0.54(4)$

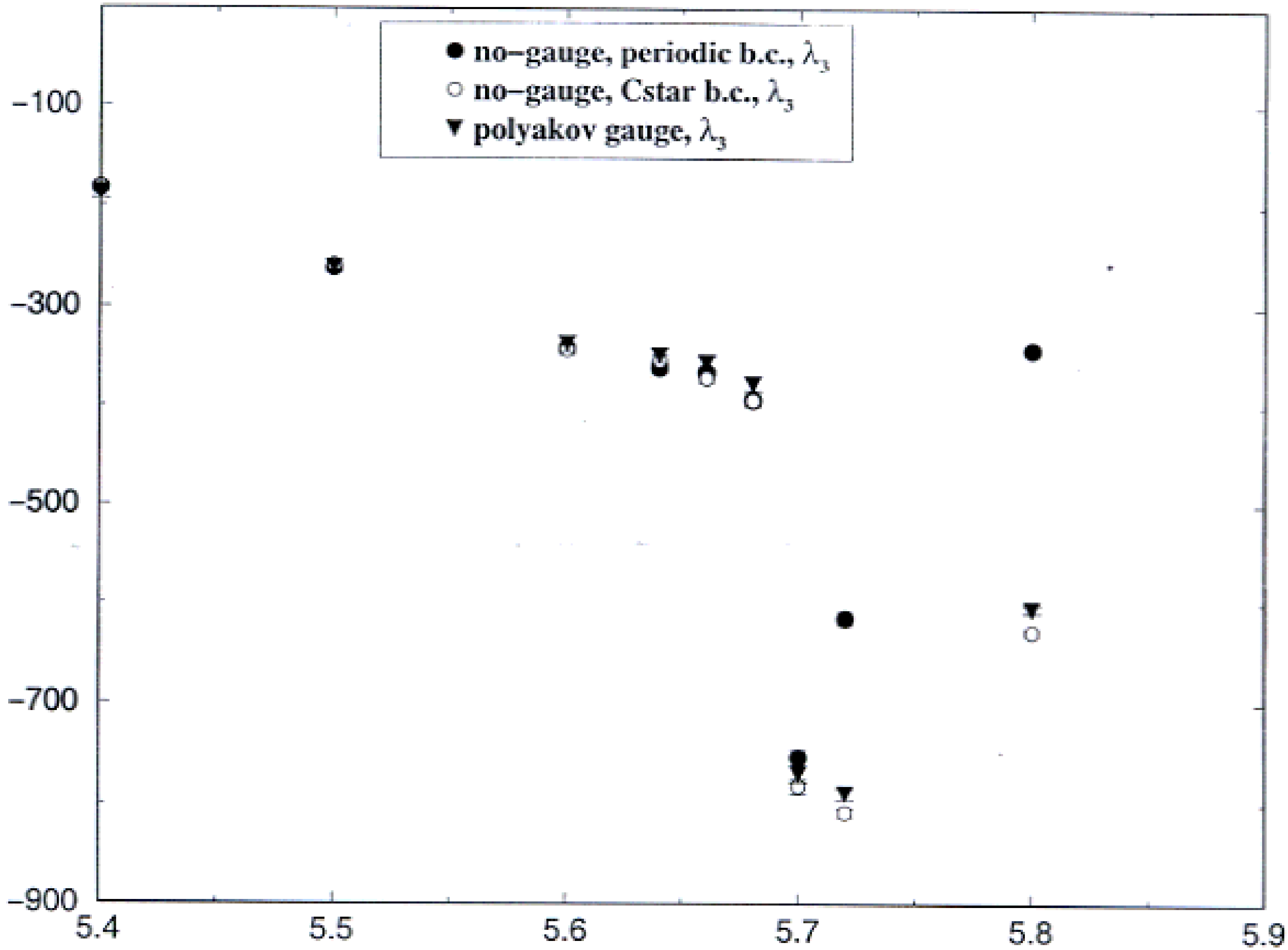
$\nu = .33$

SU(3) 2 flavors
$12^3 \times 4$ am=.02



ρ with and without gauge fixing

SU(3) pure gauge theory, lattice 16^3 x 4



INTRODUCTION VORTICES VS MONOPOLES

I 1. 3+1 d [Polyakov '77, t'Hooft '78]

VORTICES ARE STRINGLIKE TOPOLOGICAL DEFECTS

$B(C)$ CREATION OPERATOR OF A VORTEX ON THE LINE C

$W(C')$ WILSON LOOP ON THE LINE C'

$$W(C')\,B(C) = B(C)\,W(C')\exp\left(\frac{2\pi i}{N_c}\,n_{CC'}\right) \qquad (1)$$

$n_{CC'}$ LINKING # OF C C'.

IT FOLLOWS FROM EQ (1) THAT

	CONFINED	DECONFINED
$\langle W(C')\rangle$	AREA LAW	PERIMETER LAW
$\langle B(C)\rangle$	PERIMETER LAW	AREA LAW

2+1 d C IS A POINT $B(C) \to \mu(x)$

$\mu(x)$ CARRIES A CONSERVED TOPOLOGICAL CHARGE [# 2d INSTANTONS − # ANTIINSTANTONS]

– (2+1)d $\langle\mu(x)\rangle$ IS A DISORDER PARAMETER (DUALITY)

$\langle\mu(x)\rangle \neq 0$ SIGNALS BREAKING OF A SYMMETRY

(3+1)d $\langle B(C)\rangle$ SUGGESTED AS A DISORDER PARAMETER

$\langle B(C)\rangle \neq 0$ DOES NOT CORRESPOND TO ANY SYMMETRY PATTERN

I2. MONOPOLES & CONFINEMENT [t Hooft 81]

- MONOPOLES DEFINED BY ABELIAN PROJECTION
- MONOPOLES CARRY A CONSERVED TOPOLOGICAL CHARGE

$\mu(x)$ CREATION OPERATOR OF A MONOPOLE
[A. DI GIACOMO, B. LUCINI, G. PAFFUTI, L. MONTESI PRD 2005]

RESULTS:

$\langle \mu \rangle \neq 0 \quad T < T_c$

$\langle \mu \rangle = 0 \quad T > T_c$

$$\langle \mu \rangle \underset{T \to T_c}{\simeq} \left(1 - \frac{T}{T_c}\right)^{\delta}$$

- FINITE SIZE SCALING ANALYSIS $\rightarrow \delta, T_c, \nu$

$\nu \equiv$ CORRELATION LENGTH INDEX

- CONDENSATION INDEPENDENT OF THE ABELIAN PROJECTION
- $\langle \mu \rangle \neq 0$ SIGNALS DUAL SUPERCONDUCTIVITY
- $\langle \mu \rangle$ IS A GOOD ORDER PARAMETER ALSO IN FULL QCD, CONTRARY TO POLYAKOV LINE [$N_c \to \infty$ PHYLOSOPHY]

I3. WE SHALL COMPUTE $\langle B(C) \rangle$ BY THE SAME TECHNIQUE USED TO COMPUTE $\langle \mu \rangle$, FOR SU(2) AND SU(3) Y.M.

THE APPROACH GOES BACK TO KADANOFF [KADANOFF, CEVA 71]

AND HAS BEEN TESTED ON A NUMBER OF SYSTEMS

3+1 d COMPACT U(1)

3d XY MODEL

3d HEISENBERG MODEL

QCD.

• CREATION OPERATOR OF A VORTEX, $B(c)$

1. DEFINITION ON THE LATTICE

C A RECTANGLE R IN THE xz PLANE AT TIME t_0

$$R \equiv \{(x,y,z) : x_0 \le x < x_1,\ y = y_0,\ z_0 < z < z_1\}$$

SPECIAL CASE $x_1 \to \infty$ $z_0 \to -\infty$ $z_1 \to +\infty$:

A VORTEX AT x_0, y_0 EXTENDING FROM $z = -\infty$ TO $z \to +\infty$ "DUAL POLYAKOV LINE"

DEFINE $B(c, t_0)$

$$\langle B(c, t_0) \rangle = \tilde{Z} / Z$$

$$Z = \int [dU] \exp\{-\beta S[U]\} \qquad S[U] = \sum_{x,\mu\nu} \mathrm{Re}\left(\mathrm{Tr}\,[1 - P_{\mu\nu}(x)]\right)$$

WILSON'S ACTION

$$\tilde{Z} = \int [dU] \exp\{-\beta \tilde{S}[U]\}$$

$\tilde{S}[U]$ OBTAINED FROM $S[U]$ BY THE CHANGES

$$P_{oy}(t_0, x_0<x<x_1, y_0, z_0<z<z_1) \to e^{i\frac{2\pi}{N_c}} P_{oy}(t_0, x_0<x<x_1, y_0, z_0<z<z_1)$$

$SU(2)$: $e^{i\pi n} = -1$

$SU(3)$ $e^{\pm\frac{2\pi i}{3}}$

FOR THE SAKE OF SIMPLICITY DUAL POLYAKOV LINE & $SU(2)$

$\mu(x_0, y_0, t_0) \equiv B(c, t_0)$

$\tilde{S}[U]$ $P_{oy}(t_0, x_0<x, y_0, z) \to -P_{oy}(t_0, x_0<x, y_0, z)$ $\forall z$

$\Gamma(t) = \langle \mu(t_0, x_0, y_0)\, \mu(t_0+t, x_0, y_0) \rangle$ SAME CHANGE ALSO AT $t_0 + t$

2. $\mu(x_0, y_0, t_0)$ DOES CREATE A VORTEX.

$$\tilde{Z} = \int [dU] \exp\{-\beta \tilde{S}[U]\}$$

- CHANGE VARIABLE $\Downarrow$

$U_y(t_0+1, x>x_0, y_0 z) \Rightarrow -U_y(t_0+1, x>x_0, y_0, z) \quad \forall z$

$$\langle \mu \rangle = Z^{-1} \int [dU] \exp\{-\beta \tilde{S}^{(1)}[U]\}$$

$\tilde{S}^{(1)}[U]$ OBTAINED FROM $S[U]$ BY THE CHANGE

$$\begin{cases} P_{oy}(t=t_0+1, x>x_0, y_0 z) \to -P_{oy}(t=t_0+1, x>x_0, y_0, z) \quad \forall z \\ P_{xy}(t=t_0+1, x_0, y_0 z) \to -P_{xy}(t=t_0+1, x_0, y_0, z) \quad \forall z \\ P_{xy}(t=t_0+1, N_s-1, y_0, z) \to -P_{xy}(t=t_0+1, N_s-1, y_0, z) \quad \forall z \end{cases}$$

ALL OTHER PLAQUETTES ARE NOT TOUCHED OR CHANGE SIGN TWICE

A VORTEX HAS APPEARED AT t_0+1 $x_0 y_0$ (FIG. 1)
IN COMPUTING $\mu W(C')$, WITH C' A CLOSED LINE IN THE XY PLANE A CHANGE OF SIGN OCCURS WITH RESPECT TO $W(C')\mu$ IF THE # OF U_y LINKS IN C' WITH $x>x_0$ IS ODD.

t'HOOFT-POLYAKOV ALGEBRA IS OBEYED

THE VORTEX AT THE BORDER (N_s-1, y_0) IS DUE TO P.B.C. IF $N_s \gg$ CORRELATION LENGTH THE TWO VORTICES ARE INDEPENDENT

- CHANGE VARIABLE AGAIN

$$U_y(t_0+2, x>x_0, y_0 z) \to -U_y(t_0+2, x>x_0, y_0 z) \quad \forall z$$

$$\langle \mu \rangle = Z^{-1} \int [dU] \exp\{-\beta \tilde{S}^{(2)}[U]\}$$

$\tilde{S}^{(2)}[U]$ IS OBTAINED FROM $S[U]$ BY THE CHANGES

$$\begin{aligned} &P_{oy}(t=t_0+2, x>x_0, y_0, z) \to -P_{oy}(t=t_0+2, x>x_0, y_0, z) \quad \forall z \\ &P_{xy}(t=t_0+1, x=x_0, y_0, z) \to -P_{xy}(t=t_0+1, x_0, y_0, z) \quad \forall z \\ &P_{xy}(t=t_0+2, x_0, y_0, z) \to -P_{xy}(t=t_0+2, x_0, y_0, z) \quad \forall z \\ &P_{xy}(t=t_0+1, N_s-1, y_0 z) \to -P_{xy}(t=t_0+1, N_s-1, y_0, z) \quad \forall z \\ &P_{xy}(t=t_0+2, N_s-1, y_0, z) \to -P_{xy}(t=t_0+2, N_s-1, y_0, z) \quad \forall z \end{aligned}$$

|| THE VORTICES ARE NOW ALSO AT TIME t_0+2

|| THE PROCEDURE CAN BE REPEATED AT $t=t_0+3$ AND SO ON

. . . .

THE EFFECT OF $\mu(t_0, x_0, y_0)$ IS THAT A VORTEX HAS APPEARED AT ALL TIMES $t > t_0$

IF AN ANTIVORTEX IS CREATED AT t_0+t

$$\Gamma(t) = \langle \bar{\mu}(t_0+t, x_0, y_0)\, \mu(t_0, x_0, y_0) \rangle$$

THE PROCEDURE STOPS, AND THE NET EFFECT IS A VORTEX PROPAGATING FROM t_0 TO t_0+T.

$$\Gamma(t) \sim A\, e^{-mt} + \langle \mu \rangle^2$$

WHENCE $\langle \mu \rangle$ CAN BE COMPUTED

- AT FINITE TEMPERATURE THERE IS NO PROPAGATION IN TIME. ONE SINGLE VORTEX MUST BE CREATED $\Rightarrow$ C^* BOUNDARY CONDITIONS IN TIME

• CONDENSATION OF "DUAL POLYAKOV LINES" AND CONFINEMENT

1. MEASURE $\langle \mu \rangle$ ON THE LATTICE

$$\rho = \frac{d}{d\beta} \ln \langle \mu \rangle = \langle S \rangle_S - \langle \tilde{S} \rangle_{\tilde{S}}$$

EASIER TO DETERMINE & CONTAINS ALL RELEVANT INFORMATION

$$\langle \mu \rangle = \exp\left[\int_0^\beta \rho(\beta')\, d\beta'\right]$$

FIG. 2 { – ρ vs β $\quad N_T \times N_S^3$ $\quad N_T = 4$, $N_S = 12, 16, 20, 24, 32$ $\quad SU(2)$

– $\rho_{MONOPOLE}$ PLOTTED FOR COMPARISON

– LARGE β $\quad$ EXPECTED: $\rho \simeq -2 \times N_T \times L \times 2 \simeq -16L\,+\ldots$

* $\frac{\text{ACTION}}{\text{PLAQUETTE}} \simeq \frac{2}{\beta}$

MEASURED $\simeq -4L + c$ (FIG 3)

$\langle \mu \rangle \to 0$ $\quad T > T_c$ IN THE THERMODYNAMICAL LIMIT

– LOW β's $\quad \rho$ BOUNDED FROM BELOW $\Rightarrow \langle \mu \rangle \neq 0$ (FIG 4)

β's $\sim \beta_c$ $\quad \langle \mu \rangle \simeq (\beta_c - \beta)^\delta$

$$\langle \mu \rangle = (\beta_c - \beta)^\delta\, \Phi\left(\frac{N_S}{\xi}\right) \qquad \xi \simeq (\beta_c - \beta)^{-\nu}$$

$$\langle \mu \rangle = L^{-\delta/\nu}\, \tilde{\Phi}\left(L^{1/\nu}(\beta_c - \beta)\right) \Rightarrow \rho / L^{1/\nu} = f\left(L^{1/\nu}(\beta_c - \beta)\right)$$

SCALING (FIG 5)

$$X \equiv L^{1/\nu}(\beta_c - \beta)$$

$$\frac{\rho}{L^{1/\nu}} = -\frac{\delta}{X} + c$$

FIT TO THE DATA TO EXTRACT β_c, ν, δ

$\beta_c = 2.30(1)$, $\delta \simeq .5(1)$ $\nu = .7(1)$

IN AGREEMENT WITH OTHER DETERMINATIONS

δ equal within errors to δ of monopoles

- PRELIMINARY DATA FOR SU(3) SHOW SIMILAR BEHAVIOUR fig 6, 7

"DUAL POLYAKOV LINE IS A GOOD DISORDER PARAMETER FOR CONFINEMENT"

2. AT T=0 WE HAVE STUDIED THE CORRELATOR

$$\Gamma(t) = \langle \bar{\mu}(t_0+T, x_0, y_0)\, \mu(t_0, x_0, y_0) \rangle \simeq \begin{cases} a e^{-mt} + \langle\mu\rangle^2 \\ \frac{a}{t} e^{-mt} \end{cases}$$

DATA SHOW A SLIGHT PREFERENCE FOR SIMPLE EXPONENTIAL , AND THE CORRELATION LENGTH IS ~ 1 LATTICE SPACING

THIS LEGITIMATES THE TREATMENT OF THE VORTEX AT $x_0 y_0$ AND OF THE VORTEX AT $N_s - 1$, y_0 INDUCED BY PERIODIC B.C AS INDEPENDENT , I4 ($x_0 = \frac{N_s}{2}$ $N_s = 12, 16, 20, 24, 32$)

• VORTICES VS MONOPOLES: DISCUSSION & OUTLOOK

(i) DUAL SUPERCONDUCTIVITY IS AT WORK IN CONFINEMENT

μ A MAGNETICALLY CHARGED OPERATOR

$$\begin{cases} \langle\mu\rangle \neq 0 & T < T_c \\ \langle\mu\rangle = 0 & T > T_c \\ \langle\mu\rangle \underset{T \to T_c}{\sim} \left(1 - \frac{T}{T_c}\right)^{\delta} & \end{cases}$$

- INDEPENDENT OF THE ABELIAN PROJECTION
- ALSO IN THE PRESENCE OF QUARKS
- CONTRARY TO <Polyakov line> AND $\langle\bar\psi\psi\rangle$, $\langle\mu\rangle$ EXISTS BOTH FOR PURE GT AND FOR QCD. $N_c \to \infty$ IDEAS SUPPORTED

(ii) VORTICES ARE ALSO RELATED TO CONFINEMENT: AT LEAST IN PURE G.T.
"DUAL POLYAKOV LINE" IS A VORTEX, AND ACTS AS A DISORDER PARAMETER. DUAL Z_N IS A SYMMETRY RELATED TO CONFINEMENT, AT LEAST IN PURE GAUGE

(iii) THE SYMMETRY OF THE CONFINED PHASE OF QCD SHOULD BE DESCRIBED BY A SET OF DUAL FIELDS, CORRESPONDING TO TOPOLOGICAL STRUCTURES. WHAT WE KNOW IS THAT THEY MUST BE MAGNETICALLY CHARGED IN ANY ABELIAN PROJECTION, AND, IN THE SPIRIT OF $N_c \to \infty$ IDEAS SHOULD BE DEFINED BOTH IN THE PRESENCE AND IN THE ABSENCE OF QUARKS.

OUTLOOK

(1) WE ARE DETERMINING THE BEHAVIOUR OF THE DUAL POLYAKOV LINE AT T_c FOR SU(3).

(2) WE ARE DETERMINING THE CRITICAL INDICES OF THE DECONFINING TRANSITION IN FULL QCD BY THE MAGNETIC MONOPOLE DISORDER PARAMETER

(3) WE ARE TRYING TO UNDERSTAND IF AND HOW VORTICES CAN BE ADAPTED TO FULL QCD

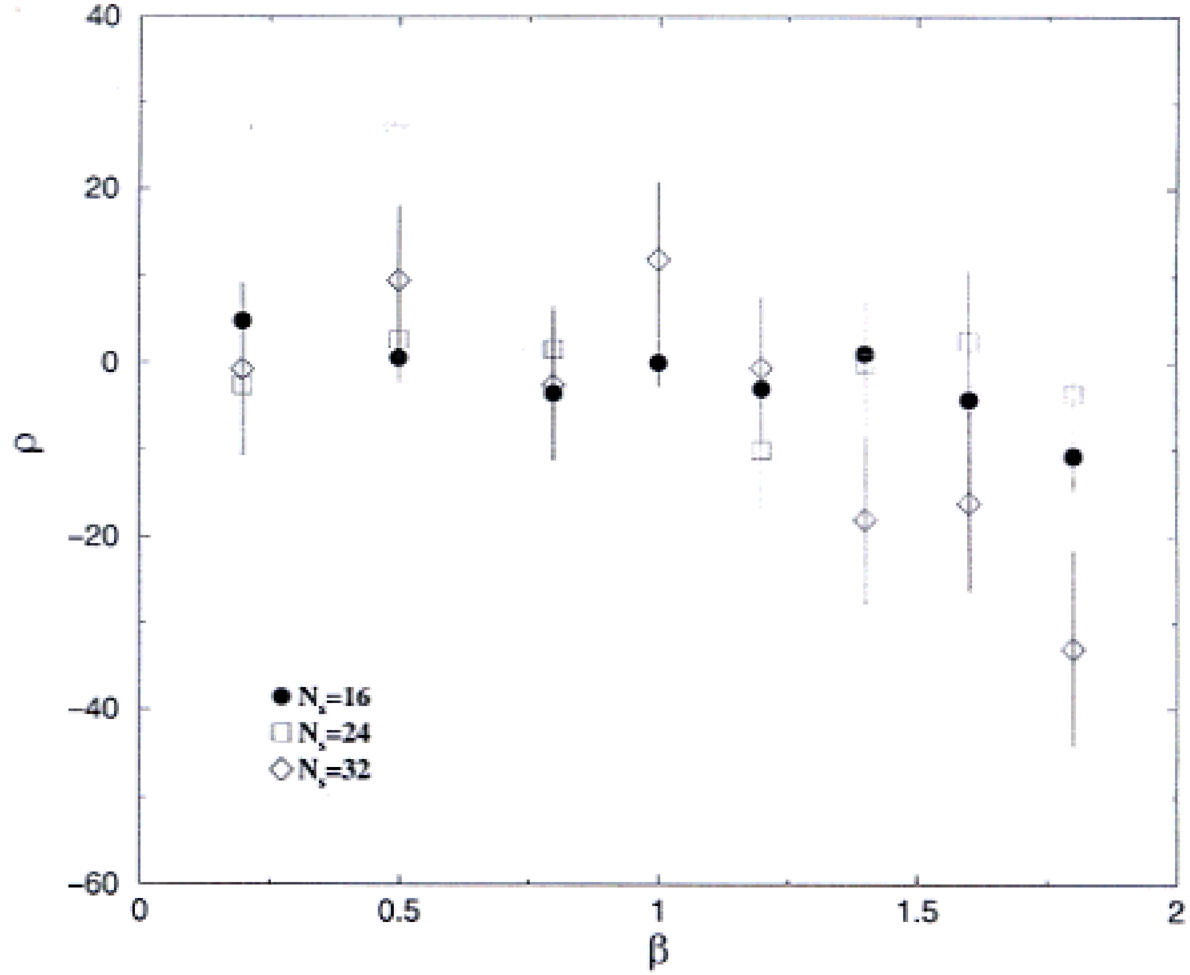
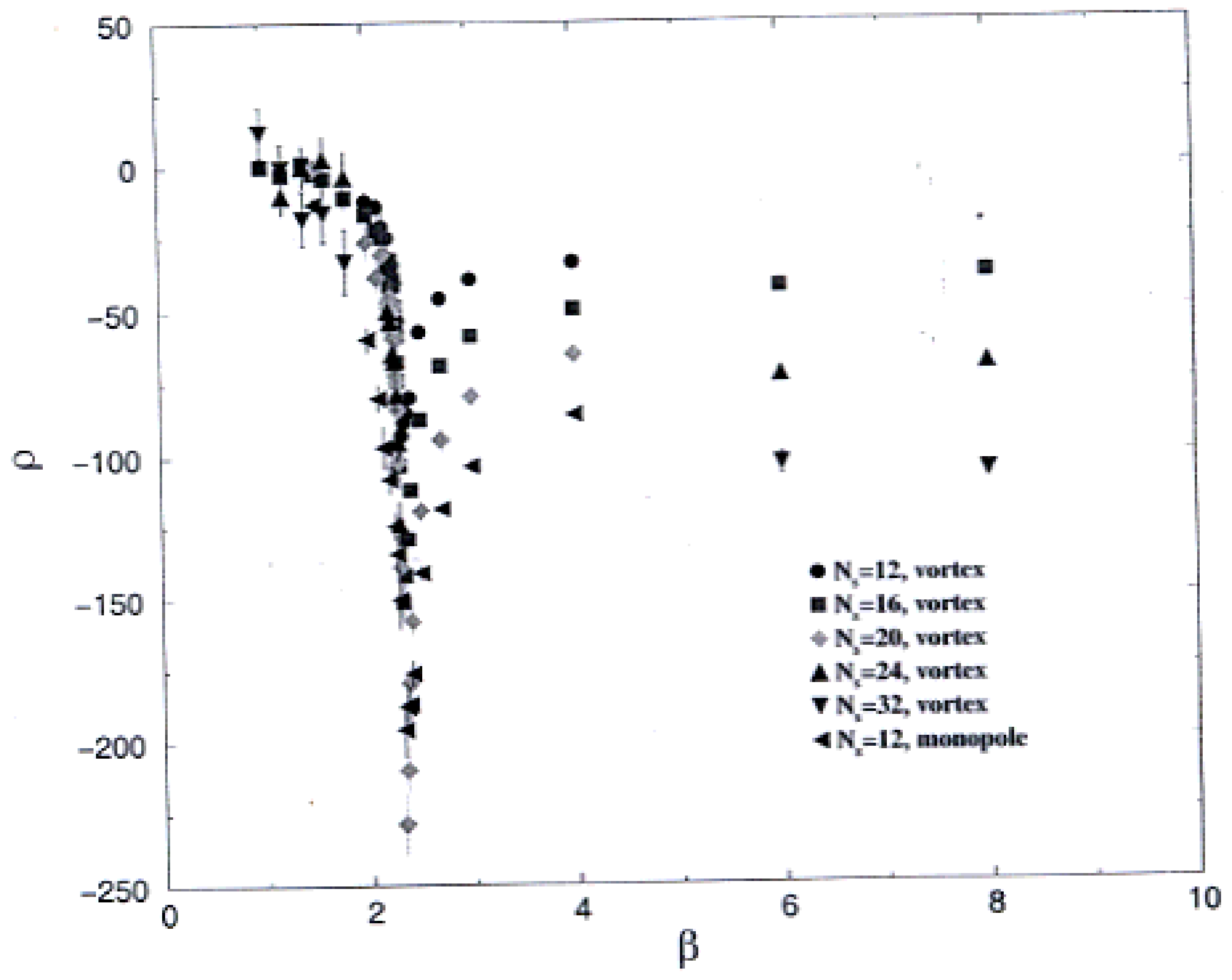


FIG 4



N_s=12, vortex
N_s=16, vortex
N_s=20, vortex
N_s=24, vortex
N_s=32, vortex
N_s=12, monopole
ρ
β

Fig 2

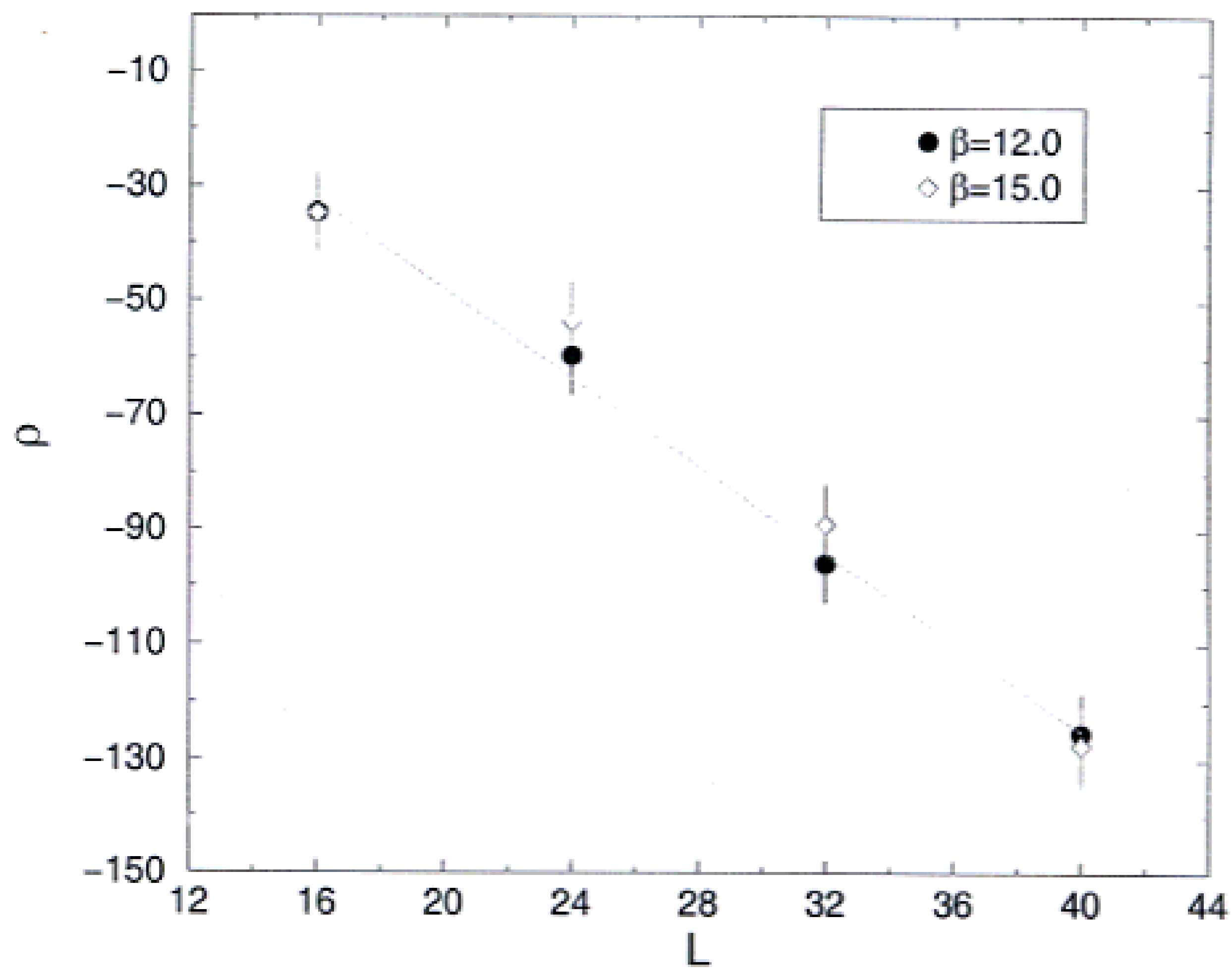


FIG. 3

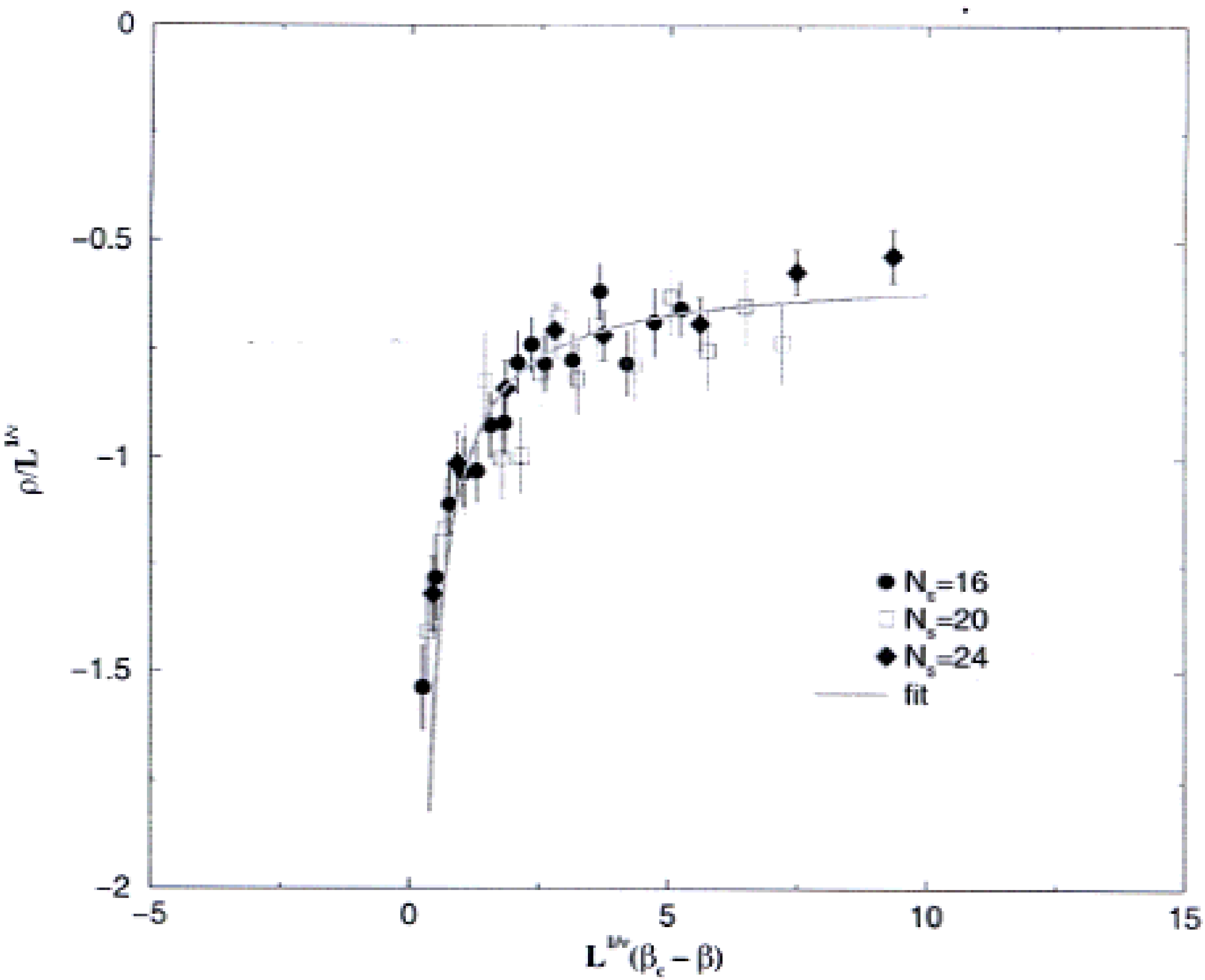
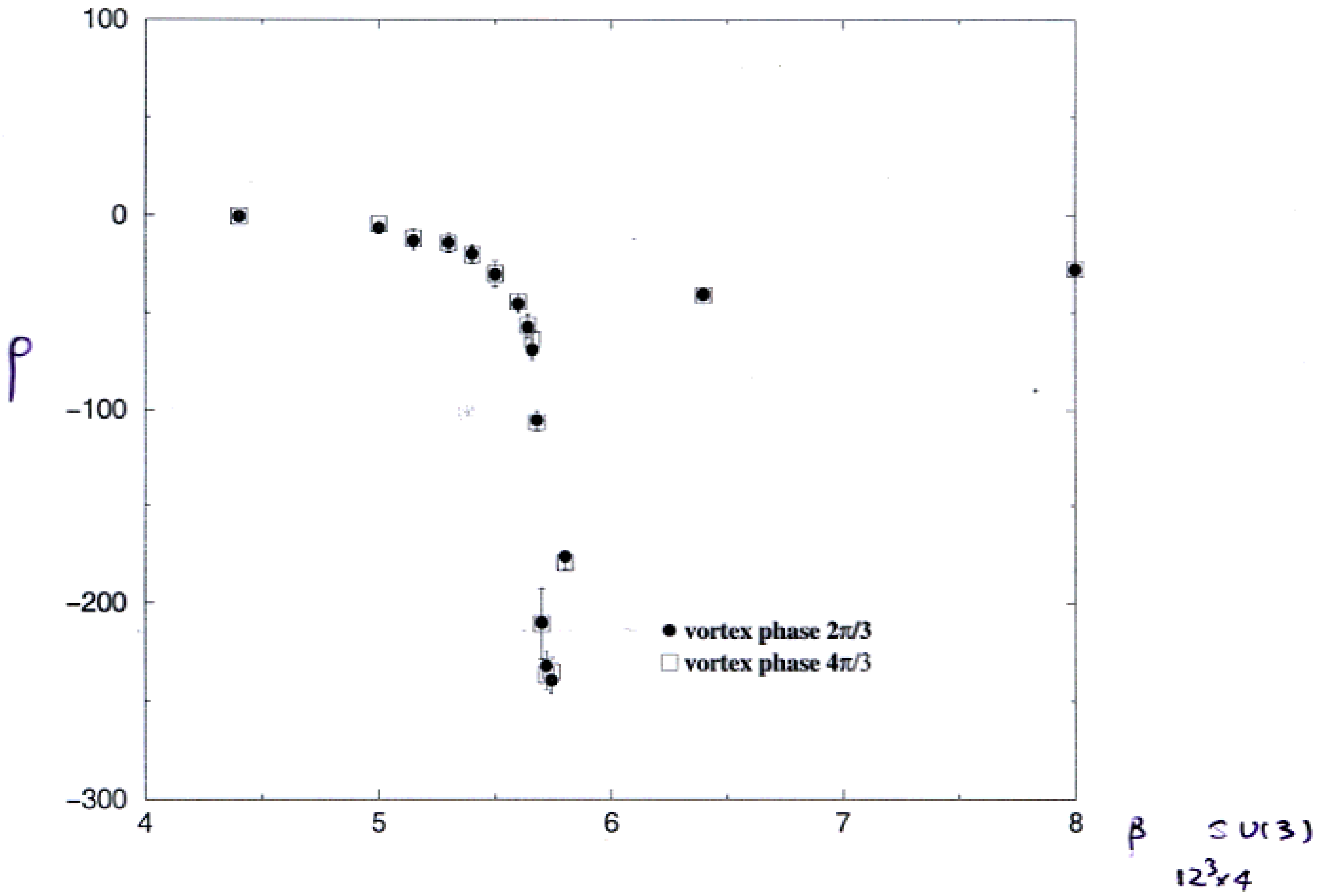


fig. 5



ρ
100
0
−100
−200
−300
4
5
6
7
8
β
SU(3)
12³×4
vortex phase 2π/3
vortex phase 4π/3

FIG 6