Hamiltonian study of the lattice two-dimensional Wess-Zumino model

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Plan of the talk

- Hamiltonian lattice field theory
- Green function Monte Carlo
- The model
- Numerical results

The usual discretized approach to Quantum Field Theory is the (Euclidean) Lagrangian approach [Wilson, 1974]. We discretize both space and time, obtaining a $d+1$-dimensional classical statistical system.

Alternatively, we can adopt the (Minkowskian) Hamiltonian approach [Kogut, Susskind, 1975], discretizing space but keeping time continuous, obtaining to a d-dimensionals quantum system.

For the purpose of numerical simulations, the Lagrangian approach is by far the most popular, and it has notable advantages over the Hamiltonian approach:

Lagrangian	Hamiltonian
very effective Monte Carlo techniques are available;	Monte Carlo techniques are more complicated and less effective;
a discrete subgroup of the (Euclidean) Lorentz group is preserved;	Lorentz invariance is completely broken;
many fermionic systems do not suffer from the sign problem.	most fermionic systems suffer from the sign problem.

On the other hand, the Hamiltonian approach has several advantages:

Lagrangian	Hamiltonian
fermions must be integrated out, leading to a non-local action;	fermions are implemented directly;
properties of the vacuum must be obtained indirectly;	the vacuum (ground-state) wavefunction is directly accessible;
supersymmetry must be broken completely;	a 1-d supersymmetry algebra can be preserved;
it is difficult to exploit knowledge of the vacuum	approximations to the vacuum wavefunction can be used to improve the simulations

Checks of universality between the Lagrangian and the Hamiltonian formalism are also very welcome.

Many powerful many-body techniques developed for condensed-matter physics can be applied to Hamiltonian lattice models.

Among many Monte Carlo techniques available to compute the vacuum wavefunction, I will focus on the Green Function Monte Carlo (GFMC) algorithm [Kalos, 1962; Trivedi, Ceperley, 1989. Reviews: von der Linden, 1992; Ceperley, Kalos, 1992].

The basic ingredient of the GFMC algorithm is the projection of a generic state $|i\rangle$ over the ground state

$$|\Psi_0\rangle = \lim_{t\to\infty} \exp[-tH]|i\rangle, \qquad \langle\Psi_0|i\rangle \neq 0 \tag{1}$$

(apart from a t-dependent normalization).

We implement Eq. (1) stochastically: set

$$|\psi_0\rangle = |i\rangle, \qquad |\psi_n\rangle = G|\psi_{n-1}\rangle, \qquad G = \exp[-\beta H],$$

so that $|\Psi_0\rangle = \lim_{n\to\infty} |\psi_n\rangle$; choose a basis $\{|m\rangle\}$; decompose G into a stochastic matrix P and a weight W:

$$G(l,m) = P(l,m)\, W(m),$$
$$0 \leq P(l,m) \leq 1, \quad \textstyle\sum_l P(l,m) = 1;$$

represent ψ_n as an index m_n plus a weight w_n:

$$|\psi_n\rangle = \overline{w_n |m_n\rangle}; \tag{2}$$

$$\mathrm{Prob}(m_{n+1}{=}m) = P(m, m_n), \quad w_{n+1} = W(m_n)\, w_n. \tag{3}$$

The vacuum expectation value of a generic observable can be computed as

$$\begin{aligned}\langle\Psi_0|O|\Psi_0\rangle &= \lim_{t,\tau\to\infty}\frac{\langle f|\exp[-\tau H]\,O\,\exp[-tH]|i\rangle}{\langle f|\exp[-(\tau+t)H]|i\rangle} \\ &= \lim_{n,l\to\infty}\frac{\overline{w_{n+l}\langle f|O|m_n\rangle}}{\overline{w_{n+l}\langle f|m_n\rangle}} \end{aligned} \tag{4}$$

(forward walking), where $|f\rangle$ is a generic state non-orthogonal to $|\Psi_0\rangle$; in practice, the choice of $|f\rangle$ is very important.

The ground state energy can be obtained simply as

$$\begin{aligned}E_0\equiv\langle\Psi_0|H|\Psi_0\rangle &= \lim_{t\to\infty}\frac{\langle f|H\exp[-tH]|i\rangle}{\langle f|\exp[-tH]|i\rangle} \\ &= \lim_{n\to\infty}\frac{\overline{w_n\langle f|H|m_n\rangle}}{\overline{w_n\langle f|m_n\rangle}}.\end{aligned}$$

$\exp[-tH]$ is typically implemented using the formula

$$\exp[-\beta(A+B)] = \exp[-\tfrac{1}{2}\beta A]\exp[-\beta B]\exp[-\tfrac{1}{2}\beta A]+O(\beta^3),$$

therefore an extrapolation to $\beta\to 0$ is required.

The procedure given above does not actually work: at each step, ever more small w_n are generated, and the variance of any observable diverges.

This problem is solved by branching [Trivedi, Ceperley, 1989]:

- use an ensemble of K walkers
  $\mathrm{ens}_n = m_n(1), w_n(1); \ldots; m_n(K), w_n(K)$;
- evolve each configuration independently, according to Eq. (3);
- periodically, eliminate walkers with a small weight and duplicate walkers with a large weight, respecting Eq. (2) and keeping the variance finite.

A typical reconfiguration: take $\mathrm{int}(cw(i) + \xi)$ copies of $m(i)$, and set $w(i)$ to 1.

In practice, Eq. (4) is respected only in the $K \to \infty$ limit.

The variance can be reduced dramatically by using a guiding wavefunction Ψ_G approximating Ψ_0: write

$$\langle m|\psi_n\rangle = \langle m|\Psi_G\rangle\langle m|\widetilde{\psi}_n\rangle,$$

and represent $\widetilde{\psi}_n$ as an index m_n plus a weight w_n:

$$|\widetilde{\psi}_n\rangle = \overline{w_n|m_n\rangle},$$

$$\mathrm{Prob}(m_{n+1}{=}m) = P(m,m_n), \quad w_{n+1} = W(m_n)\,w_n,$$

$$\widetilde{G}(l,m) = \frac{\langle l|\Psi_G\rangle\,G(l,m)}{\langle m|\Psi_G\rangle} = P(l,m)\,W(m),$$

$$0 \le P(l,m) \le 1, \quad \textstyle\sum_l P(l,m) = 1;$$

notice that, if $\Psi_G = \Psi_0$, the sampling is perfect.

In practice, Ψ_G will depend on a set of optimization parameters $\{\alpha\}$; traditionally, they are determined by a variational computation; we prefer to optimize the αs adaptively during the Monte Carlo simulation [Beccaria, 2000], by minimizing the variance of E_0 estimated over the walker ensemble:

$$\alpha_i(n+1) = \alpha_i(n) - \eta(n)\frac{\partial}{\partial\alpha_i}\mathrm{Var}_{\mathrm{ens}_n}(E_0)$$

and $\eta(n)$ is suitably chosen.

The optimal values of $\{\alpha\}$ can give direct information on the vacuum wavefunction.

Let us remind the (continuum) $N = 1$ supersymmetry algebra

$$\{Q_\alpha, Q_\beta\} = 2(\not{P}C)_{\alpha\beta}; \qquad (5)$$

since P_i are not conserved on the lattice, a lattice formulation of a supersymmetric model must break (5) explicitly.

A very important advantage of the Hamiltonian formulation is the possibility of conserving exactly a 1-d subalgebra of (5) [Elitzur, Rabinovici, Schwimmer, 1982]; specializing to $1+1$ dimensions, in a Majorana basis $\gamma_0 = C = \sigma_2$, $\gamma_1 = \mathrm{i}\sigma_3$, Eq. (5) becomes

$$Q_1^2 = Q_2^2 = P^0 \equiv H, \qquad \{Q_1, Q_2\} = 2P^1 \equiv 2P, \qquad (6)$$

On the lattice, since H is conserved but P is not, we can hold, e.g., $Q_1^2 = H$, and expect to recover the rest of (6) in the continuum limit.

Observe that $Q_1^2 = H$ is enough to guarantee that $E_0 \equiv \langle\Psi_0|H|\Psi_0\rangle \geq 0$, that all eigenstates of H with $E > 0$ are paired in doublets ($|b\rangle = Q_1/\sqrt{E}|f\rangle$, $|f\rangle = Q_1/\sqrt{E}|b\rangle$), and that $E_0 = 0$ if and only if supersymmetry is unbroken, i.e., the ground state is annihilated by Q_1.

The continuum two-dimensional Wess-Zumino model is defined by

$$Q_{1,2} = \int \mathrm{d}x \left[\pi(x)\psi^{1,2}(x) - \frac{\partial\phi}{\partial x}\,\psi^{2,1}(x) \pm V(\phi(x))\,\psi^{2,1}(x)\right],$$

where $V(\phi)$ is an arbitrary polynomial, $\phi(x)$ is a real scalar field, $\pi(x)$ is its conjugate momentum, $\psi(x)$ is a Majorana fermion, $H = Q_1^2 = Q_2^2$ as given by Eq. (6), and canonical (anti)commutation rules hold.

We will adopt the lattice formulation [Ranft, Schiller, 1984]

$$Q \equiv Q_1 = \sum_{n=1}^{L} \left[\pi_n\psi_n^1 - \left(\frac{\phi_{n+1} - \phi_{n-1}}{2} + V(\phi_n)\right)\psi_n^2\right],$$

with canonical (anti)commutation rules.

The choice of a symmetric difference leads to doubling of both boson and fermions, but allows the clever transformation

$$\psi_n^{1,2} = \frac{(-1)^n \mp \mathrm{i}}{2\mathrm{i}^n}(\chi_n^\dagger \pm \mathrm{i}\chi_n),$$
$$\{\chi_n, \chi_m^\dagger\} = \delta_{nm}, \qquad \{\chi_n, \chi_m\} = \{\chi_n^\dagger, \chi_m^\dagger\} = 0,$$

leading to

$$H = Q^2 = \frac{1}{2}\sum_{n=1}^{L}\Bigg[\pi_n^2 + \left(\frac{\phi_{n+1}-\phi_{n-1}}{2} + V(\phi_n)\right)^2 - (\chi_n^\dagger\chi_{n+1} + h.c.) + (-1)^n V'(\phi_n)\left(2\chi_n^\dagger\chi_n - 1\right)\Bigg]; \tag{7}$$

choosing for the fermions the occupation number basis

$$|n_1, n_2, \ldots\rangle = (\chi_1^\dagger)^{n_1}(\chi_2^\dagger)^{n_2}\ldots|0\rangle,$$

the Hamiltonian (7) is free of sign problems (all non-diagonal matrix elements are non-positive), apart from boundary problems: with periodic boundary conditions, there is a sign problem when the (conserved) fermion number is even, which is the sector where the supersymmetric ground state is expected to lie. Therefore we choose free boundary conditions.

In the free ($V = 0$) case, the Hamiltonian is quadratic and splits into a bosonic part and a fermionic part; the ground state $|\Psi_0\rangle = |\Psi_0^B\rangle \otimes |\Psi_0^F\rangle$ is computed exactly; when $L \equiv 2 \pmod 4$ it is unique and supersymmetric ($Q|\Psi_0\rangle = 0$), and it contains $L/2$ fermions; we will always study this sector in the following.

We will adopt the guiding wave function

$$\langle\phi, n|\Psi_G(\alpha)\rangle = \exp[S_B(\phi) + S_F(\phi, n)]\, \langle\phi|\Psi_0^B\rangle\, \langle n|\Psi_0^F\rangle,$$

$$S_B = \sum_m \sum_{k=1}^{d_B} \alpha_k^B \phi_m^k,$$

$$S_F = \sum_m (-1)^m \left(n_m - \tfrac{1}{2}\right) \sum_{k=1}^{d_F} \alpha_k^F \phi_m^k,$$

with $d_B = d_F = 4$. We expect Ψ_G to work better and better as we approach the continuum limit, since it is related to the free ground state.

For odd V, the model enjoys the symmetry $\phi_m \to -\phi_m$, and odd αs can be set to zero.

For even V, the model enjoys the approximate symmetry $\phi_m \to -\phi_m$, $n_m \to 1 - n_m$ (it is broken by irrelevant terms and by boundary terms), and we verified that odd α^B and even α^F can be set to zero.

Supersymmetry implies many non-trivial Ward identities: if $Q|\Psi_0\rangle = 0$, for each observable X we have

$$\langle\Psi_0|\{Q,X\}|\Psi_0\rangle = 0.$$

In order to study supersymmetry breaking, we will look not only at E_0, but also at $\langle Y_q\rangle$, with

$$Y_q \equiv \Big\{Q, \sum_n \phi_n^q\, \psi_n^2\Big\}.$$

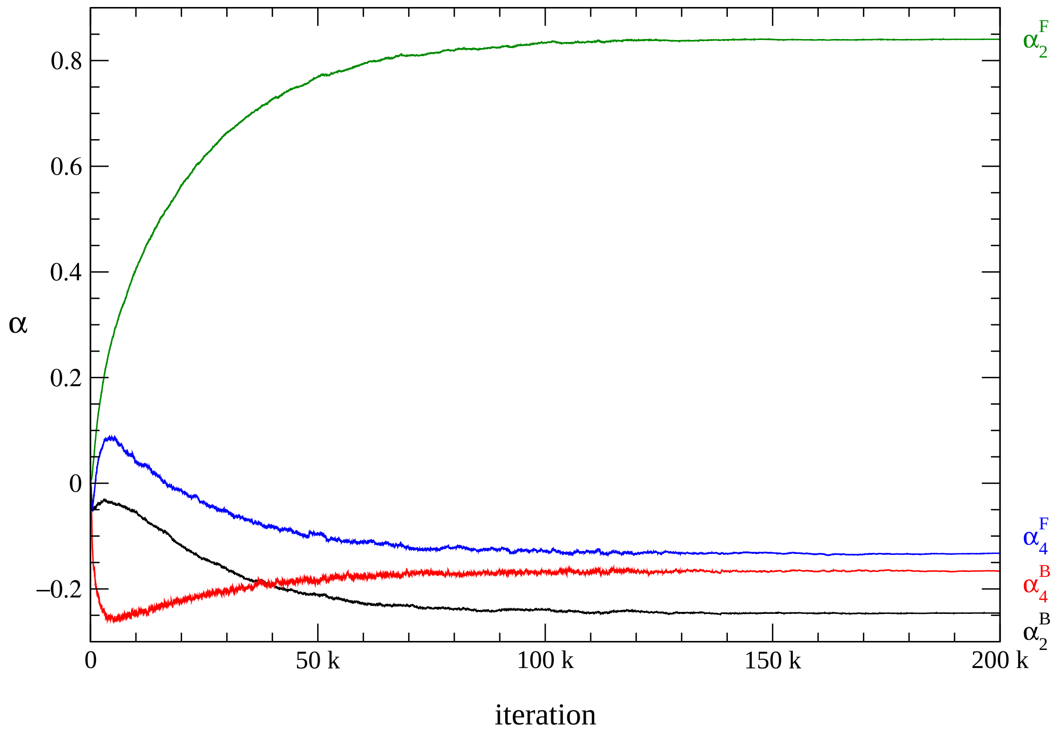
$\langle Y_q\rangle$ is computed easily, since for

$$X = \sum_n F(\phi_n)\, \psi_n^2$$

we have

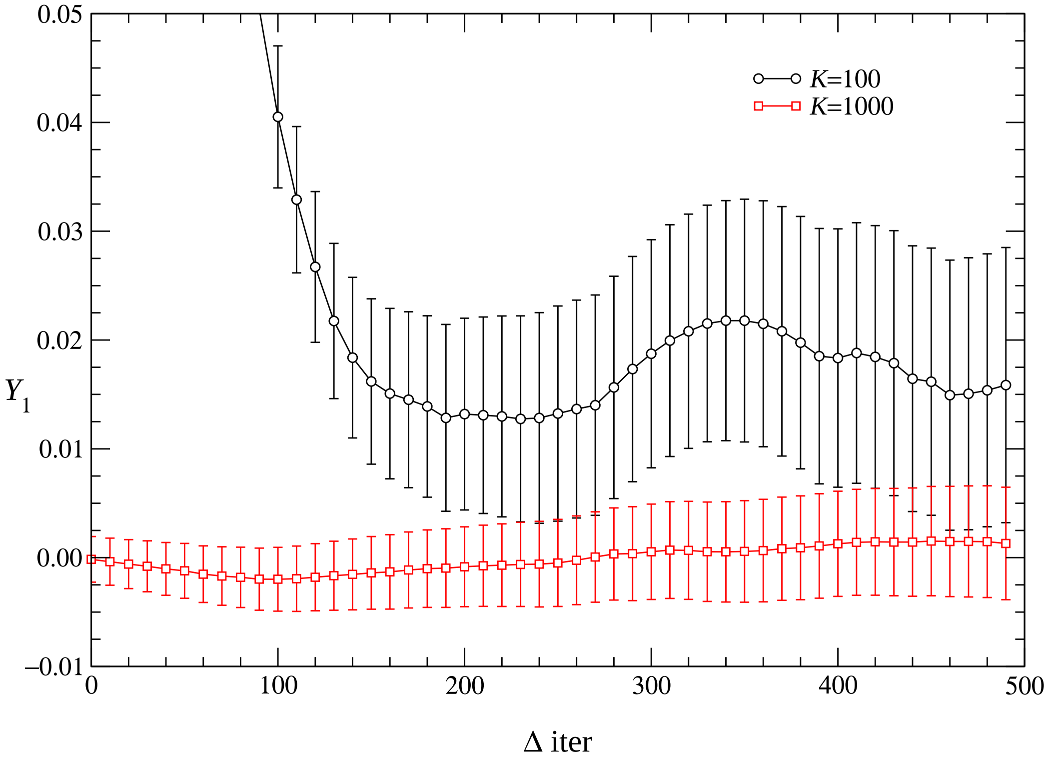
$$\{Q,X\} = \sum_n \Big\{F(\phi_n)\Big[\frac{\phi_{n+1}-\phi_{n-1}}{2} + V(\phi_n)\Big] + F'(\phi_n)(-1)^n\Big(\chi_n^\dagger\chi_n - \frac{1}{2}\Big)\Big\}.$$

A typical example of the behaviour of the optimization parameters α is the following plot, taken from a run of 10^6 iterations at $V = \phi^3$, $K = 100$, $L = 10$, $\beta = 0.01$; odd αs are set to 0. (This run required 3 hours on a PC).



The optimal values of the αs are quite insensitive to K; therefore they can be determined in a run at moderate values of K (i.e., 100 or 200) and given as initial approximations (or even kept fixed) for runs at higher values of K.

We plot two typical examples of the behaviour of an observable, obtained from the above run and from a run of $5{\times}10^5$ iterations at $K = 1000$: it is easy to measure and to extrapolate to large $\Delta\,\mathrm{iter}$ the quantity $Y_1 = \{Q, \sum_n \phi_n\, \psi_n^2\}$.



Let us now turn to the physical properties of the two-dimensional Wess-Zumino model.

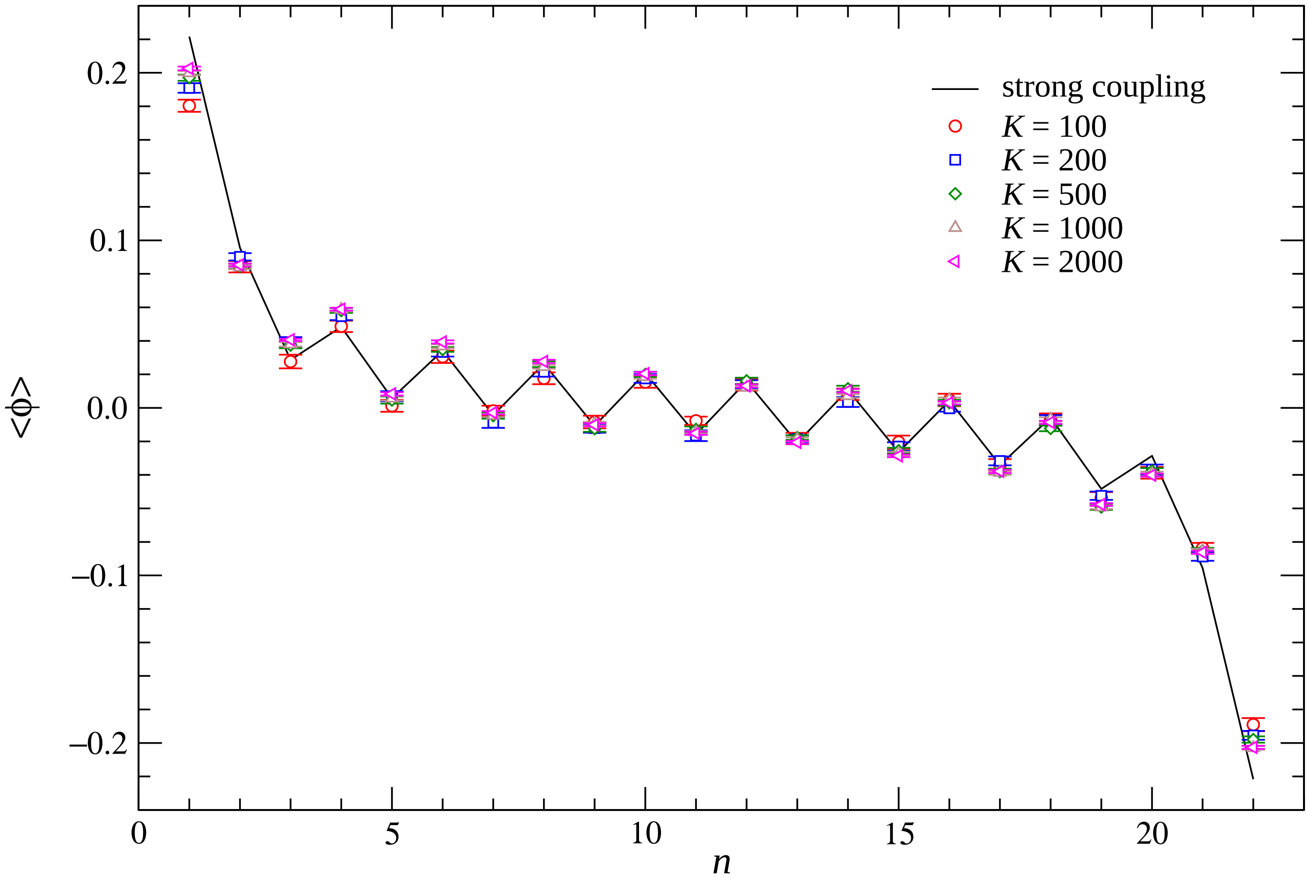
- In strong coupling, the model reduces to a supersymmetric quantum mechanics for each site; supersymmetry is broken if and only if the degree of V is even.
- In the continuum (and on the lattice in weak coupling), supersymmetry is broken at tree level if and only if V has no zeroes.

The predictions of strong coupling and weak coupling can be quite different, and it is interesting to study both numerically and analytically the crossover from strong to weak coupling. We will focus on $V(\phi) = \lambda_2\phi^2 + \lambda_0$ with $\lambda_2 > 0$; for $\lambda_0 \leq 0$, supersymmetry is unbroken at tree level.

Let us remind that, for models with only scalars and fermions in 2 dimensions, radiative corrections can change the pattern of supersymmetry breaking, unlike in 4 dimensions [Alvarez-Gaumé, Freedman, Grisaru, 1981].

We performed a strong-coupling computation of $\langle\phi_n\rangle$, E_0, and other quantities; the results are fully compatible with the Monte Carlo simulations for large V; e.g., for $V(\phi) = \lambda\phi^2$, the agreement is very good when $\lambda \gtrsim 2$.

$\langle\phi_n\rangle$ vs. n for $V = 2\phi^2$



The two-dimensional Wess-Zumino model is superrenormalizable; a 1-loop perturbative computation shows that ϕ and ψ do not renormalize, and only $V(\phi)$ needs to be normal-ordered; the dependence on the renormalization scale μ is

$$\mu \frac{\partial}{\partial \mu} \frac{V(\phi, \mu)}{\mu} = -\frac{1}{4\pi} \frac{\partial^2}{\partial \phi^2} \frac{V(\phi, \mu)}{\mu}.$$

In the case $V = \lambda_2 \phi^2 + \lambda_0 \phi$, on the lattice, write $V = \widehat{\lambda}_2 \phi^2 + \widehat{\lambda}_0 \phi$, with $\widehat{\lambda}_i = a\lambda_i$; near the continuum limit,

$$\begin{aligned} \widehat{\lambda}_2 &\approx a\lambda_2^{\mathrm{ren}} \\ \widehat{\lambda}_0 &\approx a\lambda_0^{\mathrm{ren}} + a\lambda_2^{\mathrm{ren}} \frac{1}{2\pi} \log(aM), \end{aligned}$$

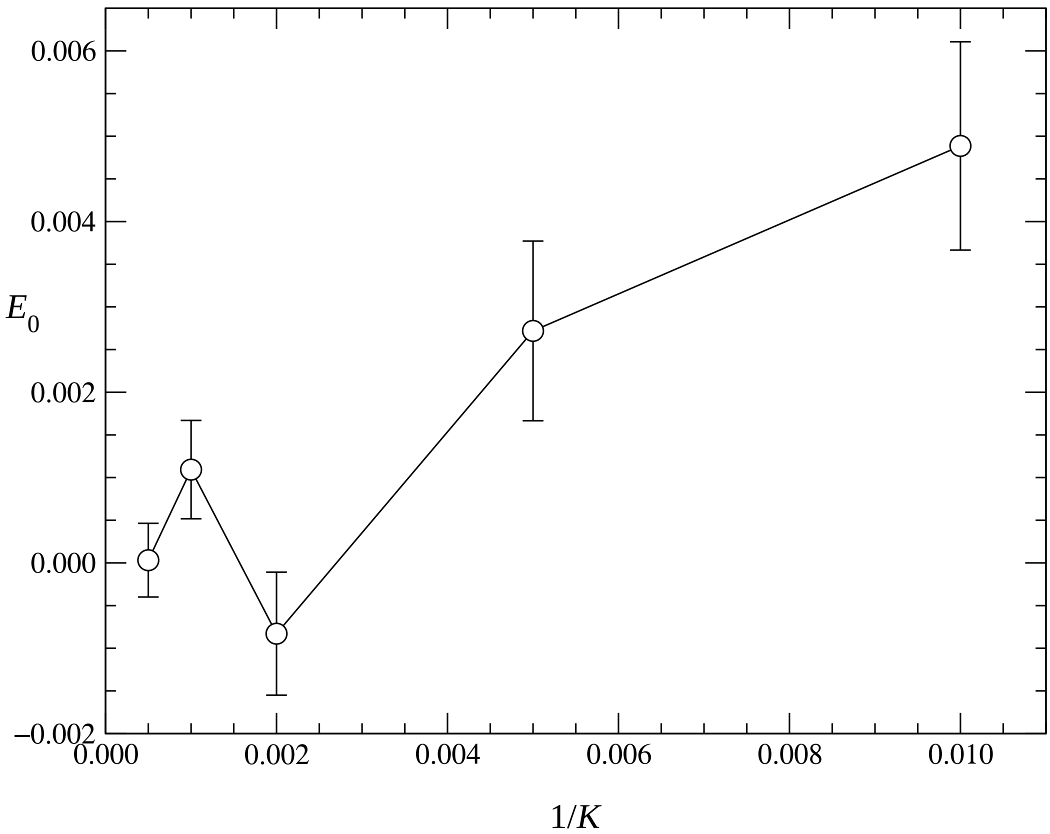
where M is the mass scale at which the renormalized couplings are defined.

For the odd $V(\phi) = \lambda_3\phi^3 + \lambda_1\phi$, both strong coupling and weak coupling predict supersymmetry to be unbroken. The Witten index

$$I_W = \mathrm{Tr}(-1)^F = \sum_E \bigl(n_B(E) - n_F(E)\bigr)$$

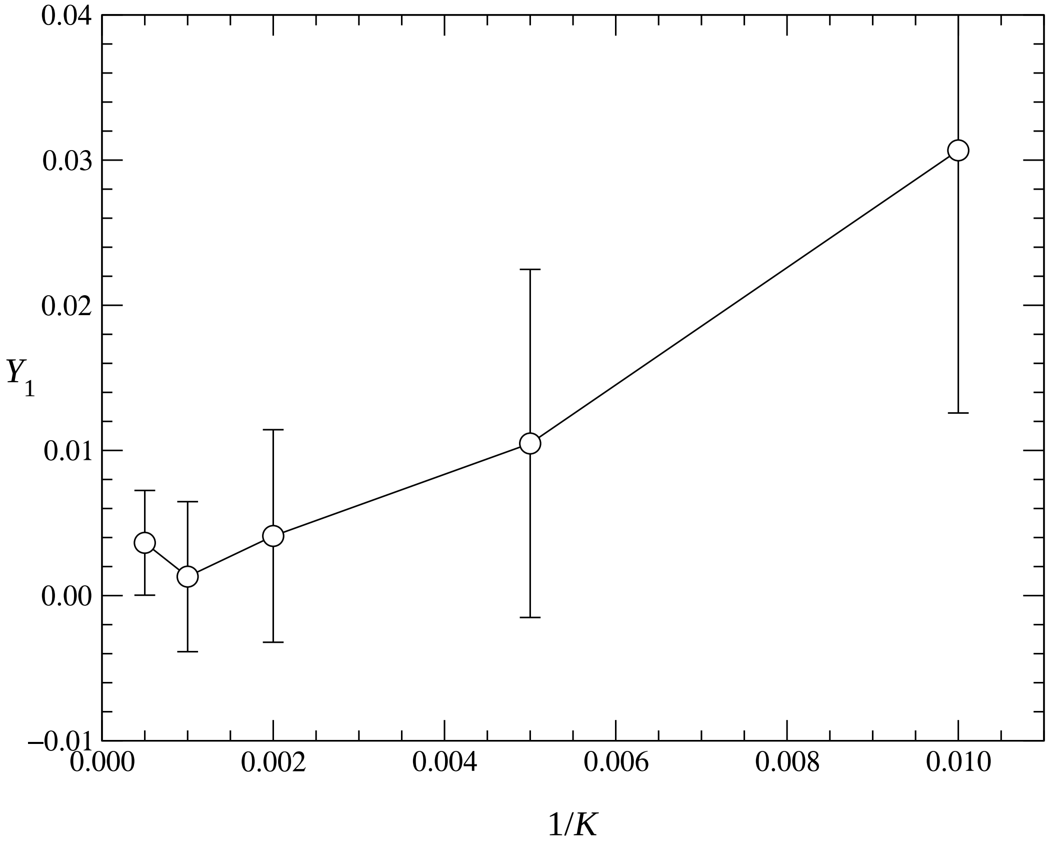
is a topological invariant and $I_W \neq 0$ implies unbroken supersymmetry; since at strong coupling $I_W = 1$, supersymmetry remains unbroken as we move towards the continuum limit (unless a phase transition occurs).

For $V = \phi^3$ we plot the ground state energy E_0 as a function of $1/K$; the parameters are $L = 10$, $\beta = 0.01$, odd αs set to 0; the statistic is $5{\times}10^5$ iterations (10^6 for $K = 100$).



The evidence for unbroken supersymmetry is convincing. The bosonic and fermionic contribution to E_0 are $\simeq \pm 7.4$: we are observing a cancellation of the order of 10^{-4}.

For the same runs, we plot the quantity $Y_1 = \{Q, \sum_n \phi_n \psi_n^2\}$.

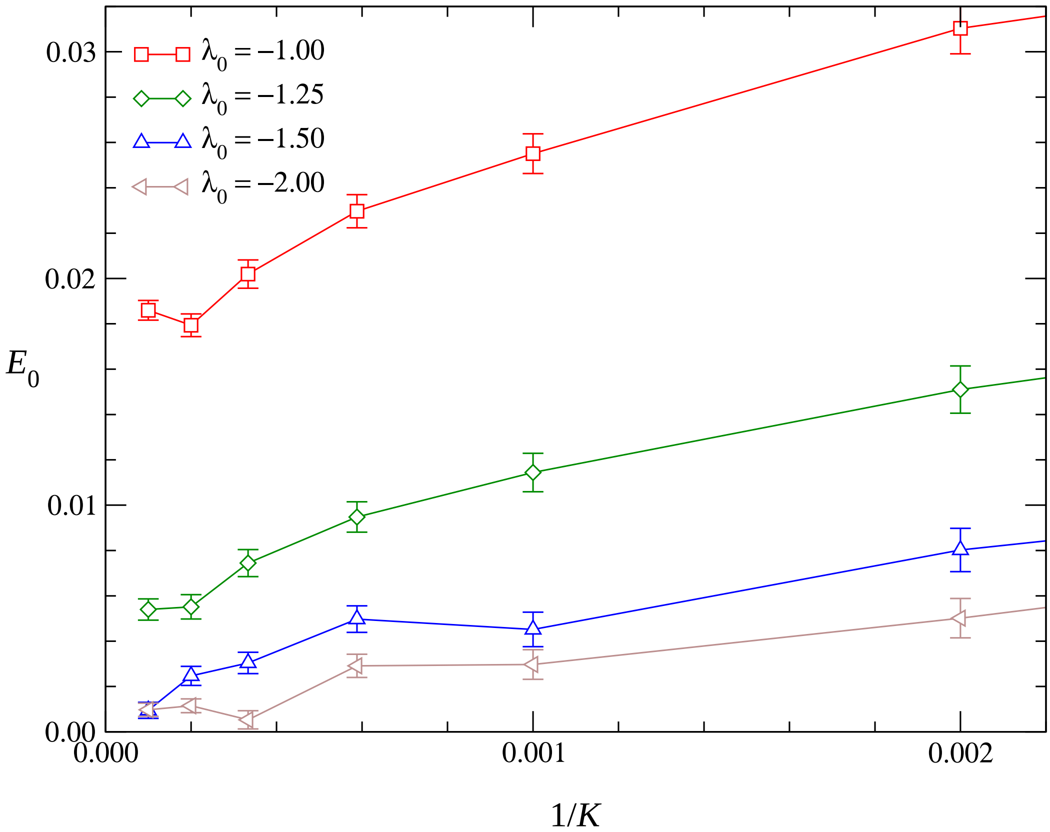


Again, the evidence for unbroken supersymmetry is convincing.

More interesting is the case of even $V(\phi) = \lambda_2\phi^2 + \lambda_0$. Strong coupling predicts supersymmetry breaking and $I_W = 0$, while weak coupling predicts unbroken supersymmetry for $\lambda_0 < 0$. We expect the actual transition value λ_0^t to be shifted by renormalization effects, and to depend on λ_2.

According to Witten (1982), supersymmetry should always be broken at finite L, and only in the $L \to \infty$ limit supersymmetry can be restored. Moreover, in the scaling region, unbroken supersymmetry should be accompanied by a nonzero $\langle\phi\rangle$, breaking the (approximate) $\phi \to -\phi$ symmetry.

At intermediate coupling, we observe $E_0 > 0$ for all λ_0; I present the plot of E_0 vs. K for $L = 10$, $\lambda_2 = 1$, and several values of λ_0, with $5{\times}10^5$ iterations for each point.



An extrapolation to $K \to \infty$ gives

$$E_0/L \propto \exp(5.5\lambda_0) \qquad \text{for } \lambda_0 \lesssim -0.5.$$

In order to confirm Witten's scenario, we study the restoring of supersymmetry and the breaking of the $\phi \to -\phi$ symmetry, varying λ_0 at fixed λ_2.

Supersymmetry breaking is studying by extrapolating E_0/L to $L \to \infty$ according to

$$\frac{E_0}{L} = \mathcal{E}\left(1 + \frac{c_L}{L} + \frac{c_K}{K}\right).$$

The $\phi \to -\phi$ symmetry breaking is studied by:

- The Binder cumulant

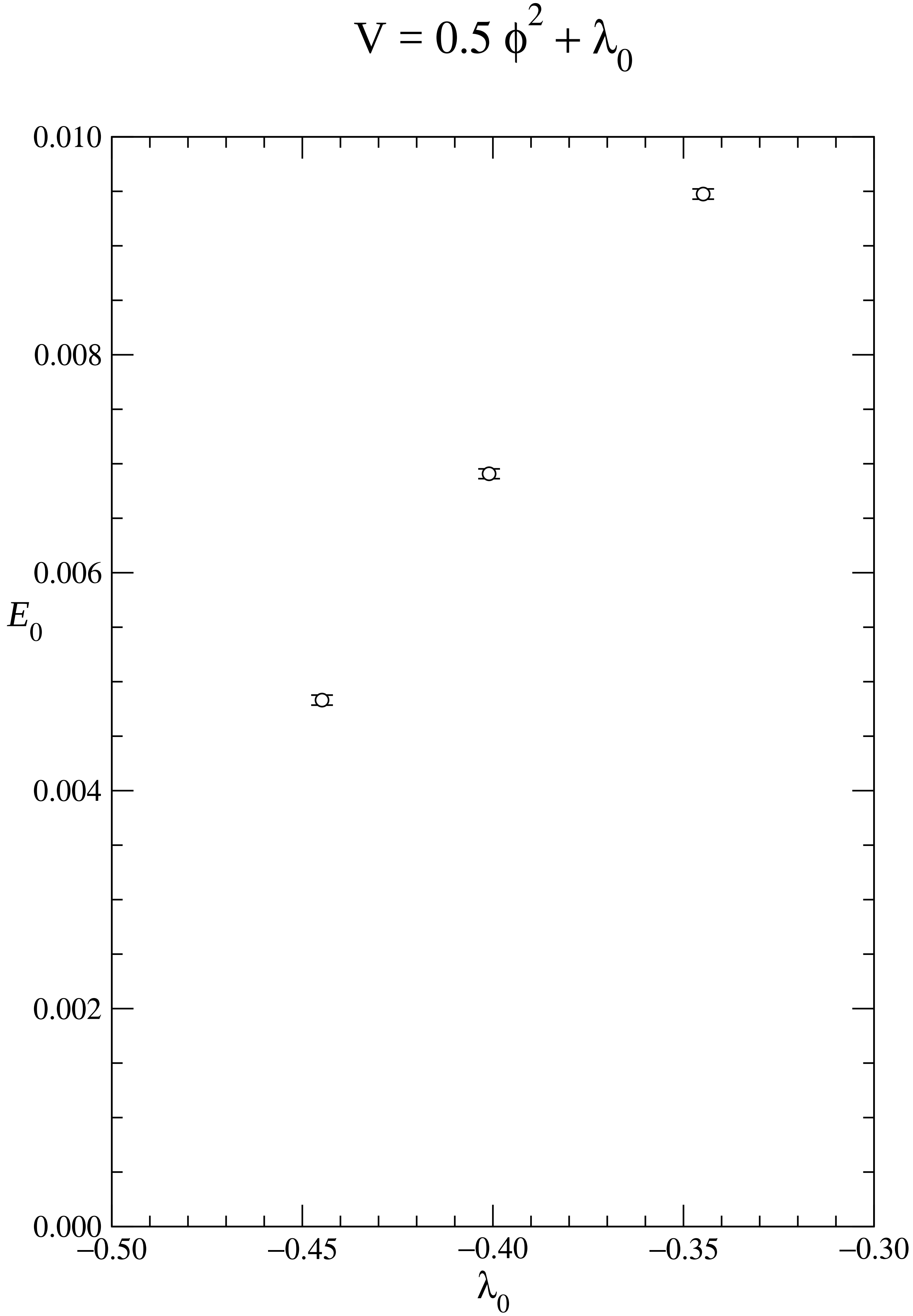
  $$Q = \frac{\langle M^4 \rangle}{\langle M^2 \rangle^2}, \qquad M = \sum_n \phi_n,$$

  where the sum excludes sites closer to the border than (typically) 6; a good estimate of the transition point is the intersection of the curves Q vs. λ_0 obtained at different values of L. Q appears to be too noisy to draw any definite conclusion.

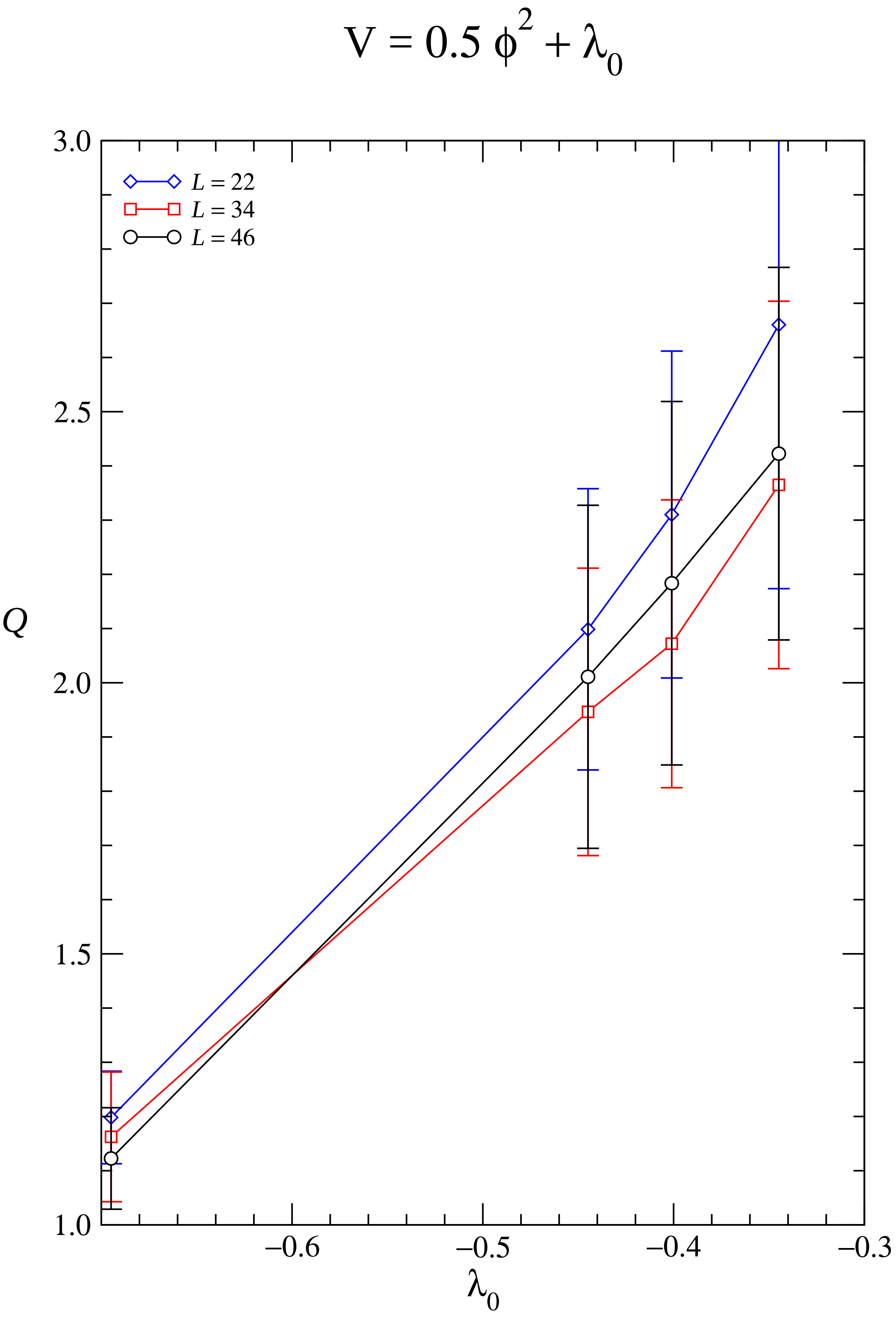
- The bosonic part of the optimized guiding wavefunction

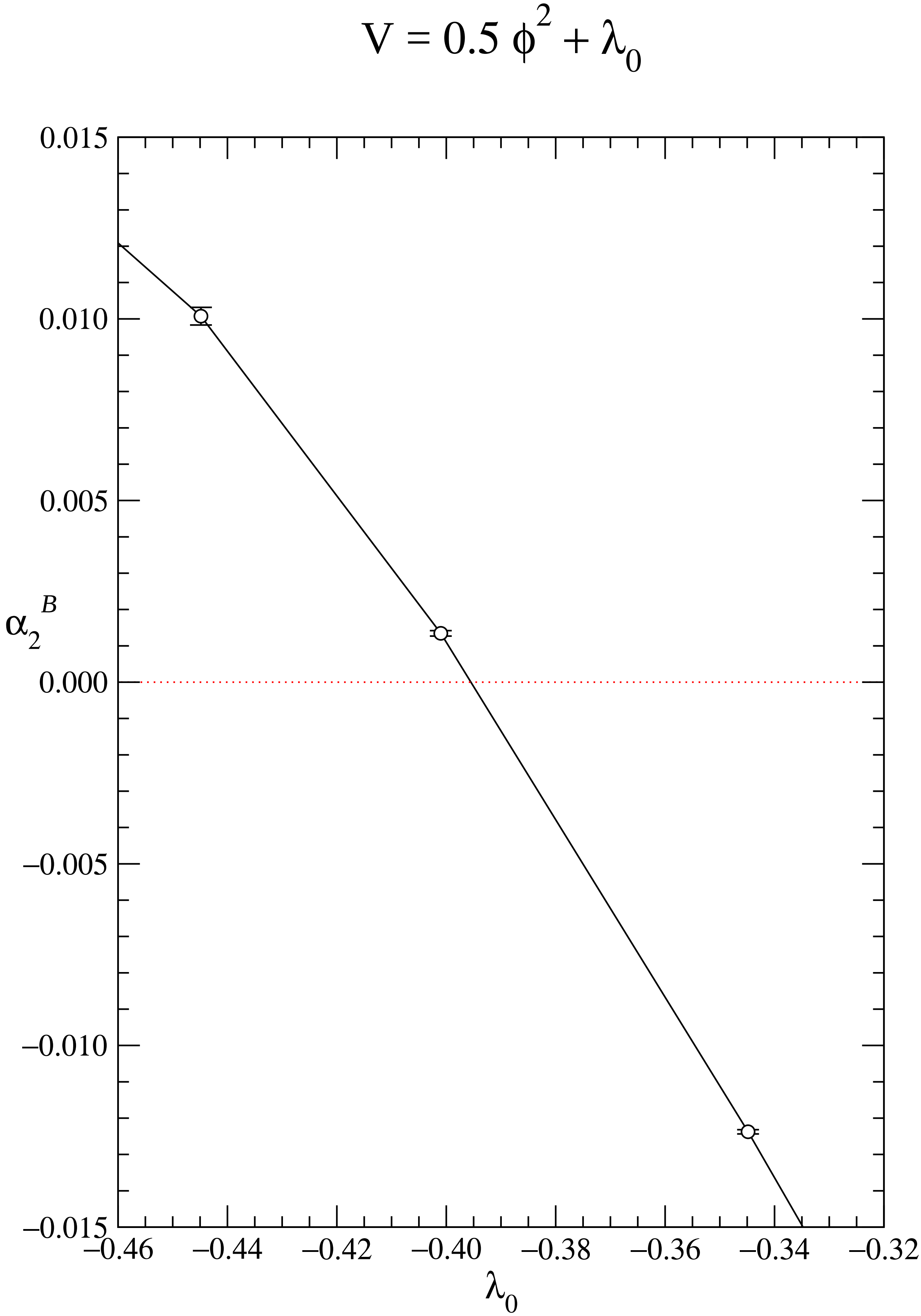
  $$\exp\left[\sum_n S_B(\phi_n)\right] \langle \phi | \Psi_0^B \rangle, \qquad S_B(\phi) = \alpha_2^B \phi^2 + \alpha_4^B \phi^4,$$

  which is related to the effective potential of the field ϕ: we expect $S_B(\phi)$ to have a double-well shape ($\alpha_2^B < 0$) for broken symmetry and a single-well shape ($\alpha_2^B > 0$) for unbroken symmetry.



V = 0.5 φ^2 + λ_0
0.010
0.008
0.006
0.004
0.002
0.000
E_0
−0.50
−0.45
−0.40
−0.35
−0.30
λ_0


V = 0.5 φ^2 + λ_0
L = 22
L = 34
L = 46
Q
λ_0
3.0
2.5
2.0
1.5
1.0
−0.6
−0.5
−0.4
−0.3


V = 0.5 φ^2 + λ_0
0.015
0.010
0.005
0.000
−0.005
−0.010
−0.015
α_2^B
−0.46
−0.44
−0.42
−0.40
−0.38
−0.36
−0.34
−0.32
λ_0

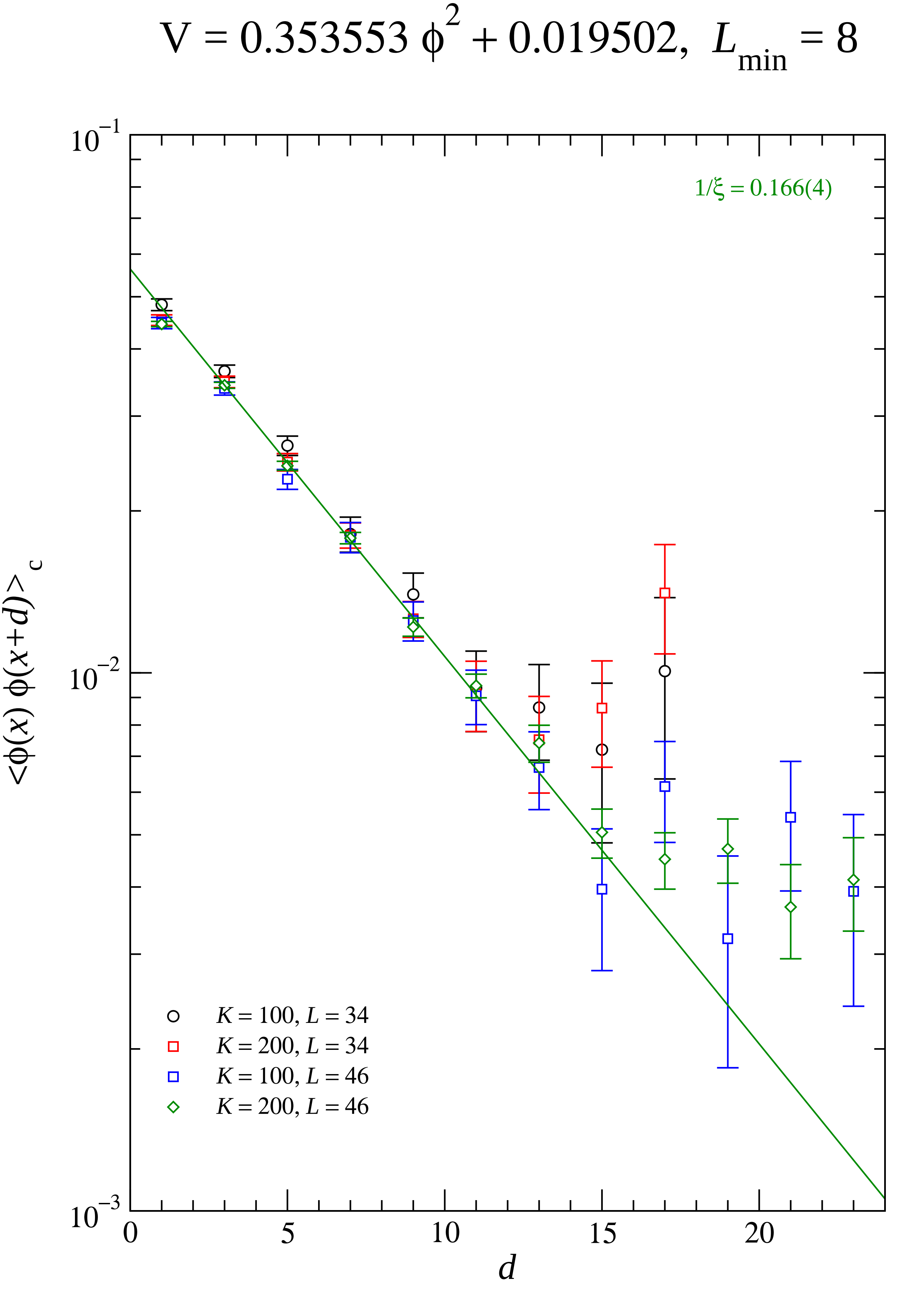
What happens in the continuum limit?

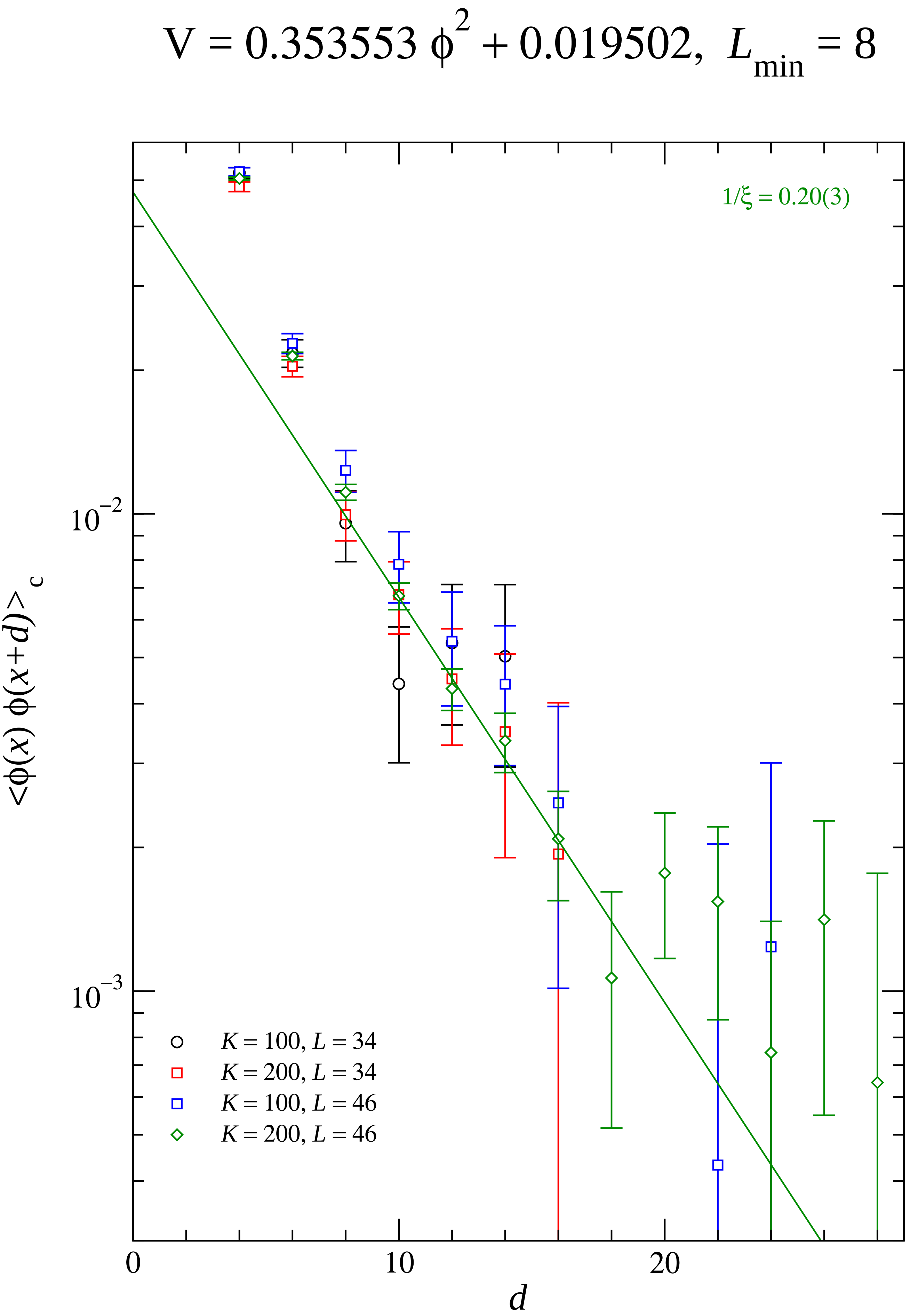
We study the trajectory

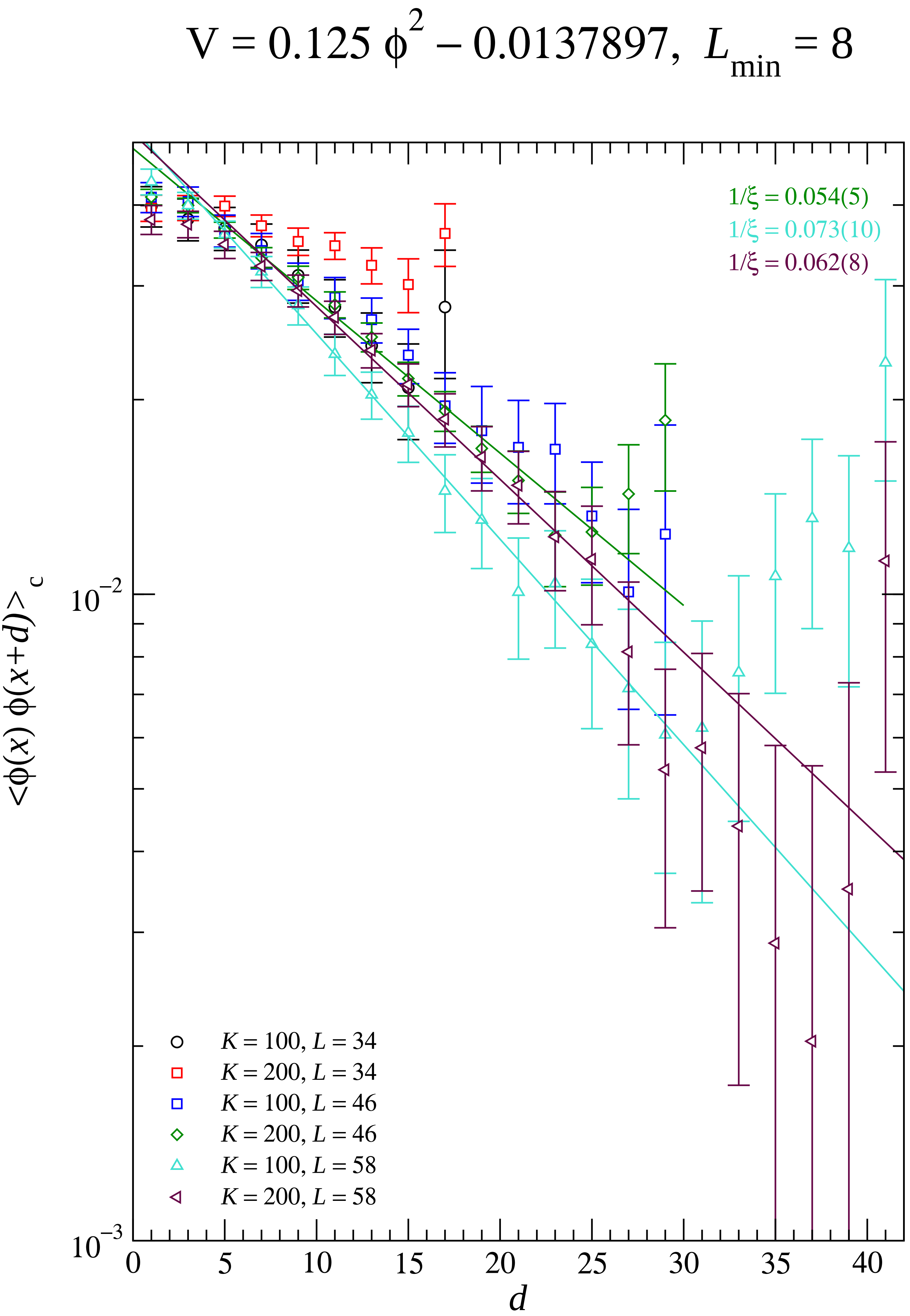
$$\lambda_0 = \frac{\lambda_2}{2\pi} \ln(4\lambda_2),$$

corresponding to a 1-loop RG trajectory; the effect of λ_0 is small in the range we considered, therefore we expect this to be a reasonable approximation to a RG trajectory.

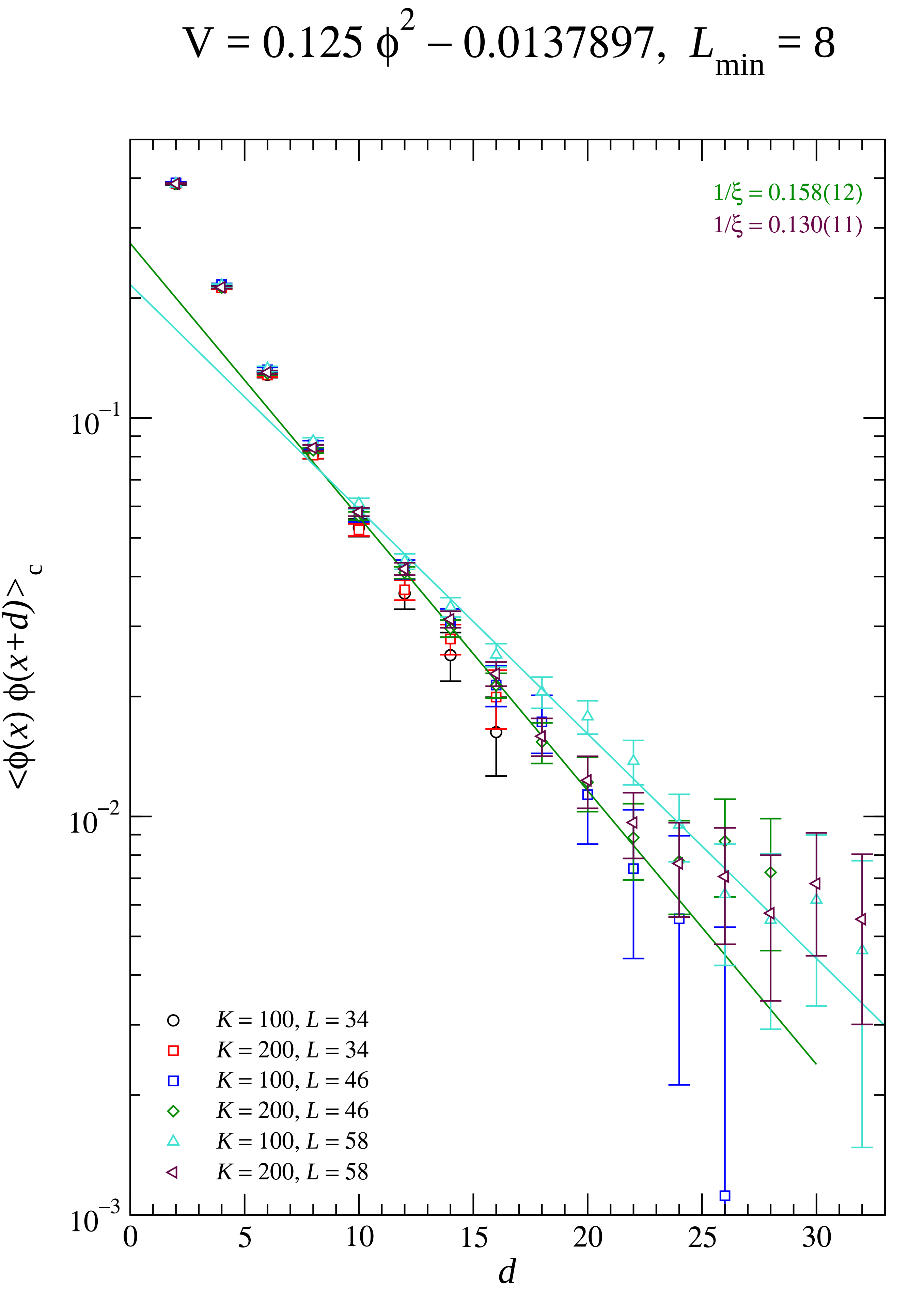
We estimate the correlation length from the exponential decay of the connected correlation function $G_d = \langle\phi_n\phi_m\rangle_c$ averaged over all n, m pairs with $|m - n| = d$, excluding pairs for which m or n is closer to the border than (typically) 8. In our formulation, fermions are staggered and even/odd d correspond to different channels.


V = 0.353553 φ^2 + 0.019502, L_min = 8
1/ξ = 0.166(4)
<φ(x) φ(x+d)>_c
d
K = 100, L = 34
K = 200, L = 34
K = 100, L = 46
K = 200, L = 46
10^-1
10^-2
10^-3
0
5
10
15
20


V = 0.353553 φ^2 + 0.019502, L_min = 8
1/ξ = 0.20(3)
<φ(x) φ(x+d)>_c
10^−2
10^−3
0
10
20
d
K = 100, L = 34
K = 200, L = 34
K = 100, L = 46
K = 200, L = 46



V = 0.125 φ^2 − 0.0137897, L_min = 8
1/ξ = 0.054(5)
1/ξ = 0.073(10)
1/ξ = 0.062(8)
K = 100, L = 34
K = 200, L = 34
K = 100, L = 46
K = 200, L = 46
K = 100, L = 58
K = 200, L = 58
<φ(x) φ(x+d)>_c
10^−2
10^−3
0
5
10
15
20
25
30
35
40
d

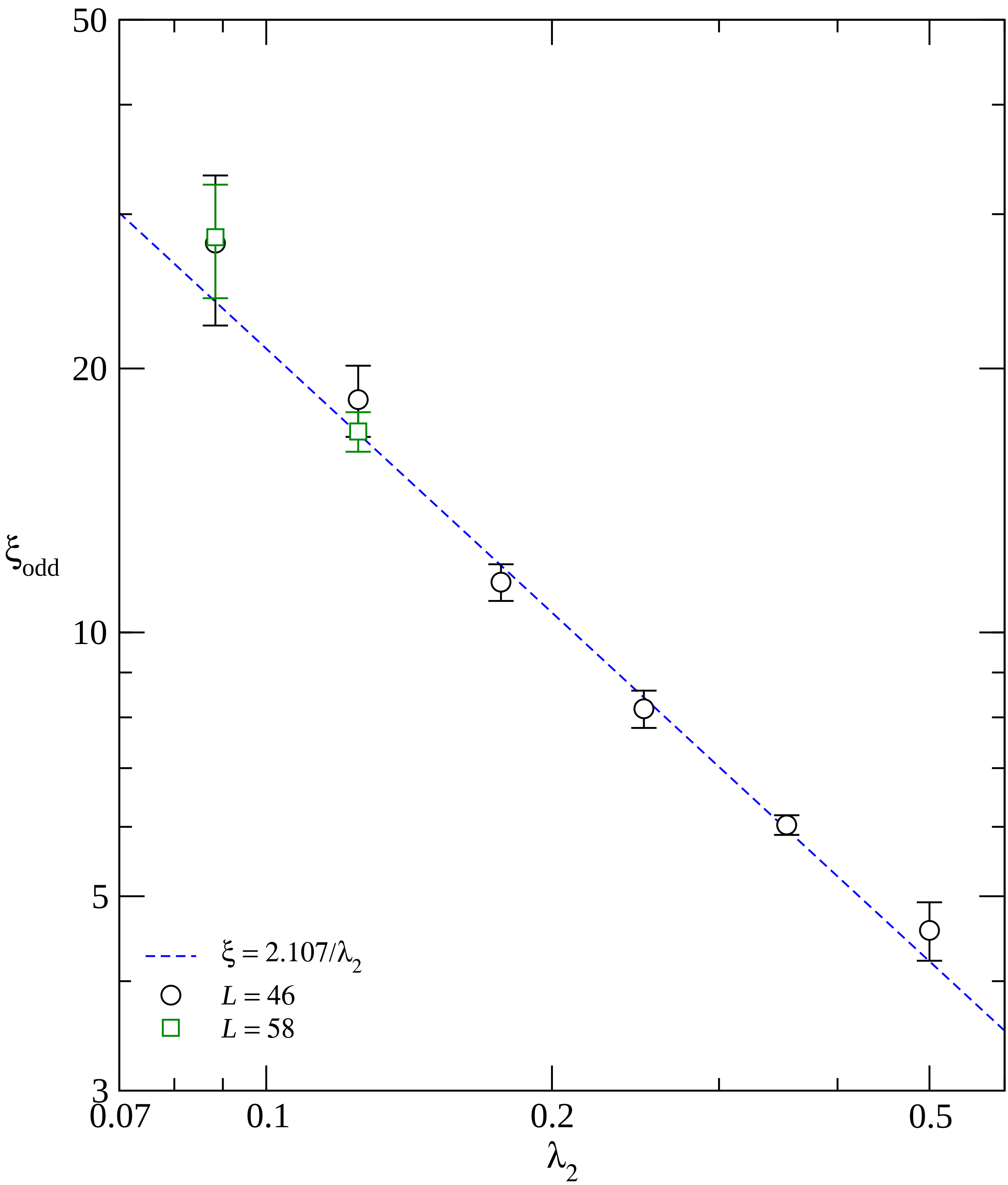

V = 0.125 φ² – 0.0137897, L_min = 8
1/ξ = 0.158(12)
1/ξ = 0.130(11)
<φ(x) φ(x+d)>_c
d
K = 100, L = 34
K = 200, L = 34
K = 100, L = 46
K = 200, L = 46
K = 100, L = 58
K = 200, L = 58

I show in detail the case $V = 0.35\,\phi^2 + 0.02$, for which we have obtained the statistics of 4×10^6 GFMC iterations. It is very difficult to extract a correlation length from the even-d channel, presumably because ϕ has a very small overlap with the lightest state of the channel, and the value $1/\xi = 0.20 \pm 0.03$ should be considered tentative. The odd-d channel is much cleaner, and it is possible to estimate ξ with a good precision.

For the other values of λ_2, the situation is similar but with slightly larger errors. The measured values of $\xi_{\rm odd}$ follow the naïve scaling behavior

$$\xi \propto 1/\lambda_2.$$

The entire range $0.088 \le \lambda_2 \le 0.35$ seem to be in the scaling region, with $\lambda_2 = 0.5$ a borderline case. The values of $\xi_{\rm even}$ have very large errors, and it is hard to draw any conclusion from them.


50
20
10
5
3
0.07
0.1
0.2
0.5
ξ_{odd}
λ_2
$\xi = 2.107/\lambda_2$
$L = 46$
$L = 58$

The GFMC algorithm gives a very accurate measurement of the ground-state energy E_0; typical results are:

$$E_0(\lambda_2{=}0.044, L{=}46, K{=}200) = (1.28 \pm 0.01)\times 10^{-3};$$
$$E_0(\lambda_2{=}0.5, L{=}46, K{=}200) = (69.44 \pm 0.05)\times 10^{-3}.$$

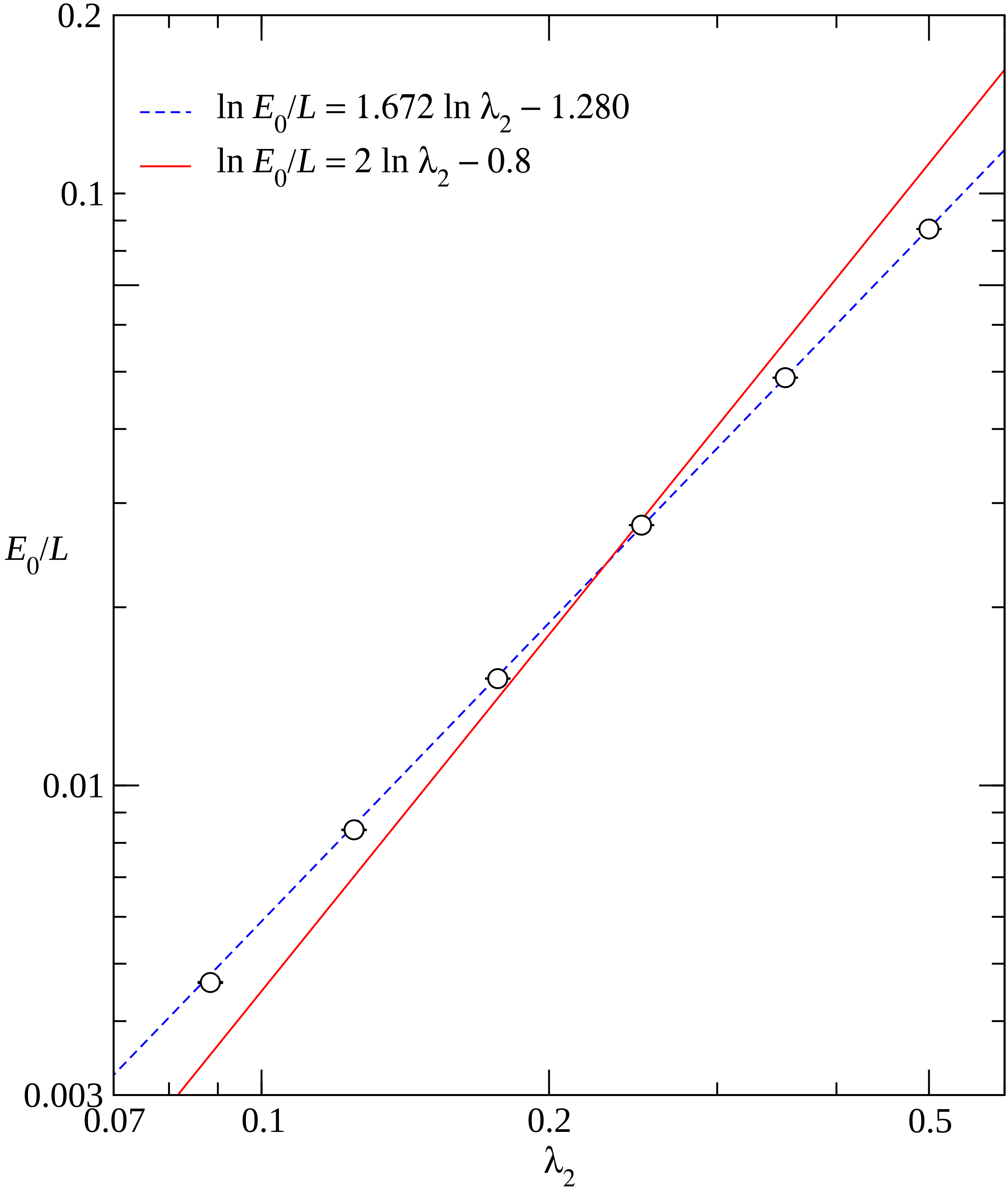
We extrapolate to $L \to \infty$ and $K \to \infty$ fitting E_0 to

$$\frac{E_0}{L} = \mathcal{E}\left(1 + \frac{c_L}{L} + \frac{c_K}{K}\right);$$

$\chi^2/\#\mathrm{d.o.f}$ is typically 2, indicating that higher corrections are very small, but not completely negligible.

For $\lambda_2 \to 0$,
$c_K \sim \mathrm{const}$ (the algorithm is performing well),
$c_L\mathcal{E} \sim \mathrm{const}$.

E_0/L seems to behave $\propto \lambda_2^{5/3}$, while naïve scaling would predict $\propto \lambda_2^2$. The value of E_0/L (disregarding this puzzling exponent) and the lack of any signal for a breakdown of parity (like a double-peaked distribution of ϕ) show that the trajectory we are considering belongs to the phase with broken supersymmetry and zero $\langle\phi\rangle$.



ln E0/L = 1.672 ln λ2 − 1.280
ln E0/L = 2 ln λ2 − 0.8
E0/L
λ2
0.2
0.1
0.01
0.003
0.07
0.1
0.2
0.5

Algorithms like GFMC can be parallelized effectively in MIMD machines, by putting a sub-ensemble of walkers of each node: communications are mainly needed for branching, to balance the number of walkers on each node. We developed a parallel code using explicit MPI calls, and reached 90% efficiency on a network of PCs, connected through a dedicated fast ethernet.

The two-dimensional lattice Wess-Zumino model can be simulated to very high accuracy using the GFMC algorithm.

The vacuum energy can be measured with a very high accuracy; in order to establish the pattern of supersymmetry breaking, an extrapolation to $L \to \infty$ is needed.

The optimal values of the guiding wavefunction parameters are very useful probes of the properties of the vacuum wavefunction.

The correlation length of ϕ can be measured with a good accuracy.

Fermions in 2 dimensions can be simulated (in many instances) at no extra cost. In 3 or more dimensions, we must tackle the sign problem; several algorithms are being tested for condensed matter physics (mainly) in 3 dimensions.