Bari Sept 2002

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COLOR CONFINEMENT – RECENT PROGRESS

Adriano DI GIACOMO (PISA)

ABSTRACT. RECENT RESULTS ON THE DECONFINING TRANSITION IN FULL QCD ARE REVIEWED. THE INDEPENDENCE OF MONOPOLE CORRELATORS ON THE ABELIAN PROJECTION IS DISCUSSED.

J. M. CARMONA, L. DEL DEBBIO, M. D'ELIA, B. LUCINI, L. MONTESI
G. PAFFUTI, C. PICA

CEA COSMAI

1. CONFINEMENT

QUARKS AND GLUONS EXIST BEYOND ANY DOUBT AS CONSTITUENTS OF HADRONS. NEVER OBSERVED AS FREE PARTICLES (CONFINEMENT)

QUARKS HUNTED SINCE 1961 [GELL-MANN]: NONE FOUND

Experiment (PDG)	Expectation for no confinement	
$\frac{n_q}{n_p} < 10^{-27}$	$n_q/n_p \simeq 10^{-12}$	C.S.M
$\sigma(p+p \to q(\bar{q})+x) < 10^{-40}\,\mathrm{cm}^2$	$\sigma \sim 10^{-25}\,\mathrm{cm}^2$	P.T

$\varepsilon < 10^{-15}$

$\Downarrow$

$\varepsilon = 0 \equiv$ CONFINEMENT AN ABSOLUTE PROPERTY BASED ON SYMMETRY.

DECONFINING TRANSITION IS ORDER-DISORDER

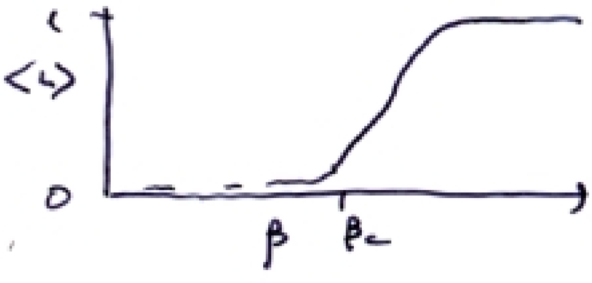
2. CONFINEMENT AND LATTICE SIMULATIONS.

- DECONFINEMENT OBSERVED IN PURE GAUGE (QUENCHED) THEORY

ORDER PARAMETER

$\langle L \rangle$ (POLYAKOV LINE)

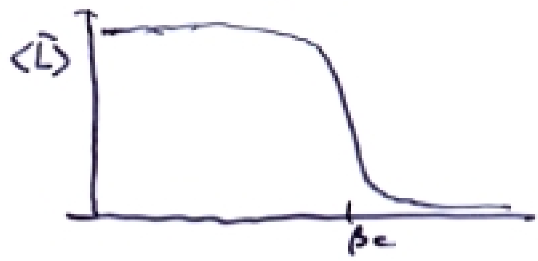
SYMMETRY Z_N



DISORDER PARAMETER

$\langle \bar{L} \rangle$ ('t HOOFT LINE)

SYMMETRY $\tilde{Z}_N$



$$D(x) = \langle L(\vec{x})\, \bar{L}(0) \rangle \simeq \exp(-T\sigma x) + \langle L \rangle)^2$$

$$V(x) = -T \ln D(x) \underset{x\to\infty}{\simeq} \begin{cases} \sigma x & \beta < \beta_c \\ \text{const} & \beta > \beta_c \end{cases}$$

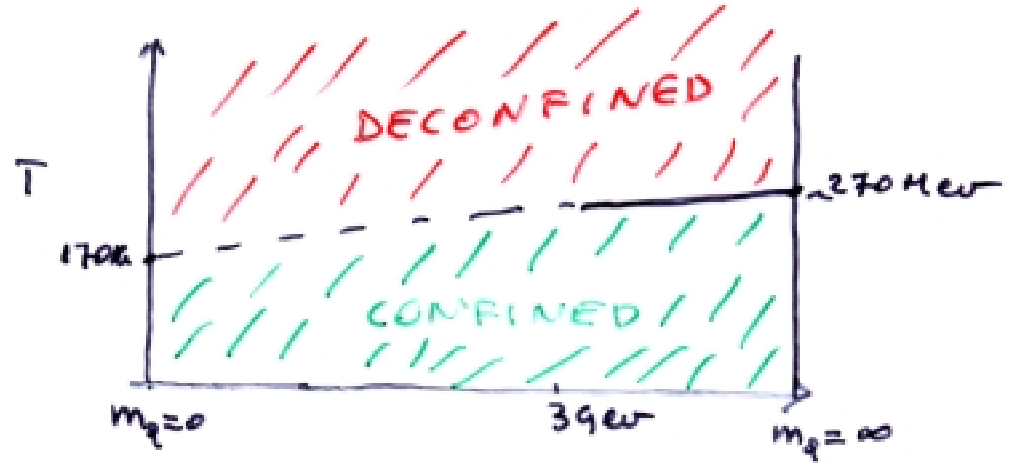
$\chi_{L\bar{L}} \equiv \int d^3x \, \langle L(\vec{x}) \bar{L}(0) \rangle$ MAXIMUM AT β_c

$\chi_{L\bar{L}}(\beta_c, V) \underset{V\to\infty}{\sim} V^{1/3\nu}$

- $N_c = 2$ $\nu = .63$ 2nd ORDER 3d ISING
- $N_c = 3$ $\nu = 1/3$ 1st ORDER (WEAK)

|| SYMMETRY DETERMINES THE UNIVERSALITY CLASS (GINZBURG-LANDAU).

- FULL QCD (UNQUENCHED) $N_f = 2$ $m_u = m_d \equiv m_q$



$\langle L \rangle$ $\langle \bar{L} \rangle$ NOT ORDER PARAMETERS $Z_N(\bar{Z}_N)$ BROKEN BY QUARKS

$\langle \bar{\psi}\psi \rangle$ AN ORDER PARAMETER AT $m_q = 0$ CHIRAL SYMMETRY BROKEN BY QUARK MASS TERMS

BIELEFELD

$\chi_{LL}(\beta, m_q, V) = \int d^3x \, \langle L(\vec{x}) \bar{L}(0) \rangle$

$\chi_{\bar{\psi}\psi,\bar{\psi}\psi}(\beta, m_q, V) = \int d^3x \, \langle \bar{\psi}\psi(\vec{x}) \, \bar{\psi}\psi(0) \rangle$

. . . .

MAXIMUM AT β_c $\chi(\beta_c, m_q, V) \underset{V\to\infty}{\simeq} V^{1/3\nu}$

$m_q > 3$ GeV $\chi_{LL}(\beta_c, m_q, V) \sim V$ $\nu = \frac{1}{3}$ 1st ORDER

$0 < m_q < 3$ GeV NONE OF THE χ's (β_c, m_q, V) DIVERGE AS $V \to \infty$

$\Rightarrow$ CROSSOVER? (NOT ORDER DISORDER!) $\boxed{\text{BOR 3}}$

DENSITY OF FREE ENERGY (EFFECTIVE LAGRANGEAN) $\mathcal{L}$

CONSTRAINED BY SYMMETRY AND DIMENSIONAL ANALYSIS [LANDAU-GINZBURG]

ONLY THE SUSCEPTIBILITIES OF THE ORDER PARAMETERS LEGITIMATE TO DETERMINE ν.

$\cancel{\langle L \rangle}$ $\cancel{\langle \bar{Q} \rangle}$ $\cancel{\langle \bar{\Psi}\Psi \rangle}$ LOOK FOR TRUE ORDER PARAM.'S.

QCD: CONFINED PHASE IS DISORDERED (STRONG COUPLING)

$T \sim \Lambda \exp(b\beta)$ [ASYMPTOTIC FREEDOM]. SYMMETRY OF DISORDERED PHASE?

- **DUALITY** [KRAMERS, WANNIER 43]

SYSTEMS ϕ WITH NONTRIVIAL TOPOLOGICAL EXCITATIONS μ. TWO COMPLEMENTARY DESCRIPT. FULLY EQUIVALENT

DIRECT	DUAL
• FIELDS ϕ	• LOCAL FIELDS μ
• ORDER PARAMETERS $\langle \phi \rangle$	• $\langle \mu \rangle$ DISORDER PARAM'S (ORDER PARAM'S OF DUAL)
• NON LOCAL EXCITATIONS μ	• ϕ NON LOCAL EXCITATIONS
• CONVENIENT FOR WEAK COUPLING (ORDERED PHASE) $g \ll 1$	• CONVENIENT FOR $g \gg 1$ (DISORDERED PHASE)
	$g_D \sim \frac{1}{g}$
	WEAK COUPLING MAPPED IN STRONG COUPLING AND VICEVERSA

- ISING MODEL (KADANOFF-CEVA 1972) σ σ^* $\beta^* \sim \frac{1}{\beta}$ $K(\sigma, \beta) = K(\sigma^*, \beta^*)$
- 3d XY MODEL θ, VORTICES
- N=2 SUSY QCD [Seiberg Witten 92]
- M STRING THEORIES [Maldacena 96]

WHAT ARE THE DUAL EXCITATIONS OF QCD?

MONOPOLES ['tHooft 75, 81] [Mandelstam 75] VORTICES ['tHooft 81]

• MONOPOLES IN QCD. ABELIAN PROJECTION

• GENERAL IDEA: MONOPOLES CONDENSE IN THE DISORDERED PHASE $\langle \mu \rangle \neq 0$. CHARGES CONFINED BY DUAL MEISSNER EFFECT:

$E = \sigma R$

• 'tHOOFT POLYAKOV MONOPOLES $SU(2)$

$$\mathcal{L} = -\frac{1}{4}\vec{G}_{\mu\nu}\vec{G}_{\mu\nu} + \frac{1}{2}(D_\mu\vec{\Phi})^+(D_\mu\vec{\Phi}) - V(\Phi)$$

$$V(\Phi) = -\frac{m^2}{2}\vec{\Phi}^2 + \frac{\lambda}{4}(\vec{\Phi}^2)^2$$ HIGGS PHASE $m^2 > 0$, $\langle\vec{\Phi}\rangle \neq 0$

A STATIC SOLUTION $\vec{A}_0 = 0$ $\vec{E} = 0$

ANSATZ
$$\phi^i(\vec{r}) = f(r)\, r^i$$
$$A^a_i = \frac{1}{2g}\varepsilon_{iak}\frac{r_k}{r^2}\cdot g(r)$$

HEDGEHOG

A SOLUTION EXISTS WITH ENERGY $\sim m$ $\begin{cases} f(r) \sim 1 \\ g(r) \sim 1 \end{cases}$ $r > \frac{1}{m}$

KEY QUANTITY ['tHooft 74]

$$F_{\mu\nu} = \hat{\varphi}\,\vec{G}_{\mu\nu} - \frac{1}{g}\hat{\varphi}\,(D_\mu\hat{\varphi}\wedge D_\nu\hat{\varphi})$$

$\hat{\varphi} = \frac{\vec{\varphi}}{|\vec{\varphi}|}$

$D_\mu\varphi = (\partial_\mu - g\vec{A}_\mu\wedge)\vec{\varphi}$

$\Downarrow$

$$= \partial_\mu(\hat{\varphi}\cdot\vec{A}_\nu) - \partial_\nu(\hat{\varphi}\cdot\vec{A}_\mu) - \frac{1}{g}\hat{\varphi}\,(\partial_\mu\hat{\varphi}\wedge\partial_\nu\hat{\varphi})$$

GAUGE $\hat{\varphi} = (0,0,1)$ $F_{\mu\nu} = \partial_\mu A^3_\nu - \partial_\nu A^3_\mu$ (Abelian projection)

MONOPOLE SOLUTION

$E_i = \vec{F}_{0i} = 0$ $\vec{H} = \frac{1}{g}\frac{\vec{r}}{r^3}$ + STRING

$F^*_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\rho\sigma}F_{\rho\sigma}$ $j^\nu = \partial_\nu F^*_{\mu\nu}$ $\partial_\nu j^\nu = 0$ [U(1) Maxwell]

A GEOMETRICALLY CONSERVED MAGN. CHARGE $j^0 = \frac{1}{g}\delta^3(r)$

IN THE COULOMB PHASE $[\langle\vec{\Phi}\rangle = 0]$ A CONSERVED j^μ CAN BE DEFINED, EVEN IF THERE EXIST NO STABLE FINITE ENERGY SOLUTION.

IN QCD NO HIGGS FIELD EXISTS. A CONSERVED MAGNETIC CURRENT CAN BE ASSOCIATED TO ANY FIELD $\vec{\Phi}$ IN THE ADJOINT REPRESENTATION.

$N_c > 2$

$$\boxed{F_{\mu\nu} = Tr\{\phi\, G_{\mu\nu}\} - \frac{1}{g} Tr\{\phi\, [D_\mu\phi, D_\nu\phi]\}}$$

$$\phi = \Sigma\, \phi^a T^a \qquad A_\mu = \Sigma\, A^a_\mu T^a \qquad G_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]$$

FOR WHAT CHOICE OF ϕ $F_{\mu\nu}$ BECOMES ABELIAN IN A "UNITARY" GAUGE OR,

$$F_{\mu\nu} = \partial_\mu Tr\{\phi \cdot A_\nu\} - \partial_\nu Tr\{\phi A_\mu\} - \frac{1}{g} Tr\{\phi\, [\partial_\mu\phi, \partial_\nu\phi]\}$$

AND A GAUGE EXISTS IN WHICH $\partial_\mu\phi = 0$

GENERAL SOLUTION'S $\quad N_c - 1 \quad \phi^a \quad a = 1, \dots, N_c - 1$

$$\phi^a = U^+(x)\, \phi^a_{diag}\, U(x) \qquad U(x) \text{ ARBITRARY}$$

$$\phi^a_{diag} = diag\Big(\underbrace{\frac{N_c - a}{N_c} \dots \frac{N_c - a}{N_c}}_{a \text{ times}}, \underbrace{\frac{-a}{N_c} \dots -\frac{a}{N_c}}_{N_c - a \text{ Times}}\Big) \qquad a = 1 \dots, N_c - 1$$

$$SU(2) \cdot \left(\tfrac{1}{2}, \tfrac{-1}{2}\right) \qquad SU(3) = \left(\tfrac{2}{3}, -\tfrac{1}{3}, -\tfrac{1}{3}\right), \; \left(\tfrac{1}{3}, \tfrac{1}{3}, -\tfrac{2}{3}\right)$$

$$F^a_{\mu\nu} = \partial_\mu Tr\{\phi^a A_\nu\} - \partial_\nu Tr\{\phi^a \cdot A_\mu\} - \frac{1}{g} Tr\{\phi^a\, [\partial_\mu\phi^a, \partial_\nu\phi^a]\}$$

GAUGE TRANSF $U(x)$ $\qquad \phi^a \longrightarrow \phi^a_{diag}$

$$F^a_{\mu\nu} = \partial_\mu Tr\{\phi^a_{diag} A_\nu\} - \partial_\nu Tr\{\phi^a_{diag} A_\mu\}$$

$$A^{diag}_\mu = \sum_b A^b_\mu C^b \qquad Tr\{\phi^a C^b\} = \delta^{ab}$$

$$C^b = diag\,(0, 0, \overset{a}{1}, \overset{a+1}{-1}, 0, 0 \dots)$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu$$

(ABELIAN PROJECTION)

BOCA

USUAL PROCEDURE :

- CHOOSE ANY ϕ IN THE ADJOINT REPRESENTATION

$$\phi = U^{+}(x)\,\phi_{diag}(x)\,U(x)$$

$$\phi_{diag}(x) = \sum_a c_a(x)\,\phi^a_{diag}(x)$$

$c_a(x)=0$ TWO EIGENVALUES COINCIDE : $U(x)$ IS SINGULAR

Def $\phi^a(x) = U^{+}(x)\,\phi^a_{diag}\,U(x)$

Def
$$F^a_{\mu\nu} = \mathrm{Tr}\{\phi^a(x)\,G_{\mu\nu}(x)\} - \frac{1}{g}\mathrm{Tr}\{\phi^a(x),[D_\mu\phi^a(x), D_\nu\phi^a(x)]\}$$

$j^a_\nu = \partial_\mu F^{a*}_{\mu\nu}$ IS A CONSERVED MAGNETIC CURRENT : $U(1)^{N-1}_{MAGN.}$ GAUGE SYMMETRY

U(1) SYMMETRY HIGGS BROKEN $\longrightarrow$ DUAL SUPERCONDUCTIVITY

WIGNER $\longrightarrow$ SUPERSELECTED MAGNETIC CHARGE

PROBLEM 1 CONSTRUCT A DISORDER PARAMETER WHICH DETECTS DUAL SUPERCONDUCTIVITY

MAGNETICALLY CHARGED μ^a $\langle\mu^a\rangle$ DISORDER PARAMETER

$\langle\mu^a\rangle \neq 0$ CONFINED PHASE (Dual Superconductivity)

$\langle\mu^a\rangle = 0$ DECONFINED PHASE (Superselected sectors)

PROBLEM 2 THE GAME CAN BE REPEATED FOR ANY CHOICE OF ϕ, OR OF THE GAUGE TRANSF. $U(x)$

IS THERE A PRIVILEGED CHOICE FOR ϕ [MAX ABELIAN]

WHAT IS THE RELATION BETWEEN $\langle\mu^a\rangle$'S DEFINED BY DIFFERENT ABELIAN PROJECTIONS?

CONSTRUCTION OF μ

[L. Del Debbio et al 95, A. DiG & Paffuti 98, A. DiGiacomo 93, A DiGia et al 2000, 2001]

$e^{ipa}|x\rangle = |x+a\rangle$

$$\mu^a(x) = \exp\left(\int d^3y\, \vec{b}_\perp(\vec{x}-\vec{y})\, \mathrm{Tr}\{\vec{E}(\vec{y},x^0)\,\Phi^a(y)\}\right)$$

$\vec{b}_\perp(x-y)$ VECTOR POTENTIAL AT $\vec{y}$ PRODUCED BY A MONOPOLE SITTING AT $\vec{x}$

$\vec{\nabla}\vec{b}_\perp(x-y) = 0$ (transverse)

$\Phi^a(x) = U^+(x)\Phi^a_{diag}U(x)$

μ IS GAUGE INVARIANT.

GAUGE TRANSFORM BY $U(x)$ $\Phi^a \to \Phi^a_{diag}$ AND

$$\mu^a(x) = \exp\left(\int d^3y\, \vec{b}_\perp(x-y)\, E^a_\perp(\vec{y},x^0)\right)$$

$\mu^a(x)$ Translates $A^a_\perp(x)$ by $\vec{b}_\perp(\vec{x}-\vec{y})$: $E^a_\perp$ IS IN ANY GAUGE THE CONJUGATE MOMENTUM TO $A^a_\perp(x)$

$$\mu^a(x)\,|\vec{A}^a_\perp(\vec{x},x^0)\rangle = |\vec{A}^a_\perp(\vec{x},x^0) + \vec{b}_\perp(\vec{x}-\vec{x}')\rangle$$

THE ARGUMENT CAN BE ADAPTED TO THE COMPACT FORMULATION OF THE LATTICE :

μ CREATES A DEGRAND–TOUSSAINT MONOPOLE

COMPUTATION OF THE CORRELATORS

$$\langle \mu^{a_1}(x_1)\dots\mu^{a_k}(x^k)\rangle = \frac{1}{Z}\int dM\, e^{-\int d^3y_1\, \vec{b}_\perp(\vec{x}_1-\vec{y}_1)\mathrm{Tr}\{\vec{E}(\vec{y}_1,x^0)\Phi^a(y_1)\}} \times \dots\dots$$

$\Phi^a(y_1) = U^+(y_1)\,\Phi^a_{diag}\,U(y)$

$Z = \int dM = \int \pi dU \exp(-\beta S)$

dM GAUGE INVARIANT.

$\langle\mu\rangle$ THE DISORDER PARAMETER IS A PARTICULAR CASE.

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$$\mu^a(x) = \exp\left\{-\int d^3y\, \vec{b}_\perp(\vec{x}-\vec{y})\, \mathrm{Tr}\left(\vec{E}(\vec{y},x^0)\,\phi^a(\vec{y},x^0)\right)\right\} =$$

$$= \exp\left\{-\int d^3y\, \vec{b}_\perp(\vec{x}-\vec{y})\, \mathrm{Tr}\left\{\vec{E}(\vec{y},x^0)\, U^\dagger(\vec{y},x^0)\,\phi^a_{diag}\, U(\vec{y},x^0)\right\}\right\}$$

$$= \exp\left\{-\int d^3y\, \vec{b}_\perp(\vec{x}-\vec{y})\, \mathrm{Tr}\left(U(\vec{y},x^0)\,\vec{E}(\vec{y},x^0)\, U^\dagger(\vec{y},x^0)\,\phi^a_{diag}\right)\right\}$$

CHANGE VARIABLES BY A GAUGE TRANSF.

$$A_\mu \to U A_\mu U^\dagger + i\partial_\mu U U^\dagger$$; MEASURE INVARIANT

$$\mu^a(x) = \exp\left\{-\int d^3y\, \vec{b}_\perp(\vec{x}-\vec{y})\, \mathrm{Tr}\left\{\vec{E}(\vec{y},x^0)\,\phi^a_{diag}\right\}\right\}$$

$U(x)$, THE MEMORY OF THE ABELIAN PROJECTION, DISAPPEARS.

$\langle \mu^a_1(x_1) \ldots \mu^a_n(x_n)\rangle$ IS INDEPENDENT ON THE CHOICE OF THE ABELIAN PROJECTION WHICH DEFINES THE MAGNETIC $U(1)$'S OF THE MONOPOLES

$\langle\mu\rangle \neq 0$ OR $\langle\mu\rangle = 0$ IS AN INTRINSIC PROPERTY, INDEPENDENT OF THE ABELIAN PROJECTION WHICH DEFINES THE MONOPOLES [A.D.G 2002]

SUPPORTED BY NUMERICAL RESULTS [Carmona et al 2001]

NOTE: $U(x)$ IS SINGULAR AT THE LOCATIONS OF THE MONOPOLES $[\phi(x) = \Sigma c_a \phi^a(x)\ ;\ c_a = 0]$

- THE SET OF POINTS $\{x\ ;\ c_a(x) = 0\}$ HAS ZERO MEASURE

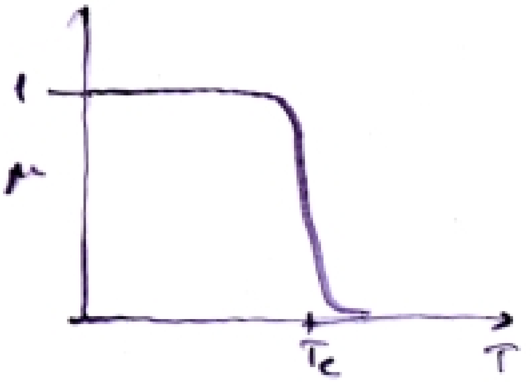
• NUMERICAL RESULTS

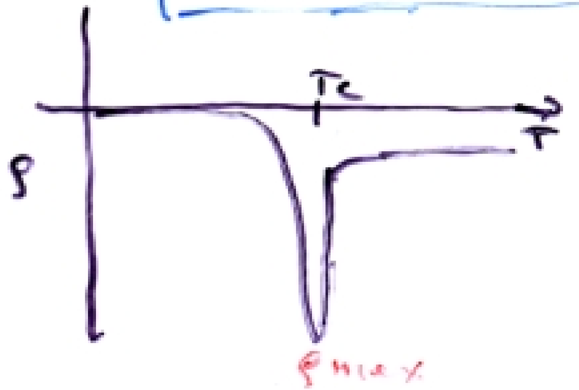
$$\langle \mu \rangle = \frac{\tilde{Z}}{Z} \implies \rho = \frac{d}{d\beta} \ln \langle \mu \rangle = \langle S \rangle_S - \langle \tilde{S} \rangle_{\tilde{S}}$$

A SUSCEPTIBILITY

$$Z = \int dM\, e^{-\beta S} \quad \tilde{Z} = \int dM\, e^{-\beta \tilde{S}}$$

$$\boxed{\langle \mu \rangle = \exp\left(\int_0^\beta \rho(\beta)\, d\beta \right)}$$





$V \to \infty$ [Lee Yang 52]

$T < T_c \qquad \rho \xrightarrow[V\to\infty]{} \bar{\rho}$ V INDEPENDENT [Fig 1 $N_f = 2$ $m_u = m_d = m_q$]

$T > T_c \qquad \rho \underset{V\to\infty}{\sim} -K V^{1/3} + \text{const}$

μ IS STRICTLY ZERO $T > T_c$ - SUPERSELECTION fig 2.

$T \sim T_c \qquad \langle \mu \rangle = \tau^\delta \phi\left(\frac{a}{\xi}, \frac{V^{1/3}}{\xi}, m_q V^\gamma\right) \qquad \tau = \left(1 - \frac{T}{T_c}\right)$

$\beta \rightleftarrows \xi \qquad \xi \underset{\tau\to 0}{\sim} \tau^{-\nu} \qquad \frac{a}{\xi} \simeq 0 \qquad (\beta, m_q, L_s)$

$\frac{N}{\xi} \rightleftarrows L_s^{1/\nu}\tau \qquad \langle \mu \rangle = \tau^\delta \tilde{\Phi}(\tau L_s^{1/\nu}, m_q L_s^{3\gamma})$

QUENCHED — NO m_q $\qquad \langle \mu \rangle = \tau^\delta f(\tau L_s^{1/\nu})$

$\implies \rho / L_s^{1/\nu} = F(\tau L_s^{1/\nu})$ FINITE SIZE SCALING

$\rho_{max} \simeq L_s^{1/\nu}$ $\quad \begin{cases} SU(2) & \nu = .62(1) \\ SU(3) & \nu = 1/3 \end{cases}$

XD, Grossi 2000

IN FULL QCD A TWO SCALE PROBLEM.

Bzz 10

γ KNOWN (KARSCH et AL) : KEEP $m_q L_s^{3\gamma}$ CONST

AGAIN

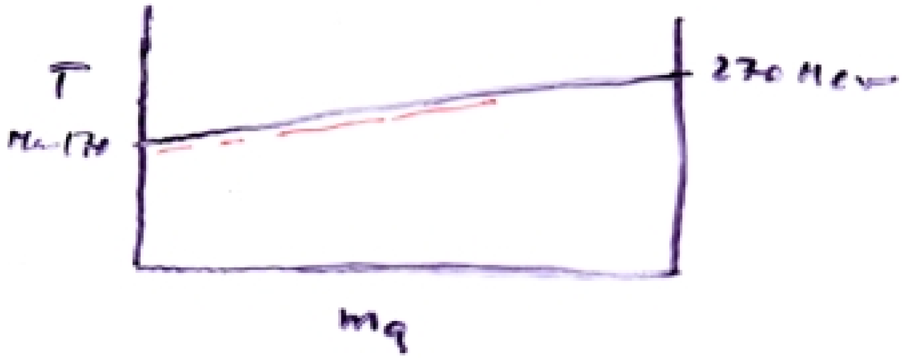
$$\rho / L_s^{1/\nu} = \phi(\tau L_s^{1/\nu}) \quad m_q L_s^{3\gamma} \text{ const}$$

$\Downarrow$

fig.'s

$\nu = 1/3$ FIRST ORDER TRANSITION

NOT A CROSSOVER



THIS RESULT IS STILL PRELIMINARY: A MORE DETAILED ANALYSIS ON THE WAY.

$\langle\mu\rangle$ IS THE DISORDER PARAMETER.

ITS SUSCEPTIBILITIES DETERMINE THE ORDER OF THE TRANSITION.

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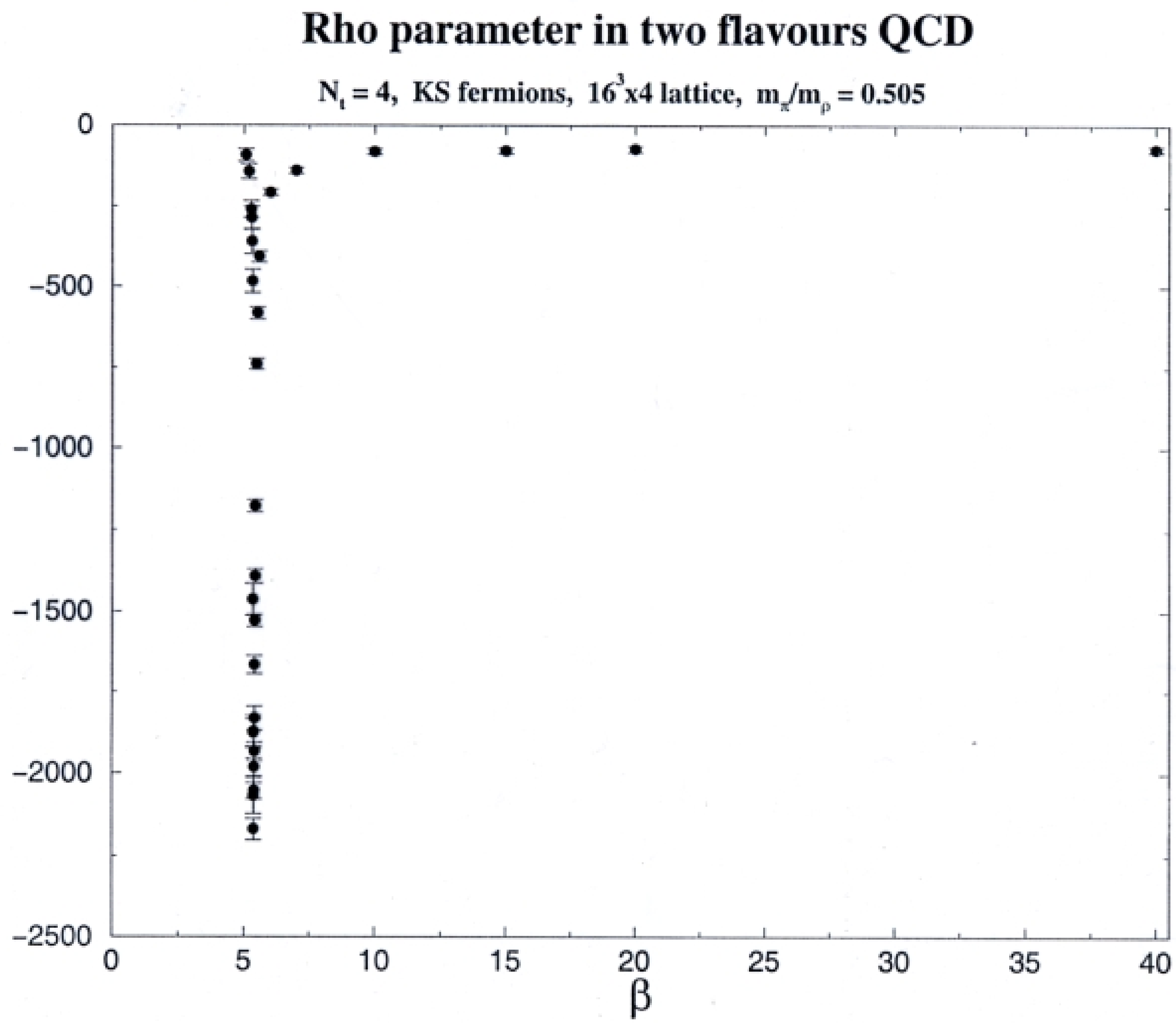
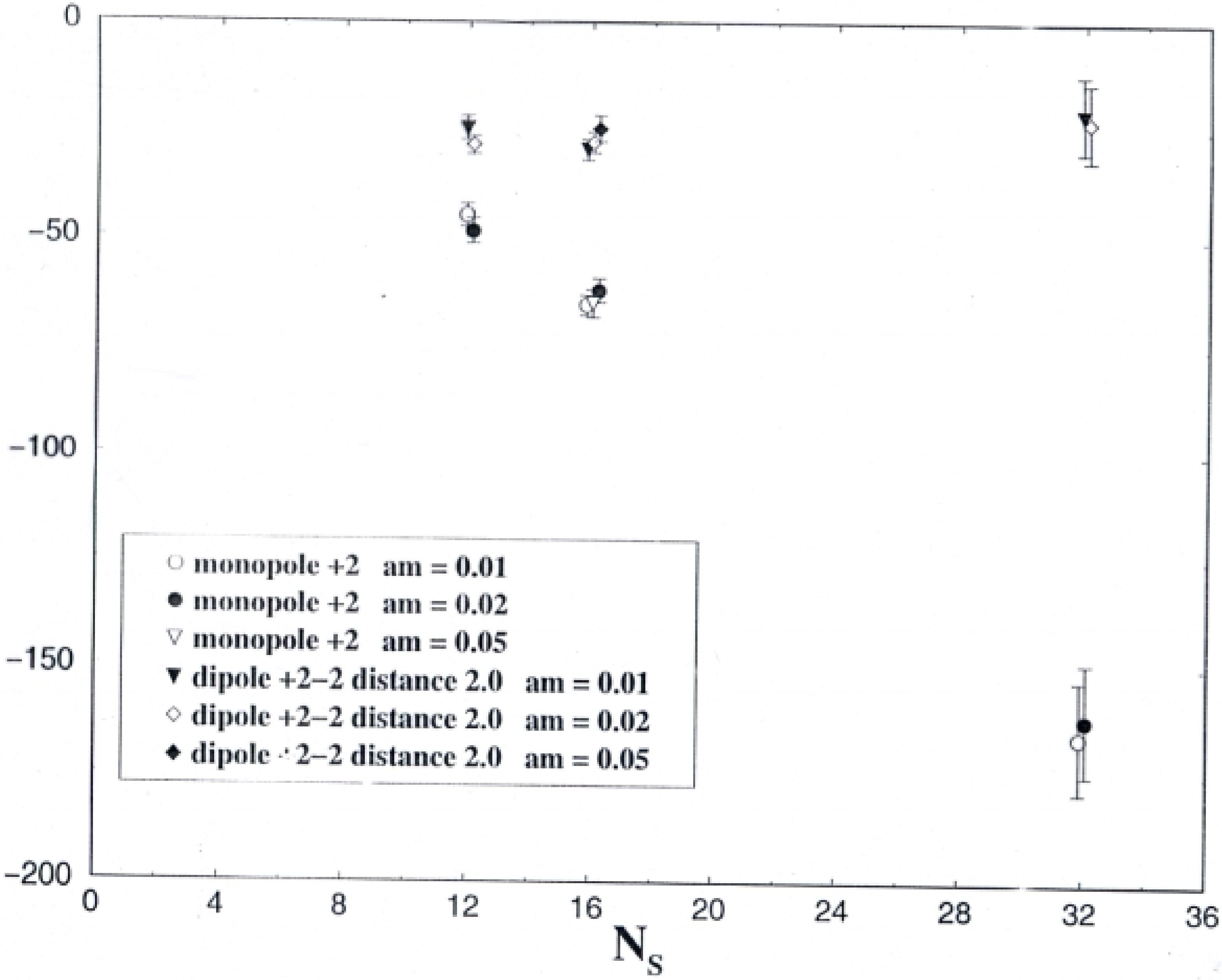

Rho parameter in two flavours QCD
$N_t = 4$, KS fermions, 16^3x4 lattice, $m_\pi/m_\rho = 0.505$
0
-500
-1000
-1500
-2000
-2500
0
5
10
15
20
25
30
35
40
β



0
-50
-100
-150
-200
0
4
8
12
16
20
24
28
32
36
N_S
monopole +2 am = 0.01
monopole +2 am = 0.02
monopole +2 am = 0.05
dipole +2−2 distance 2.0 am = 0.01
dipole +2−2 distance 2.0 am = 0.02
dipole ·2−2 distance 2.0 am = 0.05

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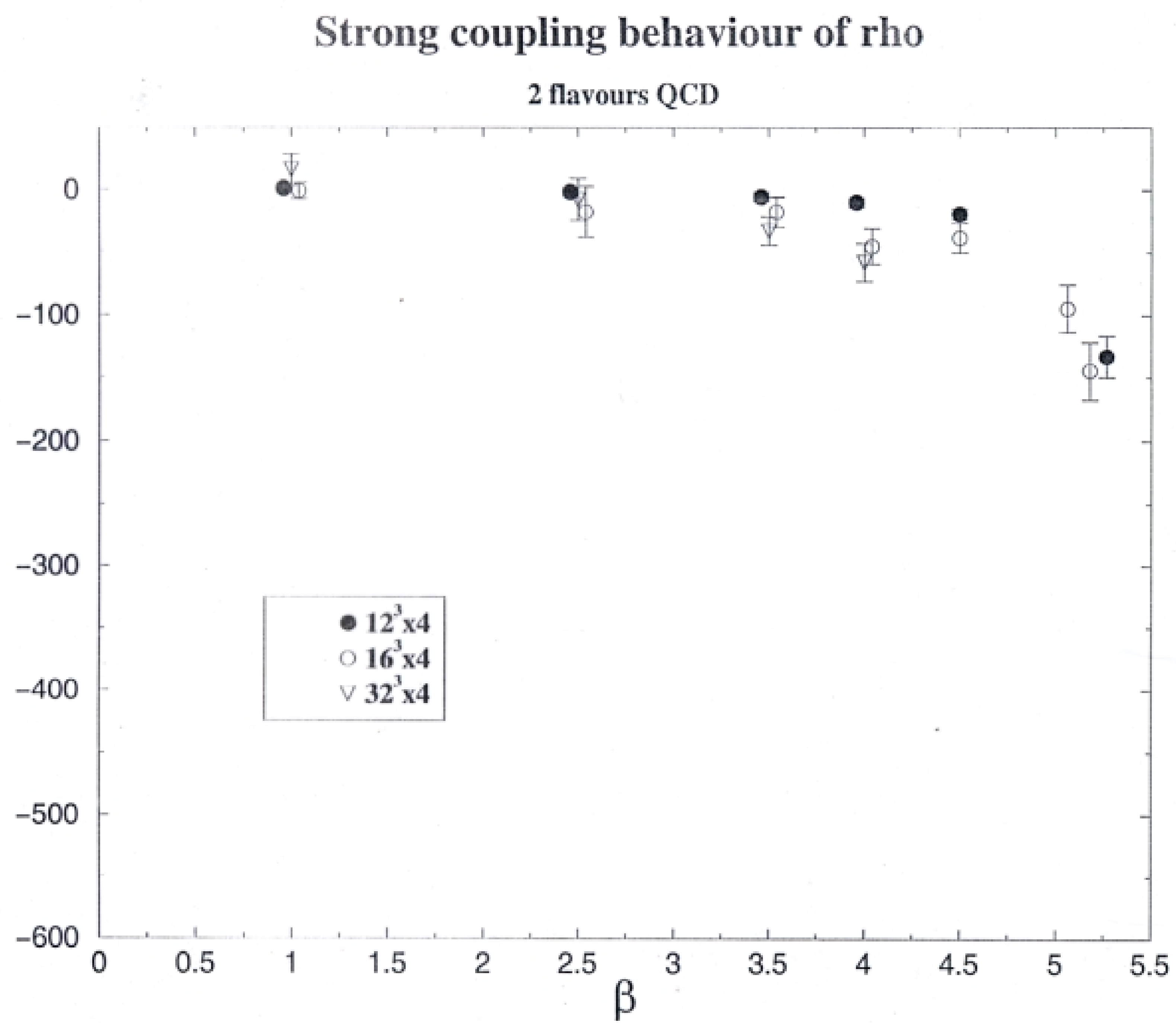
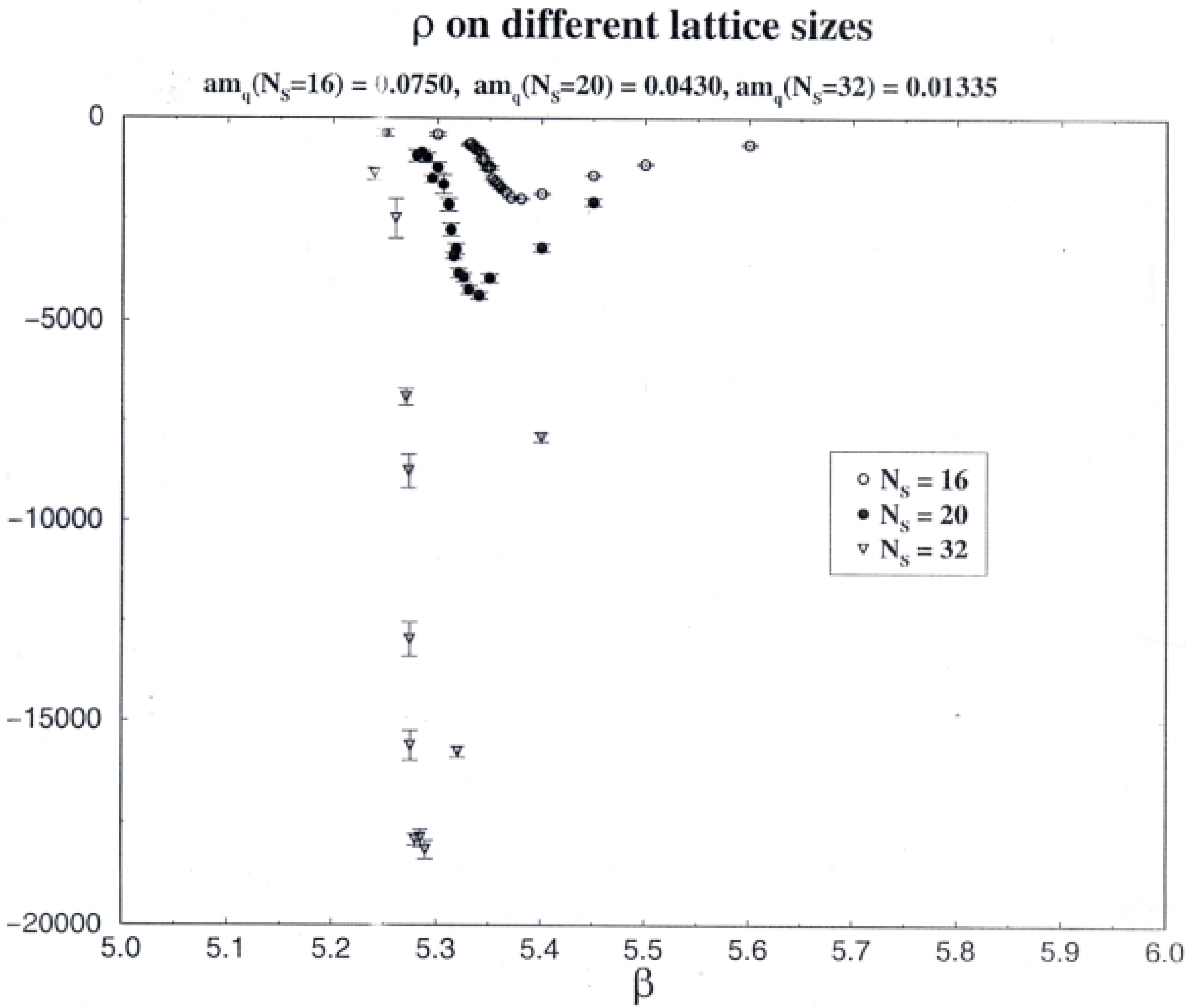
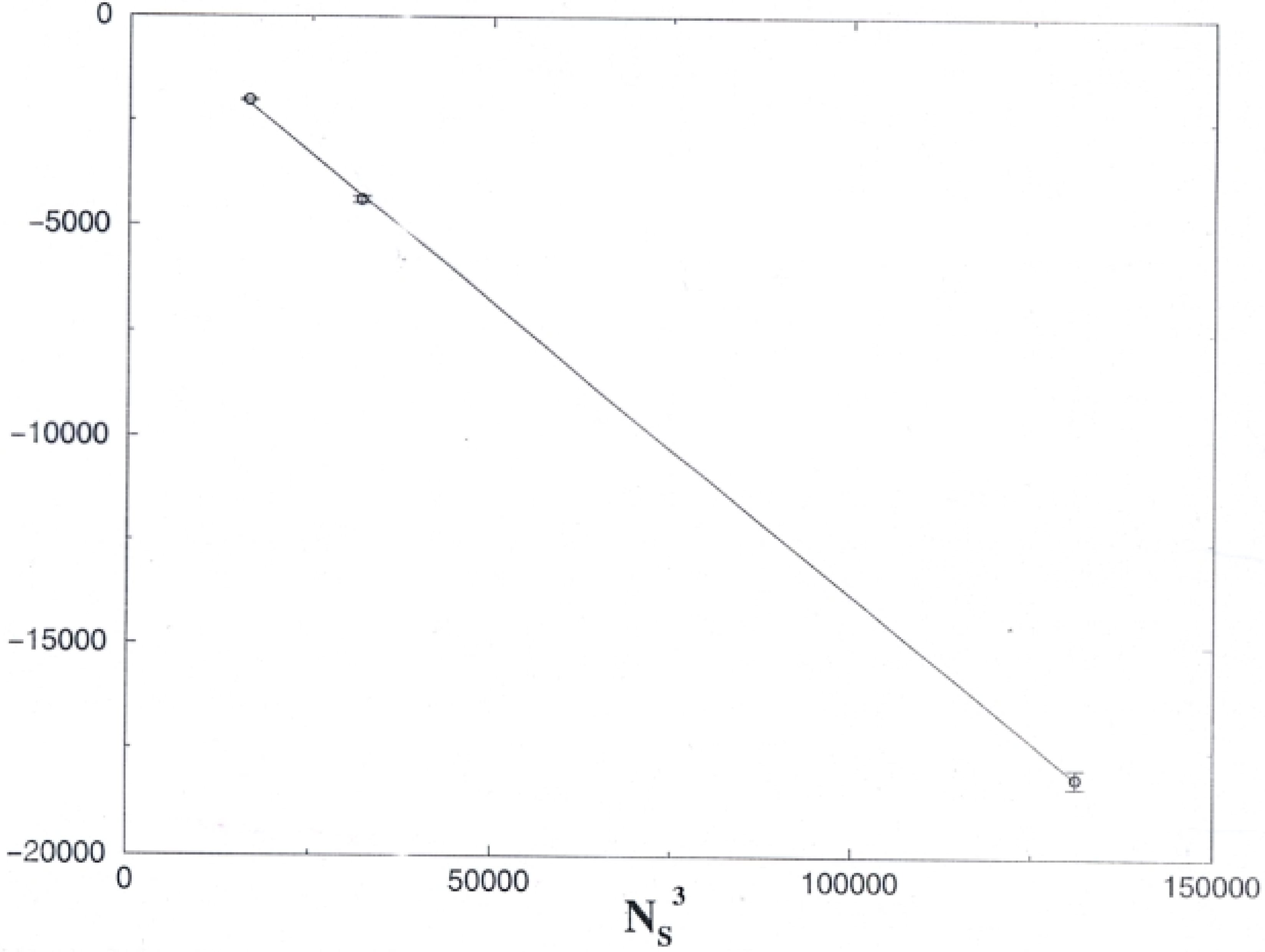


Strong coupling behaviour of rho
2 flavours QCD
0
−100
−200
−300
−400
−500
−600
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5
β
12³x4
16³x4
32³x4

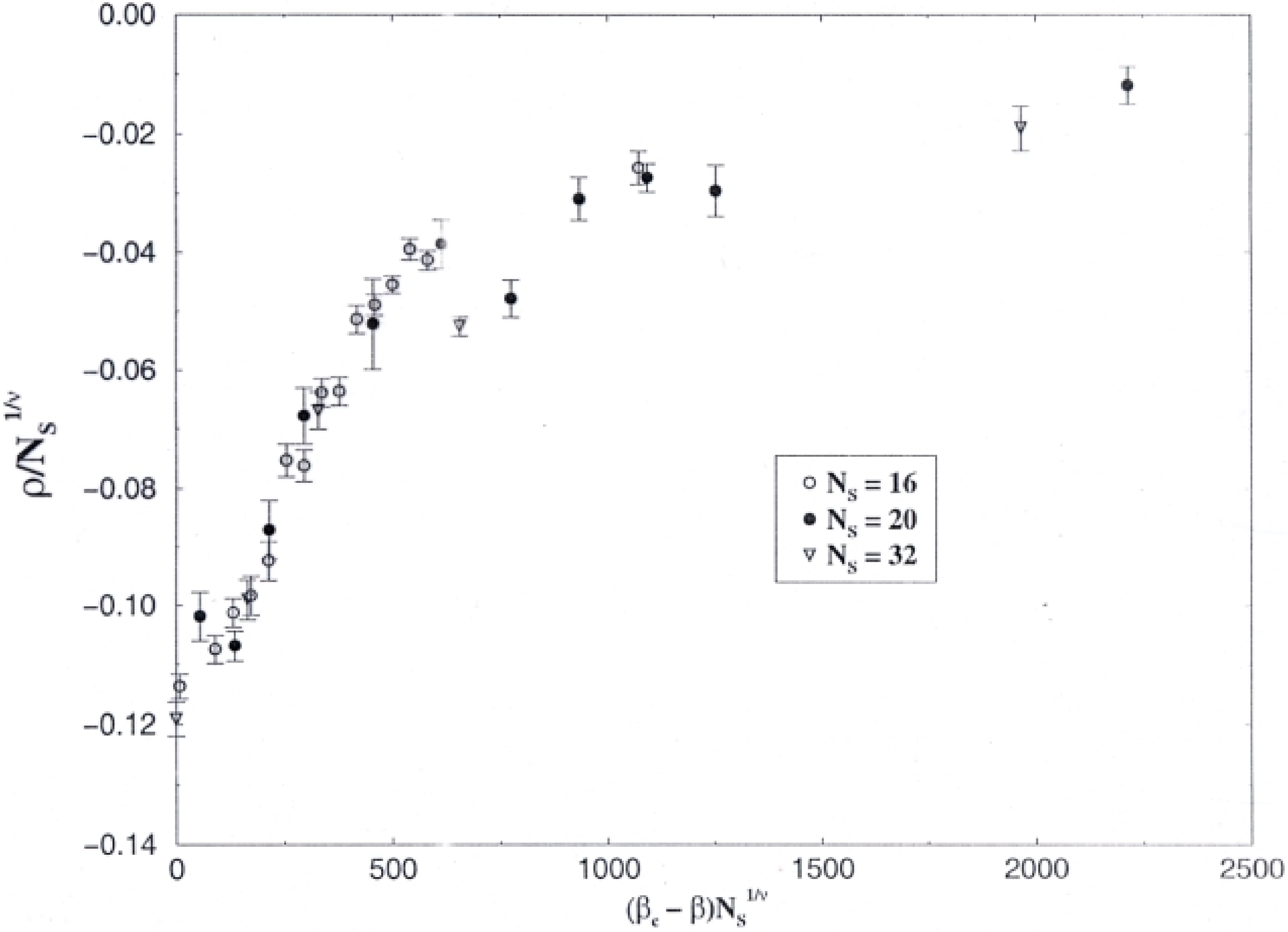

ρ on different lattice sizes
am_q(N_S=16) = 0.0750, am_q(N_S=20) = 0.0430, am_q(N_S=32) = 0.01335
0
-5000
-10000
-15000
-20000
5.0
5.1
5.2
5.3
5.4
5.5
5.6
5.7
5.8
5.9
6.0
β
N_S = 16
N_S = 20
N_S = 32

Scaling of the ρ peak

The continuous line is the result of a linear fit



Finite size scaling analysis with $\nu = 1/3$



CONCLUSIONS.

1) LATTICE RESULTS ! QCD VACUUM IS A DUAL SUPERCONDUCTOR FOR $T < T_c$, SUPERSELECTED MAGNETIC CHARGE FOR $T > T_c$, BOTH IN QUENCHED AND FULL QCD

2) STATEMENT INDEPENDENT ON THE CHOICE OF THE ABELIAN PROJECTION: ANALYTIC ARGUMENT & NUMERICAL CHECK

3). CORRECT INFORMATION ON THE ORDER OF THE PHASE TRANSITION

4.) THE DUAL EXCITATIONS OF QCD ARE STILL UNKNOWN. WE ONLY KNOW THAT THEY CARRY NON ZERO MAGNETIC CHARGE.