Critical Behavior of Frustrated Spin Systems With Non-Collinear Order

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P. Calabrese and PP, Phys. Rev. B 64, 184408 (2001);

P. Calabrese, PP, A. I. Sokolov, cond-mat/0205046;

P. Calabrese,E. V. Orlov, PP and A. I. Sokolov, cond-mat/0207187;

P. Calabrese, PP and A. I. Sokolov, in preparation;

PP, in preparation;

SMFT 2002

Bari, 26-29/09/2002

Outline

- What is frustration

- Why frustrated spin models are so interesting?

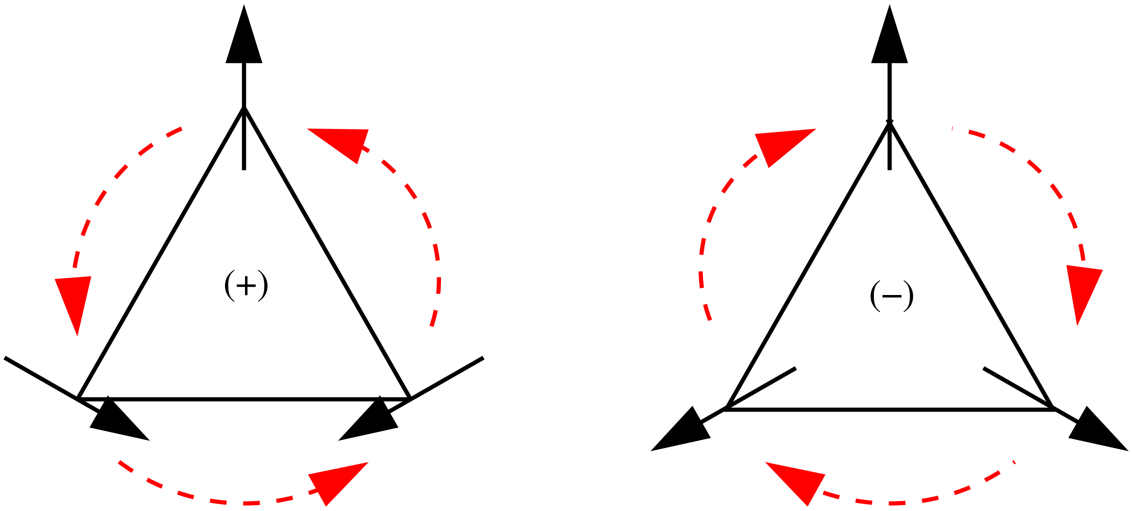
- Fixed dimension field theory

- 3 and 2-dimensional systems with planar and non-planar orderings

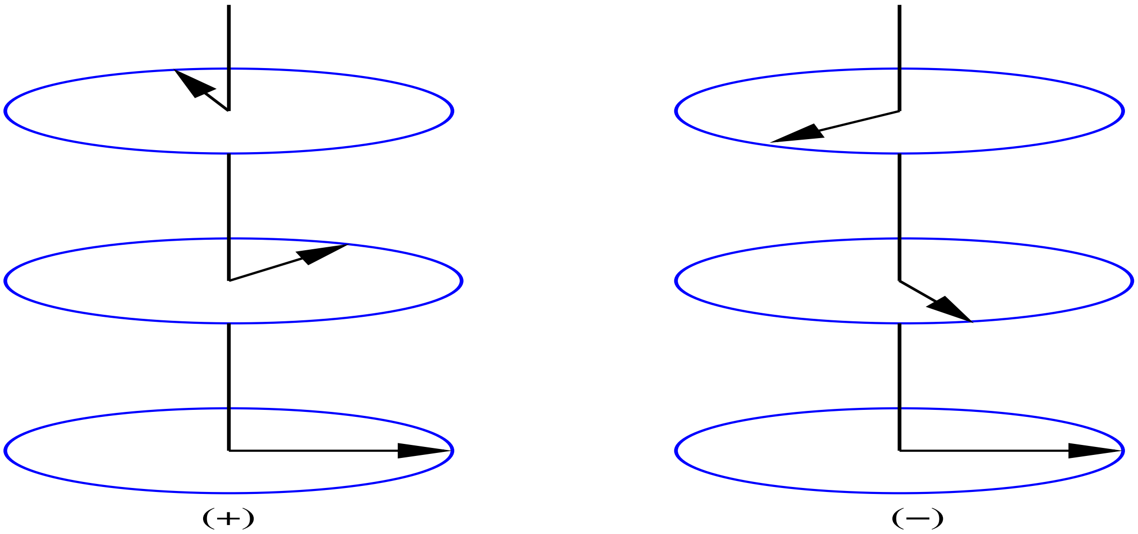
Frustration

Frustration can arise from:

- the special geometry of the lattice:



- the competition of different kinds of interactions:



A consequence of such spin structures is the appearance of a "chiral" degree of freedom

$$k_p = \frac{2}{3\sqrt{3}} \sum_{<ij>}^{p} (S_i^x S_j^y - S_i^y S_j^x)$$

Why frustrated spin models with non-collinear ordering are so interesting?

1. In 1980s the existence of a new continuous phase transition was conjectured [Kawamura].

2. There are many physical systems:

   - 3D: stacked-triangular antiferromagnets ($CsMnBr_3$, $CsVBr_3$), helimagnets (rare-earth metals Ho, Dy and Tb), etc..

   - 2D: triangular antiferromagnets, superconducting Josephson junctions array in a transverse magnetic field, etc..

3. In 3d experimental results are doubtful about a continuous phase transition and a weak first order one.

4. Field theoretical results are based on:

   - $O(n) \times O(m)$ effective LGW Hamiltonian:

     – fixed dimension expansion;

     – ϵ-expansion;

     – $1/n$ expansion;

     – ERG

   - Monte Carlo simulations

Effective $O(n) \times O(m)$ Landau-Ginzburg-Wilson Hamiltonian

$$\mathcal{H} = \int d^d x \left\{ \frac{1}{2} \sum_a \left[(\partial_\mu \phi_a)^2 + r \phi_a^2 \right] + \frac{1}{4!} u_0 \left(\sum_a \phi_a^2 \right)^2 \right.$$
$$\left. + \frac{1}{4!} v_0 \sum_{a,b} \left[(\phi_a \cdot \phi_b)^2 - \phi_a^2 \phi_b^2 \right] \right\} \qquad (1)$$

where ϕ_a $(1 \leq a \leq m)$ are m sets of n-component vectors. For $v_0 > 0, u_0 > 0$ the Hamiltonian (1) describes m-spin orderings in isotropic n-spin space[Kawamura-88]:

- $m = 2$, $n = 2, 3$ correspond to XY and Heisenberg frustrated spin systems which possess a non-collinear but planar ordering (STA, helimagnets, etc..)

- $m = 3$, $n = 3$ corresponds to systems with non-planar spin ordering (E.g. Pyrochlore antiferromagnets as FeF_3)

Fixed-Dimension Perturbative Expansion

The theory is renormalized by a set of zero-momentum conditions

$$\Gamma^{(2)}_{ai,bj}(p) = \delta_{ai,bj}\, Z_\phi^{-1}[m^2 + p^2 + O(p^4)]$$
$$\Gamma^{(4)}_{ai,bj,ck,dl}(0) = m Z_\phi^{-2}[\frac{u}{3} S_{ai,bj,ck,dl} + \frac{v}{6} C_{ai,bj,ck,dl}]$$
$$\Gamma^{(1,2)}_{ai,bj}(0) = \delta_{ai,bj}\, Z_t^{-1}$$

The fixed points of the theory are given by the common zeros of

$$\beta_u = m \left.\frac{\partial u}{\partial m}\right|_{u_0,v_0} \qquad \beta_v = m \left.\frac{\partial v}{\partial m}\right|_{u_0,v_0}$$

The stability properties are controlled by

$$\Omega = \begin{pmatrix} \frac{\partial \beta_u(u,v)}{\partial u} & \frac{\partial \beta_u(u,v)}{\partial v} \\ \frac{\partial \beta_v(u,v)}{\partial u} & \frac{\partial \beta_v(u,v)}{\partial v} \end{pmatrix}$$

A fixed point is stable if all the eigenvalues possess a positive real part.

Introducing

$$\eta_\phi(u,v) = \frac{\partial \log Z_\phi}{\log m}, \qquad \eta_t(u,v) = \frac{\partial \log Z_t}{\log m}$$

the critical exponents are derived

$$\eta = \eta_\phi(u^*, v^*) \qquad \nu = [2 - \eta_\phi(u^*, v^*) + \eta_t(u^*, v^*)]^{-1}$$

Resummation

The perturbative 6-loop (5-loop) series for RG functions in 3d (2d) are divergent but asymptotic

$$\Downarrow$$

A resummation procedure is needed to extract physical information

$$\Downarrow$$

Borel-Leroy transformation

$$+$$

Conformal Mapping for the analytical extension of the B.t.

This resummation procedure needs the knowledge of the singularity closest to the origin of the B.t., which is related to the large order behavior of the perturbative series:

$$\begin{aligned} \frac{1}{\overline{u}_b} &= -a\, R_{mn} \quad \text{for} \quad 0 < z < \frac{2m}{m-1}, \\ \frac{1}{\overline{u}_b} &= -a\, R_{mn} \left[1 - \left(1 - \frac{1}{m} \right) z \right] \text{for} \quad z < 0, \quad z > \frac{2m}{m-1}, \end{aligned}$$

where $z = \bar{v}/\bar{u}$, $a = 0.147777422$ in 3d and $a = 0.238659217$ in 2d.

[Calabrese,PP-01]

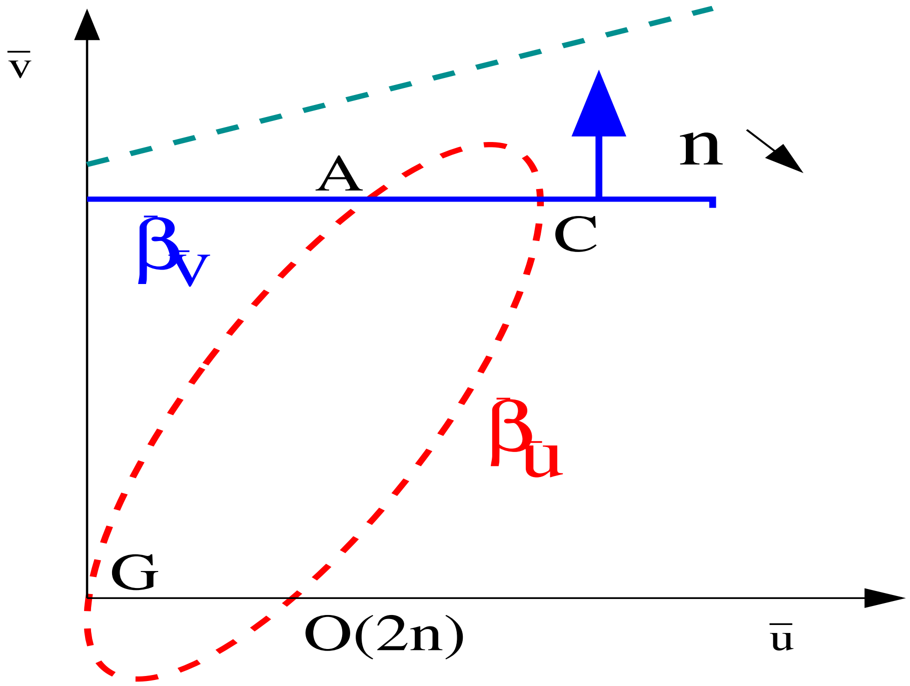
[Vicari,Pelissetto,Rossi-01]

$O(2) \times O(n)$ symmetry in 3d

- ϵ-expansion to $O(\epsilon^3)$: for $n < \bar{n} \sim 5.3$ no stable chiral fixed point exists $\rightarrow$ first order phase transition [Pelissetto,Rossi, Vicari-01];

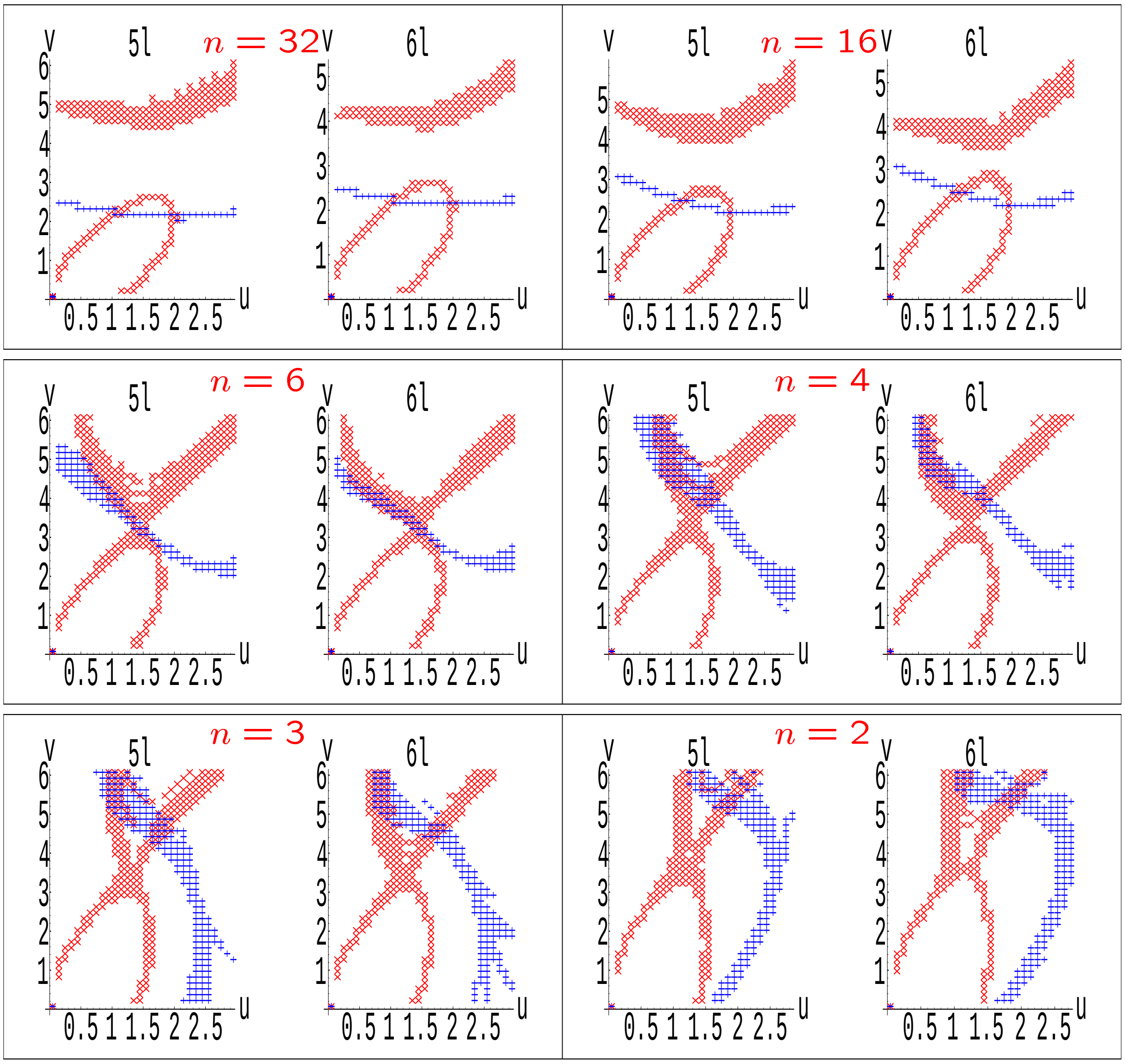
- ERG: for $n < \bar{n} \sim 4; 5$ no stable chiral fixed point exists $\rightarrow$ first order phase transition [Tissier,Mouhanna,Delamotte-00] .

For $n \rightarrow \infty$ the structure of the fixed points is:



When $n = \bar{n}$ the C. and A. fixed points merge, but it is not the end of the story!

3d 6-loop results



The lower branch of zeros of $\bar{\beta}_{\bar{u}}$ is connected with the $1/n$ results and with the ϵ-expansion ones;

$\bar{n}$ at which A. and C. merge is 6.4(6);

for $n = 2, 3$ a chiral fixed point exists [Pelissetto,Rossi,Vicari].

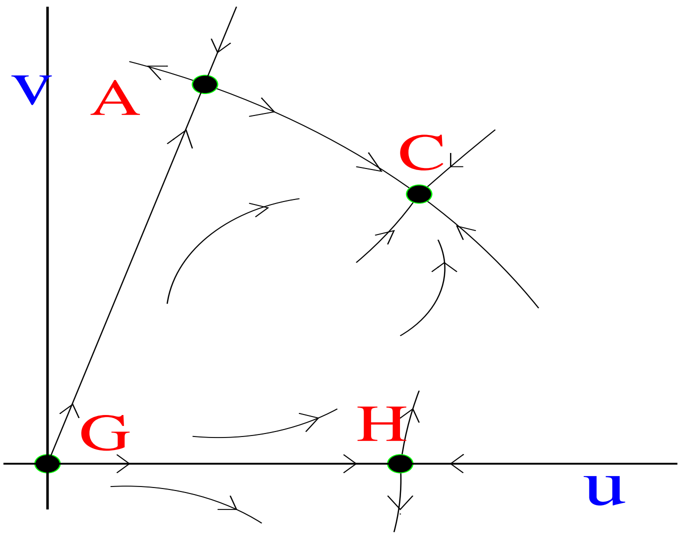
Stability of the chiral f.p. for $n = 2, 3$

For $n = 2, 3$ the stability eigenvalues possess a positive real part so the chiral fixed point is stable:

$$\omega_\pm = 1.70(40) \pm i0.90(40) \quad \text{for} \quad n = 2,$$

$$\omega_\pm = 1.00(20) \pm i0.80(25) \quad \text{for} \quad n = 3$$

The stability of the fixed point opens the possibility of a continuous phase transition if the bare parameters are in its region of attraction. If it is not the case the system will undergo a discontinuous phase transition

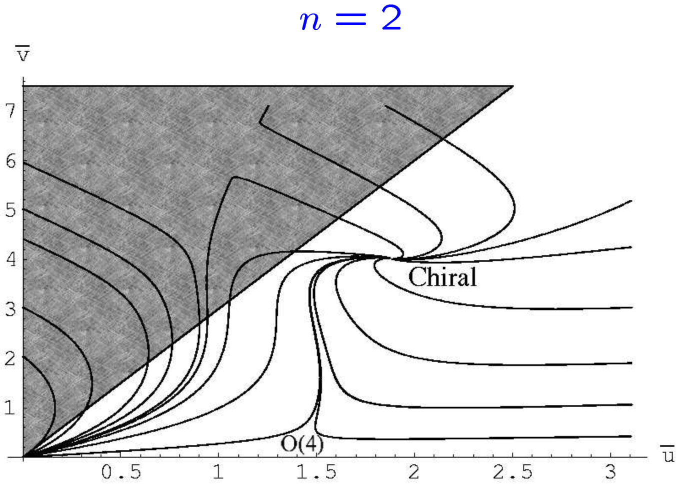


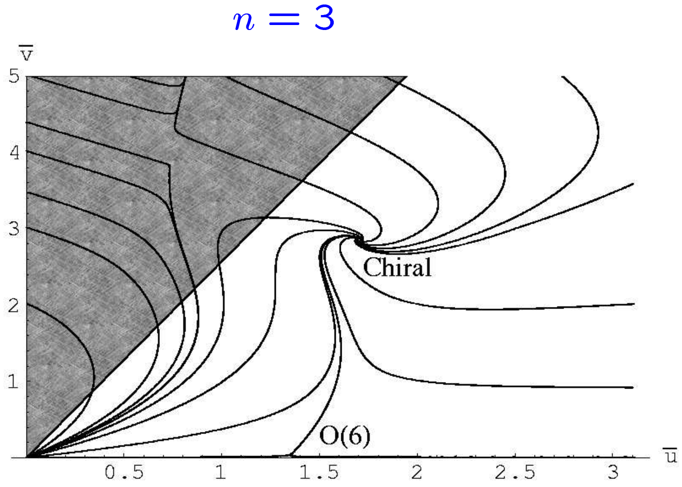
What about the complex part of the stability eigenvalues?

RG FLOW & CROSSOVER EFFECTS

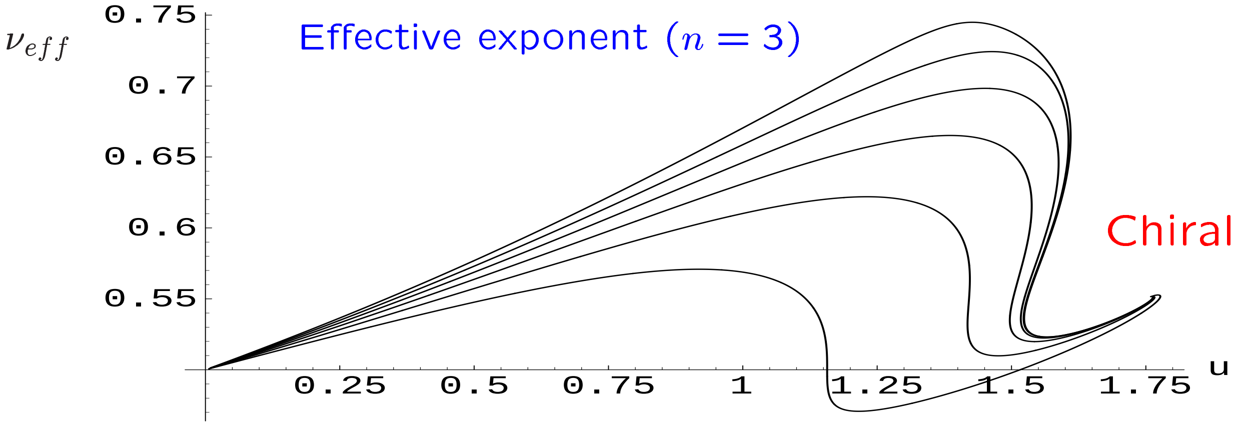
For $n = 2, 3$ the stability eigenvalues possess a non-vanishing imaginary part so the chiral f.p. is a stable focus.

Consequences





- The RG trajectories are spiral-like;
- This can be the reason of scattered results in experiments and Monte Carlo simulations;



$O(2) \times O(n)$ symmetry in 2d

(5-loop results)

- The structure of the fixed points with varying n is similar to the 3d one.

- A stable chiral fixed point exists both for $n = 2$ and $n = 3$ which is a focus with stability eigenvalues:

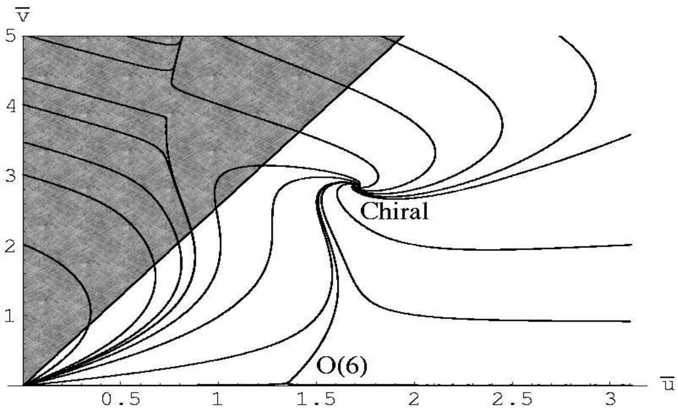
$$\omega_{\pm} = 2.05(35) \pm i0.80(55) \quad \text{for} \quad n = 2,$$

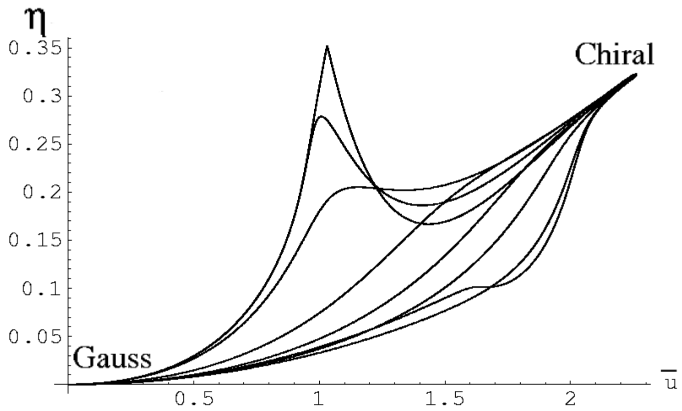
$$\omega_{\pm} = 1.55(25) \pm i0.55(35) \quad \text{for} \quad n = 3$$

$$\Downarrow$$

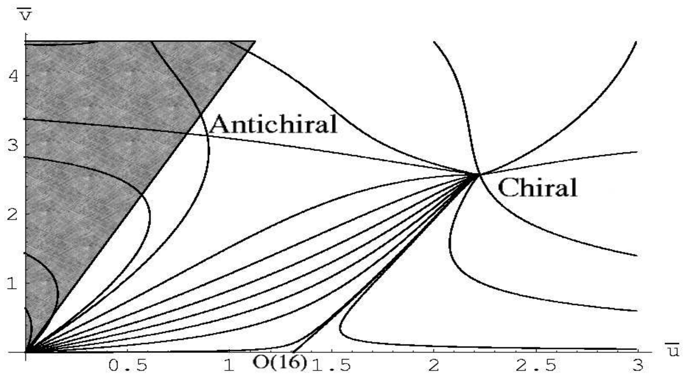
$\boxed{\text{A continuous phase transition is expected}}$

Again RG trajectories are spiral-like and can give rise to crossover effecs:





For $n > n_c$ the fixed point is not a focus, for example $n = 8$:



6-loop results vs $1/n$ expansion

- four fixed points exist: Gauss, O(2n), Antichiral & the stable Chiral
- good agreement found:

Ch. fixed point	$n = 16$	$n = 32$
C.M.(5loop)[1]	[1.963(12),2.2015(96)]	[2.000(4),2.1027(29)]
C.M.(6loop)[1]	[1.9682(59),2.2038(38)]	[2.0000(8),2.10283(57)]
Eigenvalues	$n = 16$	$n = 32$
C.M.(5loop)[1]	[0.8778(64),0.7025(134)]	[0.9307(31),0.8635(47)]
C.M.(6loop)[1]	[0.8758(38),0.7139(87)]	[0.9326(16),0.8682(23)]
$O(1/n)$[2]	[0.90, 0.80]	[0.95, 0.90]
ν	$n = 16$	$n = 32$
C.M. [1]	0.860(4)	0.935(1)
$O(1/n^2)$ [3]	0.885	0.946
γ	$n = 16$	$n = 32$
C.M.[1]	1.68(1)	1.825(2)
$O(1/n^2)$ [3]	1.75	1.88
η	$n = 16$	$n = 32$
C.M.[1]	0.0246(2)	0.01356(1)
$O(1/n^2)$ [3]	0.0245	0.01245

[1] [Calabrese,PP,Sokolov-02]

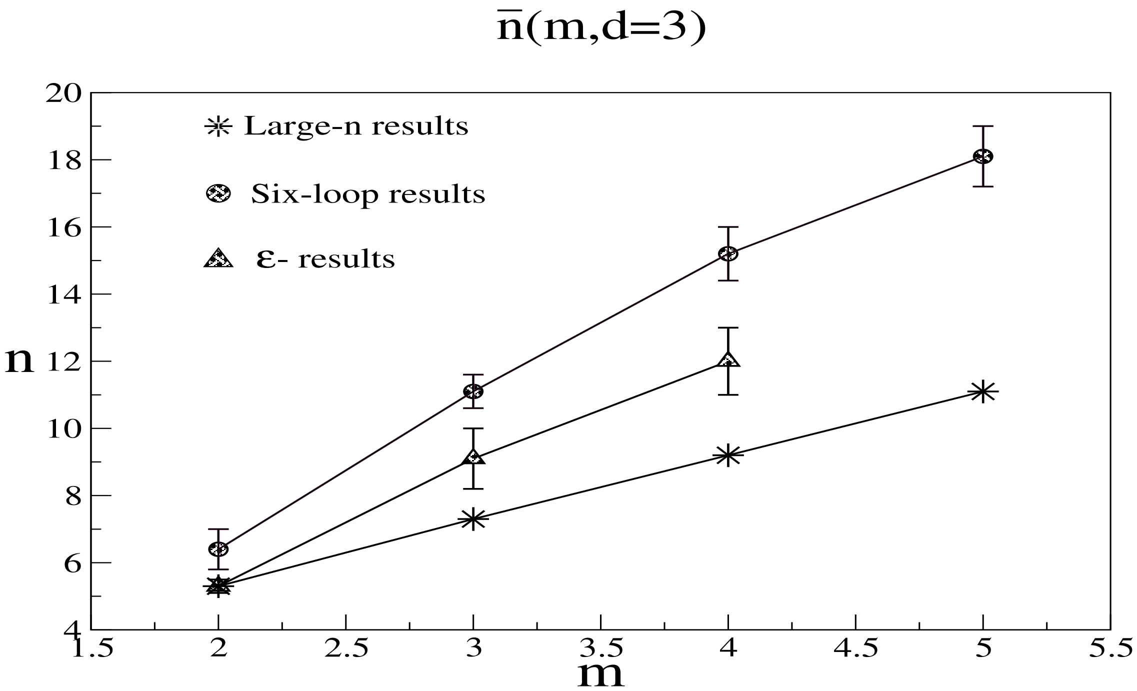
[2] [Gracey-02]

[3] [Pelissetto,Rossi,Vicari-01]

$O(n) \times O(m)$ symmetry

- $m = n = 3$, non-planar spin orderings: no stable fixed point is present so the system should undergo a first order phase transition.

- One can reconstruct the line $n = \bar{n}(m, d = 3)$ at which the chiral and antichiral fixed points merge which is important because for $n > \bar{n}(m, d = 3)$ the results are connected with the ones of the 1/n expansion and with the ones of the ϵ-expansion:



- The results in $1/n$ and ϵ-expansion are not stable with the perturbative order considered!

Conclusions

- Frustrated spin systems with planar ordering are expected to undergo a continuous phase transition with new critical exponents;

- the chiral fixed point is a stable focus both in 2 and 3 dimensions for physical systems.

- the nature of the stable fixed point can be an explanation of scattered results in experiments & numerical simulations.

- no stable fixed point is found for frustrated spin systems with non-planar orderings

- Large lattice size Monte Carlo simulations on the $O(n) \times O(m)$ Hamiltonian might give a definitive answer about the nature of the phase transitions for these models