Steady State of Random Resistor Networks Under Biased Percolation

Cecilia Pennetta

Eleonora Alfinito and Lino Reggiani

Dip. di Ingegneria dell'Innovazione, Universita' di Lecce and National Nanotechnology Laboratory of INFM, Italy

NNL National Nanotechnology Laboratory of I.N.F.M.

OUTLINE

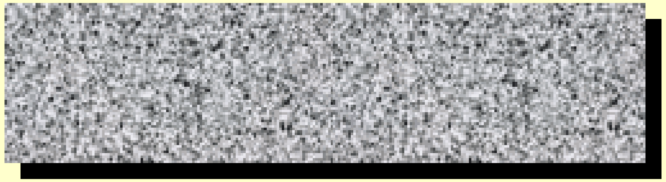
- **Aim of the work**
- **Model**
- **Results**
- **Conclusions and open questions**

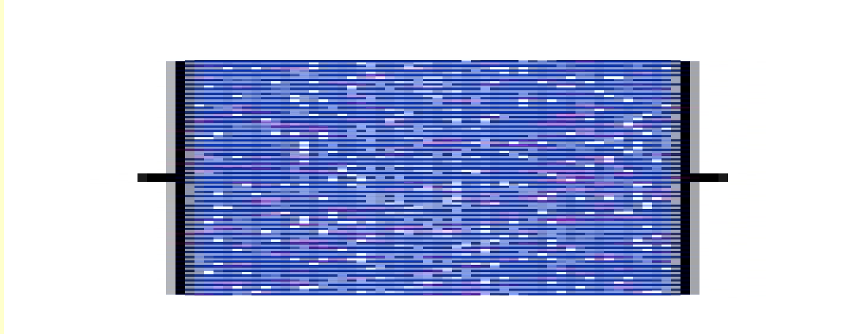
AIM OF THE WORK

- Study the electrical conduction of disordered materials over the full range of the applied stress
- Identify the failure precursors and predict electrical breakdown phenomena
- Investigate the stability of the electrical properties and electrical breakdown phenomena in conductor-insulator composites, in granular metals and in nanostructured materials

MODEL

THIN FILM OF RESISTANCE R → 2 –DIM SQUARE LATTICE RANDOM RESISTOR NETWORK





R = network resistance

I = stress current (d.c.), kept constant

T_0 = thermal bath temperature

α = temperature coeff. of the resistance

n-th resistor : $r_{reg}(T_n) = r_0 [1 + \alpha (T_n - T_{ref})]$

BIASED PERCOLATION MODEL (Gingl et all, 1996; Pennetta et all, 1999)

$$T_n = T_0 + A[\, r_n i_n^2 + (B/N_{neig}) \sum_m (r_{m,n}\, i_{m,n}^2 - r_n i_n^2)]$$

COMPETING PROCESSES :

- defect generation
- defect recovery

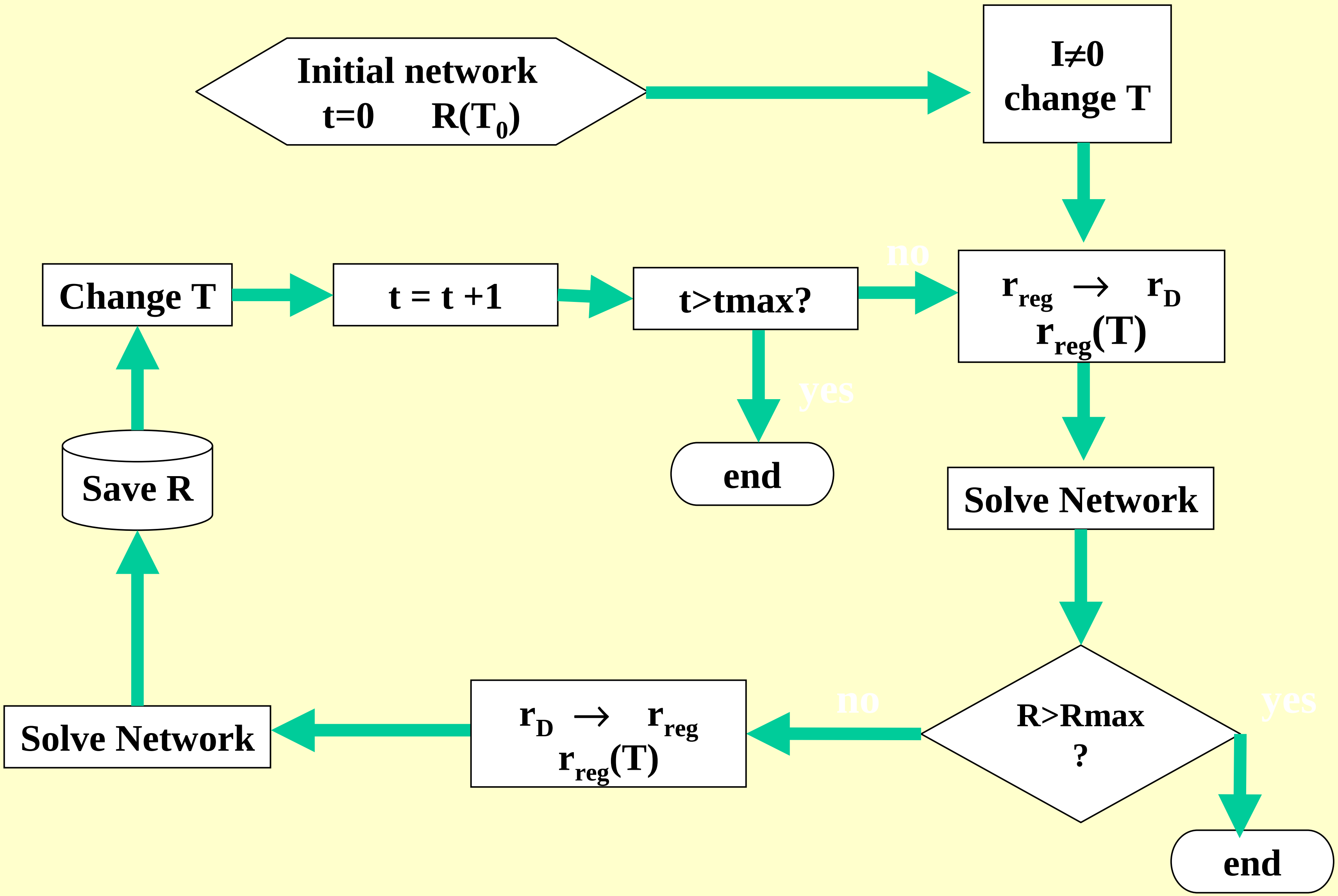
$r_{reg} \rightarrow r_D$ $W_D = \exp[-E_D/k_B T_n]$

$r_D \rightarrow r_{reg}$ $W_R = \exp[-E_R/k_B T_n]$

STEADY STATE
<p> , <R>

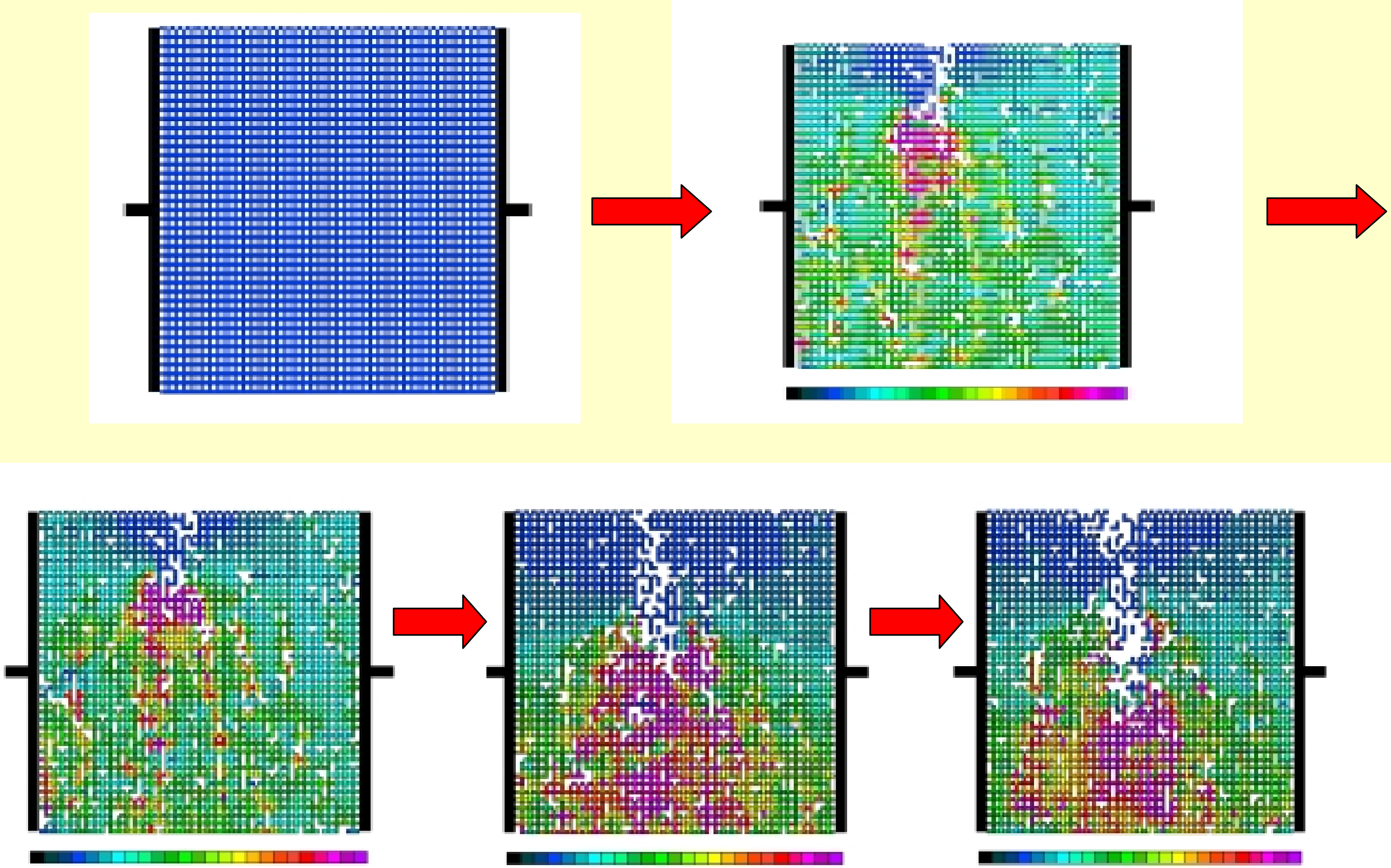
IRREVERSIBLE BREAKDOWN

Flow Chart of Computations

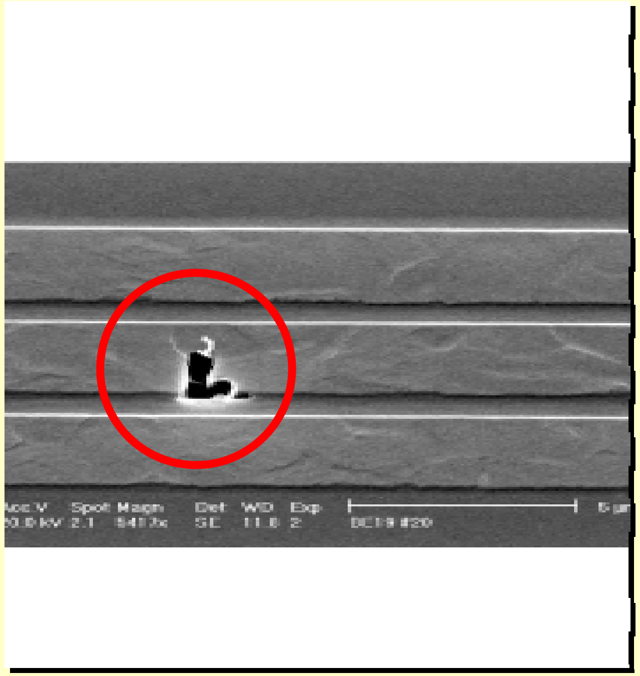


RESULTS

Network evolution for the irreversible breakdown case



Observed electromigration damage pattern



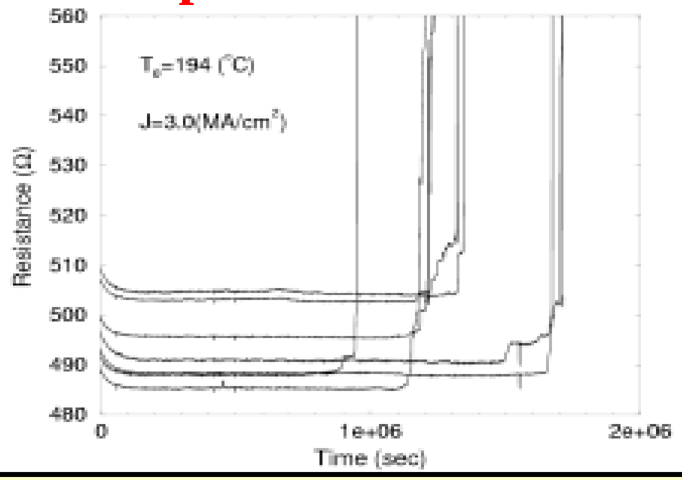
SEM image of electromigration damage in Al-Cu interconnects

- Granular structure of the material
- Atomic transport through grain boundaries dominates
- Transport within the grain bulk is negligeable
- Film: network of interconnected grain boundaries

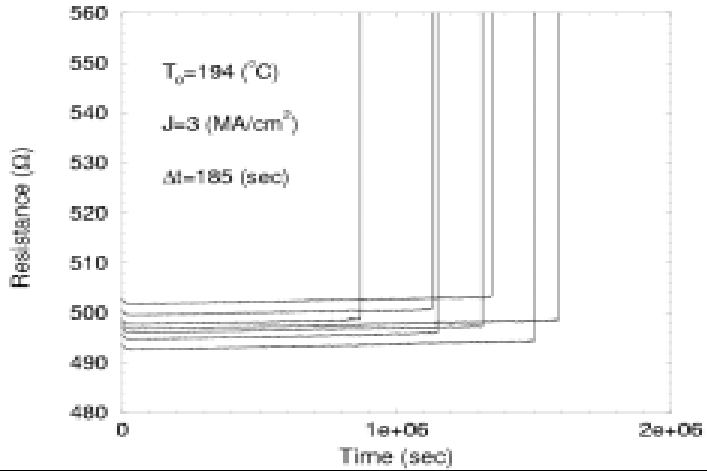
Experiments and Simulations

Evolution and TTF

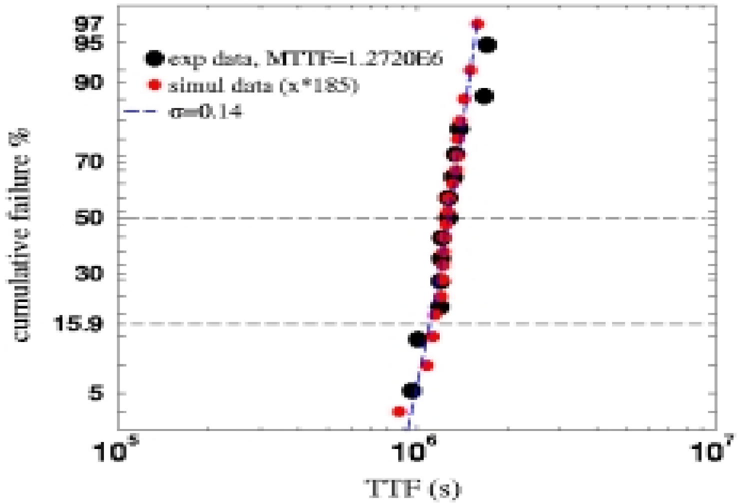
Experimental failure



Simulated Failure



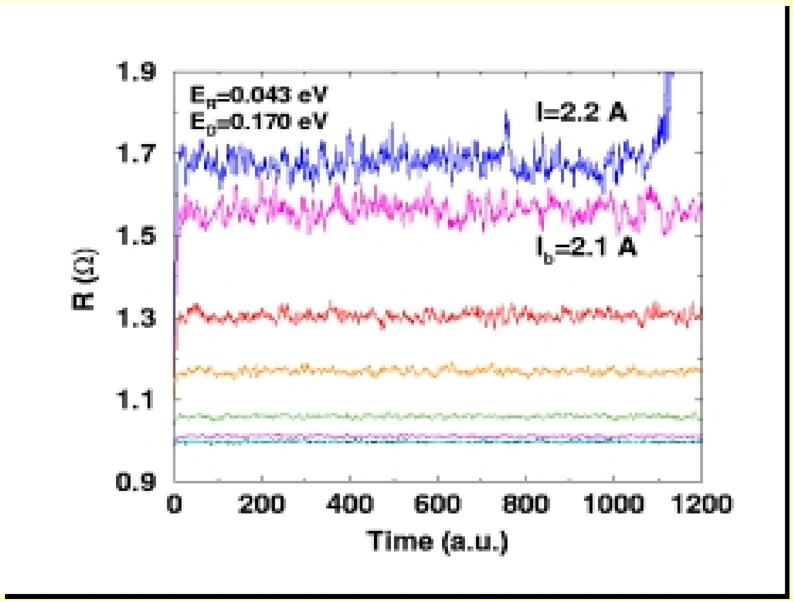
Lognormal Distribution



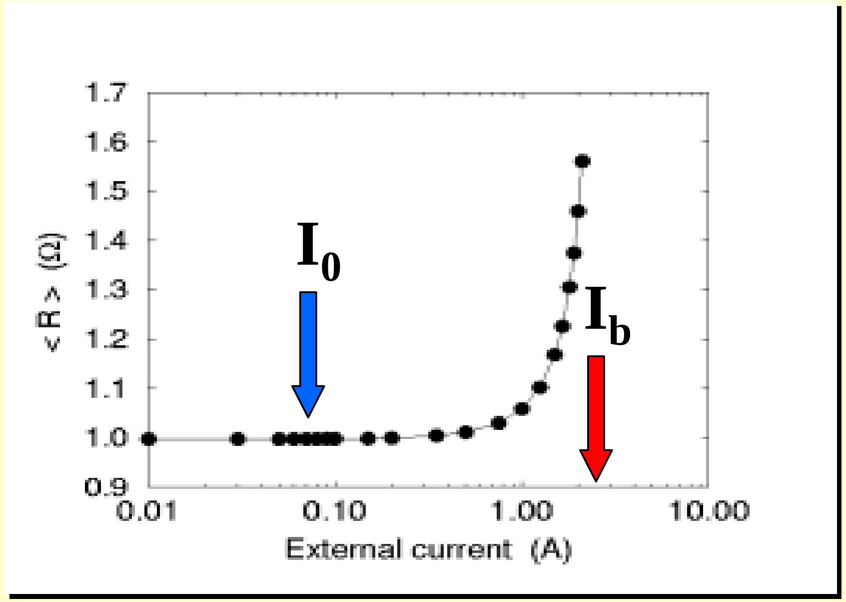
Tests under accelerated conditions

Qualitative and quantitative agreement

Resistance evolution:

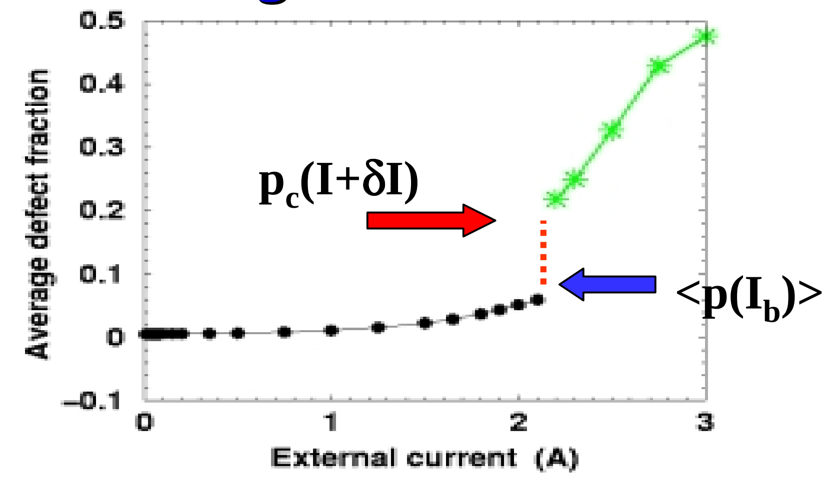


Average resistance:

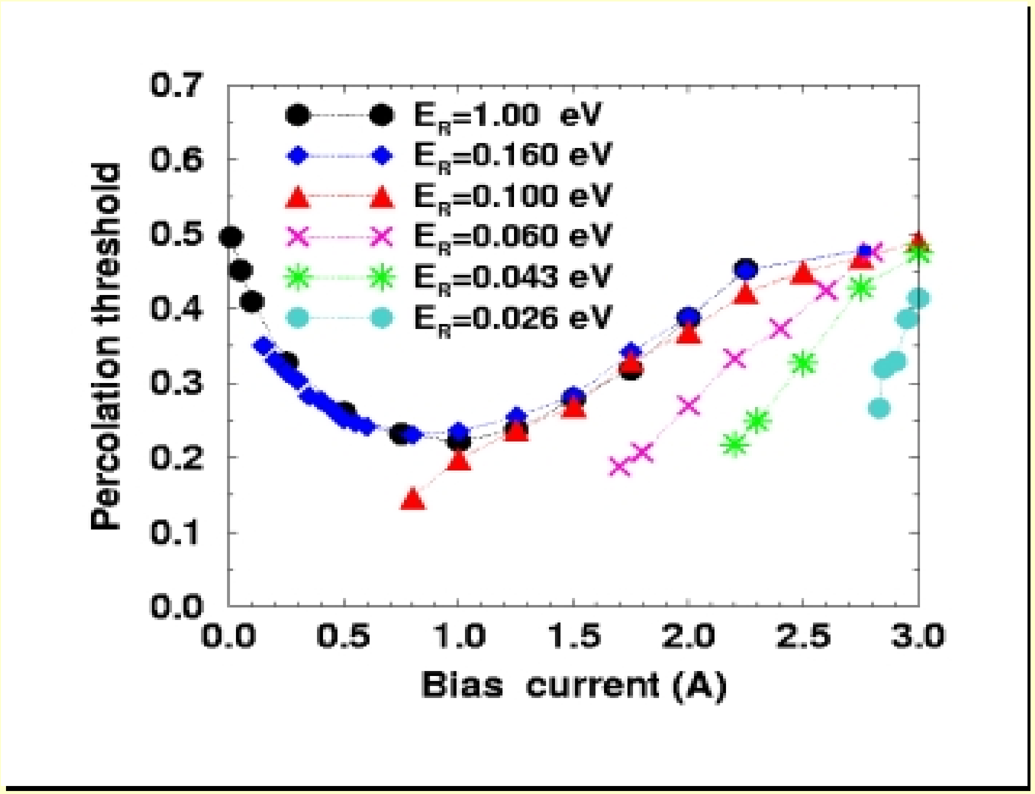


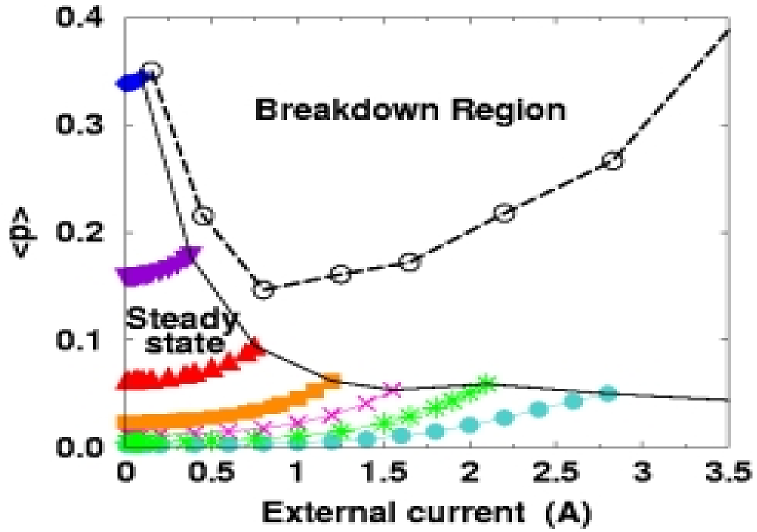
BREAKDOWN:
FIRST ORDER TRANSITION

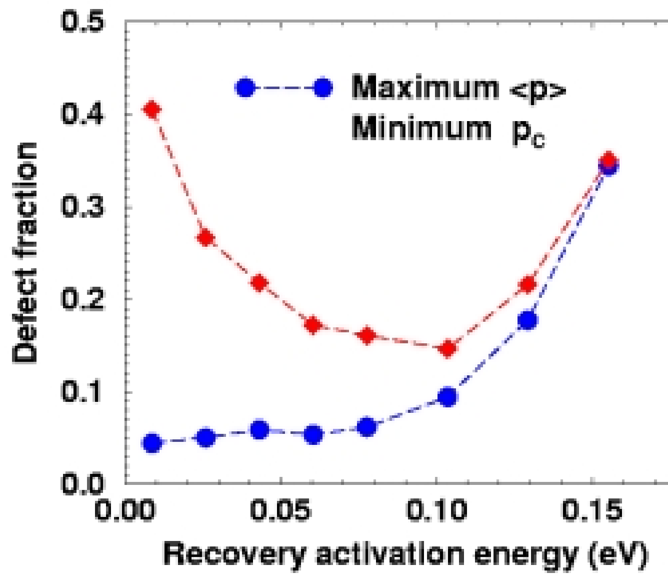
Average defect fraction:



p_c depends on the bias and on E_R

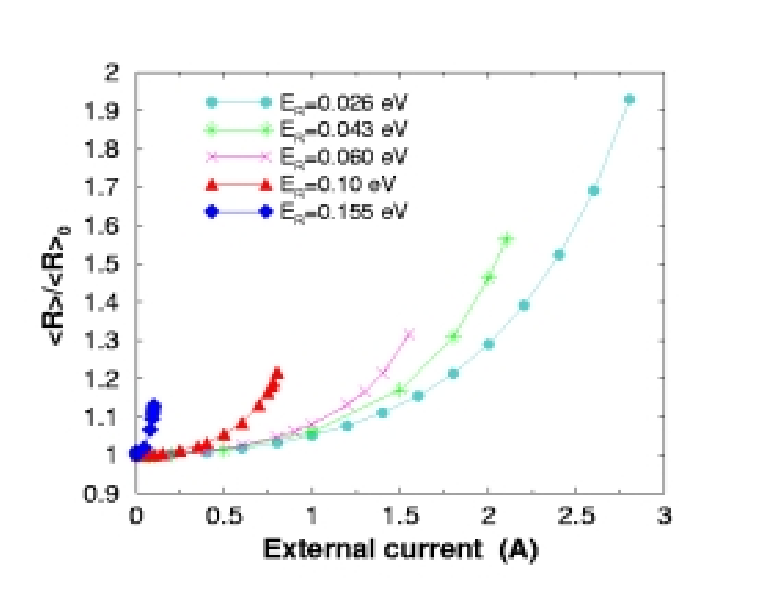




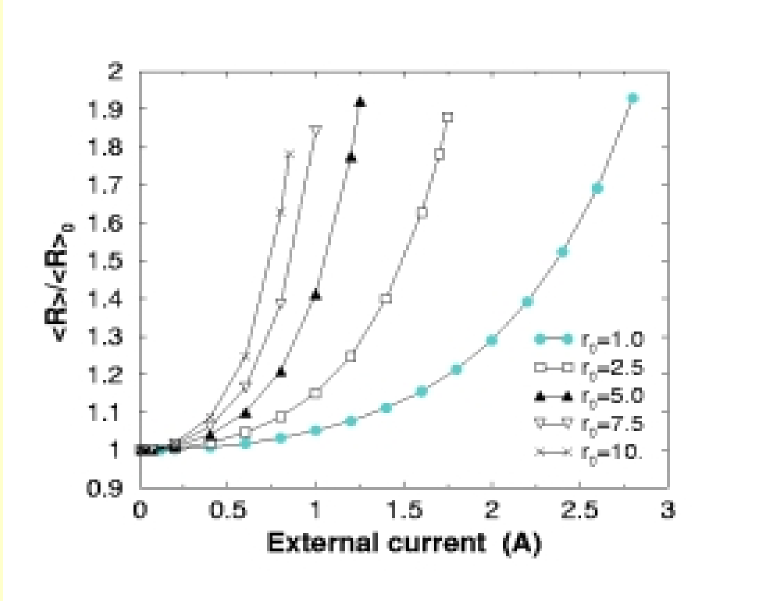


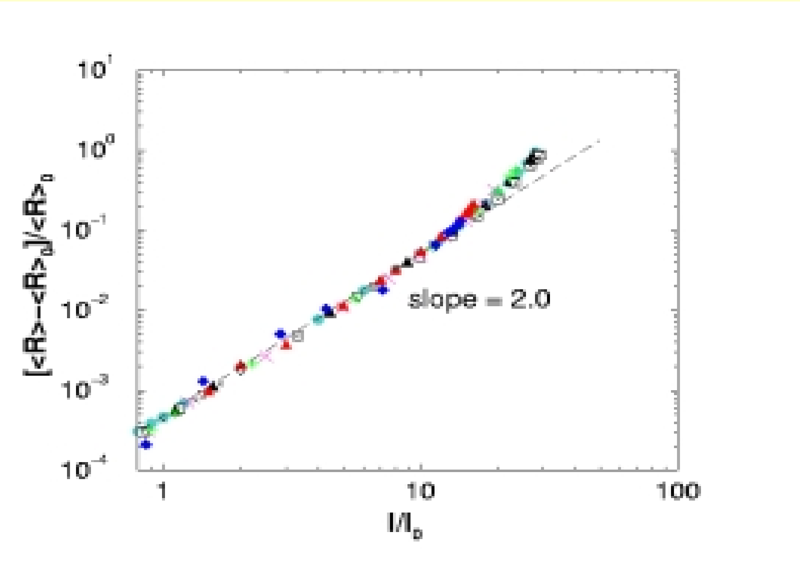
- In general: $\langle p \rangle_b \neq p_c$
- at increasing values of E_R (near the stability region) $\langle p \rangle_b \rightarrow p_c$

Effect of the recovery energy:



Effect of the initial film resistance:

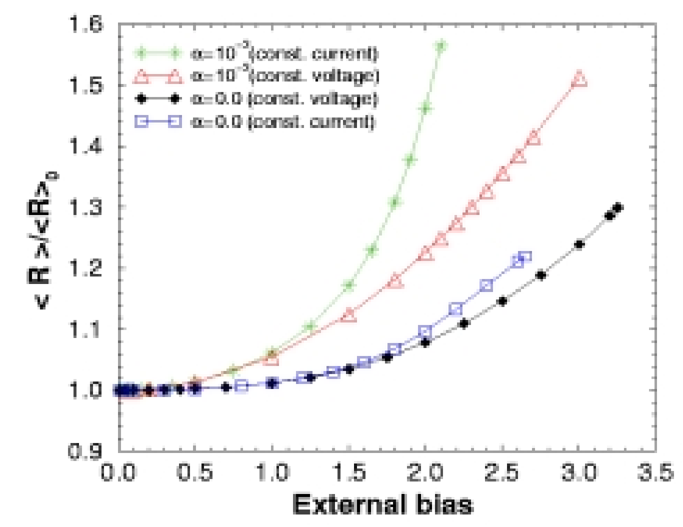


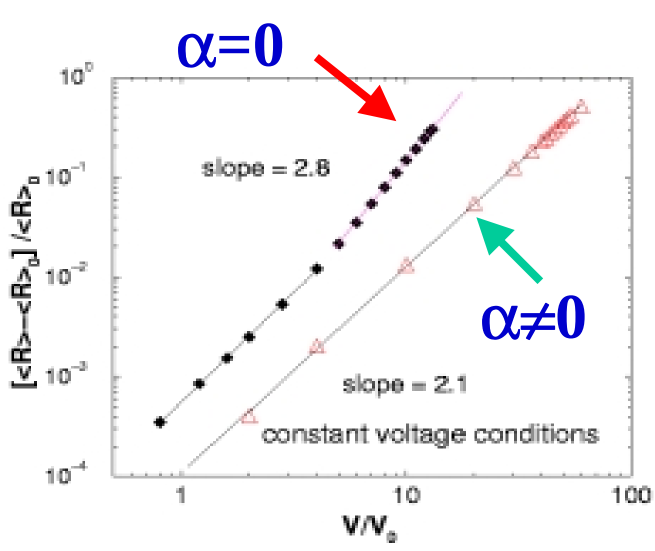


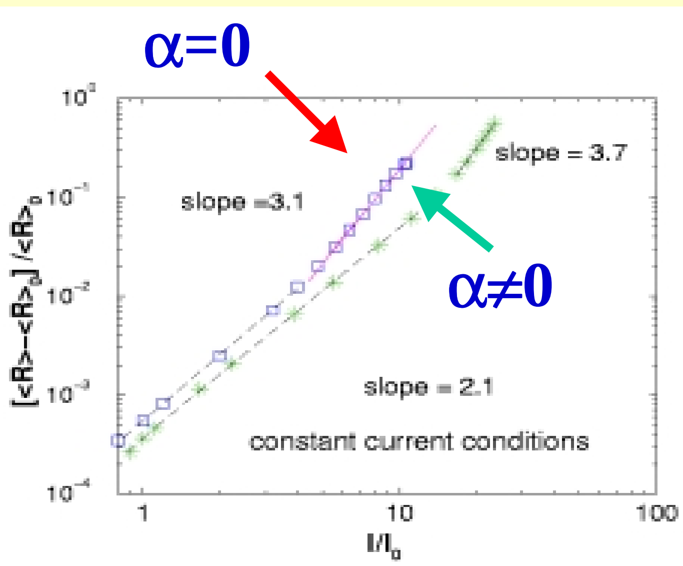
$$\frac{<R>}{<R>_0} = g\left(\frac{I}{I_0}\right) \qquad g(I/I_0) \approx 1 + a(I/I_0)^{\vartheta}$$

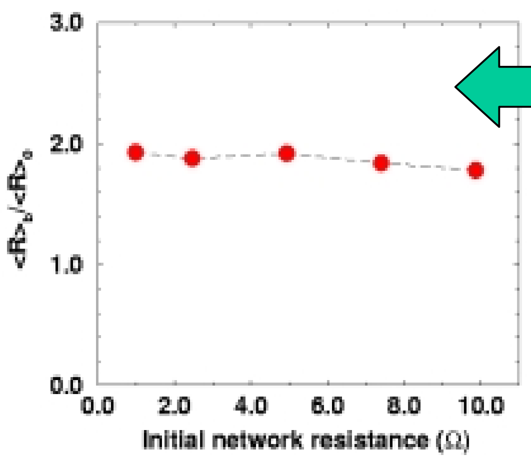
In the pre-breakdown region: $\theta_I = 3.7 \pm 0.3$

Effect on the average resistance of the bias conditions (constant voltage or constant current) and of the temperature coefficient of the resistance α





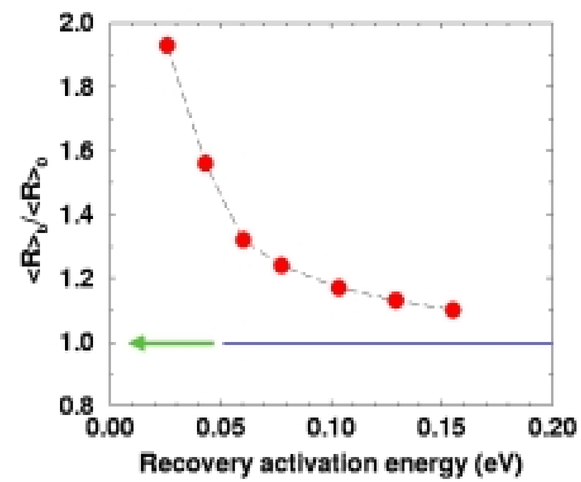




We have found that $Y \equiv \frac{<R>_b}{<R>_0}$ **is:**

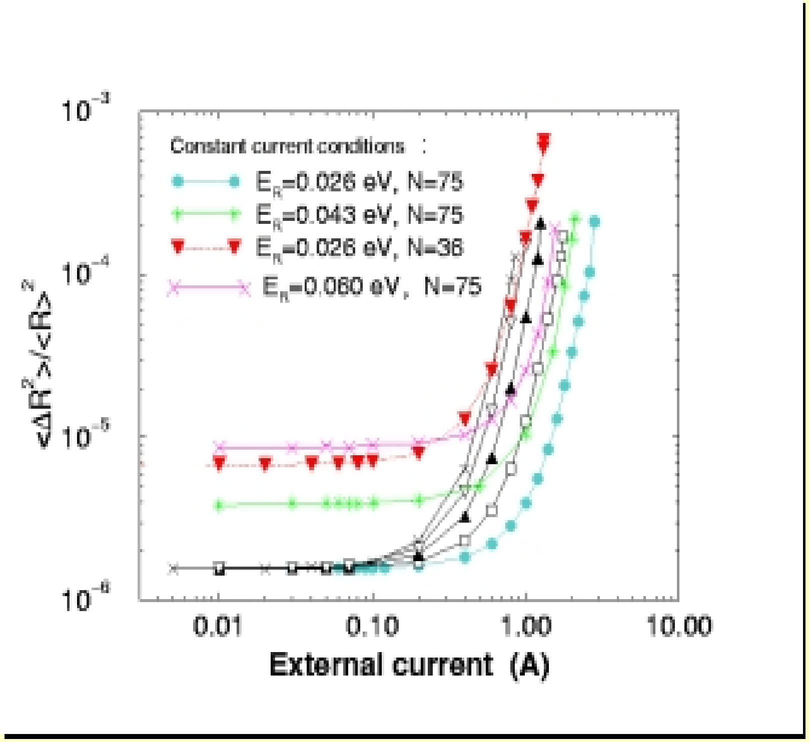
- **independent on the initial resistance of the film**
- **independent on the bias conditions**
- **dependent on the temperature coef. of the resistance**
- **dependent on the recovery activation energy**

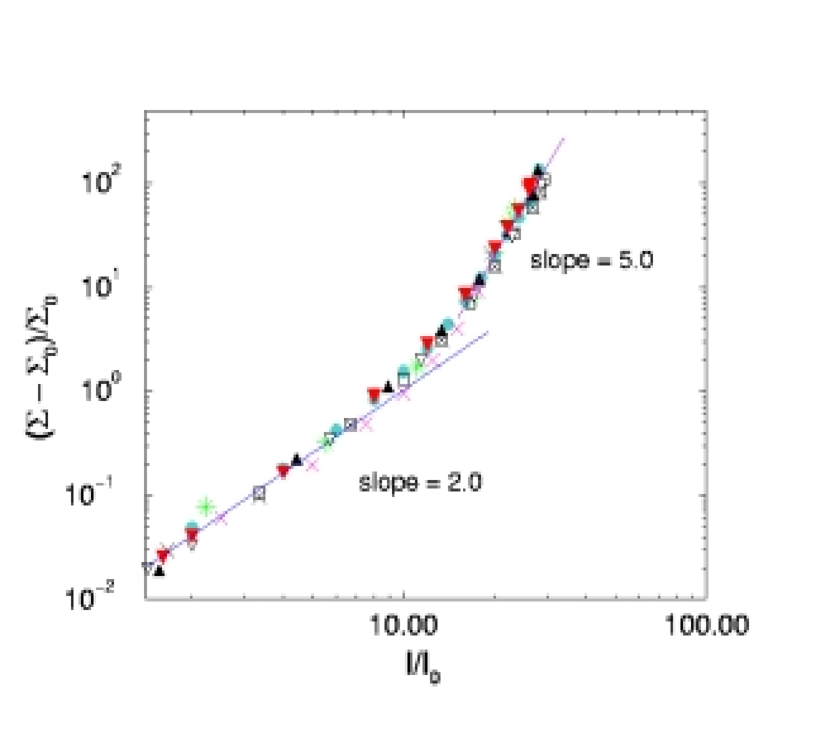
Y = 1.85 ± 0.08



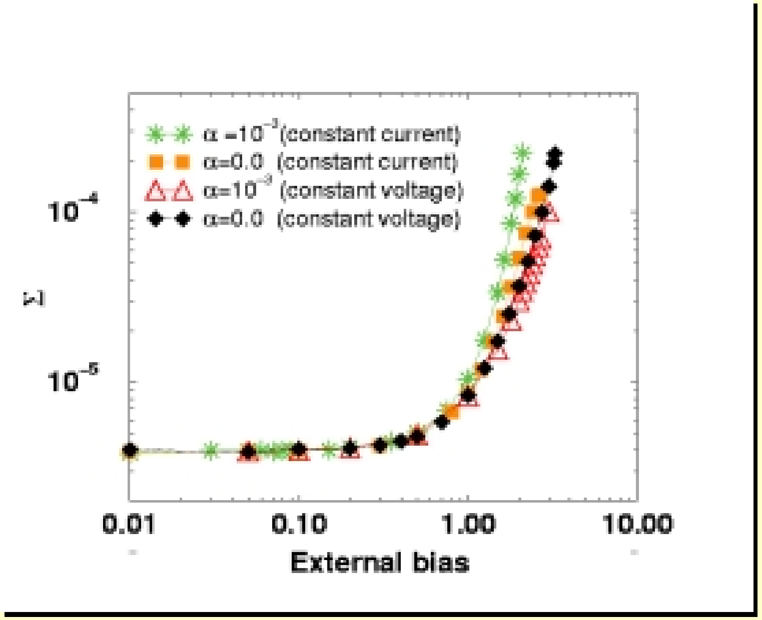
All these features are in good agreements with electrical measurements up to breakdown in carbon high-density polyethylene composites
(K.K. Bardhan, PRL, 1999)

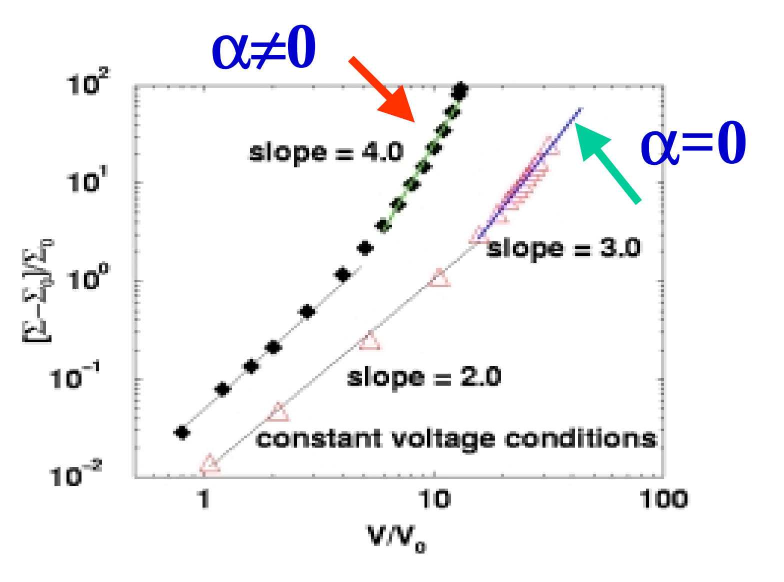
Relative variance of resistance fluctuations

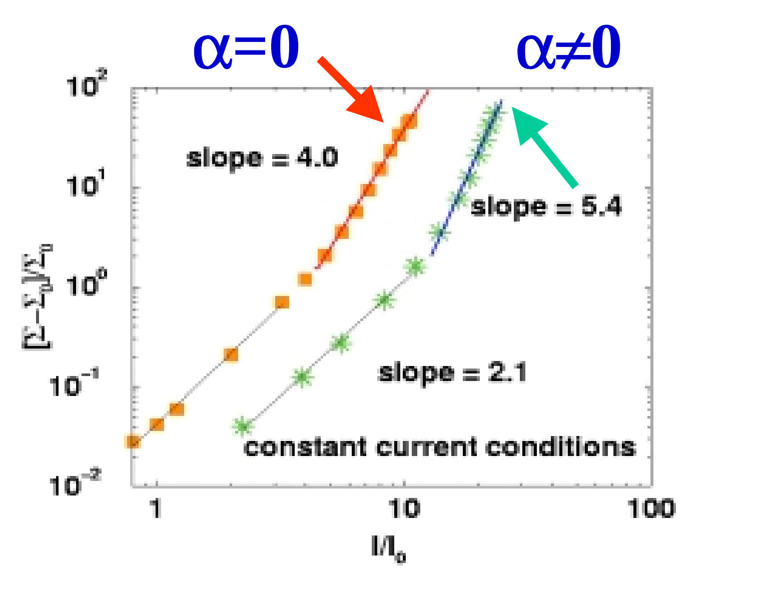




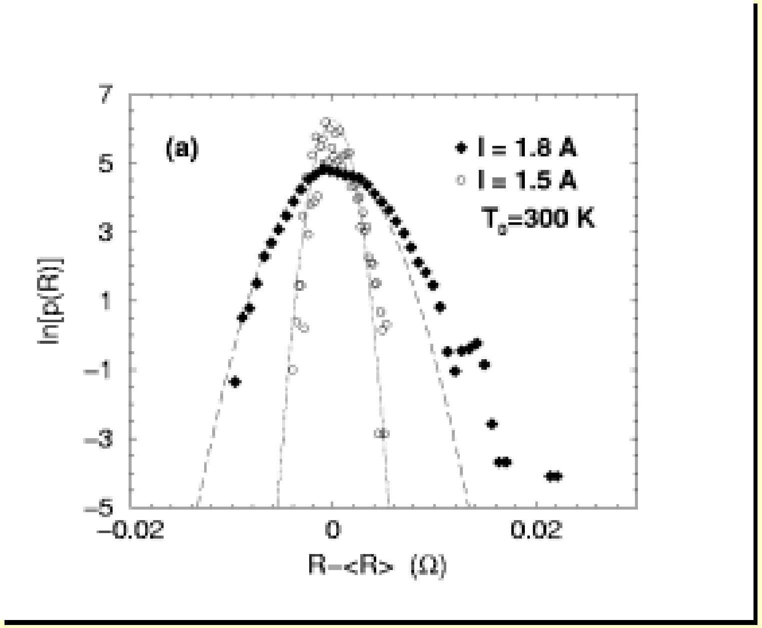
Effect on the resistance noise of the bias conditions and of the temperature coefficient of the resistance α



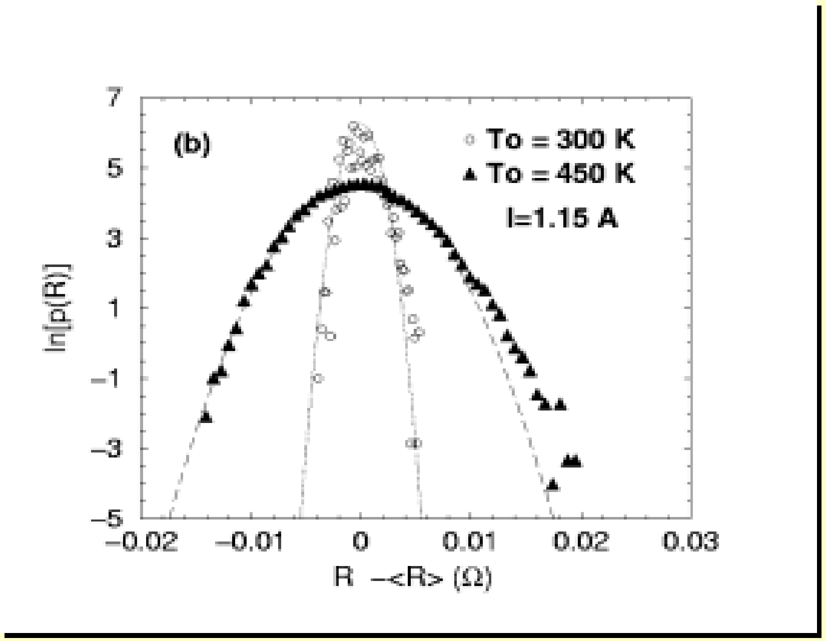




Non Gaussianity of the resistance fluctuations in the pre-breakdown region

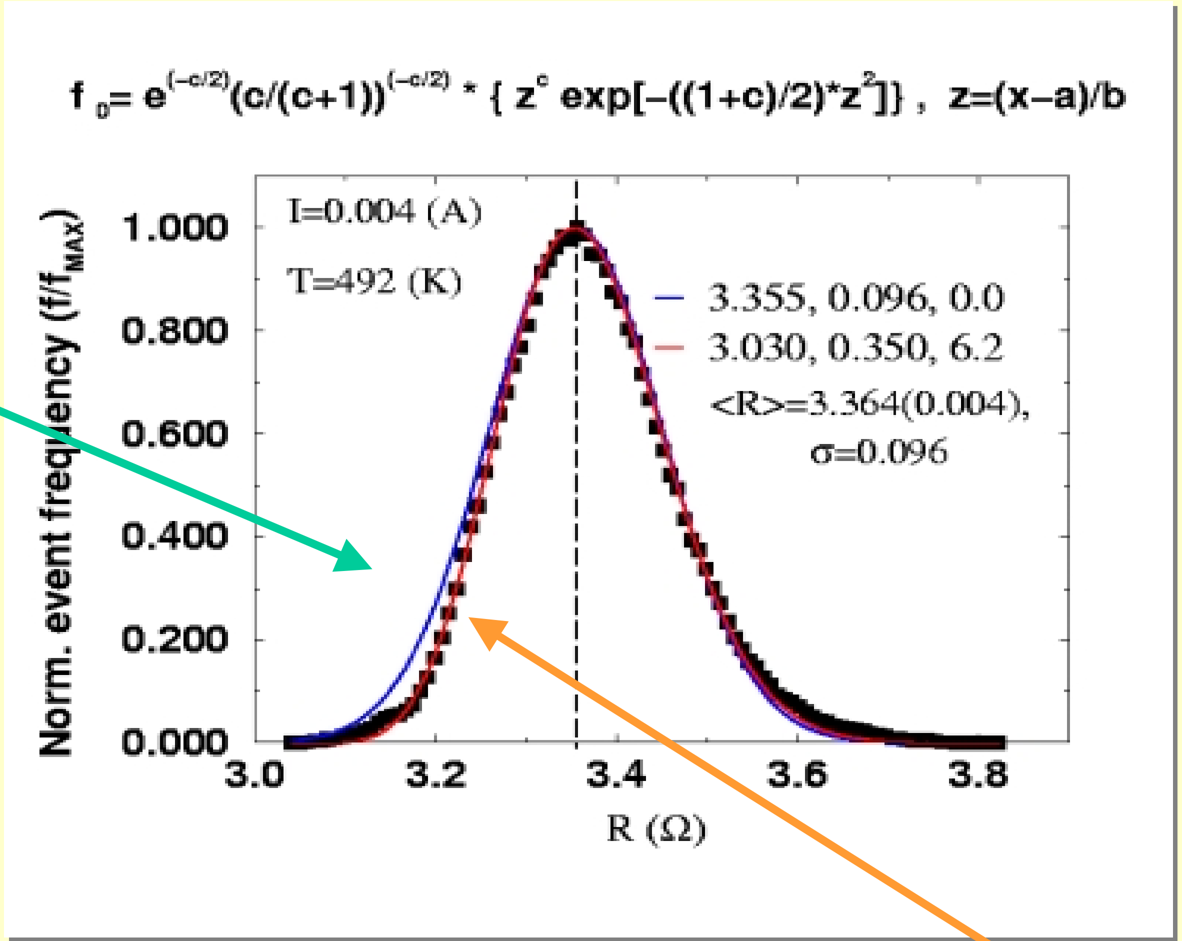


at increasing current



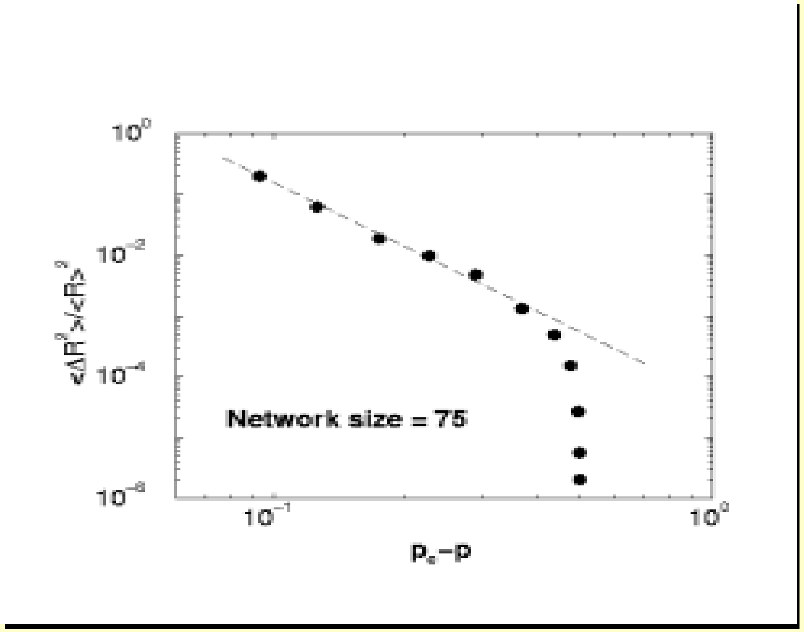
at increasing temperature

Gaussian distribtion



Nakagami distribtion

Linear regime: intrinsic noise (homogeneous processes)

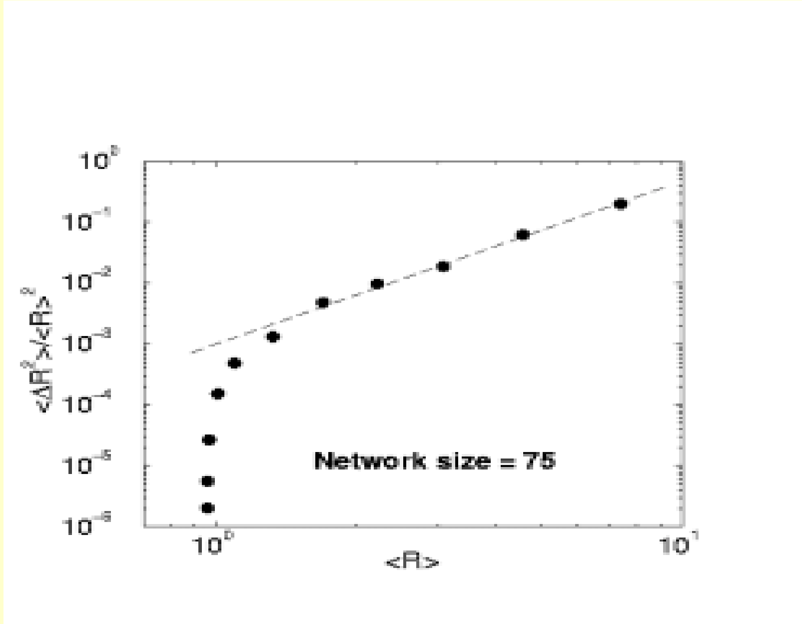


steady state condition:

$$W_R > W_D/(1+W_D)$$

$\langle \Delta R^2 \rangle / \langle R \rangle^2 \sim |p-p_C|^{-k}$ with $k = 3.1$

$\langle \Delta R^2 \rangle / \langle R \rangle^2 \sim \langle R \rangle^{s}$ with $s = 2.6$



Generalization of the model:

A network made of N_{spec} different resistors + broken resistors

The active resistors are different for:

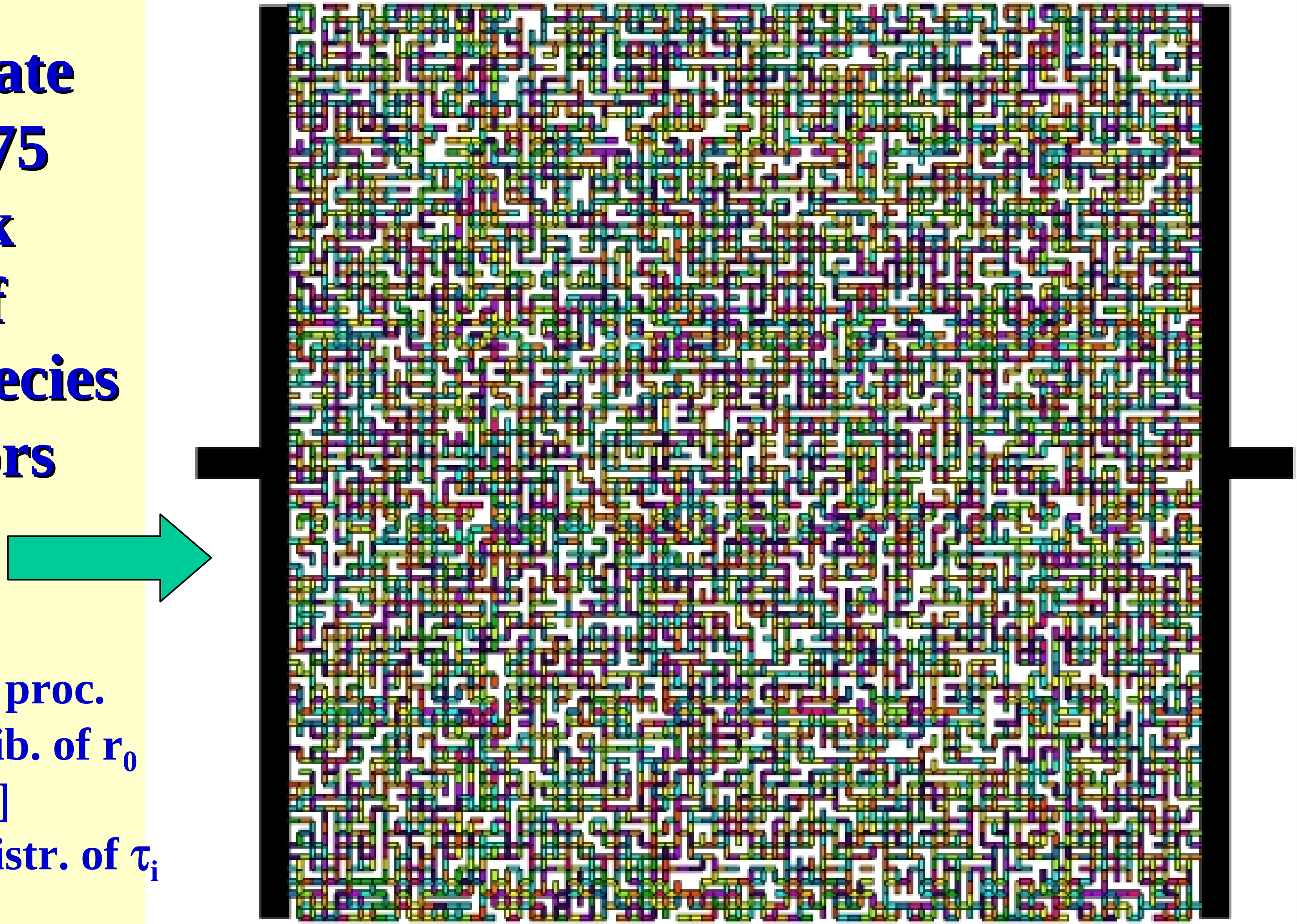
- the resistance value (and/or the TCR)
- the defect generation energy
- the defect recovery energy

Each species can:

- reach a steady-state within a caracteristic time
- extinguish

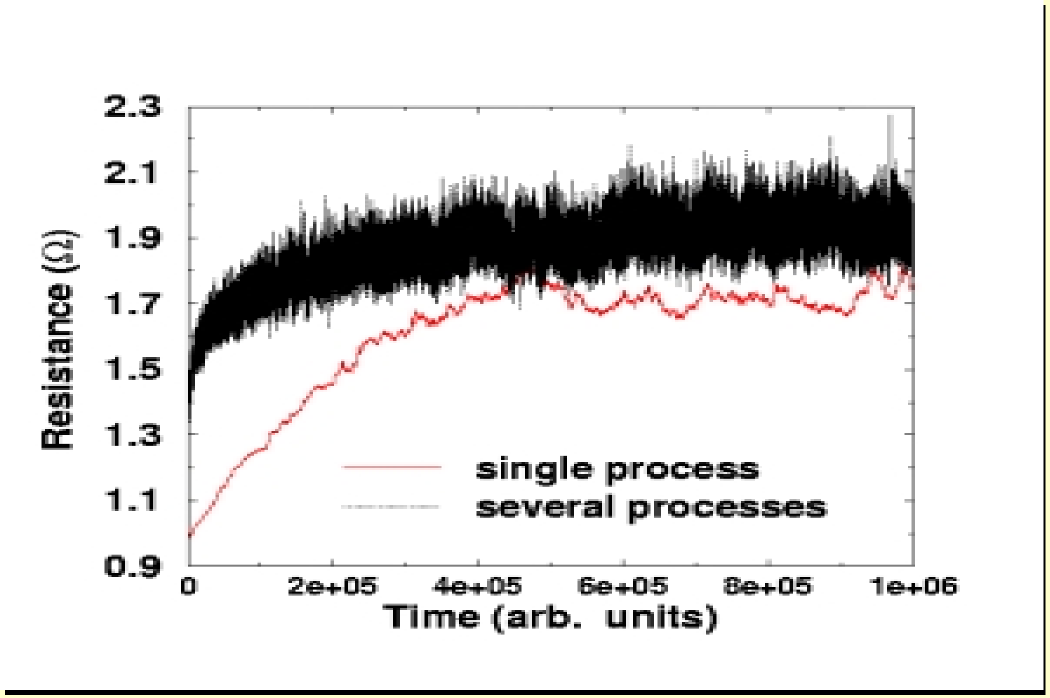
In the low-bias limit (homogeneous processes) $\tau_i \approx p_i/W_{di}$
where p_i= average fraction of broken resistors of each species

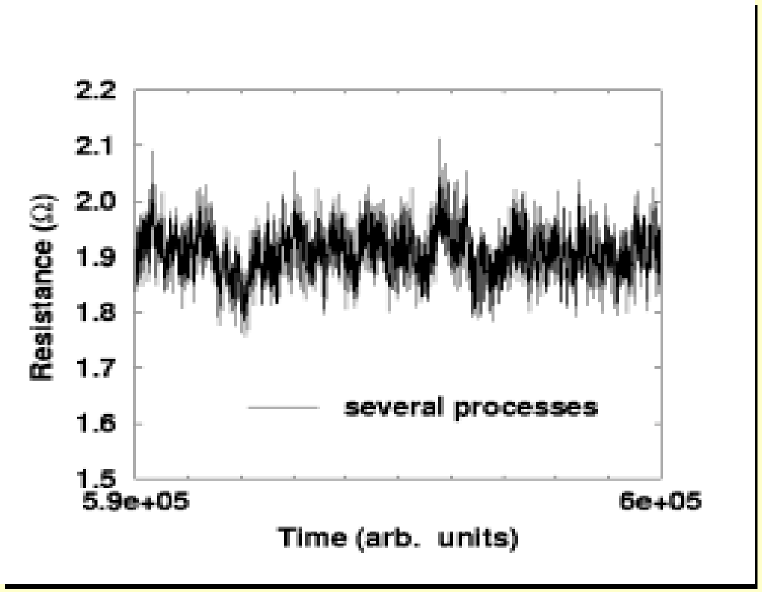
Steady-state of a 75x75 network made of several species of resistors

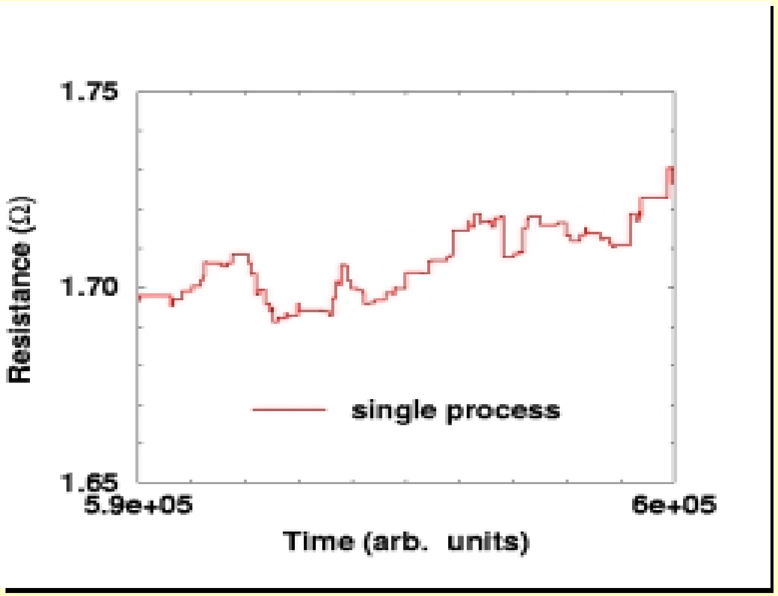


* N_{spec}=15
* homogeneous proc.
* uniform distrib. of r_0
* $r_0 \in [\, 0.5, 1.5 \,]$
* logarithmic distr. of τ_i
* $p_i \approx 0.25 \quad \forall\, i$

Resistance evolution



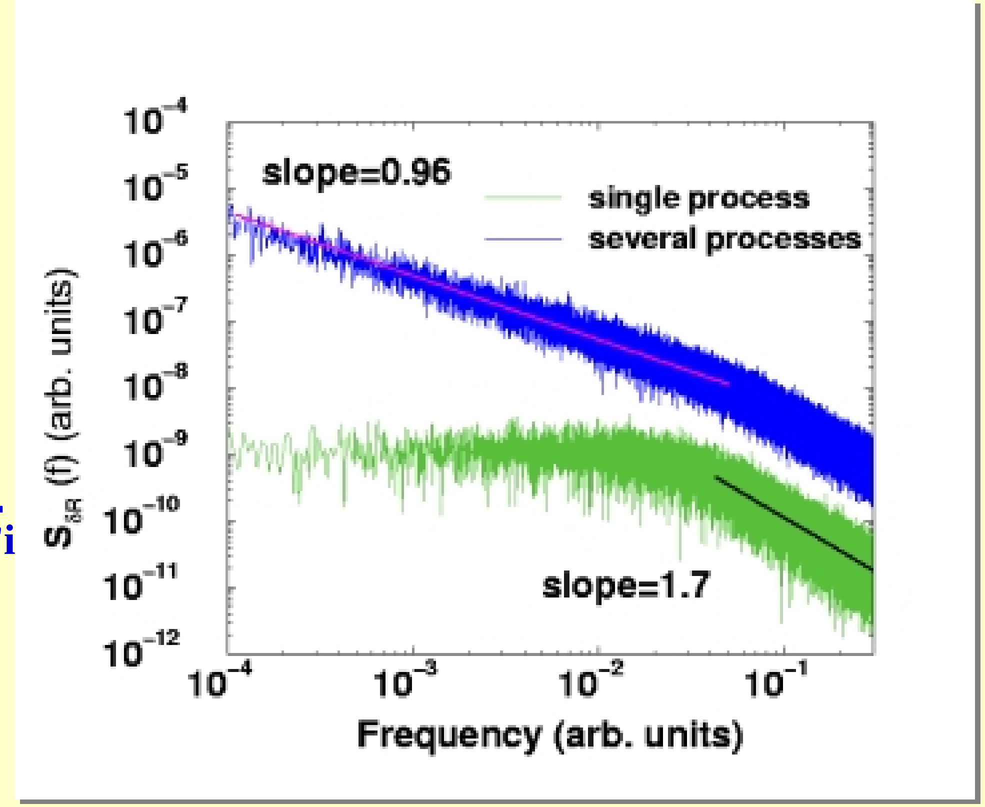




Power spectral density of resistance fluctuations

- Lorentzian spectrum in the case of a single species
- 1/f spectrum when several species are present

* N_{spec}=15
* homogeneous processes
* uniform distribution of r_0
* $r_0 \in [\ 0.5, 1.5\ ]$
* logarithmic distribution of τ_i
* $p_i \approx 0.25 \quad \forall\, i$



CONCLUSIONS

- **We have studied by MC simulations the stationary regime of a 2D RRN resulting from the competition of biased processes.**

- **The full range of the bias values, from the linear regime up to the breakdown, has been considered with the purpose of identifying precursors of failure.**

- **We have found scaling relations relating $\langle R\rangle/\langle R_0\rangle$ and $\langle \Delta R^2\rangle/\langle R\rangle^2$ with I/I_0**

- **We have analized, under different bias conditions, the role of different material parameters like: the initial resistance of the film, the TCR, the recovery activation energy.**

- **The agreement with measurements of the electrical properties of composites and nanostructured materials, and of electromigration damage in metallic lines is largely satisfactory.**

OPEN QUESTIONS

1. To what extent the comparison with experiments can be made more quantitative?

2 . Can we identify suitable parameters which act as precursors of the electrical breakdown?

3. For composites K. K. Bardhan (PRL, 1999) suggested that:

$$\Lambda = \frac{Y - 1}{\alpha \kappa \rho_0}$$

would have an universal value, where κ is the thermal conductivity of the material and ρ_0 the resistivity of the conductive component. Λ is really universal ?

4. **Is it possible to generalize the scaling relations found in the case of linear regime to the case of nonlinear regime?**

5 . **How the dimensionality, the geometry and the topology of the network would influence the results?**

6. **Concerning the extension of the model to the case of several species of resistors, we have studied only the linear regime by taking a comparable concentration of the different species. What happens in the biased case and for different initial concentrations of the different species?**

REFERENCES

- Z. Gingl, C. Pennetta, L. B. Kish, L. Reggiani, *Semicond. Sci. Technol.* 11, 1770, 1996.
- C. Pennetta, L. Reggiani , L. B. Kish, *Physica A*, 266, 214, 1999.
- C. D. Mukherijee, K. K. Bardhan, M. B. Heaney, *Phys. Rev. Lett.*, 83,1215, 1999.
- Z. Rubin, A. Sunshine, M.B. Heaney, I. Bloom, I. Balberg, *Phys. Rev.* B, 59, 12196, 1999
- C. Pennetta, L. Reggiani, G. Trefan, *Phys. Rev. Lett.* 84, 5006, 2000.
- C. Pennetta, L. Reggiani, G. Trefan, *Phys. Rev. Lett.* 85, 5238, 2000.
- C. Pennetta, L. Reggiani, E. Alfinito, G. Trefan, *J. Phys. C*, 14, 2371, 2002.
- C. Pennetta, L. Reggiani, G. Trefan, E. Alfinito, *Phys. Rev. E*, 65, 066119, 2002.
- C. Pennetta, *Fluctuation and Noise Lett.*, 2, R29, 2002.
- L. B. Kish, C. Pennetta, Z. Gingl, *Proc. of the SPIE's Annual Optical Sci. and Tech. Meeting*, Seattle, July, 2002.