Color Superconductivity in High Density QCD

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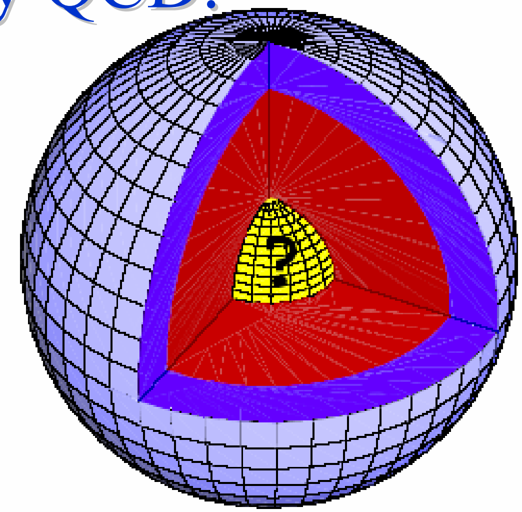
Bari, September 29 – October 1, 2004

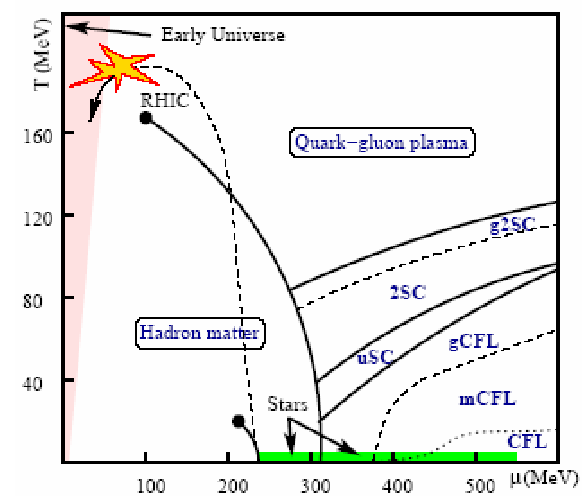
Introduction

Motivations for the study of high-density QCD:



- Understanding the interior of CSO's
- Study of the QCD phase diagram at T~0 and high μ

Asymptotic region in μ fairly well understood: existence of a CS phase. Real question: does this type of phase persists at relevant densities (~5-6 ρ_0)?



Summary

- Mini review of CFL and 2SC phases
- Pairing of fermions with different Fermi momenta
- The gapless phases g2SC and gCFL
- The LOFF phase and its phonons

CFL and 2SC

Study of CS back to 1977 (Barrois 1977, Frautschi 1978, Bailin and Love 1984) based on Cooper instability:

At T ~ 0 a degenerate fermion gas is unstable

Any weak attractive interaction leads to Cooper pair formation

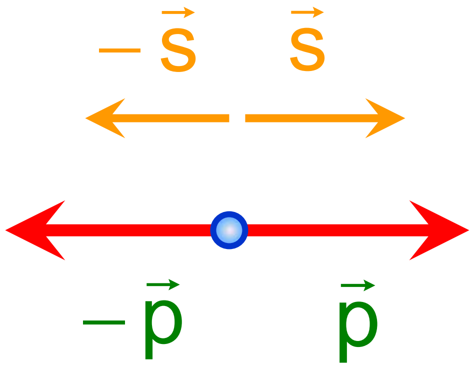
- Hard for electrons (Coulomb vs. phonons)
- Easy in QCD for di-quark formation (attractive channel $\bar{3}$) $(3 \otimes 3 = \bar{3} \oplus 6)$

In QCD, CS easy for large μ due to asymptotic freedom

At high μ, m_s, m_d, $m_u \sim 0$, 3 colors and 3 flavors

Possible pairings: $\langle 0 | \psi^{\alpha}_{ia} \psi^{\beta}_{jb} | 0 \rangle$

- Antisymmetry in color (α, β) for attraction
- Antisymmetry in spin (a,b) for better use of the Fermi surface
- Antisymmetry in flavor (i, j) for Pauli principle



Only possible pairings

LL and RR

Favorite state CFL (color-flavor locking)

(Alford, Rajagopal & Wilczek 1999)

$$\langle 0 | \psi^{\alpha}_{aL} \psi^{\beta}_{bL} | 0 \rangle = -\langle 0 | \psi^{\alpha}_{aR} \psi^{\beta}_{bR} | 0 \rangle = \Delta \varepsilon^{\alpha\beta C} \varepsilon_{abC}$$

Symmetry breaking pattern

$$SU(3)_c \otimes SU(3)_L \otimes SU(3)_R \Rightarrow SU(3)_{c+L+R}$$

What happens going down with μ? If $\mu << m_s$ we get

3 colors and 2 flavors (2SC)

$$\langle 0 | \psi^{\alpha}_{aL} \psi^{\beta}_{bL} | 0 \rangle = \Delta \varepsilon^{\alpha\beta 3} \varepsilon_{ab}$$

$$SU(3)_c \otimes SU(2)_L \otimes SU(2)_R \Rightarrow SU(2)_c \otimes SU(2)_L \otimes SU(2)_R$$

But what happens in real world ?

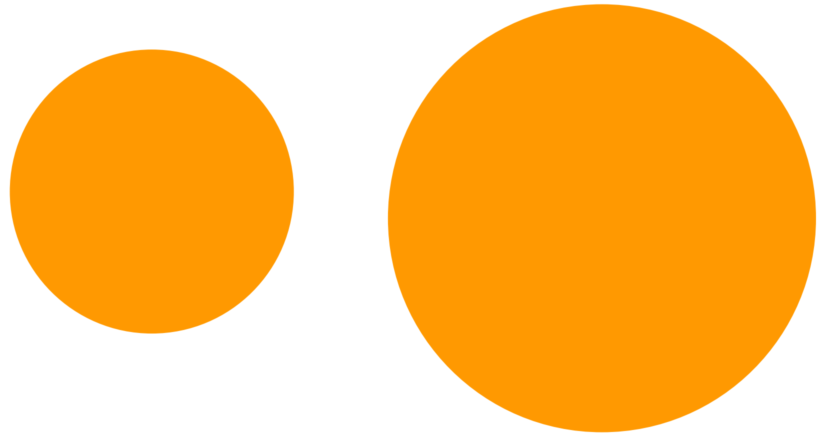
- M_s not zero
- Neutrality with respect to em and color
- Weak equilibrium

(no free energy cost in neutral -> singlet, Amore et al. 2003)

All these effects make Fermi momenta of different fermions unequal causing problems to the BCS pairing mechanism

Consider 2 fermions with $m_1 = M$, $m_2 = 0$ at the same chemical potential μ. The Fermi momenta are

$$p_{F1} = \sqrt{\mu^2 - M^2}$$



$$p_{F2} = \mu$$

Effective chemical potential for the massive quark

$$\mu_{\text{eff}} = \sqrt{\mu^2 - M^2} \approx \mu - \frac{M^2}{2\mu}$$

Mismatch:

$$\boxed{\delta\mu \approx \frac{M^2}{2\mu}}$$

If electrons are present, weak equilibrium makes chemical potentials of quarks of different charges unequal:

$$d \to u e \bar{\nu} \quad \Rightarrow \quad \mu_d - \mu_u = \mu_e$$

In general we have the relation: $(\mu_i = \mu + Q\mu_Q)$

$$\boxed{\mu_e = -\mu_Q}$$

N.B. μ_e is not a free parameter

Neutrality requires:

$$\frac{\partial V}{\partial \mu_e} = -Q = 0$$

Example 2SC: normal BCS pairing when

$$\mu_u = \mu_d \Rightarrow n_u = n_d$$

But neutral matter for

$$n_d \approx 2n_u \Rightarrow \mu_d \approx 2^{1/3}\mu_u \Rightarrow \mu_e = \mu_d - \mu_u \approx \frac{1}{4}\mu_u \neq 0$$

Mismatch:

$$\delta\mu = \frac{p_F^d - p_F^u}{2} = \frac{\mu_d - \mu_u}{2} = \frac{\mu_e}{2} \approx \frac{\mu_u}{8} \neq 0$$

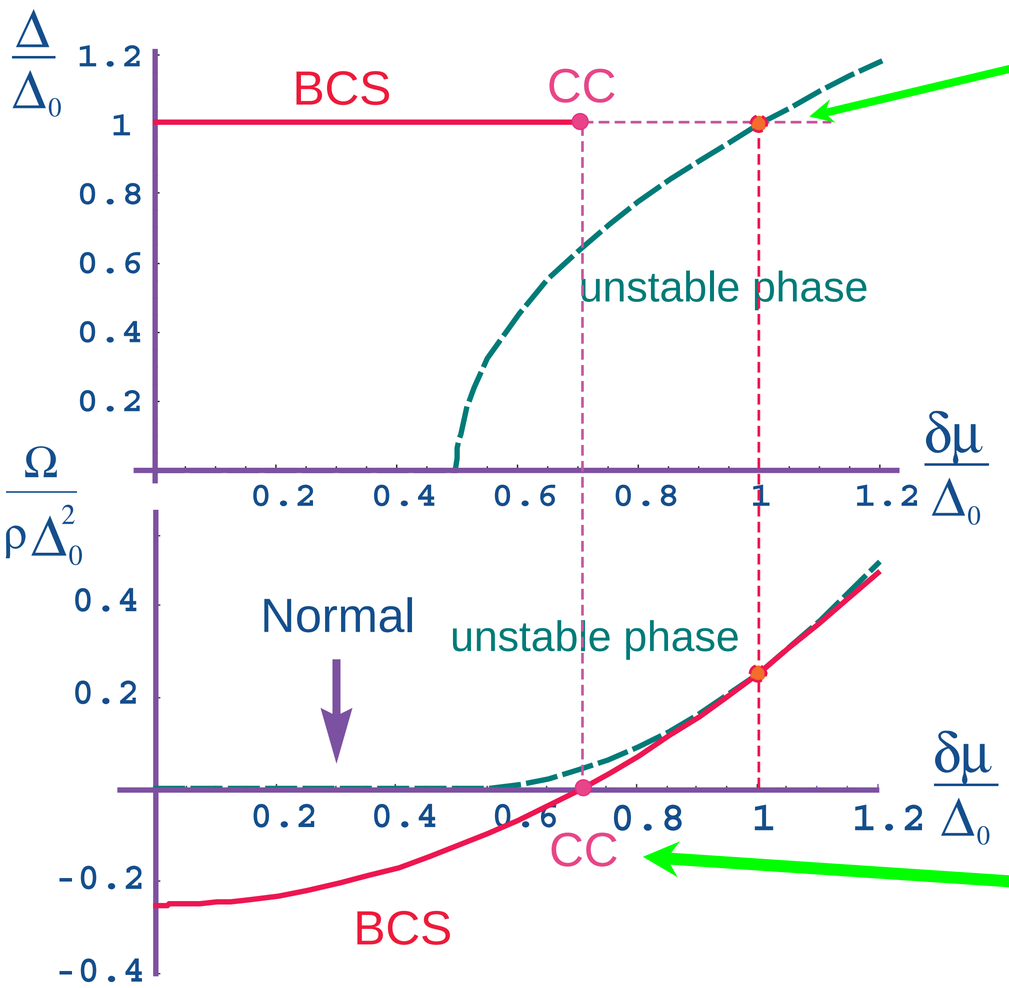
Also color neutrality requires

$$\frac{\partial V}{\partial \mu_3} = T_3 = 0, \quad \frac{\partial V}{\partial \mu_8} = T_8 = 0$$

As long as $\delta\mu$ is small no effects on BCS pairing, but when increased the BCS pairing is lost and two possibilities arise:

- The system goes back to the normal phase
- Other phases can be formed

In a simple model with two fermions at chemical potentials $\mu+\delta\mu$, $\mu-\delta\mu$ the system becomes normal at the Chandrasekhar-Clogston point. Another unstable phase exists.

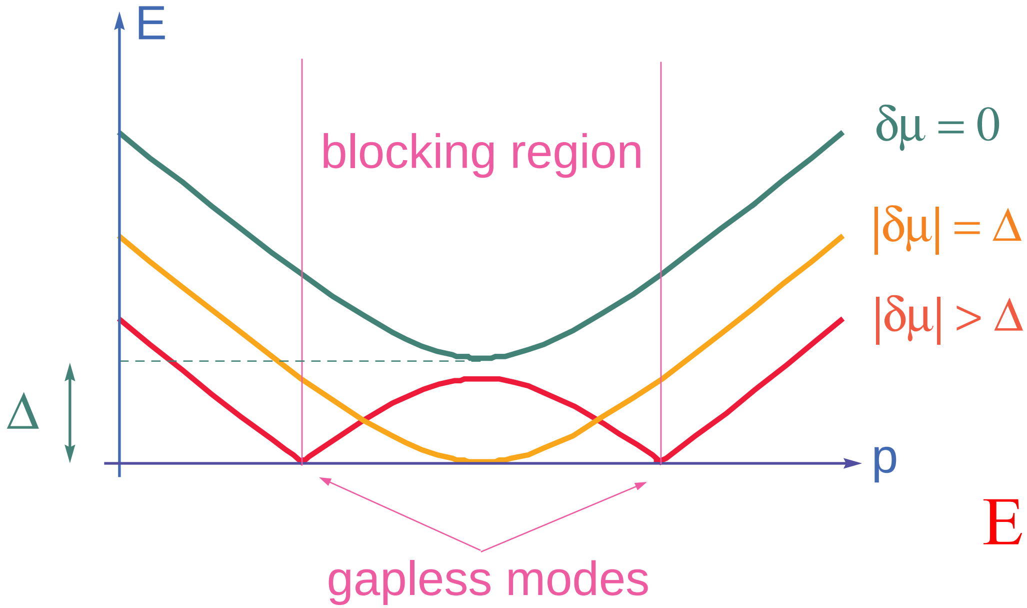


$$\delta\mu = \Delta_{BCS}$$

$$\delta\mu_1 = \frac{\Delta_{BCS}}{\sqrt{2}}$$

The point $|\delta\mu| = \Delta$ is special. In the presence of a mismatch new features are present. The spectrum of quasiparticles is

$$E(p) = \left|\delta\mu \pm \sqrt{(p-\mu)^2 + \Delta^2}\right|$$



For $|\delta\mu| < \Delta$, the gaps are $\Delta - \delta\mu$ and $\Delta + \delta\mu$

For $|\delta\mu| = \Delta$, an unpairing (blocking) region opens up and gapless modes are present.

$$E(p) = 0 \Leftrightarrow p = \mu \pm \sqrt{\delta\mu^2 - \Delta^2}$$

$2\delta\mu$ Energy cost for pairing

2Δ Energy gained in pairing

begins to unpair → $2\delta\mu > 2\Delta$

g2SC

Same structure of condensates as in 2SC

(Huang & Shovkovy, 2003)

4x3 fermions:

$$\langle 0 | \psi^{\alpha}_{aL} \psi^{\beta}_{bL} | 0 \rangle = \Delta \varepsilon^{\alpha\beta 3} \varepsilon_{ab}$$

- 2 quarks ungapped q_{ub}, q_{db}
- 4 quarks gapped q_{ur}, q_{ug}, q_{dr}, q_{dg}

General strategy (NJL model):

- Write the free energy: $V(\mu, \mu_3, \mu_8, \mu_e, \Delta)$
- Solve:

Neutrality

$$\frac{\partial V}{\partial \mu_e} = \frac{\partial V}{\partial \mu_3} = \frac{\partial V}{\partial \mu_8} = 0$$

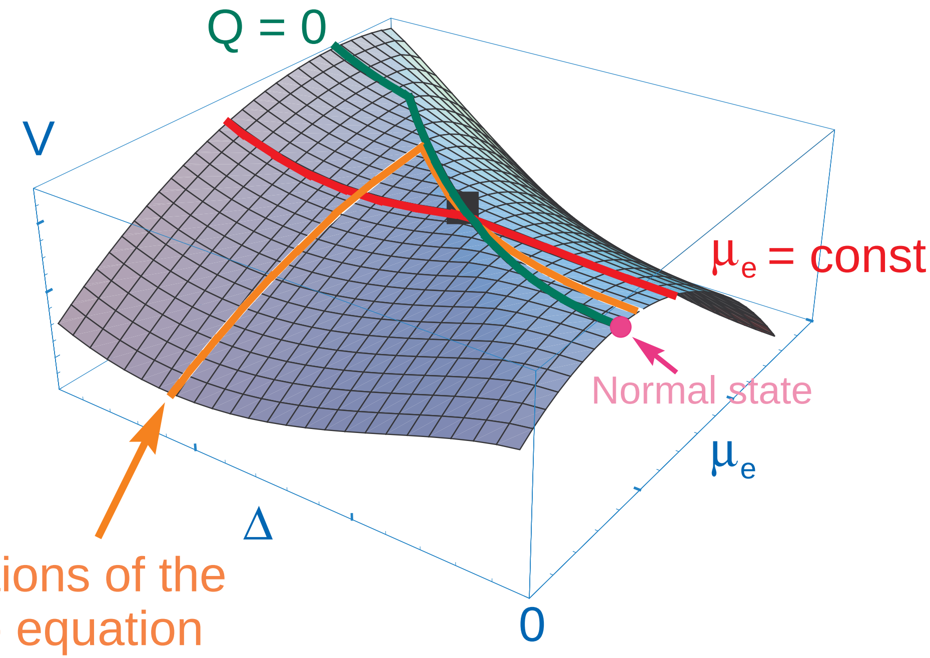
Gap equation

$$\frac{\partial V}{\partial \Delta} = 0$$

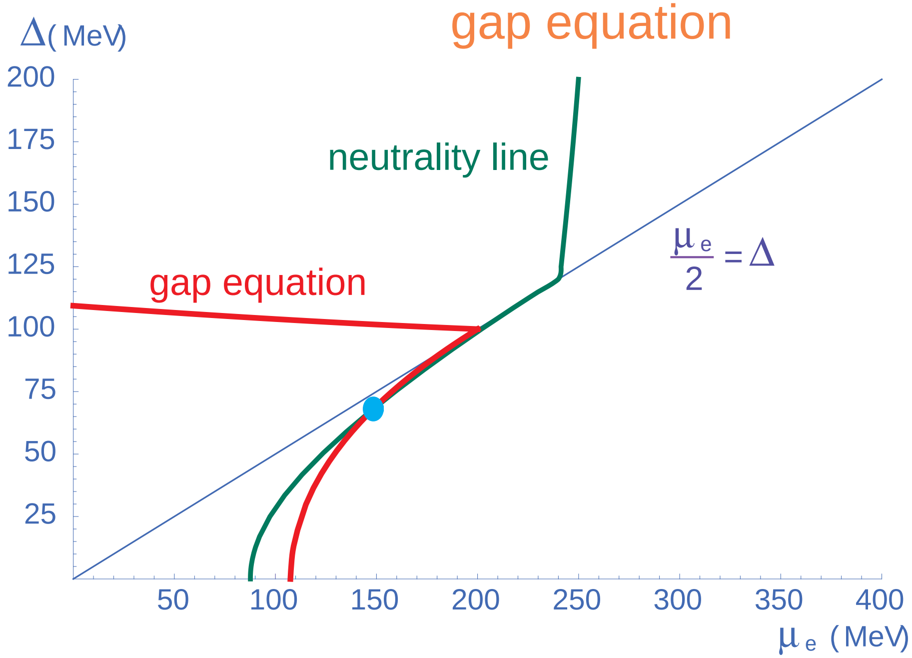
- For $|\delta\mu| > \Delta$ ($\delta\mu=\mu_e/2$) 2 gapped quarks become gapless. The gapless quarks begin to unpair destroying the BCS solution. But a new stable phase exists, the gapless 2SC (g2SC) phase.

- It is the unstable phase which becomes stable in this case (and CFL, see later) when charge neutrality is required.

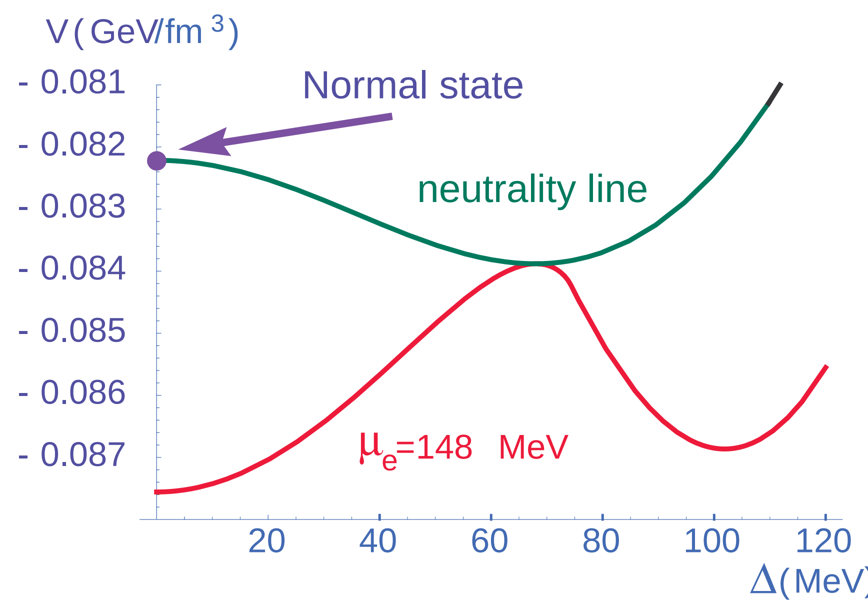
g2SC



Q = 0
V
μe = const.
Normal state
μe
Δ
0
solutions of the
gap equation



Δ(MeV)
200
175
150
125
100
75
50
25
neutrality line
gap equation
$\frac{\mu_e}{2} = \Delta$
50
100
150
200
250
300
350
400
μe (MeV)



V (GeV/fm 3)
- 0.081
- 0.082
- 0.083
- 0.084
- 0.085
- 0.086
- 0.087
Normal state
neutrality line
μe=148 MeV
20
40
60
80
100
120
Δ(MeV)

- But evaluation of the gluon masses (5 out of 8 become massive) shows an instability of the g2SC phase. Some of the gluon masses are imaginary (Huang and Shovkovy 2004).

- Possible solutions are: gluon condensation, or another phase takes place as a crystalline phase (see later), or this phase is unstable against possible mixed phases.

- Potential problem also in gCFL (calculation not yet done).

Generalization to 3 flavors → gCFL

$$\langle 0|\psi^{\alpha}_{aL}\psi^{\beta}_{bL}|0\rangle = \Delta_1\varepsilon^{\alpha\beta 1}\varepsilon_{ab1} + \Delta_2\varepsilon^{\alpha\beta 2}\varepsilon_{ab2} + \Delta_3\varepsilon^{\alpha\beta 3}\varepsilon_{ab3}$$

Different phases are characterized by different values for the gaps. For instance (but many other possibilities exist)

$$\text{CFL} : \quad \Delta_1 = \Delta_2 = \Delta_3 = \Delta$$

$$\text{g2SC} : \quad \Delta_3 \neq 0, \Delta_1 = \Delta_2 = 0$$

$$\text{gCFL} : \quad \Delta_3 > \Delta_2 > \Delta_1$$

Gaps in gCFL

$\tilde{Q}$	0	0	0	-1	+1	-1	+1	0	0
	ru	gd	bs	rd	gu	rs	bu	gs	bd
ru		Δ_3	Δ_2						
gd	Δ_3		Δ_1						
bs	Δ_2	Δ_1							
rd					$-\Delta_3$				
gu				$-\Delta_3$					
rs							$-\Delta_2$		
bu						$-\Delta_2$			
gs									$-\Delta_1$
bd								$-\Delta_1$	

Strange quark mass effects:

- Shift of the chemical potential for the strange quarks:

$$\mu_{\alpha s} \Rightarrow \mu_{\alpha s} - \frac{M_s^2}{2\mu}$$

- Color and electric neutrality in CFL requires

$$\mu_8 = -\frac{M_s^2}{2\mu}, \quad \mu_3 = \mu_e = 0$$

- gs-bd unpairing catalyzes CFL to gCFL

$$\delta\mu_{bd-gs} = \frac{1}{2}\left(\mu_{bd} - \mu_{gs}\right) = -\mu_8 = \frac{M_s^2}{2\mu}$$

$$\delta\mu_{rd-gu} = \mu_e, \quad \delta\mu_{rs-bu} = \mu_e - \frac{M_s^2}{2\mu}$$

It follows:

$\frac{M^2}{\mu}$ Energy cost for pairing

2Δ Energy gained in pairing

begins to unpair

$$\frac{M^2}{\mu} > 2\Delta$$

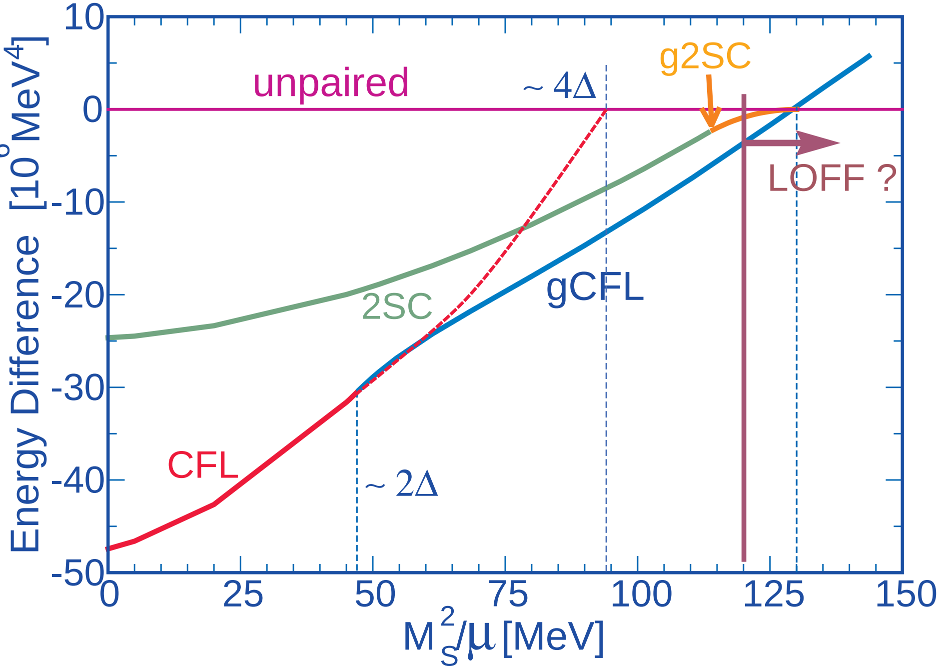
Again, by using NJL model (modelled on one-gluon exchange):

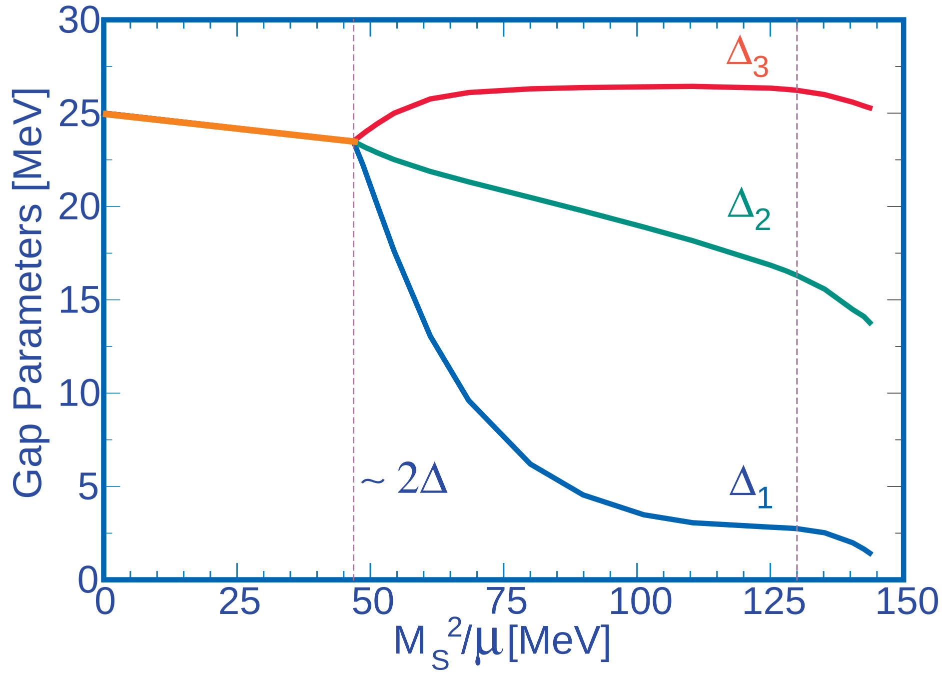
- Write the free energy: $V(\mu, \mu_3, \mu_8, \mu_e, M_s, \Delta_i)$
- Solve:

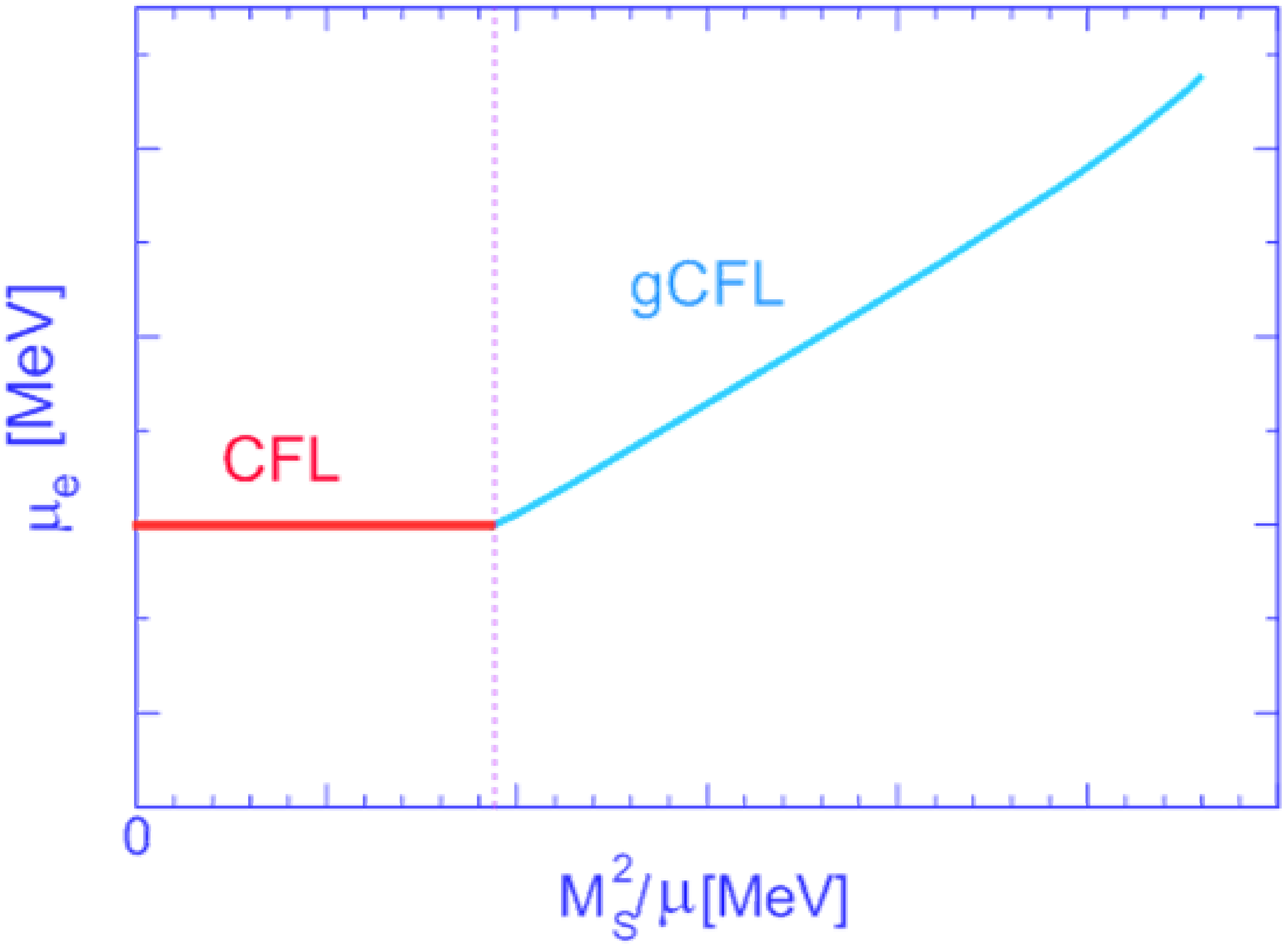
Neutrality $$\frac{\partial V}{\partial \mu_e} = \frac{\partial V}{\partial \mu_3} = \frac{\partial V}{\partial \mu_8} = 0$$

Gap equations $$\frac{\partial V}{\partial \Delta_i} = 0$$

- CFL ↦ gCFL 2nd order transition at $M_s^2/\mu \sim 2\Delta$, when the pairing gs-bd starts breaking
- gCFL has gapless quasiparticles. Interesting transport properties







- gCFL has μ_e not zero, with charge cancelled by unpaired u quarks

LOFF phase

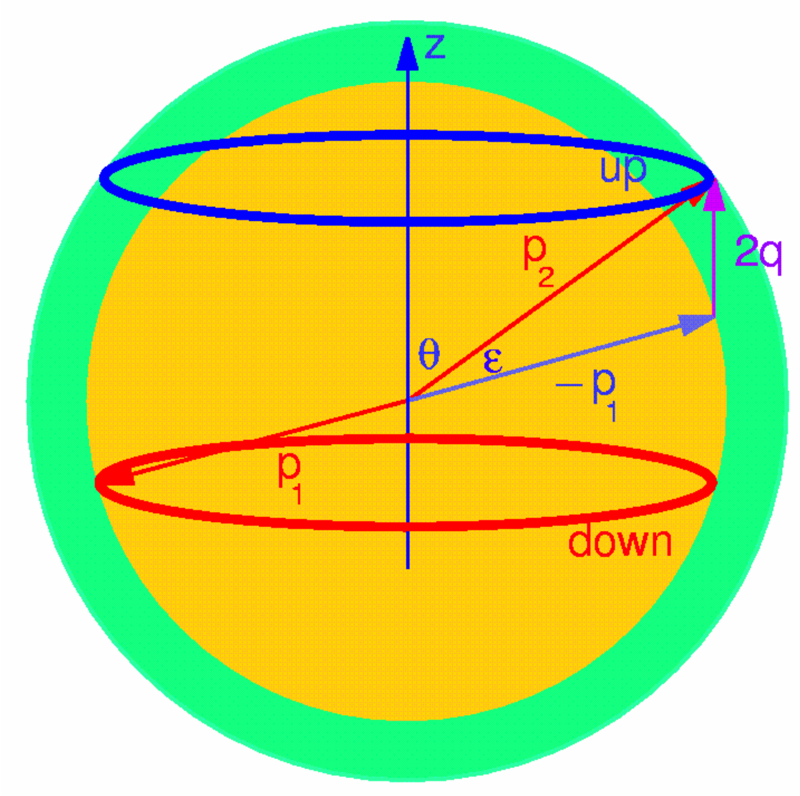
- LOFF (Larkin, Ovchinnikov, Fulde & Ferrel, 1964): ferromagnetic alloy with paramagnetic impurities.

- The impurities produce a constant exchange field acting upon the electron spins giving rise to an effective difference in the chemical potentials of the opposite spins producing a mismatch of the Fermi momenta

According to LOFF, close to first order point (CC point), possible condensation with **non zero total momentum**

$$\vec{p}_1 = \vec{k} + \vec{q} \quad \vec{p}_2 = -\vec{k} + \vec{q} \rightarrow \langle \psi(x)\psi(x) \rangle = \Delta e^{2i\vec{q}\cdot\vec{x}}$$

More generally $\longrightarrow \langle \psi(x)\psi(x) \rangle = \sum_m \Delta_m c_m e^{2i\vec{q}_m\cdot\vec{x}}$



$$\vec{p}_1 + \vec{p}_2 = 2\vec{q}$$

$|\vec{q}|$ fixed variationally

$\vec{q}/|\vec{q}|$ chosen spontaneously

Single plane wave:

$$E(\vec{p}) - \mu \to E(\pm\vec{p} + \vec{q}) - \mu \mp \delta\mu \approx \sqrt{(p-\mu)^2 + \Delta^2} \mp \bar{\mu}$$

$$\bar{\mu} = \delta\mu - \vec{v}_F \cdot \vec{q}$$

Also in this case, for $|\bar{\mu}| = \delta\mu - \vec{v}_F \cdot \vec{q} < \Delta$

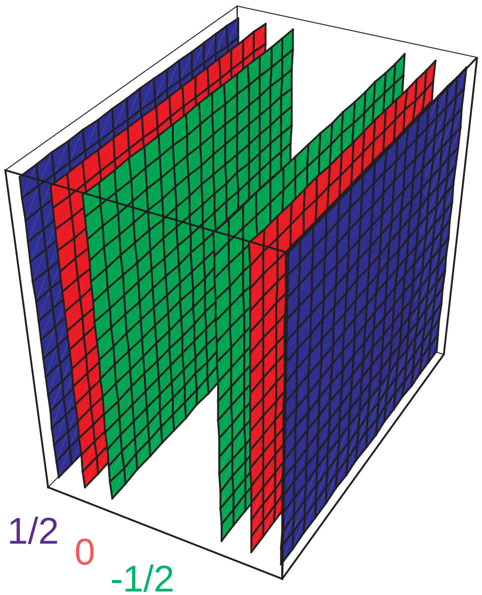
a unpairing (blocking) region opens up and gapless modes are present

Possibility of a crystalline structure (Larkin & Ovchinnikov 1964, Bowers & Rajagopal 2002)

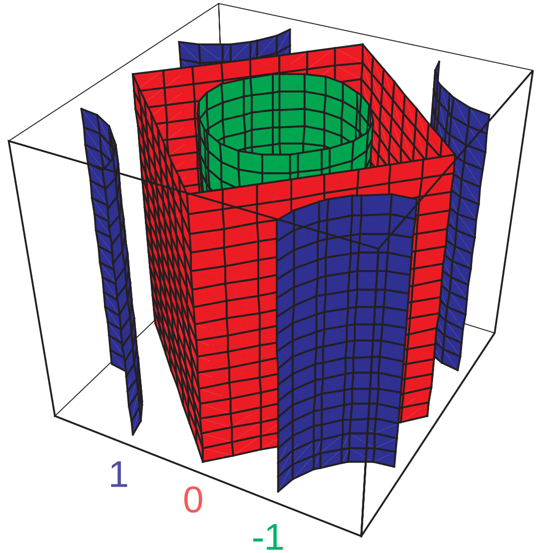
$$\langle \psi(x)\psi(x) \rangle = \Delta \sum_{|\vec{q}_i| = 1.2\delta\mu} e^{2i\vec{q}_i \cdot \vec{x}}$$

The q_i's define the crystal pointing at its vertices.

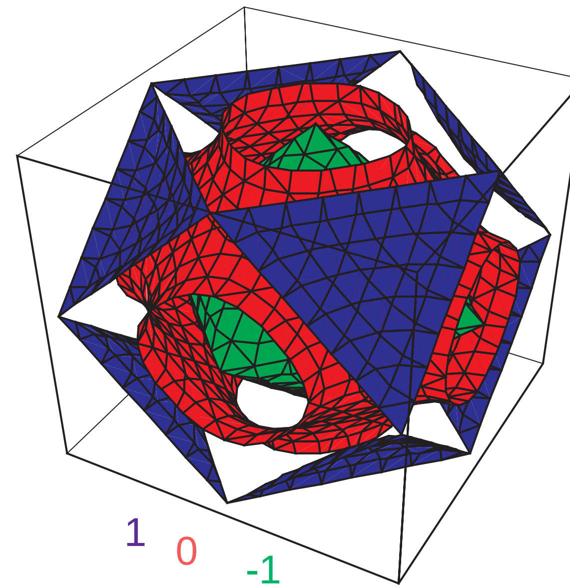
P = 2



P = 4

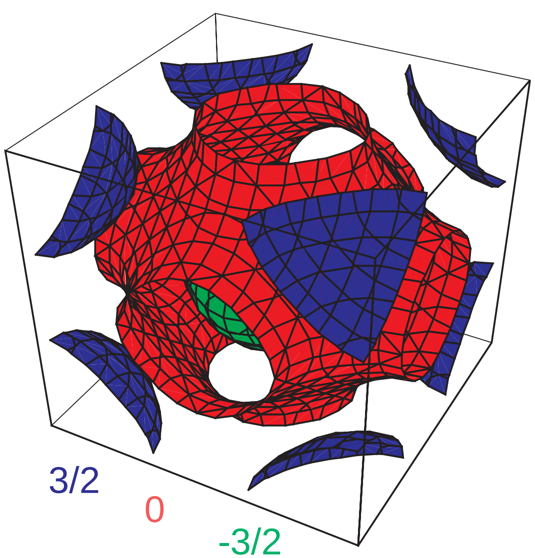


P = 6

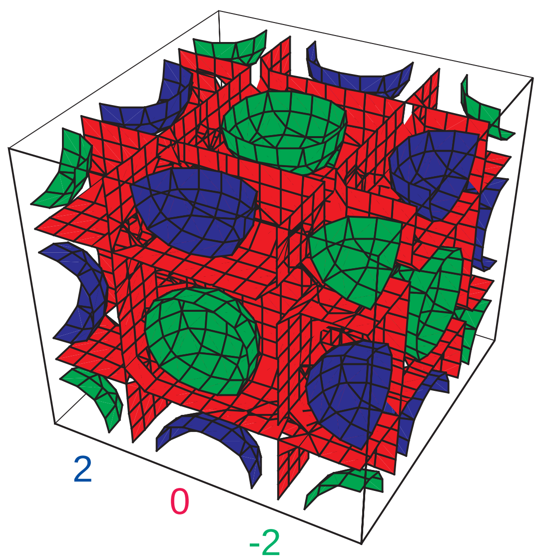


x 2 Δ

P = 6



P = 8



Crystalline structures in LOFF

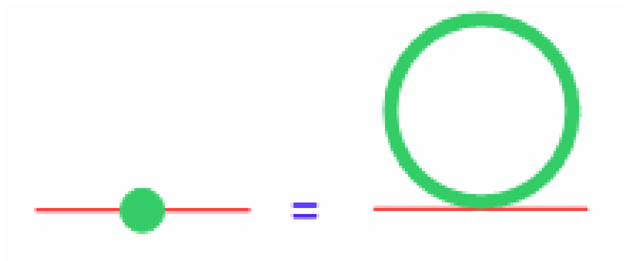
The LOFF phase is studied via a Ginzburg-Landau expansion of the grand potential

$$\Omega = \alpha\Delta^2 + \frac{\beta}{2}\Delta^4 + \frac{\gamma}{3}\Delta^6 + \cdots$$

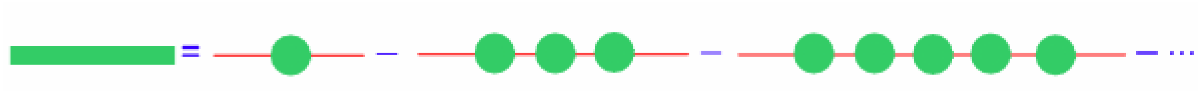
(for regular crystalline structures all the Δ_q are equal)

The coefficients can be determined microscopically for the different structures (Bowers and Rajagopal (2002))

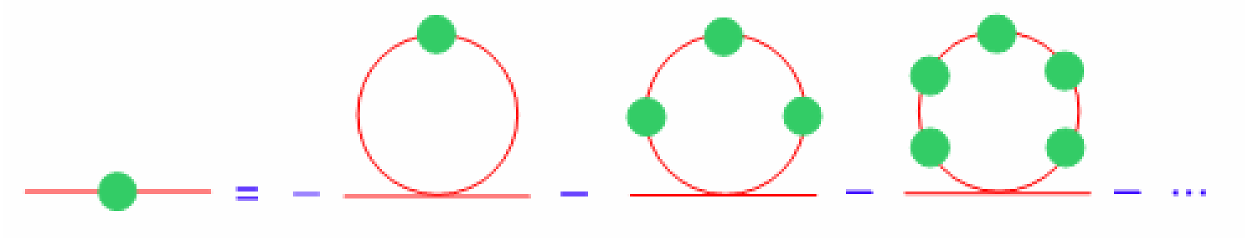
- Gap equation



- Propagator expansion



- Insert in the gap equation

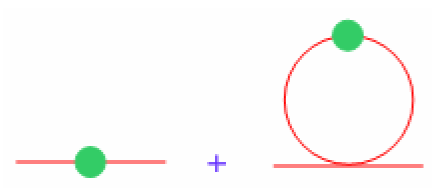


We get the equation

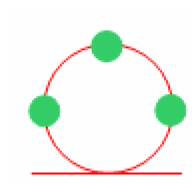
$$\alpha\Delta + \beta\Delta^3 + \gamma\Delta^5 + \cdots = 0$$

Which is the same as $$\frac{\partial\Omega}{\partial\Delta} = 0$$ with

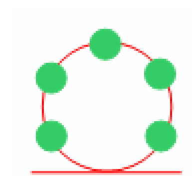
$$\alpha\Delta =$$



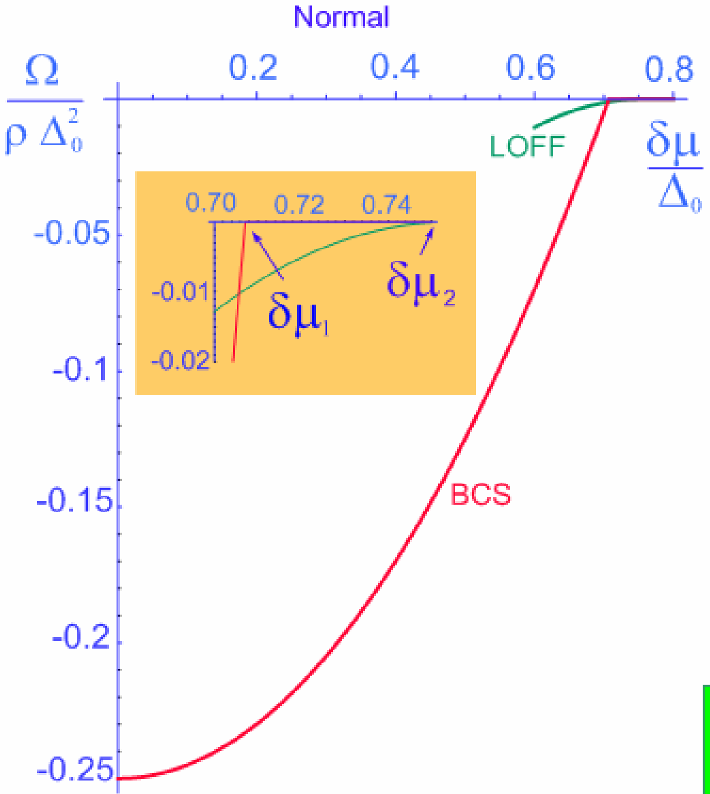
$$\beta\Delta^3 =$$



$$\gamma\Delta^5 =$$



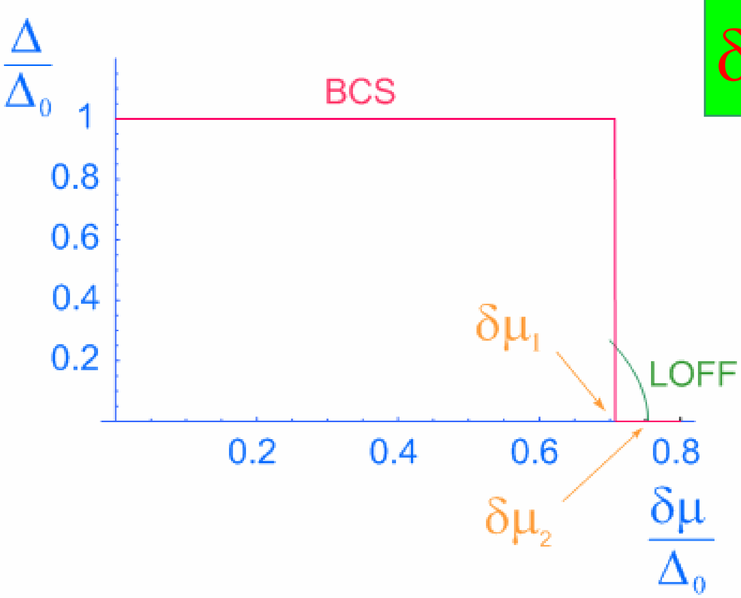
The first coefficient has universal structure, independent on the crystal. From its analysis one draws the following results



$$\Omega_{BCS} - \Omega_{normal} = -\frac{\rho}{4}\left(\Delta^2_{BCS} - 2\delta\mu^2\right)$$

$$\Omega_{LOFF} - \Omega_{normal} = -0.44\rho(\delta\mu - \delta\mu_2)^2$$

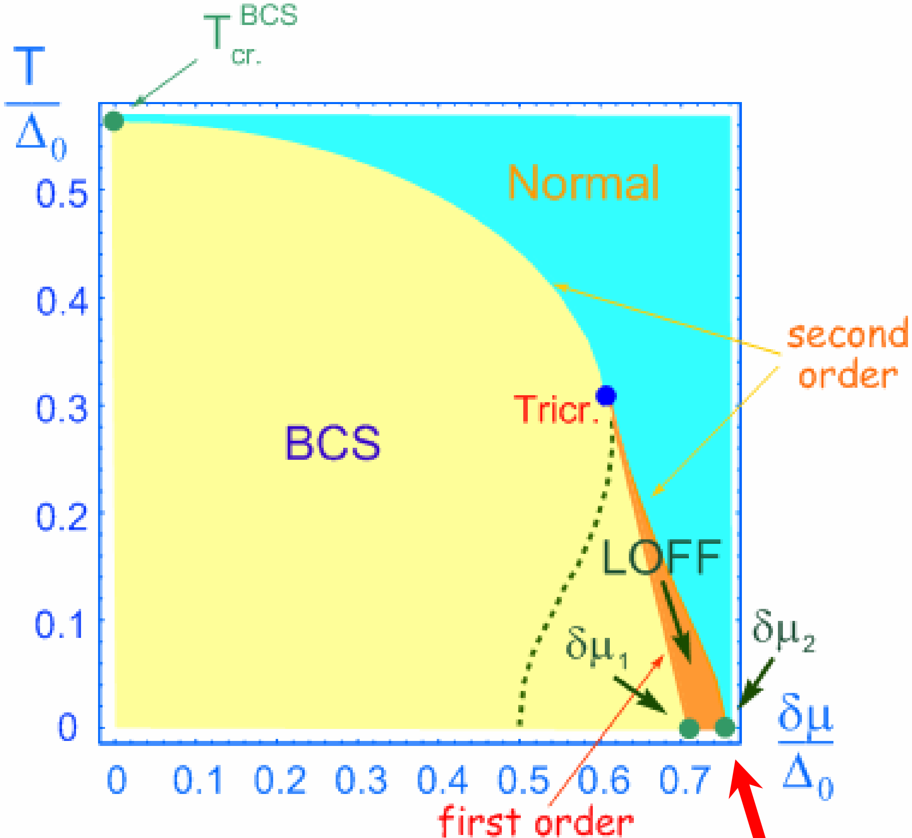
$$\Delta_{LOFF} \approx 1.15\sqrt{(\delta\mu_2 - \delta\mu)}$$



$$\delta\mu_1 = \Delta_{BCS} / \sqrt{2}$$
$$\delta\mu_2 \approx 0.754\Delta_{BCS}$$

Small window. Opens up in QCD?

(Leibovich, Rajagopal & Shuster 2001; Giannakis, Liu & Ren 2002)



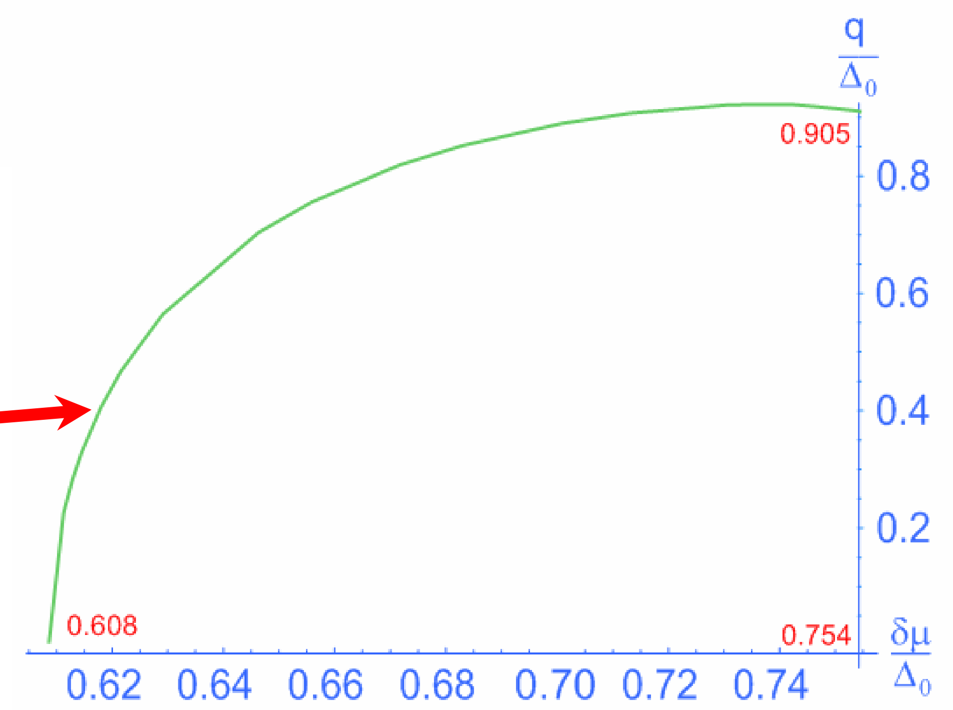
Single plane wave

Critical line from

$$\frac{\partial\Omega}{\partial\Delta}=0, \qquad \frac{\partial\Omega}{\partial q}=0$$

Along the critical line

$(\text{at } T=0,\ q=1.2\delta\mu_2)$



Structure	P	$\mathcal{G}$(Föppl)	$\bar{\beta}$	$\bar{\gamma}$	$\bar{\Omega}_{min}$	$\delta\mu_*/\Delta_0$
point	1	$C_{\infty v}(1)$	0.569	1.637	0	0.754
antipodal pair	2	$D_{\infty v}(11)$	0.138	1.952	0	0.754
triangle	3	$D_{3h}(3)$	-1.976	1.687	-0.452	0.872
tetrahedron	4	$T_d(13)$	-5.727	4.350	-1.655	1.074
square	4	$D_{4h}(4)$	-10.350	-1.538	–	–
pentagon	5	$D_{5h}(5)$	-13.004	8.386	-5.211	1.607
trigonal bipyramid	5	$D_{3h}(131)$	-11.613	13.913	-1.348	1.085
square pyramid	5	$C_{4v}(14)$	-22.014	-70.442	–	–
octahedron	6	$O_h(141)$	-31.466	19.711	-13.365	3.625
trigonal prism	6	$D_{3h}(33)$	-35.018	-35.202	–	–
hexagon	6	$D_{6h}(6)$	23.669	6009.225	0	0.754
pentagonal bipyramid	7	$D_{5h}(151)$	-29.158	54.822	-1.375	1.143
capped trigonal antiprism	7	$C_{3v}(13\bar{3})$	-65.112	-195.592	–	–
cube	8	$O_h(44)$	-110.757	-459.242	–	–
square antiprism	8	$D_{4d}(44)$	-57.363	-6.866	–	–
hexagonal bipyramid	8	$D_{6h}(161)$	-8.074	5595.528	-2.8×10^{-6}	0.755
augmented trigonal prism	9	$D_{3h}(3\bar{3}\bar{3})$	-69.857	129.259	-3.401	1.656
capped square prism	9	$C_{4v}(144)$	-95.529	7771.152	-0.0024	0.773
capped square antiprism	9	$C_{4v}(14\bar{4})$	-68.025	106.362	-4.637	1.867
bicapped square antiprism	10	$D_{4d}(14\bar{4}1)$	-14.298	7318.885	-9.1×10^{-6}	0.755
icosahedron	12	$I_h(15\bar{5}1)$	204.873	145076.754	0	0.754
cuboctahedron	12	$O_h(4\bar{4}\bar{4})$	-5.296	97086.514	-2.6×10^{-9}	0.754
dodecahedron	20	$I_h(5555)$	-527.357	114166.566	-0.0019	0.772

General analysis

(Bowers and Rajagopal (2002))

Preferred structure:

face-centered cube

Effective gap equation for the LOFF phase

(R.C., M. Ciminale, M. Mannarelli, G. Nardulli, M. Ruggieri & R. Gatto, 2004)

For the single plane wave (P = 1) the pairing region is defined by

$$\Delta_{eff} = \Delta\theta(E_u)\theta(E_d) = \begin{cases} \Delta & \text{for } (p, \vec{v}_F) \in PR \\ 0 & \text{elsewhere} \end{cases}$$

$$E_{u,d} = \pm(\delta\mu - \vec{v}_F \cdot \vec{q}) + \sqrt{\xi^2 + \Delta^2}, \quad \xi = p - \mu$$

$$\Delta = \frac{g\rho}{2}\int \frac{d\vec{v}}{4\pi}\int_0^\delta d\xi \frac{\Delta_{eff}}{\sqrt{\xi^2 + \Delta_{eff}^2}}$$

$$\rho = 4\frac{\mu^2}{\pi^2}$$

How to obtain this result starting from an effective theory for fermions close to the Fermi surface? Problem:

$$\mathcal{L} \sim \Delta e^{2i\vec{q}\cdot\vec{r}} \psi_{-v}^{T} C \psi_{v}$$

where in the Fermi fields the large part in the momentum has been extracted

$$p = \mu v_F + \ell$$

Solution: appropriate average procedure over the cell size

$$\mathcal{L} \to \Delta_{eff} \psi_{-v}^{T} C \psi_{v}$$

Average by

$$g_R(\vec{r}) = \prod_{k=1}^{3} \frac{\sin(\pi q r_k / R)}{\pi r_k}$$

When $R/\pi \sim 1$ different from zero in a region of the order of the cell size. Condition satisfied if the gap is not too small.

For P plane waves

$$\langle \psi(x)\psi(x) \rangle = \Delta \sum_{k=1}^{P} e^{2i\vec{q}_k \cdot \vec{x}}$$

an analogous average procedure gives pairing regions and effective gap given by

$$P_k = \left\{ (p, \vec{v}_F) \mid \Delta_E(p, \vec{v}_F) = k\Delta \right\}$$

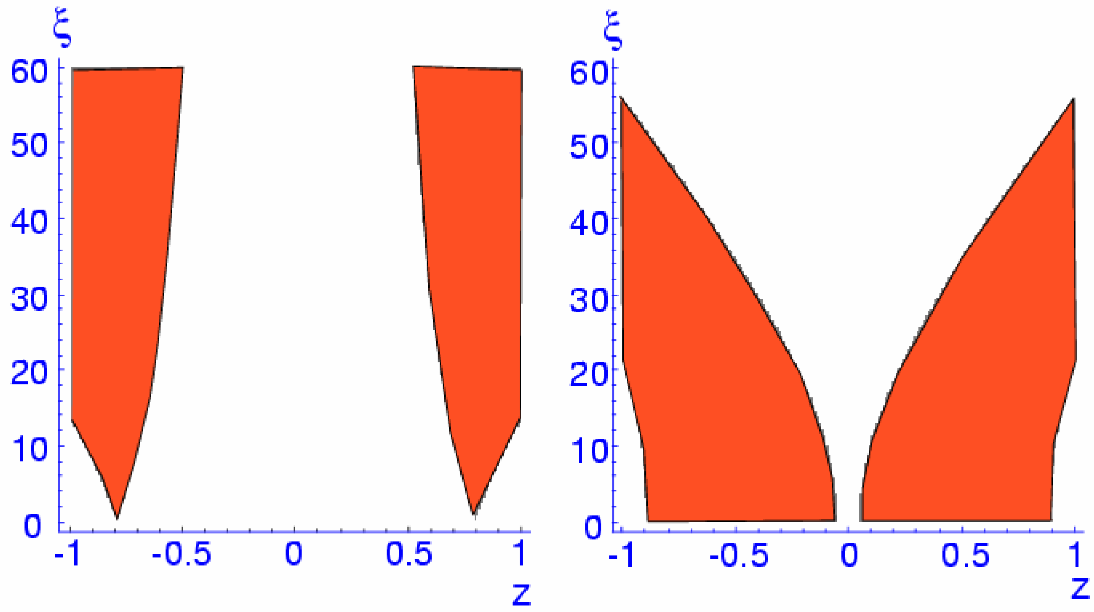
$$\Delta_E(p, \vec{v}_F) = \sum_{m=1}^{P} \Delta_{eff}(p, \vec{v}_F \cdot \vec{q}_m)$$

We obtain the following gap equation

$$P\Delta = \frac{g\rho}{2}\sum_{k=1}^{P}\iint_{P_k}\frac{d\vec{v}}{4\pi}\frac{d\xi}{2\pi}\frac{\Delta_E}{\sqrt{\xi^2+\Delta_E^2}} =$$

$$= \frac{g\rho}{2}\sum_{k=1}^{P}\iint_{P_k}\frac{d\vec{v}}{4\pi}\frac{d\xi}{2\pi}\frac{k\Delta}{\sqrt{\xi^2+k^2\Delta^2}}$$

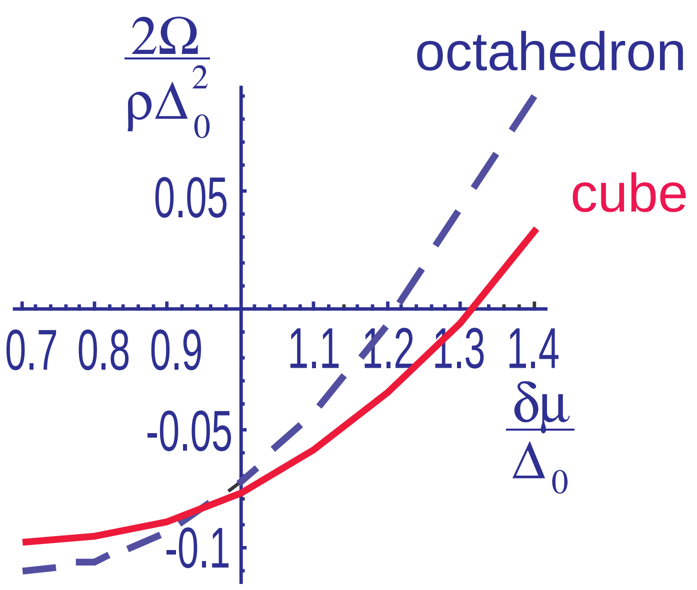
The result can be interpreted as having P quasi-particles each of one having a gap $k\Delta$, $k = 1, \ldots, P$.



The approximation is better far from a second order transition and it is exact for P = 1 (original FF case).

Evaluating the free energy at the CC point we see that the P=6 case (octahedron) is favored. Then the cube takes over at $\delta\mu_2 \sim 0.95\ \Delta$

P	z_q	$\frac{\Delta}{\Delta_0}$	$\frac{2\Omega}{\rho\Delta_0^2}$
1	0.78	0.24	-1.8×10^{-3}
2	1.0	0.75	-0.08
6	0.9	0.28	-0.11
8	0.9	0.21	-0.09



P	$\delta\mu_2/\Delta_0$	Order	z_q	Δ/Δ_0
1	0.754	II	0.83	0
2	0.83	I	1.0	0.81
6	1.22	I	0.95	0.43
8	1.32	I	0.9	0.35

Two phase transitions from the CC point ($M_s^2/\mu = 4\ \Delta_{2SC}$) up to the cube case ($M_s^2/\mu \sim 7.5\ \Delta_{2SC}$). Extrapolating to CFL ($\Delta_{2SC} \sim 30$ MeV) one gets that LOFF should be favored from about $M_s^2/\mu \sim 120$ MeV up $M_s^2/\mu \sim 225$ MeV

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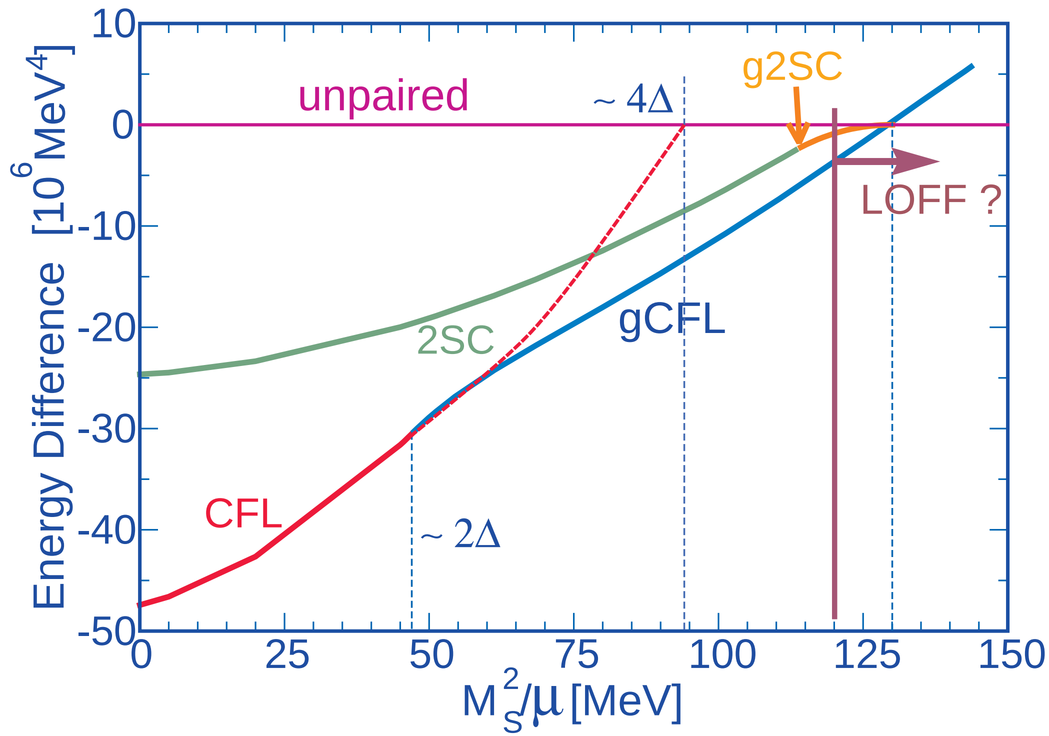

Conclusions

- Under realistic conditions (M_s not zero, color and electric neutrality) new CS phases might exist

- In these phases gapless modes are present. This result might be important in relation to the transport properties inside a CSO.