Dual superconductivity and typology of the QCD vacuum*

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Dual superconductor model of confinement

According to this model the QCD vacuum behaves like a dual superconductor, "dual" as the roles of the electric and magnetic fields are exchanged: the (chromo)electric field between two static color charges is compelled in narrow flux tubes yielding a linearly rising potential and confinement. Magnetic monopoles in the dual picture are the analogue of Cooper pairs in a superconductor: in the confined phase they condense breaking the $U(1)$ electromagnetic symmetry.

What type of superconductor is the QCD vacuum?

Two lengths characterize a superconductor:

- the penetration length λ of an external field;
- the correlation length ξ of the Higgs condensate.

These two lengths determine whether the superconductor is of type I ($\xi > \lambda$) or of type II ($\xi < \lambda$). Saying which type of superconductor the QCD vacuum is can help clarifying the dynamics of color confinement and of flux tubes interactions.

ξ and λ

Dual GL theory: let $\psi = se^{i\varphi}$, B_μ =vect. pot.,
$S = \int d^4x \left(-\frac{1}{4}\overline{F}^2 + \frac{g^2 s^2}{2}(B + \partial\varphi)^2 + \frac{1}{2}(\partial s)^2 - \frac{1}{2}b(s^2 - v^2)^2 \right)$

The fluctuations of s around $s = v$ describe a scalar particle of mass $m_H = 2v\sqrt{b}$, the photon acquires mass $m_V = gv$.

- In the London limit $s \to v$ we obtain: $E_l \propto K_0(m_V x_t) \;\Rightarrow\; \lambda = \frac{1}{m_V}$ is the penetration length.

- From the equations of motion for $s - v$ at the lowest order in $s - v$: $s - v \propto K_0(m_H x_t) \;\Rightarrow\; \xi = \frac{1}{m_H}$ is the correlation length.

type I	$\xi > \lambda$	$B > B_c$	normal state	tubes attr.
type II	$\xi < \lambda$	$B_{c1} < B < B_{c2}$	flux tubes	tubes rep.
		$B > B_{c2}$	normal state	

Abelian projection $SU(N) \to U(1)^{N-1}$

How can we obtain an abelian theory from a non-abelian one?

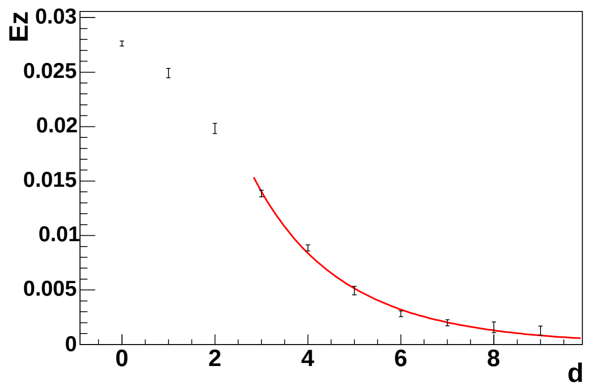
- We make a partial gauge fixing which leaves a residual invariance under the group $U(1)^{N-1}$: a gauge fixing G is identified with a 't Hooft operator ϕ^a in the adjoint representation via $G\phi G^\dagger = \phi_{diag}$.

- The gauge G is not univoquely determined since a diagonal transformation leaves ϕ_{diag} untouched.

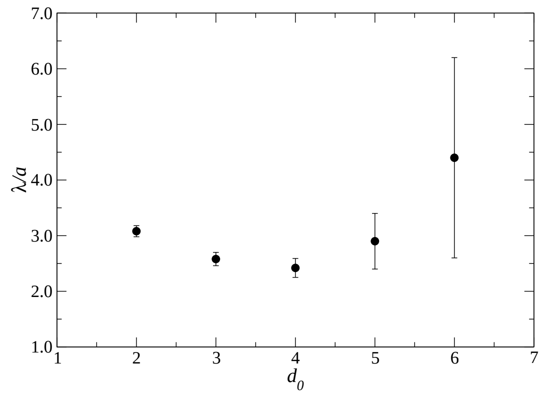
- A broader invariant subgroup remains where two or more eigenvalues of $\phi(x)$ coincide: magnetic monopoles are located there.

Determination of λ

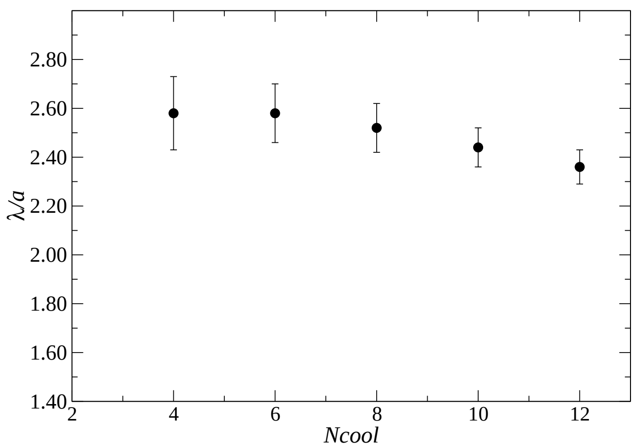
$$E_i = \frac{<\mathrm{tr}(W^{AbPr}\prod_{0i}^{AbPr})>}{<\mathrm{tr}(W^{AbPr})>} - \frac{<\mathrm{tr}(W^{AbPr})\mathrm{tr}\prod_{0i}^{AbPr}>}{2<\mathrm{tr}(W^{AbPr})>}$$



Single tube ($\beta = 2.6$)



Fit st. point dependence



Cooling dependence

β	λ (fm)
2.5115	0.163 ± 0.007
2.6	0.160 ± 0.007

Scaling

Our value $\hat{\lambda}(\beta = 2.5115) = 1.96 \pm 0.08$ is in agreement with literature.

The operator μ

The operator μ developed by the Pisa group is a magnetically charged operator detecting dual superconductivity ($<\mu>\neq 0$ in the confined phase).

$$\mu^a(\vec{x},t) = \exp\left[i\int d\vec{y}\,\mathrm{Tr}\{\phi^a(\vec{y},t)\vec{E}(\vec{y},t)\}\vec{B}(\vec{x}-\vec{y})\right]$$

with ϕ^a the adjoint field defining the projection and $\vec{B}$ the field of the monopole sitting at $\vec{x}$. We study

$$\rho(\hat{t}) = \frac{d}{d\beta}\ln\langle\overline{\mu}(\hat{t},n)\mu(0,n)\rangle = <S>|_S - <\tilde{S}(\hat{t})>\Big|_{\tilde{S}(\hat{t})}$$

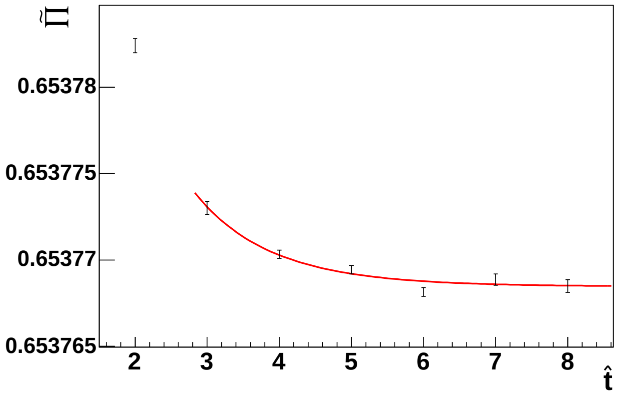
The expected behavior is

$$<\overline{\mu}(\hat{t},n)\mu(0,n)> \;=\; <\mu>^2 + \gamma\frac{e^{-\hat{t}/\hat{\xi}}}{\hat{t}^{3/2}}$$

$$\rho(\hat{t}) \;=\; A + B\frac{e^{-\hat{t}/\hat{\xi}}}{\hat{t}^{1/2}} + C\frac{e^{-\hat{t}/\hat{\xi}}}{\hat{t}^{3/2}}$$

As the Higgs field couples to μ the mass $\frac{1}{\xi_\mu}$ of the lowest state coupling to μ is greater or equal to the mass $m_H = \frac{1}{\xi}$ of the Higgs field. We assume (as logical) that the Higgs condensate is the lowest energy state ($\xi_\mu = \xi$).

Determination of ξ



$< \sum_{\mu<\nu}$ tr $\prod_{\mu\nu}^{mod}/6V >\Big|_{S_{mod}}$

fit from $\hat{t}_0 =?$	$\hat{\xi}$
1	0.99 ± 0.03
2	1.14 ± 0.10
3	1.28 ± 0.25
4	1.0 ± 0.4

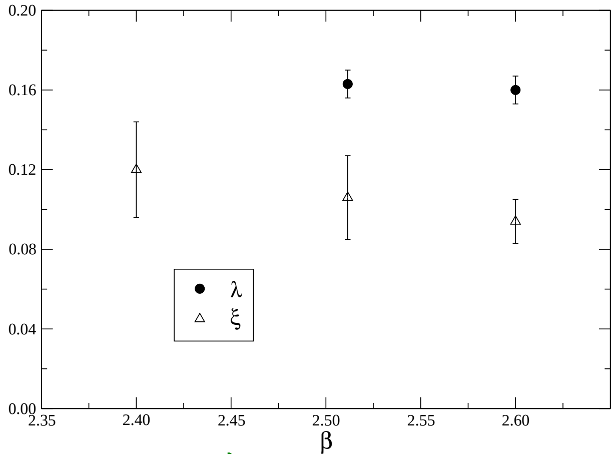
Fit st. point dependence

lattice	$\hat{\xi}(\beta = 2.5115)$
$12^3 \times 20$	1.24(21)
$16^3 \times 20$	1.28(25)

Finite size effect

β	$a(\beta)$ fm	ξ fm
2.4	0.118	0.120(24)
2.5115	0.083	0.106(21)
2.6	0.062	0.094(11)

Scaling



Our value $\hat{\xi}(\beta = 2.5115) = 1.28 \pm 0.25$ is just below $\hat{\lambda}$.
The QCD vacuum is marginally on the type II side.

The parameter $\tilde{\rho}^*$

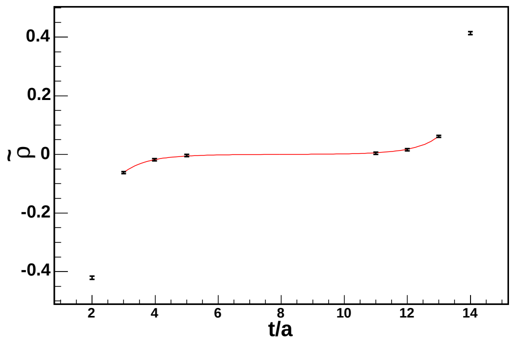
$$\rho(\hat{t}) = \frac{d}{d\beta}\ln\langle\overline{\mu}(\hat{t},n)\mu(0,n)\rangle$$

$$\tilde{\rho}(\hat{t}) = \frac{d}{d\hat{t}}\ln\langle\overline{\mu}(\hat{t},n)\mu(0,n)\rangle =$$

$$= -\left(\widehat{M}+\frac{3}{2\hat{t}}\right)\frac{\gamma e^{-\widehat{M}\hat{t}}/\hat{t}^{3/2}}{<\mu>^2+\gamma\, e^{-\widehat{M}\hat{t}}/\ \hat{t}^{3/2}}$$

- The noisy $<\mu>^2$ offset in $<\overline{\mu}(\hat{t},n)\mu(0,n)> = <\mu>^2 + \gamma\frac{e^{-\hat{t}/\hat{\xi}}}{\hat{t}^{3/2}}$ is cut away by the $\frac{d}{d\hat{t}}$ derivative;

- There are three parameters $(\widehat{M},\gamma,<\mu>^2)$ instead of four to be fitted.



We obtain $\widehat{\xi}(\beta=2.5115)=1.32\pm0.25$: the compatibility with the result from ρ $(\widehat{\xi}(\beta=2.5115)=1.28\pm0.25)$ rules out the presence of systematic effects.

*This idea was suggested by L.Tagliacozzo hep-lat/0603022

Does ξ depend on the gauge in which μ is defined?

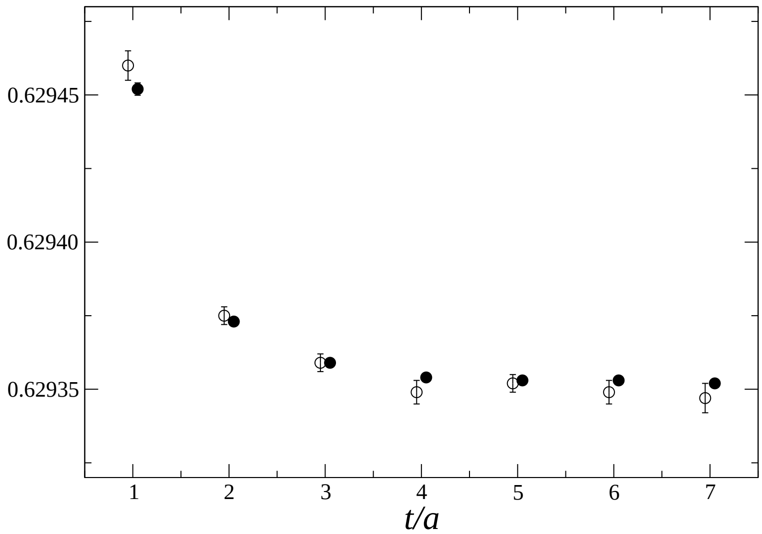
- The natural physical expectation is that one only coherence length characterize the QCD vacuum;

- This is consistent with 't Hooft ansatz that all abelian projection are equivalent to each other;

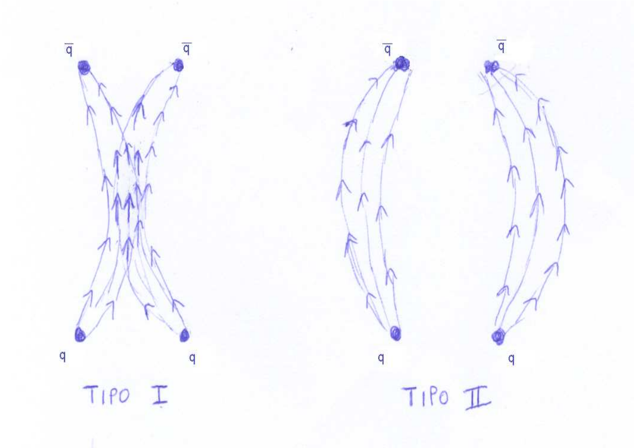
- The equivalence between different abelian projections also emerges clearly from numerical determinations of $< \mu >$;

- Theoretical argument: the operator μ defined in one particular abelian projection creates magnetic charges in every other projection, so that the lowest state coupled to μ should be universal;

- Anyway a numerical test of the issue was done giving $\hat{\xi} = 1.02(20)$(random gauge, black circles), $\hat{\xi} = 1.3(8)$(Polyakov gauge, empty circles):

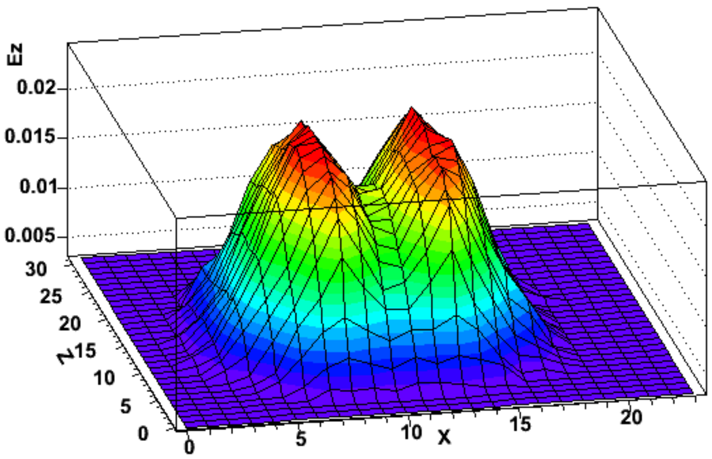


Interaction between two tubes



We study the interaction between two narrow parallel flux tubes:

$$E_i = \frac{<\mathrm{tr}\left(W_1^{AbPr} W_2^{AbPr} \prod_{0i}^{AbPr}\right)>}{<\mathrm{tr}\left(W_1^{AbPr} W_2^{AbPr}\right)>} - \frac{<\mathrm{tr}\left(W_1^{AbPr} W_2^{AbPr}\right) \mathrm{tr} \prod_{0i}^{AbPr}>}{2<\mathrm{tr}\left(W_1^{AbPr} W_2^{AbPr}\right)>}$$



D	$d_{TOP} - D$ (top-top)	$d_{Wtd} - D$ (weighted)
4	0.44(17)	0.017(9)
5	0.22(14)	-0.007(15)
Closer tubes	$\Rightarrow$	stronger repulsion

There are weak signs of repulsion as tubes are brought closer to each other.

Conclusions

The aim of our study was to have an indication on the type of dual superconductor which is realized in the QCD vacuum: with this intention we followed two strategies, a numerical determination of the parameters ξ and λ and an analysis of the interaction between parallel flux tubes.

- From the numerical determination we found $\hat{\xi}(\beta = 2.5115) = 1.28 \pm 0.25$ and $\hat{\lambda}(\beta = 2.5115) = 1.96 \pm 0.08$: the QCD vacuum is at border between type I and type II, slightly on the type II side ($\xi < \lambda$).

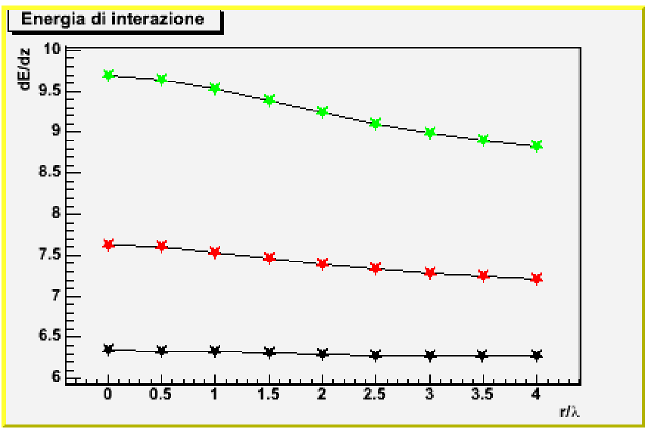
- This evidence is confirmed by our qualitative determination based on the observation of the interaction between two parallel strings: closer tubes show a stronger (still weak) repulsion.

Order of magnitude of the deflexion (classical)

1. Numeric minimization of the dual GL equation to obtain a cylindrical symmetric solution wuth n flux units;

2. Superposition of two $n = 1$ solutions at distance d apart (Abrikosov form): we obtain the interaction energy of two parallel tubes;

3. Given $\frac{dE}{dz}(d)$ we calculate $d(z)$ which minimizes the energy at fixed extrema:



$\xi/\lambda = 0.5$ $\xi/\lambda = 0.75$ $\xi/\lambda = 1$

4. We test the model in the Bogomol'nyi limit $\xi = \lambda$ (very good agreement).

The deflexion is about 1 lattice spacing at $\beta = 2.6$.