Dual Superconductivity in G_2 group

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Outline

Motivation and background

- Testing Dual Superconductor Picture of Confinement
- G_2 group

Contribution

- Simulating G_2 group
- Results on thermodynamics
- ρ operator
- Summary and outlook

Outline

Models of confinement dynamics

- Center Vortexes (Vortex free energy)
- Dual Superconductor Picture (DSP) (Monopole Condensation)
- Testing the DSP
  - Series of Papers : **"Colour confinement and dual superconductivity of the vacuum, I-II-III-IV"**
  - Proposal of an order parameter carring magnetic charge: μ
- G_2 has a trivial center $\Rightarrow$ No center vortexes
- G_2 group admits Monopole Solutions

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Dual Superconductor Picture

- Superconductors: electric charges condensation (Cooper pairs)
- Dual mechanism: condensation of magnetic charges (Monopoles, defined through Abelian Projection)
  $\Rightarrow$ electric field confined in strings
  $\Rightarrow$ linearly rising potential

DSP Vacuum state

Condensate of magnetically charged fields confining electrically charged particles (quarks)

Testing DSP with the magnetic operator μ

Strategy

- Construct an operator carrying nonzero magnetic charge
- Evaluate its v.e.v. $\leftarrow$ candidate order parameter

Operator

- $\langle \mu \rangle$ constructed adding a monopole field in the t_0 time-slice
- Direct measurement of $\langle \mu \rangle$ quite noisy
- Numerically more convenient the susceptibility:

$$\rho = \frac{\partial}{\partial \beta} \ln\langle \mu \rangle = \langle S \rangle_S - \langle S_M \rangle_{S_M}$$

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G_2 group: basic facts

Definition

- Exceptional G_2 group: AUTOMORPH. of the OCTONIONS
- Natural construction as a subgroup of $SO(7)$ (21 gen.) +

$$v = \Omega v \qquad v_k = \Omega_{ji} T_{ijk}$$

(T_{ijk} tot. antisymm.). 7 relations $\rightarrow$ 14 generators

Center

- G_2 group has real representation
- $SU(3) \subset G_2 \rightarrow \mathcal{C}(G_2) \subset \mathcal{C}(SU(3)) = \mathbb{Z}_3$
- Reality implies center of $G_2 = \{\mathbb{1}\}$ trivial

G_2 topology

Interesting homotopy groups

- Maximal Abelian (Cartan) subgroup: $U(1)^2$ as in $SU(3)$
- 't Hooft-Polyakov monopoles

$$\Pi_2(G_2/U(1)^2) = \mathbb{Z} \times \mathbb{Z}$$

- Twist-sectors ('t Hooft flux vortices if Π_1 isn't trivial)

$$\Pi_1(G_2/\mathcal{C}(G_2)) = \Pi_1(G_2/\mathbb{1}) = \mathbb{1}$$
$$\Pi_1(SU(3)/\mathbb{Z}_3) = \mathbb{Z}_3$$

- Absence of the center is not essential for twists sectors, see $SO(3)$ case

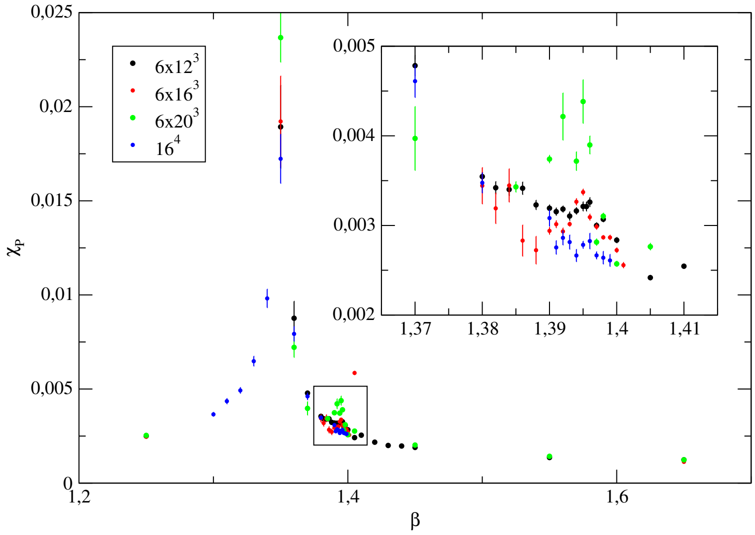
Outline

Simulations

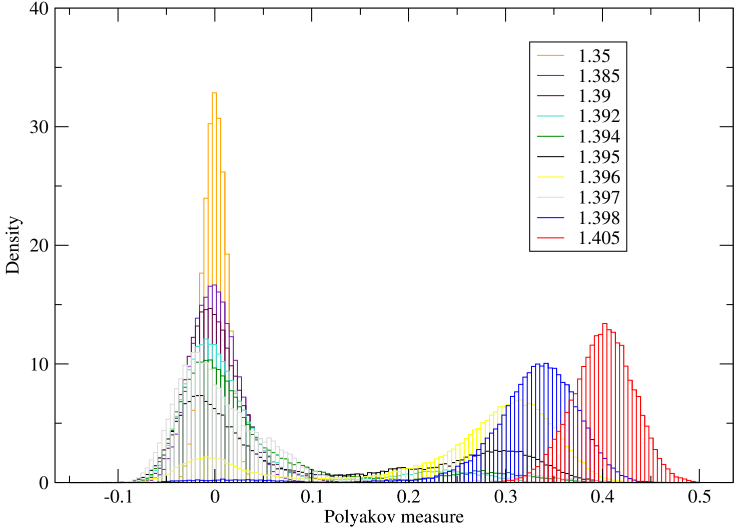
- Lot of work on this subject by the Bern group (P. Minkowski, U.-J. Wiese, M.Pepe and K. Holland)
- Our work deals with extensive finite temperature simulations at different spatial volumes $N_s = 12, 16, 20, 24, 32$ with $N_t = 6$ and a 16^4 lattice (zero temperature).
- Measurements: operator ρ, Polyakov Loop (high statistics in the 3 smaller lattices)
- Update: Cabibbo-Marinari on 3 $SU(2)$ subgroups and successive random gauge transformations.
- Fast code using directly SSE2 assembly instructions (matrix-multiplication core) and only real algebra

Outline

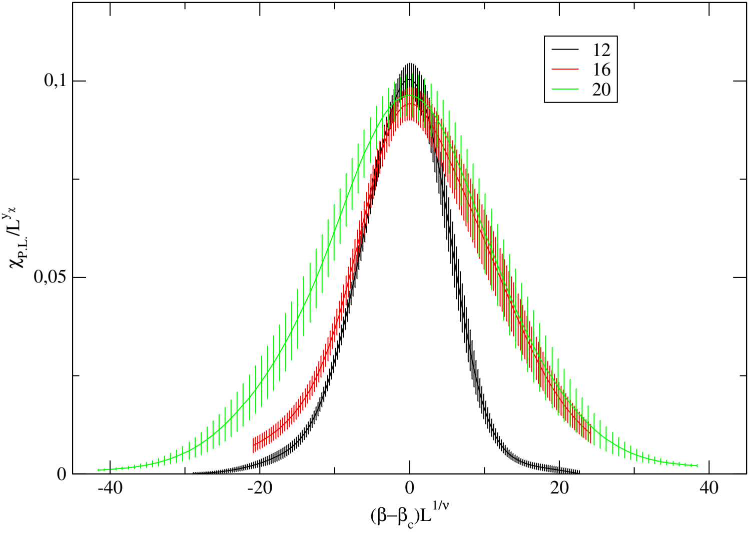
Plaquette Susceptibility



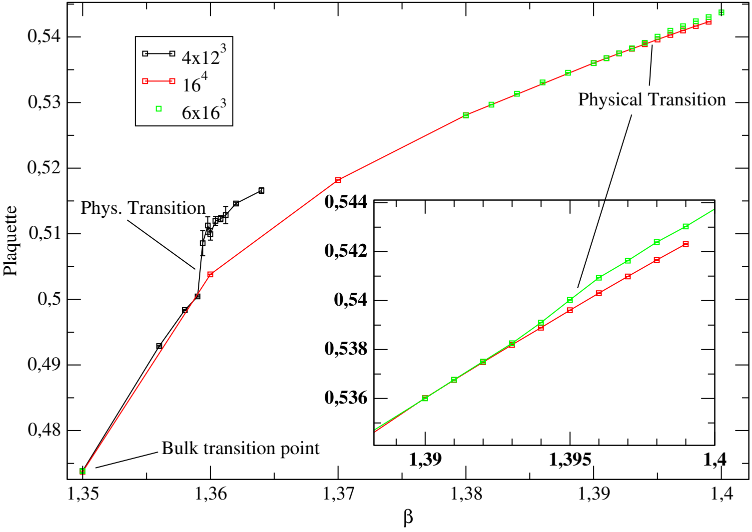
Polyakov Loop Histogram Plot - Lattice 6×20^3



Pol. Loop Susceptibility - Scaling Assuming First Order



Comparison with $T = 0$ simulations



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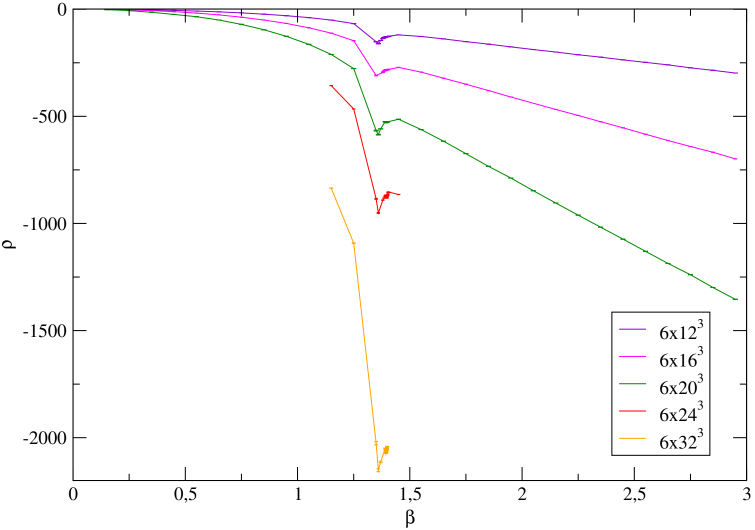
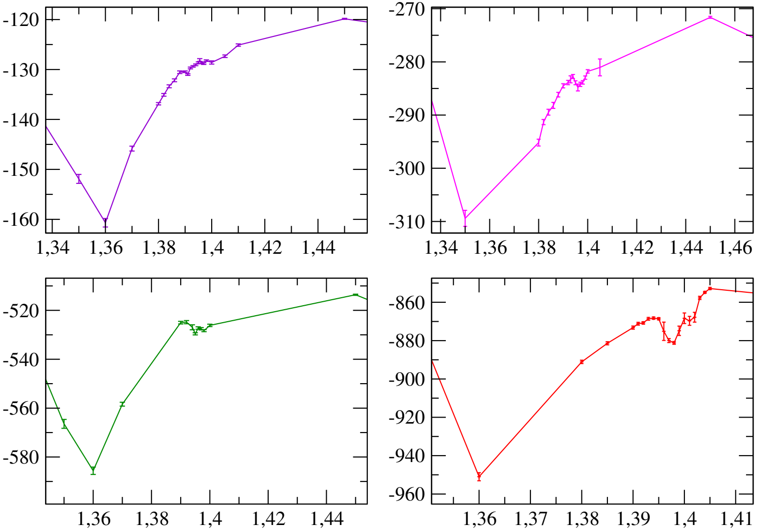
Global view of results



“Peaks” for different lattice sizes (12,16,20,24)



Outline

Results

- Our aim was to give a confinement criterion by using $\langle \mu \rangle$
- The operator is not blind to the bulk transition
- This precludes any direct scaling analysis using monopole operator.
- Moreover the operator should be volume indepentent before the physical transition.
- Our data clearly show a dip at the transition point as expected but the bulk transition obscurates the key features so...

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What is to be done?

- A possible solution for the bulk transition problem: suppress the lattice artifacts ($\mathbb{Z}_2$ monopoles) or
- consistently subtract the unphysical background: show the correct behavoiur of the magnetic operator and its scaling in the weak coupling region above bulk transition. Most promising

Thank you!

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Thank you!