ANALYTIC ESTIMATE OF THE ORDER PARAMETER FOR MAGNETIC CHARGE CONDENSATION IN QCD

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- MOTIVATIONS

- ANY QCD AMPLITUDE CAN BE WRITTEN IN TERMS OF GAUGE INVARIANT FIELD STRENGTH CORRELATORS [Schwinger gauge]

- STOCHASTIC APPROACH TO QCD [Dosch Simonov]:

  CLUSTER EXPANSION OF CORRELATORS TRUNCATED AT $n=2$.

- $$\mathcal{D}_{\mu\nu,\rho\sigma}(x,x_0) \equiv \mathrm{Tr}\left\{ U(c,x_0 x)\, G_{\mu\nu}(x)\, U^{+}(c,x_0 x)\, G_{\rho\sigma}(x_0) \right\}$$

  $U \equiv$ PARALLEL TRANSPORT FROM x_0 TO x ALONG c.

- CONFINEMENT $\tilde{\mathcal{D}}(p)$: NO POLES

- CONSTRAINTS ON $\mathcal{D}$ FROM CONFINEMENT:

  ORDER PARAMETER $\langle\mu\rangle$, μ A MAGNETICALLY CHARGED OPERATOR

  $$\begin{cases} \langle\mu\rangle \neq 0 & T < T_c \\ \langle\mu\rangle = 0 & T \geq T_c \end{cases}$$

  COMPUTE $\langle\mu\rangle$ IN TERMS OF $\mathcal{D}$'s.

1. INTRODUCTION

CONFINEMENT AS DUAL SUPERCONDUCTIVITY OF VACUUM ['t Hooft 75, Mandelstam

- FLUX TUBES OBSERVED.
  CONSISTENT WITH ABRIKOSOV-NIELSEN-OLESEN
- ORDER PARAMETER $\langle \mu \rangle$, V.E.V. OF A MAGNETICALLY CHARGED OPERATOR μ.
  [A.D.G 94]

- $\langle \mu \rangle \neq 0$ IN THE CONFINED PHASE.
- $\langle \mu \rangle = 0$ IN THE DECONFINED PHASE
- DECONFINEMENT IS AN ORDER-DISORDER TRANSITION.

[PISA 95 →]
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- SU(N) GAUGE GROUP $\langle \mu^a \rangle$ $a = 1, 2, \ldots N-1$

$$\mu^a(\vec{x}, t) = e^{i \frac{q}{2g} \int d^3y \, \vec{b}_\perp(\vec{x}-\vec{y}) \, \mathrm{Tr}\{\phi^a \vec{E}\}_{\vec{y},t}}$$

$$\vec{\nabla} \cdot \vec{b}_\perp = 0 \qquad \vec{\nabla} \wedge \vec{b}_\perp(\vec{r}) = \frac{\vec{r}}{r^3} + \text{Dirac string}.$$

- q AN INTEGER

$$U = P e^{i \int_{x_0}^{x} A_\mu dx^\mu}$$

- $\phi^a(\vec{y}, t) = U(\vec{y}, t) \, \phi^a_{diag} \, U^\dagger(\vec{y}, t)$

$$\phi^a_{diag} \equiv \Big(\underbrace{\frac{N-a}{N}, \ldots \frac{N-a}{N}}_{a}, \underbrace{-\frac{a}{N}, \ldots -\frac{a}{N}}_{N-a} \Big) \quad (a = 1, 2 \ldots N-1)$$

- $\langle \mu^a \rangle$ IS GAUGE INVARIANT
- IN THE GAUGE $\phi^a = \phi^a_{diag}$ (ABELIAN PROJECT)

$$\mu^a(\vec{x},t) = e^{\,i q/2g \int d^3y\, \vec{b}_\perp(\vec{x}-\vec{y})\, \vec{E}^a_\perp(\vec{y},t)}$$

$$\vec{E}^a \parallel T^a = (0\ 0 .. 0, \overset{a}{1}, \overset{a+1}{-1}, 0 .. 0) \qquad Tr\{\phi^a T^b\} = \delta_{ab}$$

- $e^{ipa} |x\rangle = |x+a\rangle \qquad \mu^a(\vec{x},t)\,|\vec{A}_\perp(\vec{z},t)\rangle = |\vec{A}_\perp(\vec{z},t) + \frac{q}{2g}\, b_\perp(\vec{x}-\vec{z})\rangle$

$$\langle \mu^a \rangle = \frac{Z(S+\Delta^a S)}{Z(S)} \qquad \langle \mu^a \rangle_{\beta=0} = 1 \qquad \beta = 2N/g^2$$

- $$\boxed{\rho^a \equiv \frac{\partial}{\partial\beta} \ln \langle \mu^a \rangle} \longrightarrow \boxed{\langle \mu^a \rangle = \exp\left[\int_0^\beta \rho(\beta')\, d\beta'\right]}$$

- LATTICE RESULTS $\qquad V = L_s^3$

(1) $T < T_c$ $(\beta < \beta_c)$ $\qquad \rho^a \xrightarrow[V\to\infty]{}$ finite limit $\Rightarrow \boxed{\langle \mu^a \rangle \neq 0}$

(2) $T > T_c$ $(\beta > \beta_c)$ $\qquad \rho^a \underset{V\to\infty}{\simeq} -c L_s + c' \qquad c > 0 \quad \boxed{\langle \mu^a \rangle = 0}$

(3) $T \simeq T_c$ $\quad \beta \simeq \beta_c$ $\qquad \rho^a / L_s^{1/\nu} = f(\tau L_s^{1/\nu})$ $\qquad$ (finite size scaling)

$$\tau = 1 - \frac{T}{T_c} \qquad \lambda \underset{\tau\to 0_+}{\simeq} \tau^{-\nu}$$

|| COMPACT U(1), SU(2), SU(3), $N_f = 2$ QCD.
|| INDEPENDENT OF THE ABELIAN PROJECTION.

• FROM THE DEFINITION OF $\langle \mu^a \rangle$

$$\langle \mu^a \rangle = \sum_{n=0}^{\infty} \left(\frac{ig}{2g}\right)^n \frac{1}{n!} \int d^3y_1 \ldots d^3y_n \, b_\perp^{i_1}(\vec{x}-\vec{y}_1) \ldots b_\perp^{i_n}(\vec{x}-\vec{y}_n)$$

$$\underbrace{\left\langle (\phi^a \cdot \vec{E})_{i_1, \vec{y}_1} \ldots \ldots (\phi^a \cdot \vec{E})_{i_n, \vec{y}_n} \right\rangle}$$

$$(\phi^a \cdot \vec{E}) \equiv \mathrm{Tr}\{\phi^a \cdot \vec{E}\}$$

GAUGE INVARIANT FIELD STRENGTH CORRELATORS

• WE SHALL IDENTIFY OUR G.I FIELD CORRELATORS WITH THOSE OF THE "STOCHASTIC VACUUM" APPROACH TO QCD [DOSH, SIMONOV] AND PERFORM A CLUSTER EXPANSION, AND TRUNCATE IT AT $n=2$.

2. CLUSTER EXPANSION OF $\langle \mu^a \rangle$

$$0 = \langle \Phi^a \vec{E} \rangle$$

$$\Phi^a_{i_1 i_2}(\vec{y}_1 - \vec{y}_2) \equiv \langle (\Phi^a \vec{E})_{i_1 \vec{y}_1} (\Phi^a \cdot \vec{E})_{i_2 y_2} \rangle$$

NEGLECT HIGHER CONNECTED CORRELATORS.

- $$\int d^3 y_1 \ldots \int d^3 y_{2n}\, b_\perp^{i_1}(y_1 - x) \ldots b_\perp^{i_{2n}}(y_{2n} - x) \langle (\Phi^a \vec{E})_{i_1 \vec{y}_1} \ldots (\Phi^a \vec{E})_{i_{2n} \vec{y}_{2n}} \rangle$$
$$= (2N-1)!! \left[\int d^3 y_1 \int d^3 y_2\, \Phi^a_{i_1 i_2}(\vec{y}_1 - \vec{y}_2)\, b_\perp^{i_1}(\vec{x} - \vec{y}_1)\, b_\perp^{i_2}(\vec{x} - \vec{y}_2) \right]^n$$

- ODD CORRELATORS VANISH $\qquad y_i \to y_i - x$

$$\langle \mu^a \rangle = e^{-\frac{g^2}{8g^2} \int d^3 y_1 \int d^3 y_2\, \Phi^a_{i_1 i_2}(\vec{y}_1 - \vec{y}_2)\, b_\perp^{i_1}(\vec{y}_1)\, b_\perp^{i_2}(\vec{y}_2)}$$

OR , SINCE $\beta = \frac{2N}{g^2}$

$$\boxed{\begin{aligned} \rho^a &\equiv \frac{\partial}{\partial \beta} \ln \langle \mu^a \rangle = \\ &= -\frac{g^2}{16N} \frac{\partial}{\partial \beta} \left[\beta \int d^3 y_1\, d^3 y_2\, \Phi^a_{i_1 i_2}(\vec{y}_1 - \vec{y}_2)\, b_\perp^{i_1}(y_1)\, b_\perp^{i_2}(y_2) \right] \end{aligned}}$$

- IDENTIFY $\Phi^a_{i_1 i_2}$ WITH THE 2-POINT CORRELATOR WITH A STRAIGHT LINE PARALLEL TRANSPORT
$\langle E^a E^b \rangle = \delta^{ab} \Phi \implies \rho^a$ INDEPENDENT ON a (AGREES WITH LATTICE DATA)
- CLUSTER EXPANSION JUSTIFIED AT LARGE DISTANCES (WE LOOK FOR I.R. PROPERTIES)
- CHECK : $\rho^a \propto g^2$; HIGHER CLUSTERS GIVE HIGHER POWERS OF g (FIG)

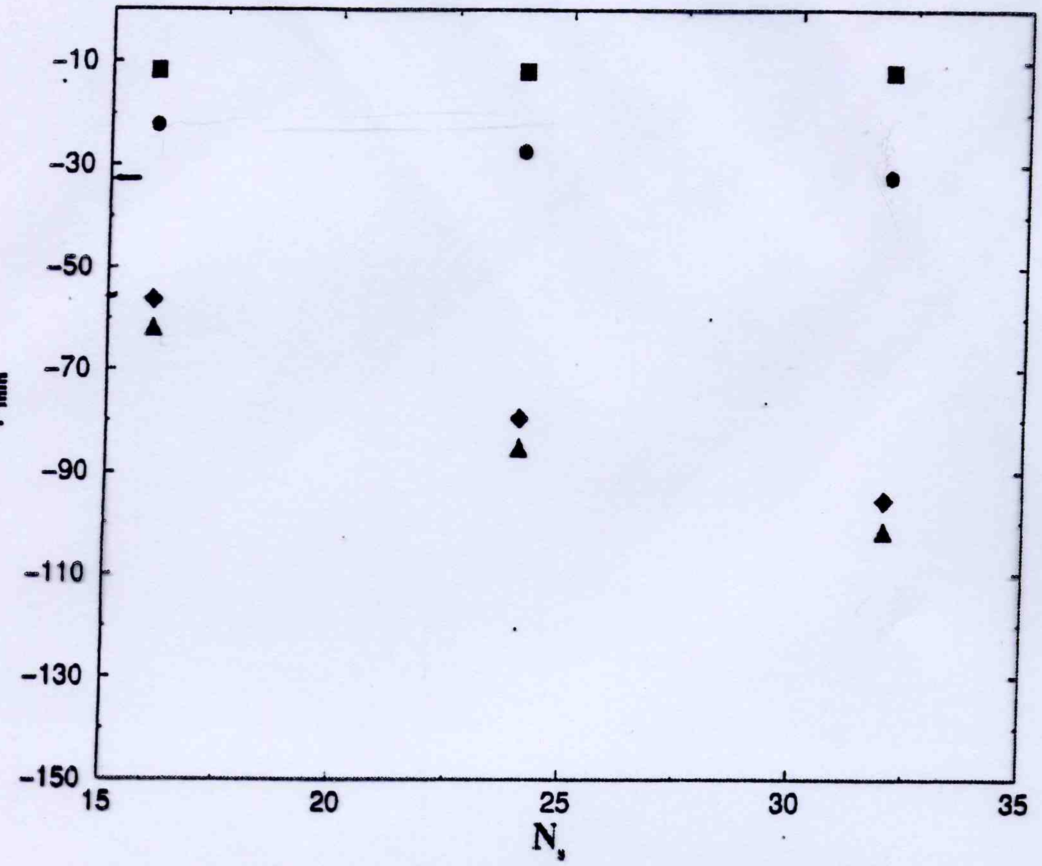

FIG. 2. ρ parameter in the weak coupling limit as a function of the spatial lattice size for different values of the whole magnetic charge in SU(2) gauge theory. Bullets refer to a single monopole of charge 2, squares refer to a dipole of zero net magnetic charge, diamonds refer to a single monopole of charge 4, and triangles refer to two monopoles of charge 2 put at a distance of 2 lattice spacings apart from each other.

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3. THE FIELD CORRELATORS

- GENERAL PARAMETRIZATION [Simonov, Dosch]

$$\Phi^{a,b}_{\mu_1\nu_1;\mu_2\nu_2} = \frac{1}{N} \mathrm{Tr} \langle F_{\mu_1\nu_1}(z_1)\, \Phi(z_1 z_2)\, F_{\mu_2\nu_2}(z_2)\, \Phi(z_2 z_1) \rangle$$

$$= \delta^{ab} (\delta_{\mu_1\mu_2}\delta_{\nu_1\nu_2} - \delta_{\mu_1\nu_2}\delta_{\mu_2\nu_1})\, D(z_2 - z_1)$$

$$+ \frac{1}{2} \Big(\frac{\partial}{\partial z_{\mu_1}} \left[(z_{\mu_2}\delta_{\nu_1\nu_2} - z_{\nu_2}\delta_{\nu_1\mu_2}) D_1(z_1 - z_2) \right]$$

$$+ \frac{\partial}{\partial z_{\nu_1}} \left[(z_{\nu_2}\delta_{\mu_1\mu_2} - z_{\mu_2}\delta_{\mu_1\nu_2}) D_1(z_1 - z_2) \right]$$

AT $T \neq 0$ $\quad D^E\ D_1^E\ ;\ D^H,\ D_1^H$

- FOR $\vec{E}$ FIELD $\quad \mu_1 = 0\ \nu_1 = i_1 \quad \mu_2 = 0\ \nu_2 = i_2\ ;\ z_1^0 - z_2^0 = 0$

$\Downarrow$

$$\Phi^{ab}_{i_1 i_2}(y_1 - y_2) = \delta \left[\delta_{i_1 i_2} \left(D^E + \frac{D_1^E}{2} \right) + \frac{\partial}{\partial i_1} \cdots \right]$$

IN THE CONVOLUTION WITH $\vec{b}_\perp$ THE DERIVATIVE TERMS GIVE 0.

FOR THE SAME REASON

$$\delta_{i_1 i_2} \Longleftrightarrow \delta_{i_1 i_2} - \frac{k_1 k_2}{k^2}$$

AND, GOING TO FOURIER TRANSFORM

$$\rho^a = - \frac{g^2}{16N} \frac{\partial}{\partial \beta} \left[\beta \int d^3y_1\, d^3y_2\, \Phi^a_{i_1 i_2}(y_1 - y_2)\, b_\perp^{i_1}(\vec{y}_1)\, b_\perp^{i_2}(y_2) \right]$$

$$= - \frac{g^2}{16} \frac{\partial}{\partial \beta} \left[\beta \int \frac{d^3k}{(2\pi)^3}\, \tilde{b}_\perp^{i_1}(k)\, \tilde{b}_\perp^{i_2}(-k) \right.$$

$$\tilde{\mathcal{D}}_E(k^2) \frac{1}{k^2} (k^2 \delta_{i_1 i_2} - k_{i_1} k_{i_2})$$

$$\tilde{\mathcal{D}}_E(k^2) = \tilde{D}^E + \tilde{D}_1^E/2$$

$$(k^2\delta_{i_1 i_2} - k_{i_1}k_{i_2})\, b_\perp^{i_1}(\vec{k})\, b_\perp^{i_2}(-k) = |\vec{H}(k)|^2$$

$$\vec{H}(k) = \vec{k} \wedge \vec{b}_\perp(k) = \frac{\vec{k}}{k^2} - \frac{\vec{n}}{(\vec{n}\vec{k}) - i\varepsilon}$$

$$\vec{H}(k^2) = -\frac{1}{k^2} + \frac{1}{k_z^2}$$

$$\boxed{\rho^a = \frac{g^2}{16}\frac{\partial}{\partial\beta}\left\{\beta \cdot \int \frac{d^3k}{(2\pi)^3}\left(\frac{1}{k^2} - \frac{1}{k_z^2}\right) f(k^2)\right\}}$$

$$f(k^2) = \frac{\tilde{\mathcal{D}}_E(k^2)}{k^2} \simeq \frac{\Phi_{ii}(k^2)}{N k^2}$$

ρ^a INDEPENDENT ON a.

ρ^a INDEPENDENT ON $U(x)$ i.e. ON ABELIAN PROJECTION

- D^E, D_1^E FROM LATTICE

$$\begin{cases} D^E = A\, e^{-x/\lambda_b} + \dfrac{b}{x^4} e^{-x/\lambda_a} \\ \mathcal{D}_1^E = A_1\, e^{-x/\lambda_b} + \dfrac{b_1}{x^4} e^{-x/\lambda_a} \end{cases} \qquad \boxed{.1\,fm \le x \le 1\,fm}$$

$\lambda_b \simeq .3\,fm \qquad \lambda_a \simeq 2\lambda_b \qquad \mathcal{D}_1^E \simeq .1\,\mathcal{D}^E$

A, A_1, b, b_1, λ_b, λ_a INDEPENDENT ON β

FOR $\beta < \beta_c$ UP TO $\frac{T}{T_c} \simeq .95 - .97$

ONLY DEPENDENCE ON β : EXPLICIT β FACTOR

PARAMETRIZATION INSPIRED BY WILSON'S

O.P.E. $\quad \frac{1}{x^4} \to I \qquad A \to \langle G_2 \rangle$

FITS THE LATTICE DATA IN THE RANGE

$.1\,fm \le |\vec{z}| \le 1\,fm$

• LARGE β $(T > T_c)$ - Pt THEORY

$$f(k) = \frac{1}{2Nk}$$

$$\rho^a = \frac{g^2}{16N} \int \frac{d^3k}{(2\pi)^3} \frac{1}{2k} \left(\frac{1}{k^2} - \frac{1}{k_z^2} \right) \qquad T >> T_c$$

U.V. CUT-OFF $1/a$ I.R. CUT-OFF $\frac{1}{L_s a}$

$$\boxed{\rho^a = \frac{g^2}{16N} \frac{1}{(2\pi)^2} \left\{ -\sqrt{2} L_s + 2 \ln L_s + \text{const} \right\}} \quad T > T_c$$

$$\rho^a \xrightarrow[L_s \to \infty]{} -c\, L_s + c' \qquad \langle \mu^a \rangle = 0$$

• $T < T_c \cdot (.96)$

$$\rho^a = \frac{g^2}{16N} \int \frac{d^3k}{(2\pi)^3} \left(\frac{1}{k^2} - \frac{1}{k_z^2} \right) f(k)$$

$$f(k) = \frac{1}{k^2} \left(\widetilde{D + \frac{D_1}{2}} \right)$$

- LATTICE PARAMETRIZATION OF CORRELATORS NOT CORRECT. AT SHORT DISTANCES

$$\text{O.P.E.} \xrightarrow[x \to 0]{} \left[\frac{b}{x^4} + c + x^2 \right] \qquad \frac{1}{x^4} \simeq \frac{1}{2} \left[\frac{1}{(x+i\xi)^4} + \frac{1}{(x-i\xi)^4} \right]$$

AND AT LARGE DISTANCES, WHERE A STRONGER I.R. CUT-OFF IS NEEDED Λ
THE TERM SINGULAR AT $x=0$ GIVES

$$\boxed{\rho^a_{sing} = \frac{g^2}{16N} \frac{1}{(2\pi)^2} \left\{ -\sqrt{2} \frac{\Lambda}{a} + 2 \ln \frac{\Lambda}{a} + \text{const} \right\}} \qquad L_s \rightleftarrows \Lambda$$

THE NON SINGULAR TERM CONTRIBUTES

$$\rho^a_{reg} = \frac{g^2}{8} \Lambda_b^3 \left(A + \frac{A_1}{2}\right) \frac{7}{3\pi} \Lambda$$

$$\rho^a = \rho_{sing} + \rho_{reg} = \frac{g^2}{16N} \left\{ \frac{1}{(2\pi)^2} \left[-\sqrt{2}\frac{\Lambda}{a} + 2 \ln \frac{\Lambda}{a} + \text{const} \right] + 2N \left(A + \frac{A_1}{2}\right) \frac{7}{3\pi} \Lambda_b^3 \Lambda \right\}$$

- ρ^a FINITE AS $L_s \to \infty \Rightarrow \langle \mu^a \rangle \neq 0$
- ρ^a U.V. DIVERGENT $\propto \frac{1}{a}$ (LIKE THE POLYAKOV LOOP)

CONSISTENT WITH LATTICE DATA (SEE FIG)

A CAREFUL STUDY PLANNED

- $T \sim T_c$ $\quad A, A_1, \Lambda$ ARE β DEPENDENT

$$\rho^a = \frac{g^2}{16N} \frac{\partial}{\partial \beta} \left[\beta \left\{ \frac{1}{(2\pi)^2} \left[-\sqrt{2}\frac{\Lambda}{a} + 2 \ln \frac{\Lambda}{a} + \text{const} \right] + 2N \left(A + \frac{A_1}{2}\right) \frac{7}{3\pi} \Lambda_b^3 \Lambda \right\} \right]$$

$$\begin{cases} \left(A + \frac{A_1}{2}\right) \simeq E^2 \xrightarrow[T \to T_c]{} 0 & [\text{f-y}] \\ \Lambda \xrightarrow[T \to T_c]{} \infty \end{cases}$$

A PRECISE STUDY NEEDED

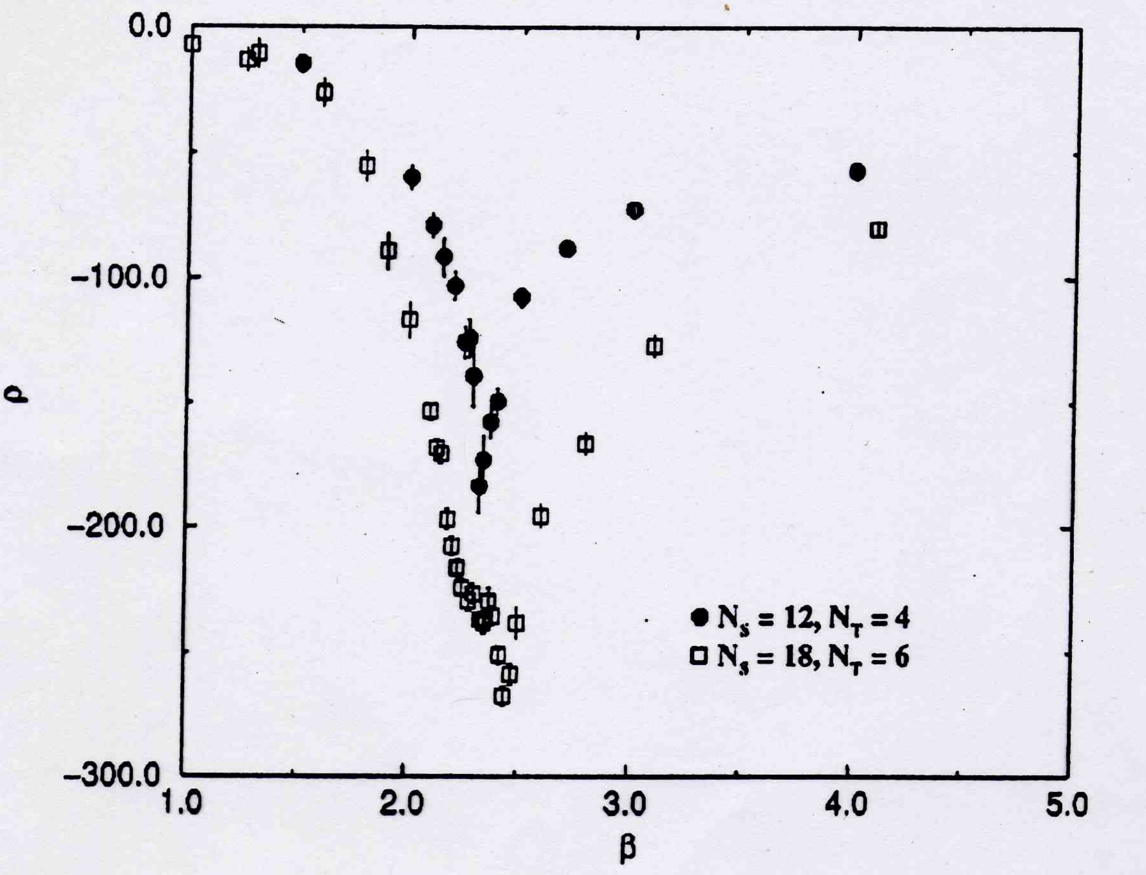

FIG. 2. ρ vs β for different lattice extensions (lattices $N_s^3 \times N_t$). Polyakov projection.

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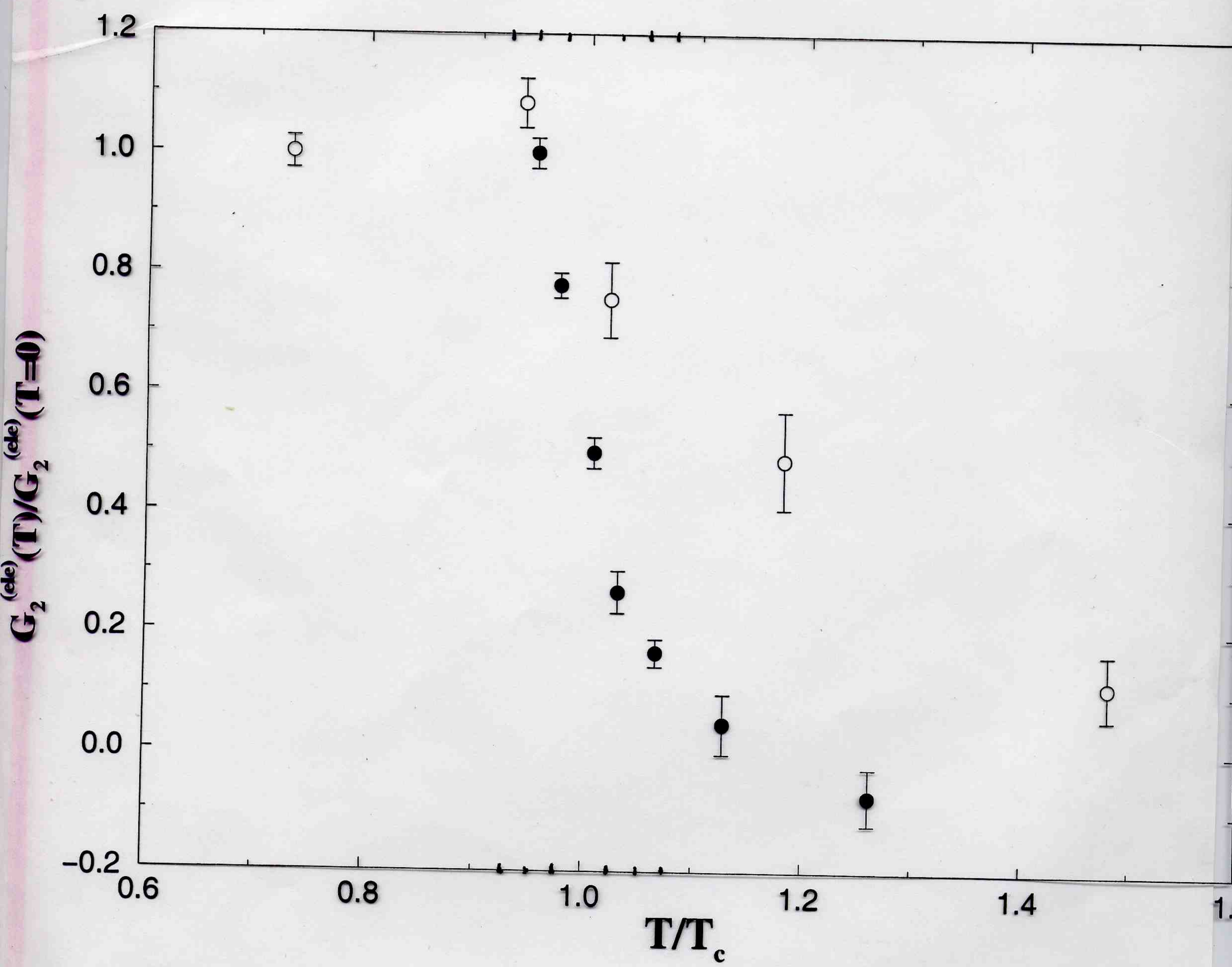

3 CONCLUSIONS

1) THE ORDER PARAMETER FOR MONOPOLE CONDENSATION CAN BE COMPUTED IN THE STOCHASTIC VACUUM APPROXIMATION

$\rho^a \propto q^2$ $\quad$ $q =$ magnetic charge.

ρ^a INDEP. ON a.

2) AT $T > T_c$ VACUUM IS NORMAL AND MAGNETIC CHARGE SUPERSELECTED

3) $T < T_c$ AN INFRARED CUT-OFF IN Λ THE CORRELATORS NEEDED TO HAVE $\langle\mu\rangle \neq 0$

- TO BE CHECKED ON LATTICE -

4) ρ U.V. DIVERGENT (LIKE POLYAKOV LINE)

- CONSISTENT WITH LATTICE DATA.

5) AT $T \simeq T_c$ $\langle \vec{E}^2 \rangle \to 0$ $\quad$ $\Lambda \to \infty$

FURTHER STUDY NEEDED TO RELATE SCALING (ORDER AND UNIVERSALITY CLASS OF THE TRANSITION) TO PROPERTIES OF CORRELATORS.

AN INTERESTING INTERPLAY OF CONFINEMENT WITH PROPERTIES OF GAUGE-INVARIANT FIELD CORRELATORS.