Exploring the QCD phase diagram

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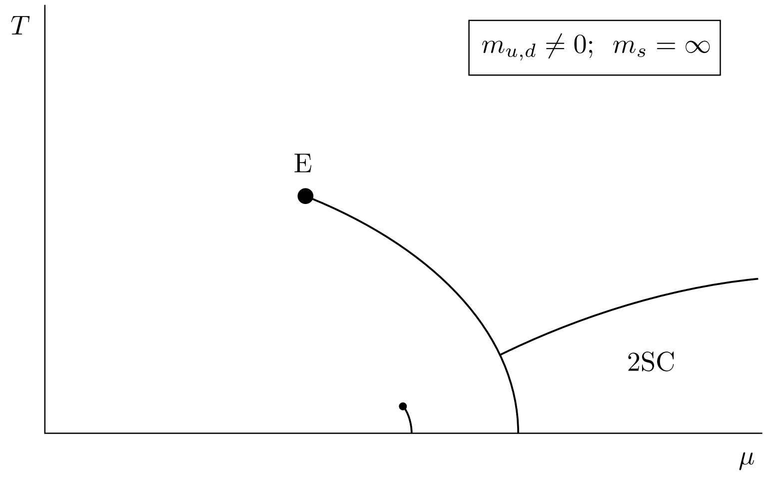
- Introduction
- Lattice QCD at finite density, the imaginary μ approach
- Numerical results for $N_f = 3$
- Numerical results for $N_f = 2 + 1$
- Conclusions

QCD: conjectured phase structure



Non-pert. problem $\Rightarrow$Lattice 1975-2001: $\mu \neq 0$ impossible $\Rightarrow$sign problem

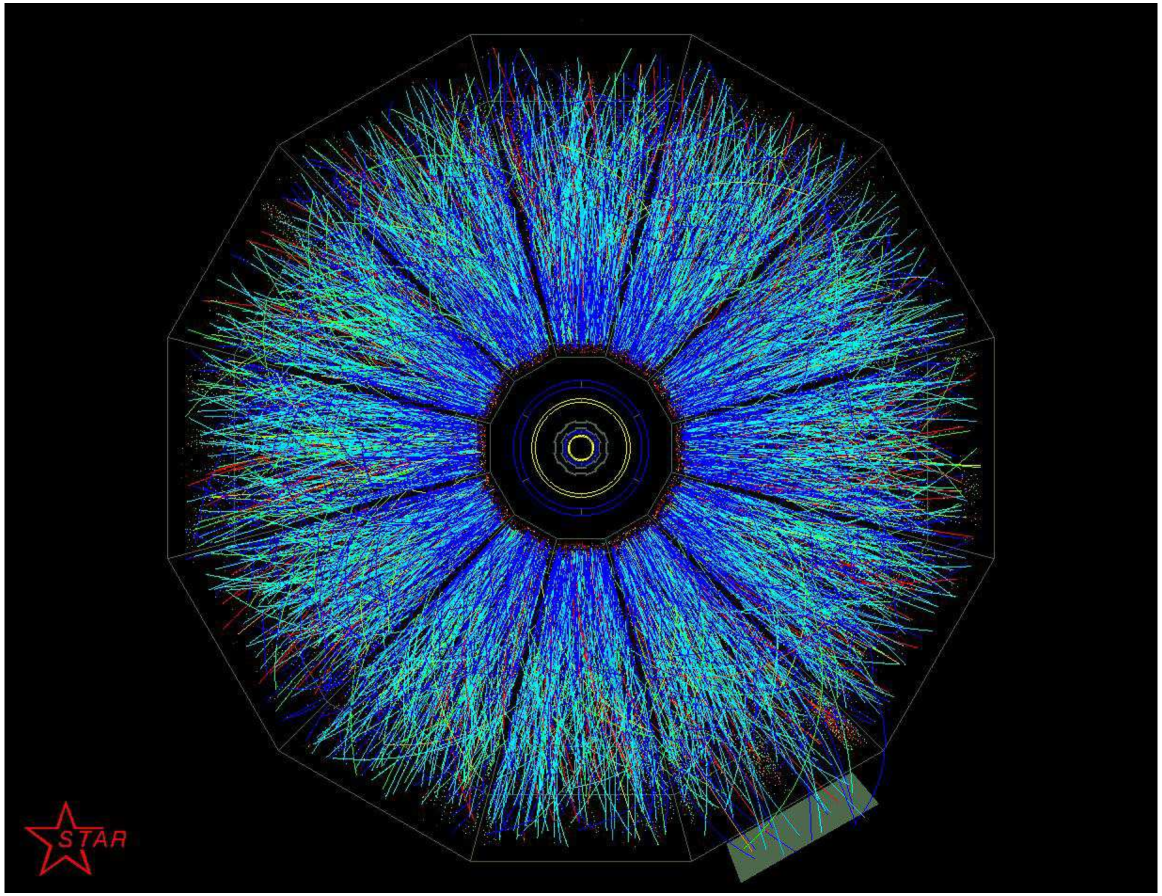
Where does this picture come from?

- Simulations on T-axis (light quarks only now)
- models for $T = 0, \mu \neq 0$

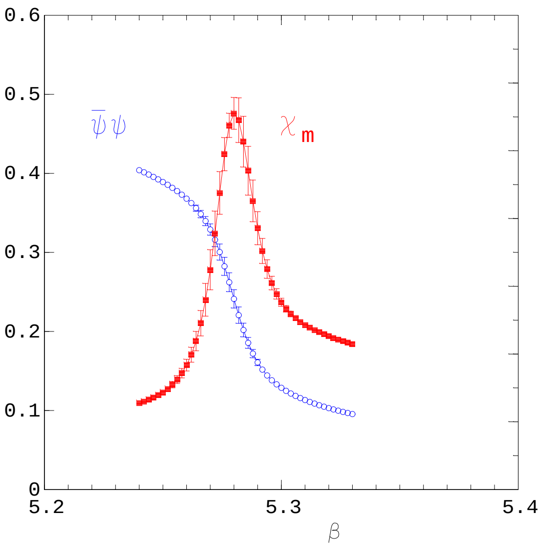
Since recently: phase diagram, $\mu_q/T \lesssim 1$: Reweighting, Taylor expansion, imaginary μ

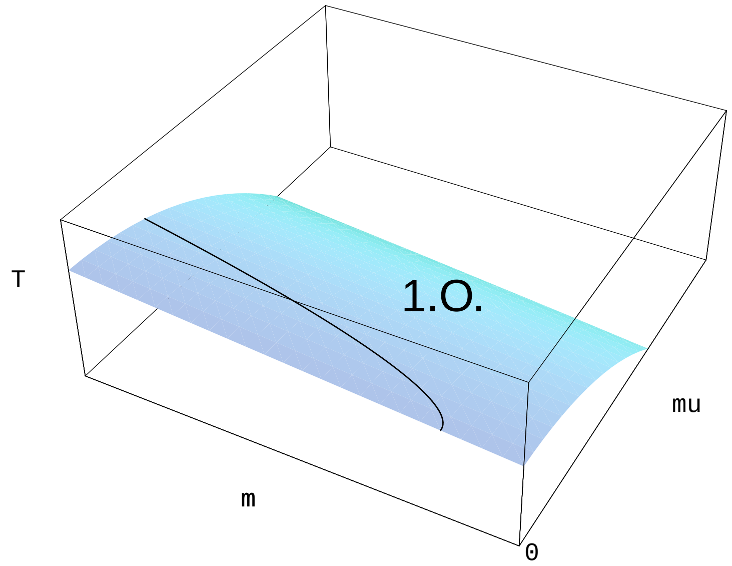
Take more general view $\Rightarrow$parameter space $\{m_{u,d}, m_s, T, \mu\}$

How experiment probes the phase transition & QGP....



$N_f = 3$ phase diagram 3d: $\{m, T, \mu\}$



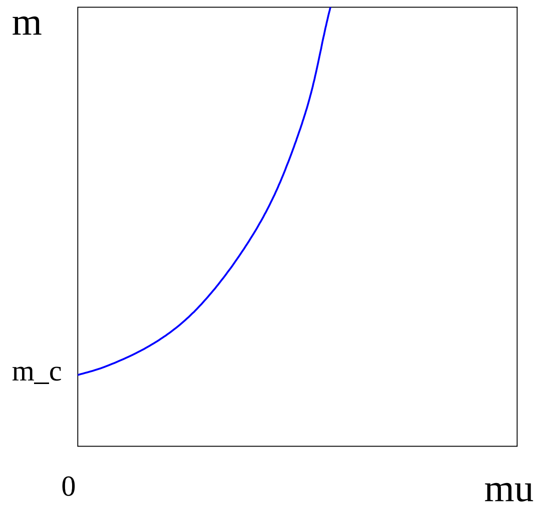


●confined/deconfined ⇒pseudo-crit. surface $T_0(\mu, m)$ (from susceptibilities)

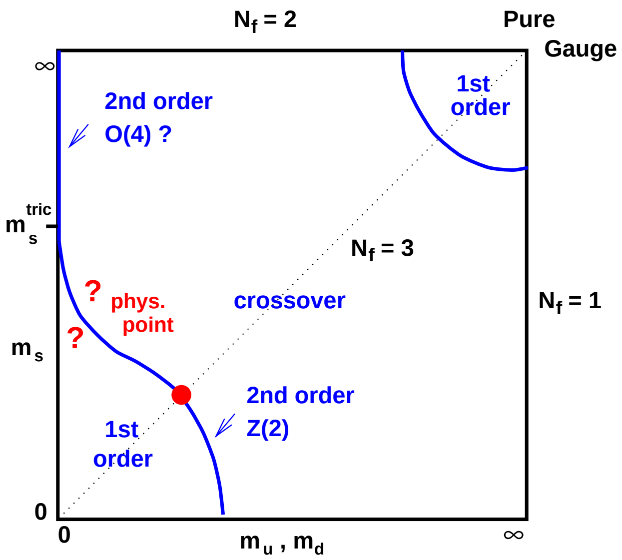
●1.O./crossover ⇒line of crit.points $T^E(\mu) = T_0(\mu, m_c(\mu))$ (from finite size scaling)

Projection onto (pseudo-) critical surface:

⇒$\mu^c(m)$ or $m_c(\mu)$



The case $N_f = 2+1, \mu = 0$:



$\Rightarrow m_c(\mu = 0)$ (unimproved KS)

Bielefeld; Columbia; de Forcrand, O.P.

$N_f = 3$ **universality:** 3d Ising model — Bielefeld

N.B: m_c has strong cut-off effects!

(factor 1/4?) — Bielefeld, MILC

$N_f = 2, m = 0$: is it O(4)/O(2) or first order?

● previous evidence inconclusive — cf. old proceedings

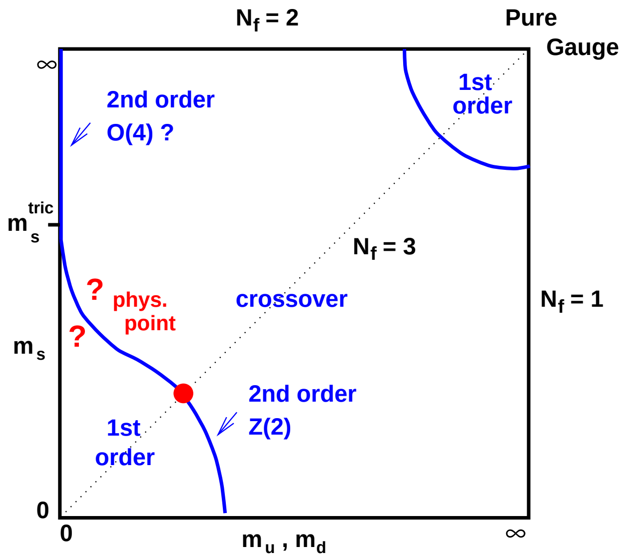
New investigations:

D'Elia, Di Giacomo, Pica: FSS on $L^3 \times 4$, $L = 16-32$, standard KS, R-algorithm, $m/T \gtrsim 0.055$

$\Rightarrow$ prefers first order

Kogut, Sinclair: FSS, χQCD, standard KS, fitting to small volume O(2) model $\Rightarrow$ O(2) scaling

The case $N_f = 2 + 1, \mu = 0$:



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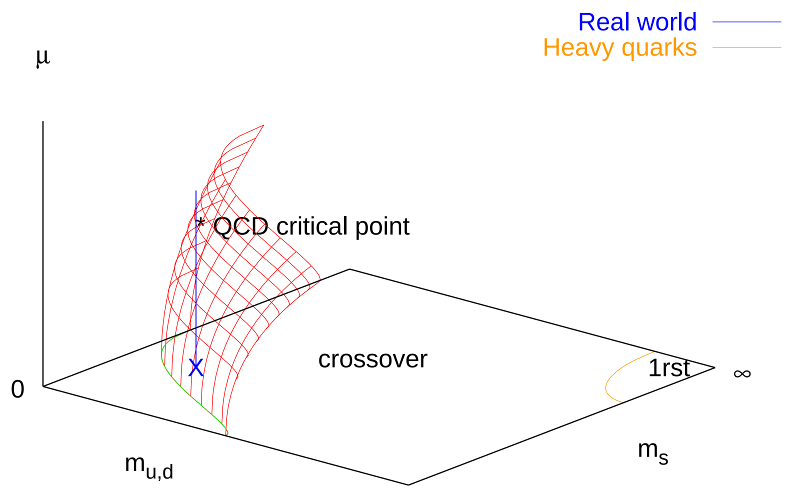
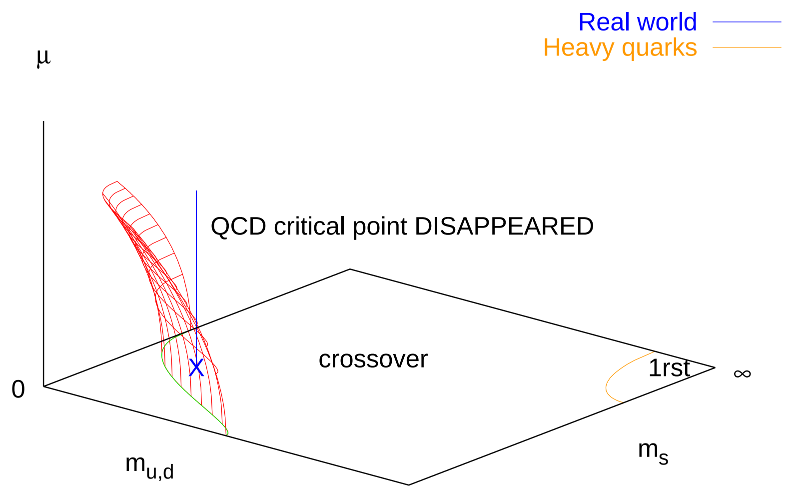
Finite density, $\mu \neq 0$:





Lattice QCD at finite temperature and density

Difficult (impossible?): **sign problem of lattice QCD**

$$Z = \int DU\, [\det M(\mu)]^f e^{-S_g[U]}, \quad S_f = \sum_f \bar{\psi} M \psi$$

$\det(M)$ complex for SU(3), $\mu = \mu_B/3 \neq 0 \Rightarrow$ no Monte Carlo importance sampling

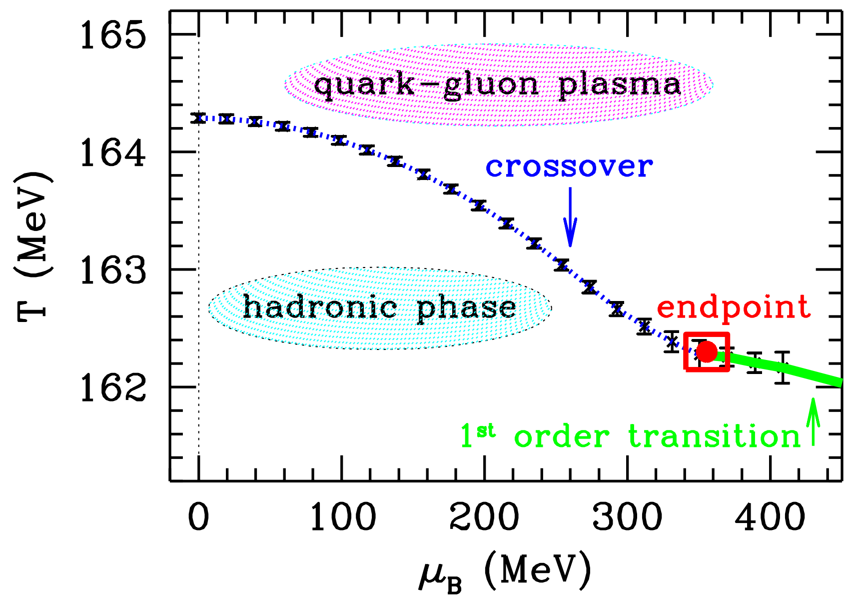
Evading the sign problem:

I. Two-parameter reweighting in (μ, β) Fodor, Katz

$$Z = \left\langle \frac{e^{-S_g(\beta)} \det(M(\mu))}{e^{-S_g(\beta_0)} \det(M(\mu = 0))} \right\rangle_{\mu=0,\beta_0}$$

$$= e^{\Delta F/T} \sim e^{-const.V}$$

idea: simulate at $\beta_0 = \beta_c(0)$, better overlap by sampling both phases; errors? ovlp.?



II. Taylor expansion

idea: for small μ/T, compute coeffs. of Taylor series $\Rightarrow$local ops.

$\Rightarrow$gain V convergence?

Bielefeld/Swansea: $N_f = 2, m/T_0 = 0.4$, improved KS, no critical signal at $O(\mu^6)$

Gavai/Gupta: $N_f = 2, m/T_0 = 0.1$, standard KS, critical point at $\mu_B^c/T = 1.1 \pm 0.2$

III.a Imaginary μ + analytic continuation

de Forcrand, O.P.

D'Elia, Lombardo

Azcoiti et al.

Chen, Luo

fermion determinant positive $\Rightarrow$no sign problem

idea: for small μ/T, fit **full** simulation results of imag. μ by Taylor series

- vary **two** parameters (μ, T) $\Rightarrow$controlled continuation?

III.b Imaginary μ + Fourier transformation

de Forcrand, Kratochvila

Alexandru et al

idea: canonical partition function at fixed baryon density

- no analytic continuation, but determinant needed $\Rightarrow$thermodynamic limit?

QCD at complex μ: general properties

$$Z(V,\mu,T) = \mathrm{Tr}\left(\mathrm{e}^{-(\hat{H}-\mu\hat{Q})/T}\right); \quad \mu = \mu_r + \mathrm{i}\mu_i; \quad \bar{\mu} = \mu/T$$

exact symmetries: μ-reflection and μ_i-periodicity — Roberge, Weiss

$$\boxed{Z(\bar{\mu}) = Z(-\bar{\mu}), \qquad Z(\bar{\mu}_r,\bar{\mu}_i) = Z(\bar{\mu}_r,\bar{\mu}_i + 2\pi/N_c)}$$

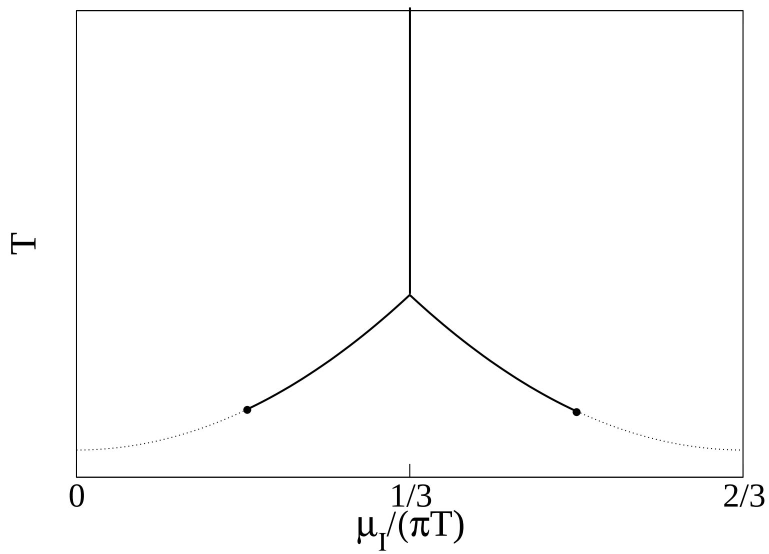
Imaginary μ phase diagram:

Z(3)-transitions: $\bar{\mu}_i^c = \frac{2\pi}{3}\left(n+\frac{1}{2}\right)$

1rst order for high T, crossover for low T

analytic continuation within:

$|\mu|/T \leq \pi/3 \Rightarrow \mu_B \lesssim 550\mathrm{MeV}$

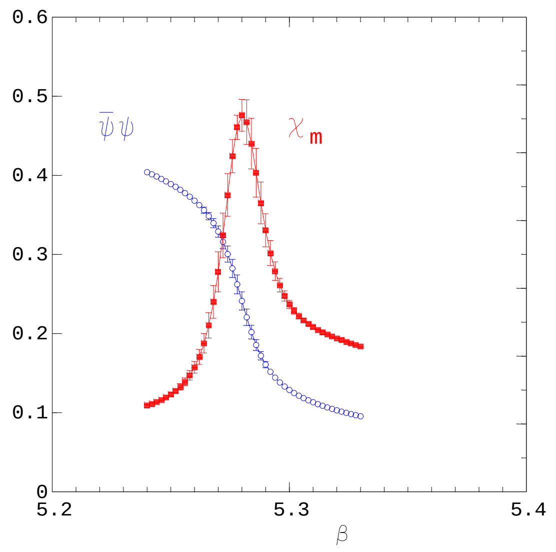


$$\langle O \rangle = \sum_n^N c_n \bar{\mu}_i^{2n} \Rightarrow \mu_i \longrightarrow -\mathrm{i}\mu_i$$

Analyticity of $T_0(\mu)$ on finite V

de Forcrand, O.P.

location of phase transition from susceptibilites:



$$\chi(\beta, a\mu, V) = V N_t \left\langle (\mathcal{O} - \langle \mathcal{O} \rangle)^2 \right\rangle ,$$

- finite volume:

suscept. always finite and analytic

Critical line $\beta_c(a\mu)$ defined by peak $\chi_{max} \equiv \chi(a\mu_c, \beta_c)$

- implicit function theorem: $\chi(\beta, a\mu)$ analytic $\Rightarrow \beta_c(a\mu)$ analytic!

symmetries: $\Rightarrow$ $\boxed{\beta_c(a\mu) = \sum_n c_n (a\mu)^{2n}}$

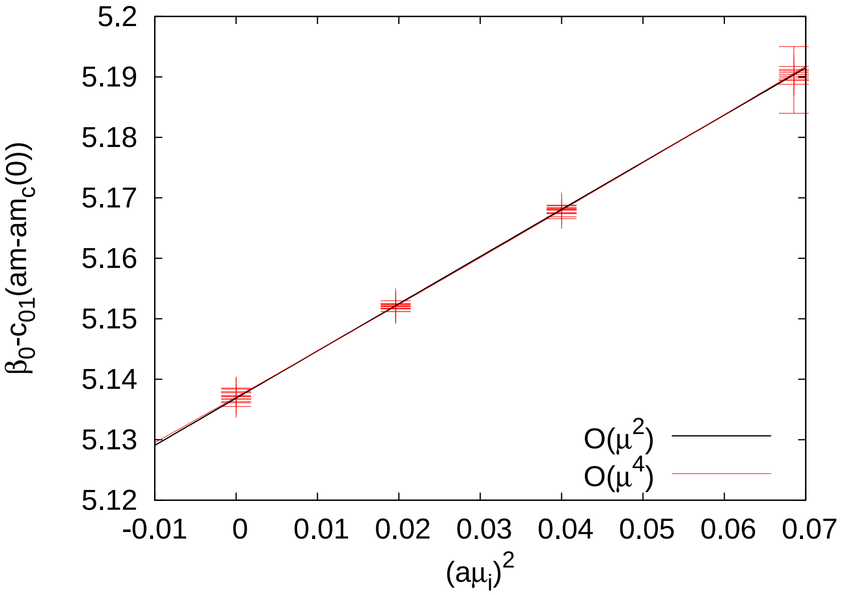
What to expect in physical units?

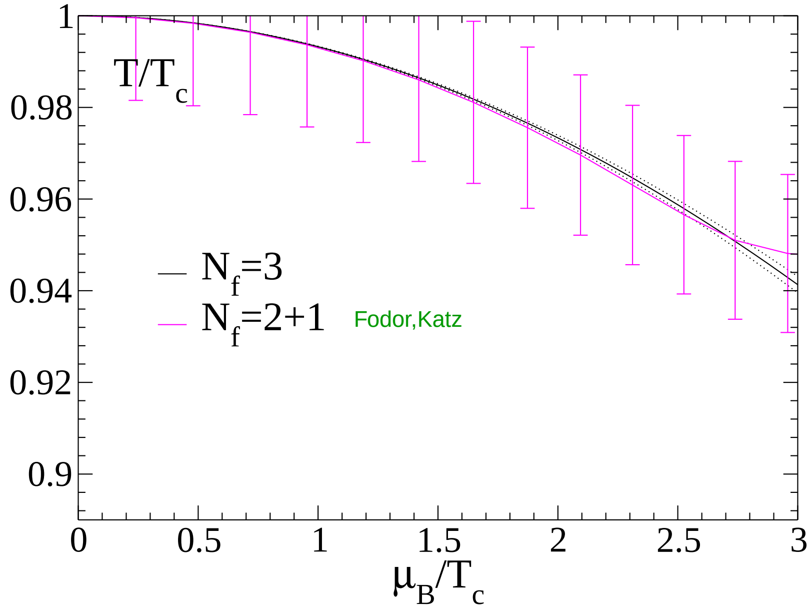
Natural expansion parameter is $\frac{\mu}{\pi T}$: • thermal perturbation theory

$$\Rightarrow \frac{T_0(\mu)}{T_0(\mu=0)} = 1 - O(1) \left(\frac{\mu}{\pi T_0(0,m)} \right)^2 + O(1) \left(\frac{\mu}{\pi T_0(0,m)} \right)^4 + \ldots$$

$T_0(\mu)$: $N_f = 3$ **results, quark mass dependence**, unimproved KS, $8^3 - 16^3 \times 4$

$$\beta_0(a\mu, am) = \sum_{k,l=0} c_{kl}\,(a\mu)^{2k}\,(am - am_c(0))^l$$





$\Rightarrow$data well described by μ^2 fit !

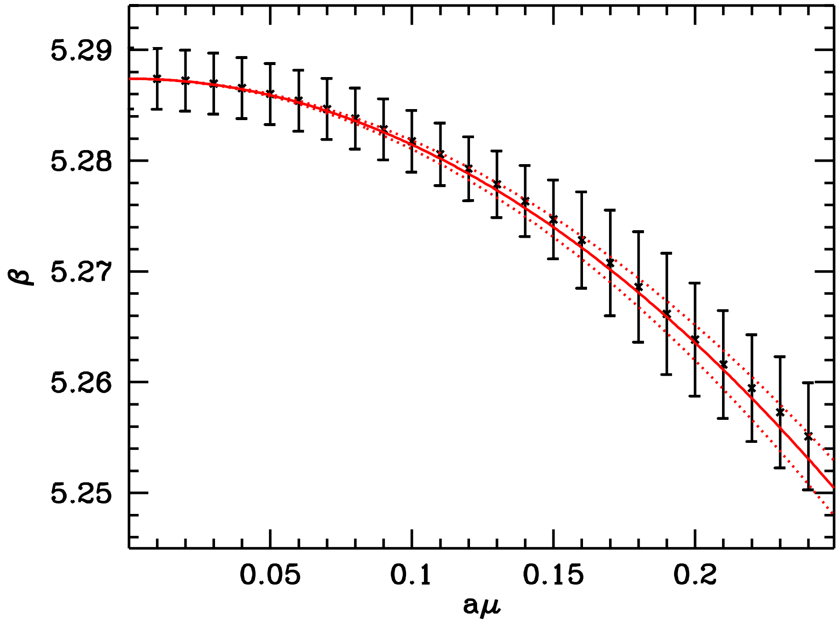
similar for pressure, screening masses

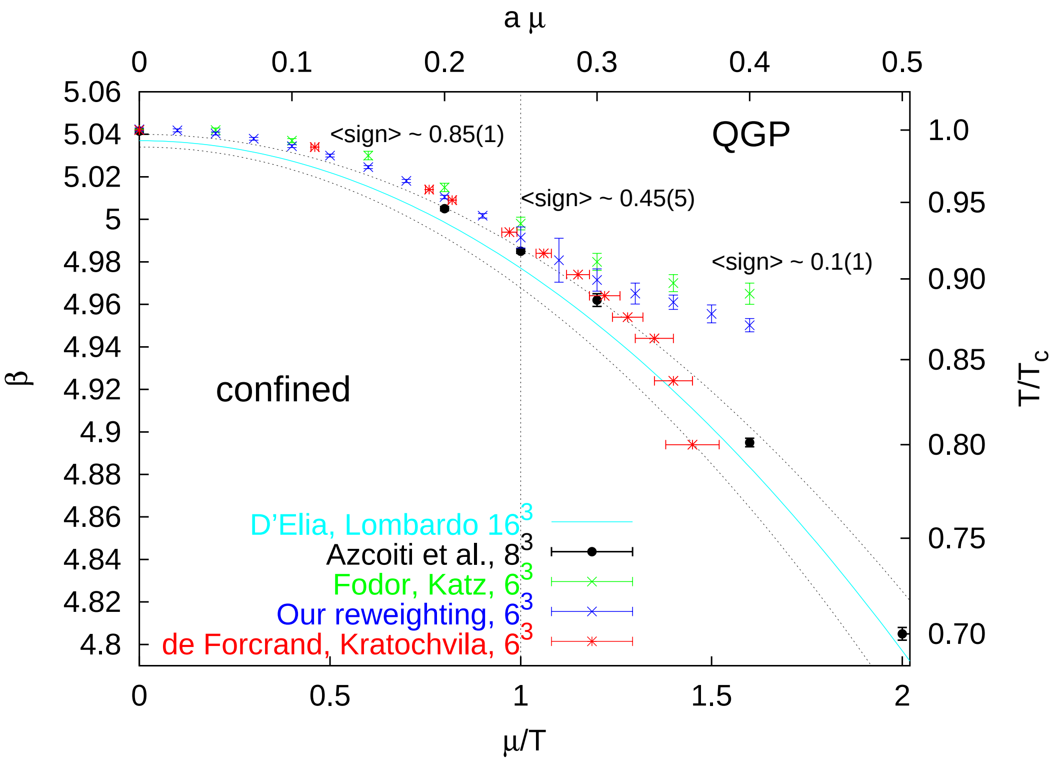
$$\frac{T_0(\mu, m)}{T_0(\mu = 0, m_c(0))} = 1 + 2.111(17)\frac{m - m_c(0)}{\pi T_0} - 0.667(6)\left(\frac{\mu}{\pi T_0(0, m)}\right)^2 + \ldots$$

Comparing different approaches: $N_f = 2$ (left), $N_f = 4$ (right):

Reweighting vs. imag. μ (FK, FP)

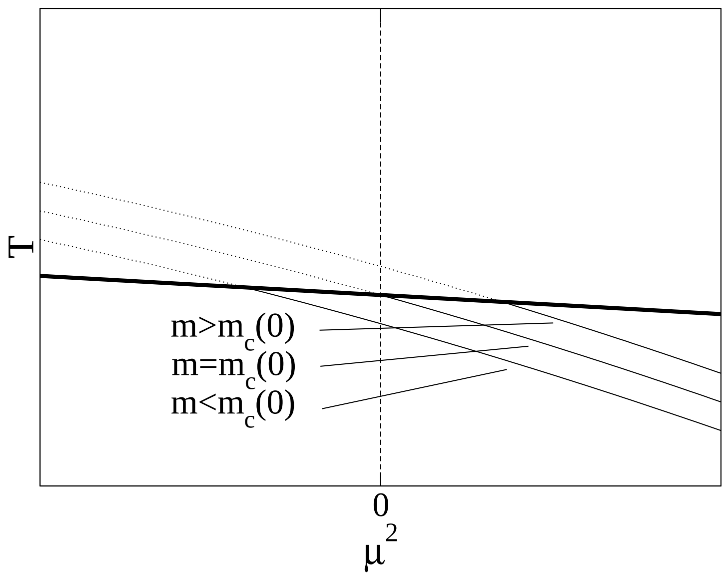
Rew., imag. μ, canonical ensemble ...

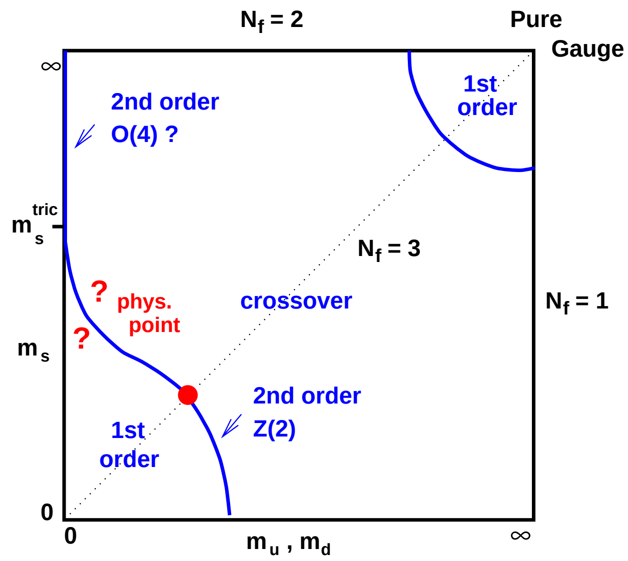


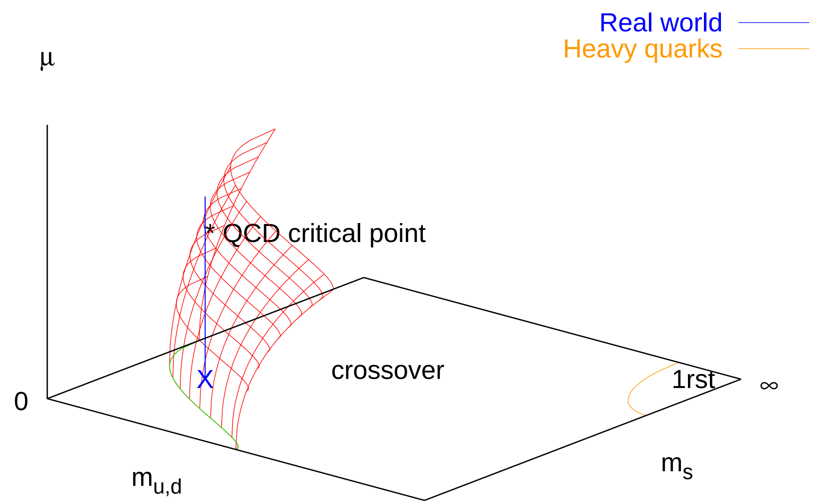


All agree on $T_0(m, \mu)$!!! $(\mu/T \lesssim 1)$

The critical endpoint and its quark mass dependence in $N_f = 3$







Expect: $$\frac{m_c(\mu)}{m_c(\mu{=}0)} = 1 + c_1 \left(\frac{\mu}{\pi T}\right)^2 + \ldots$$

Inverted: curvature of critical surface $\mu_c(m)$

Criticality: cumulant ratios, Binder cumulant

$$B_4(m,\mu) = \frac{\langle (\delta\bar{\psi}\psi)^4\rangle}{\langle (\delta\bar{\psi}\psi)^2\rangle^2}, \qquad \delta\bar{\psi}\psi = \bar{\psi}\psi - \langle\bar{\psi}\psi\rangle$$

3d Ising universality:

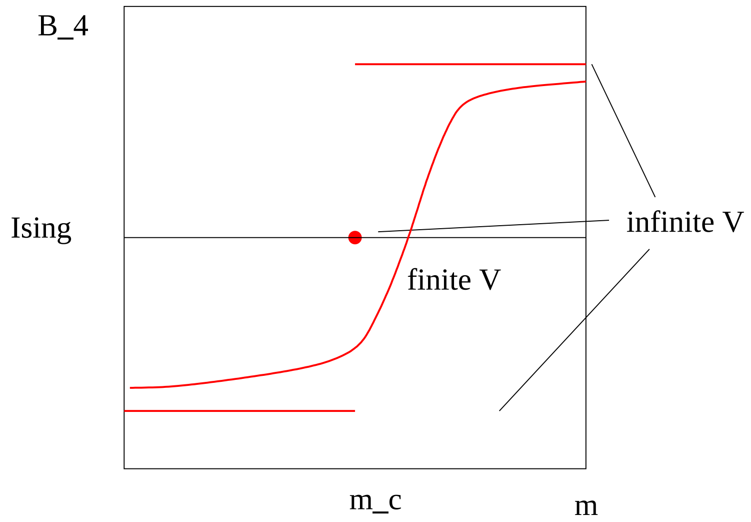
$V \to \infty$, step function

$$B_4(m,\mu) \quad \to \quad 3 \quad \text{crossover}$$

$$B_4(m_c,\mu_c) \quad \to \quad 1.604 \quad \text{critical}$$

$$B_4(m,\mu) \quad \to \quad 1 \quad \text{first order}$$

finite V + FSS

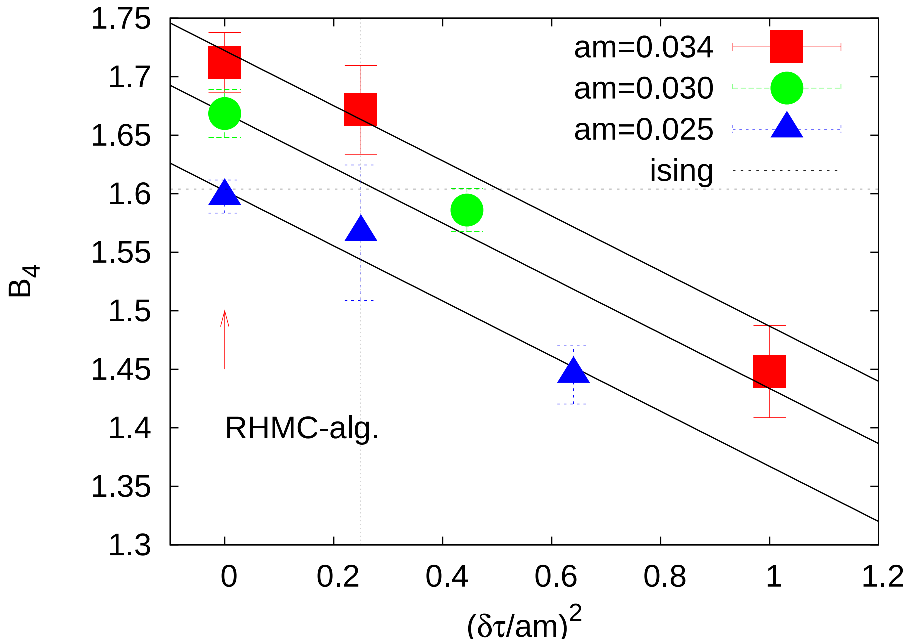


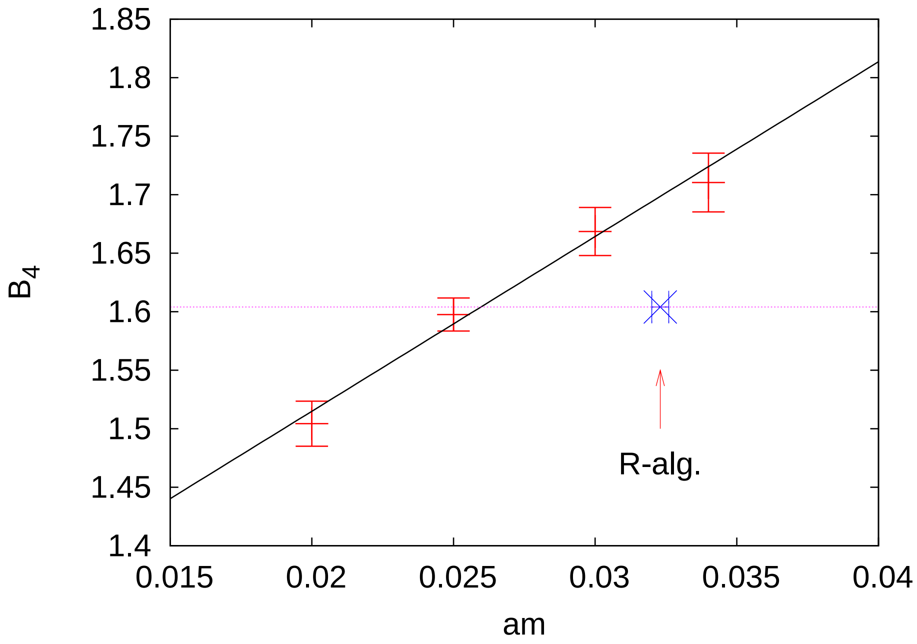
Redoing $N_f = 3$ with an exact algorithm

controlling algorithmic step size errors prohibitively expensive for small m

$\Rightarrow$**use exact algorithm** here: Rational Hybrid MC Clark, Kennedy

critical mass $m_c(0)$ with RHMC:



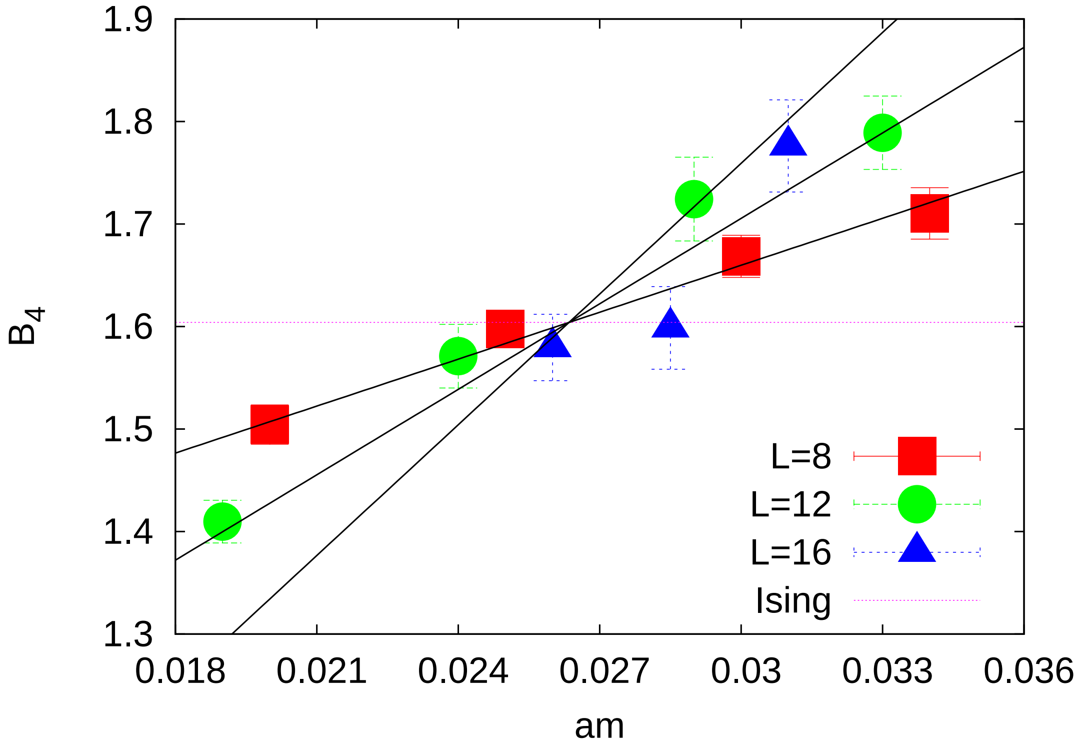


$\Rightarrow$25% change in $m_c(0)$

$\Rightarrow \sim$10% change in m_π^c

cf. Kogut, Sinclair

Finite size scaling:

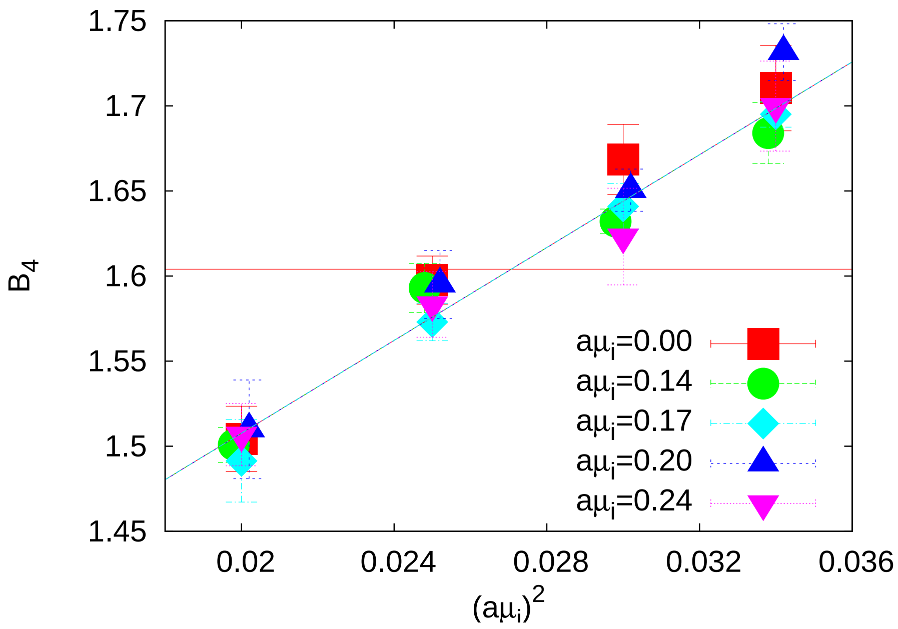


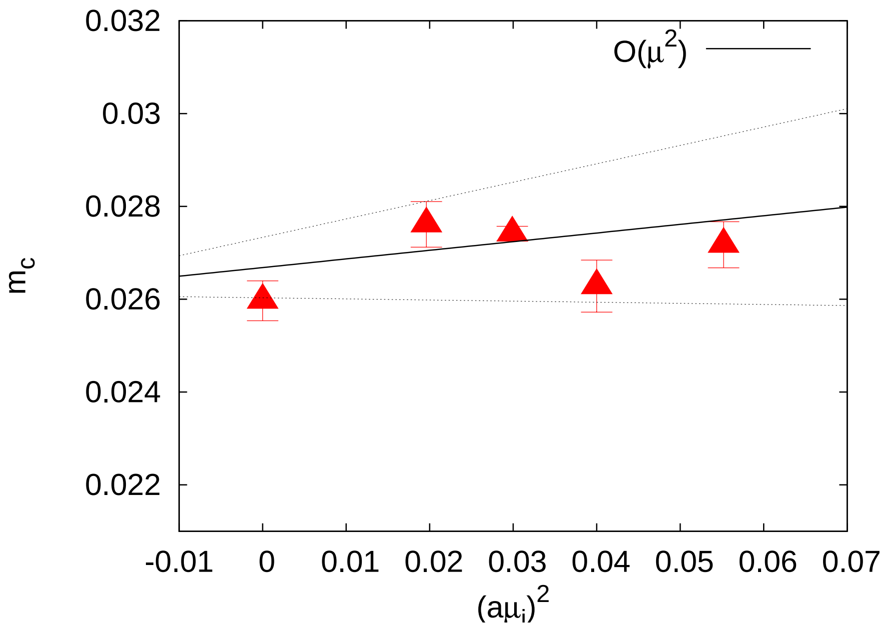
Fit: $\nu = 0.67(13)$

$$\xi \quad \sim \quad L^{-\nu}, \qquad \nu_{\text{Ising}} = 0.63$$

$$\Rightarrow B_4(m, L) \quad = \quad b_0 + bL^{1/\nu}(m - m_0^c)$$

Computing $m_c(\mu)$ for $N_f = 3$





About 300k trajectories per data point!

Fitting to Taylor series $\quad B_4(am, a\mu) = 1.604 + B\left(am - am_c(0) + A(a\mu)^2\right) + \dots$

$$\Rightarrow \frac{am^c(\mu)}{am^c(\mu = 0)} = 1 + c_1'(a\mu)^2 + \dots$$

N.B. finite a effect: $\frac{am_c(\mu)}{am_c(0)} \neq \frac{m_c(\mu)}{m_c(0)}; \qquad a = \frac{1}{T(\mu)N_t} = a(\mu)$

Continuum conversion:

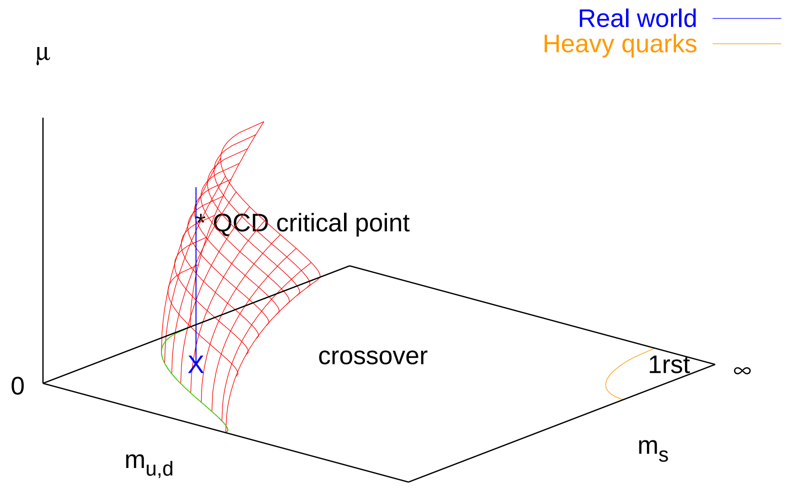
$$c_1 = \frac{1}{m_c(0)} \frac{dm_c}{d(\mu/\pi T)^2} = \frac{\pi^2}{N_t^2 (am_c)(0)} \frac{d(am_c)}{d(a\mu)^2} + \frac{1}{T_0} \frac{dT_0}{d(\mu/\pi T)^2}.$$

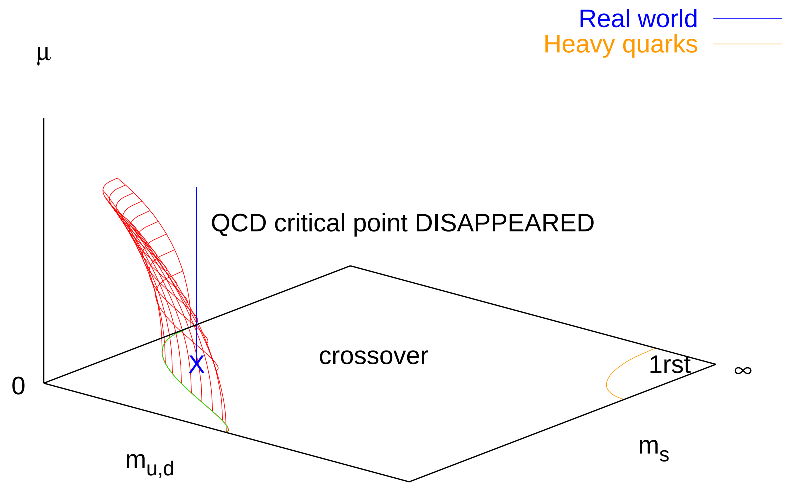
....leads to negative curvature of critical mass:

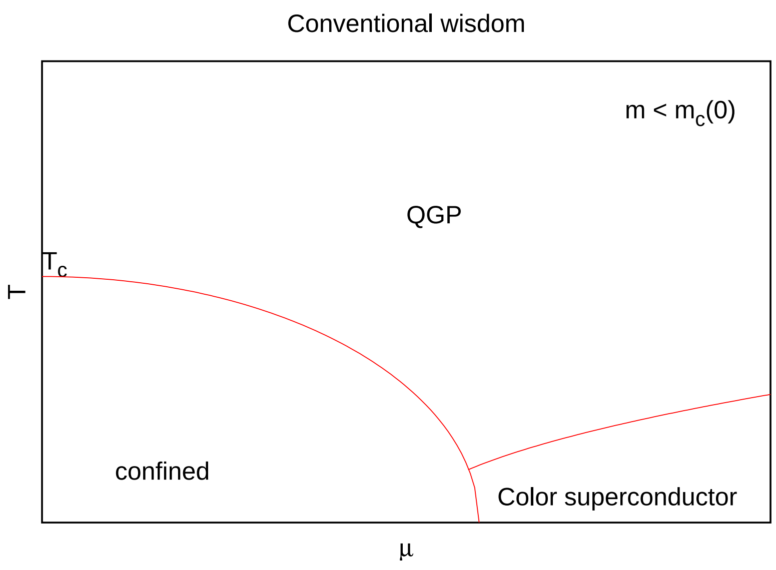
$$\Rightarrow \frac{m_c(\mu)}{m_c(\mu=0)} = 1 - 0.7(4) \left(\frac{\mu}{\pi T}\right)^2 + \ldots$$

A non-standard scenario: no critical point?

sign of $\frac{dm_c(\mu)}{d\mu^2}\Big|_{\mu=0}$







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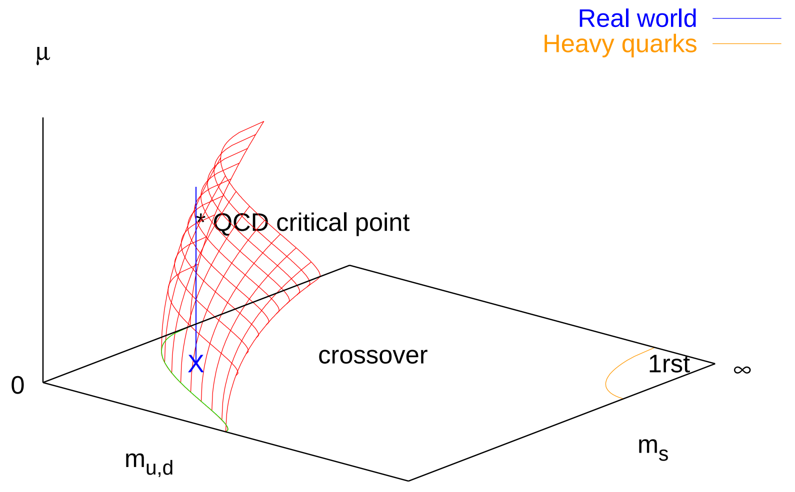

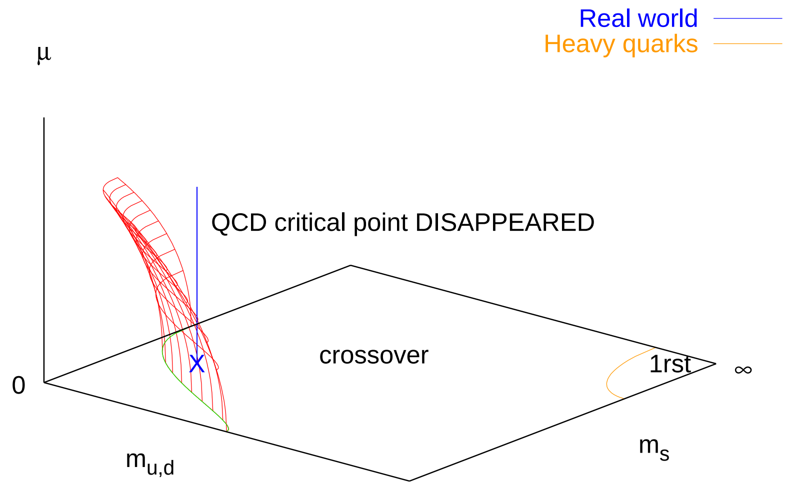

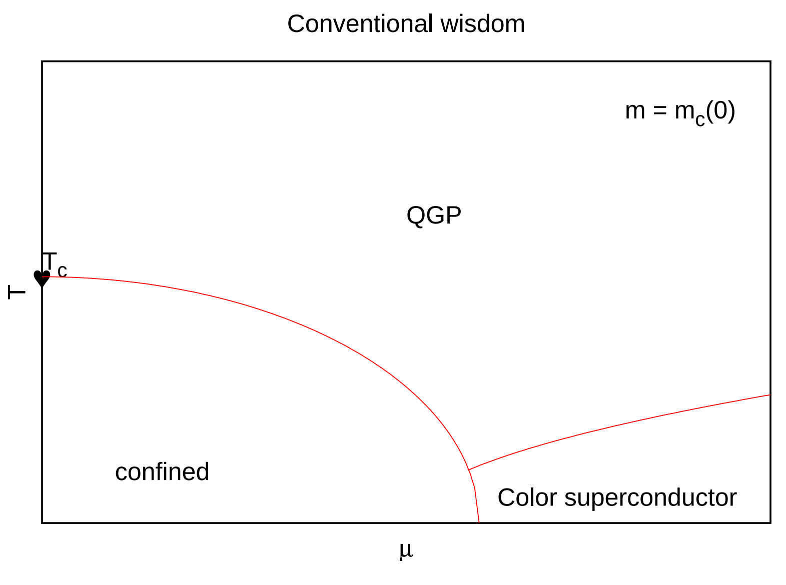

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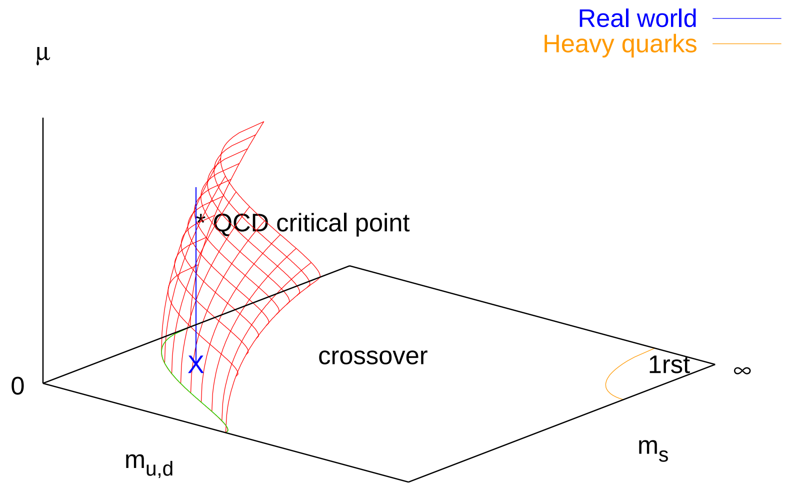

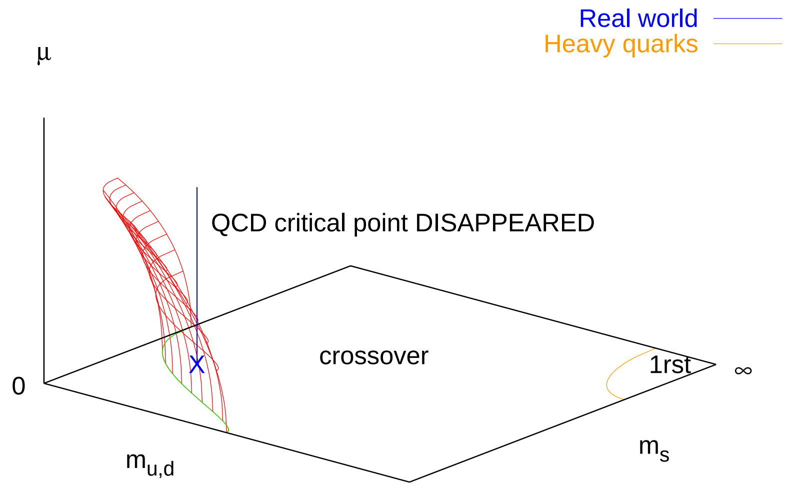

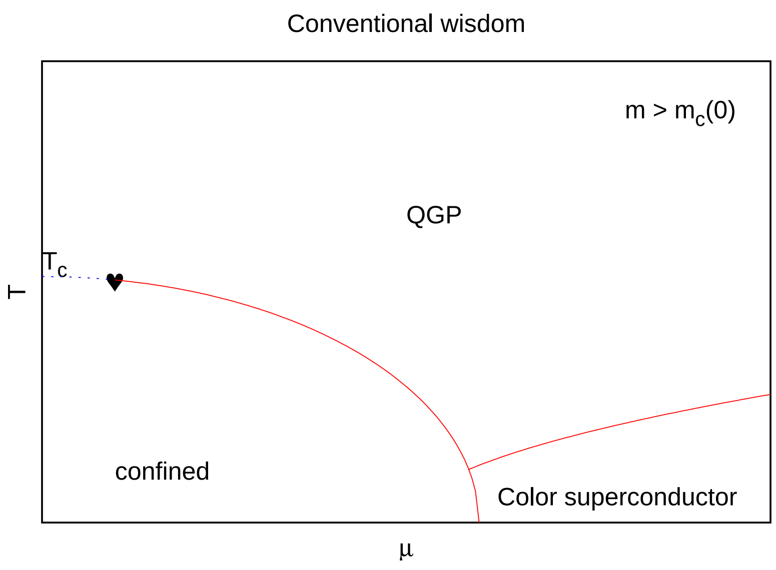

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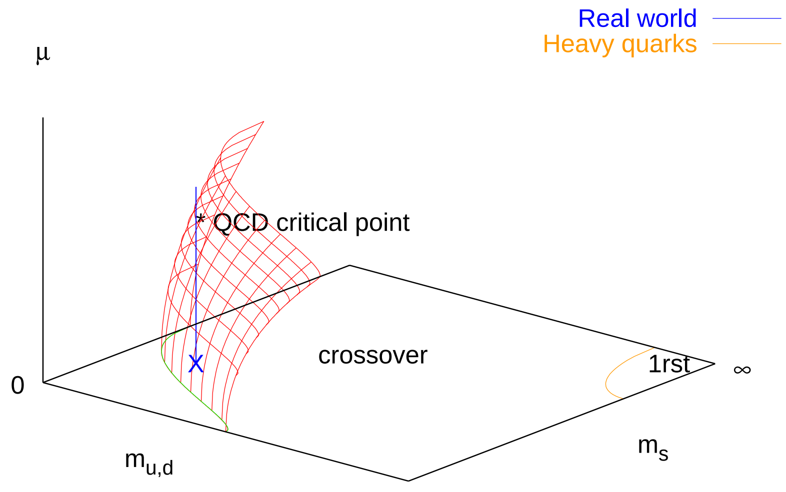

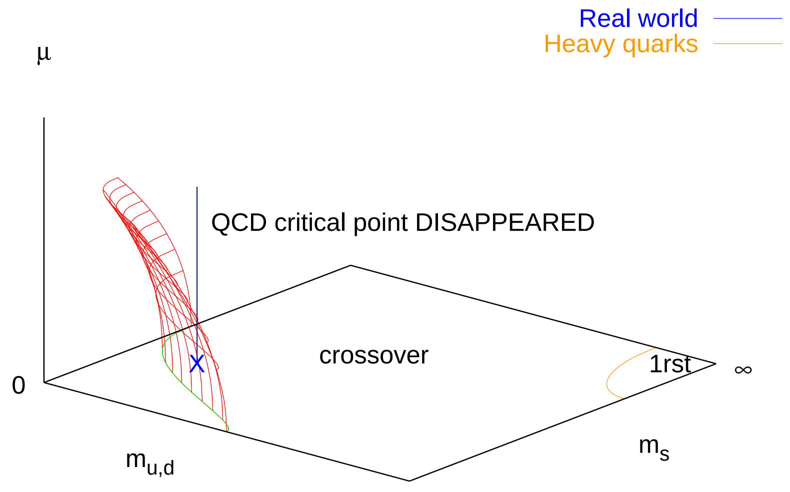

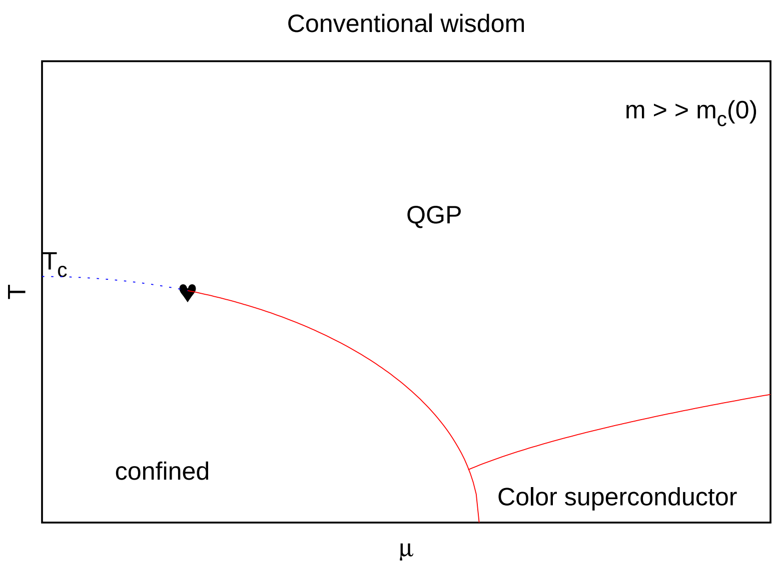

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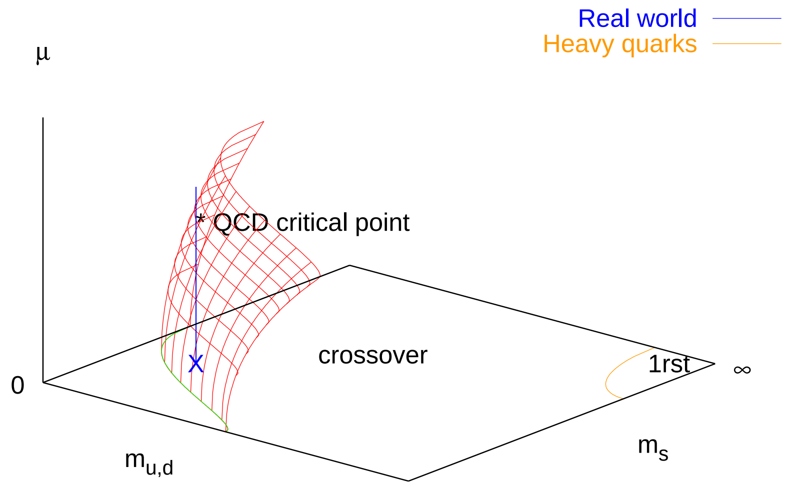

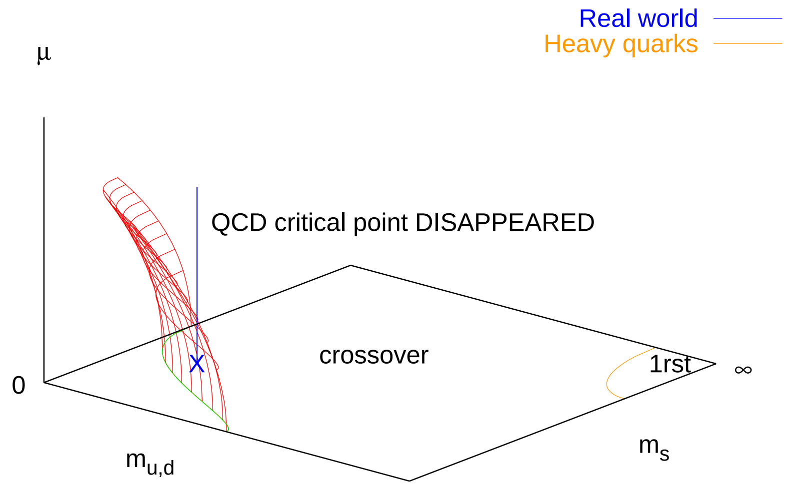

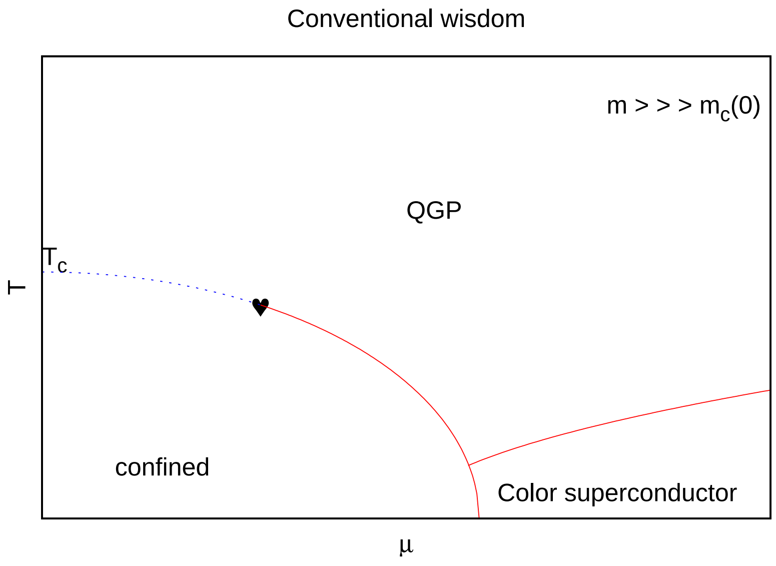

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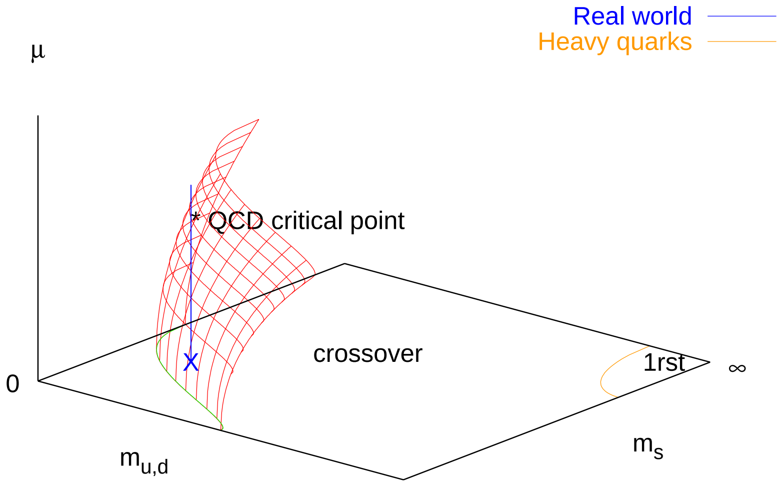

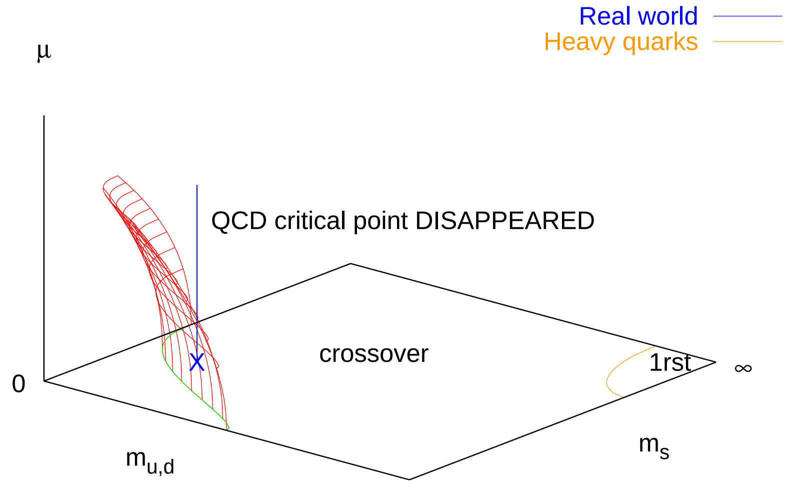

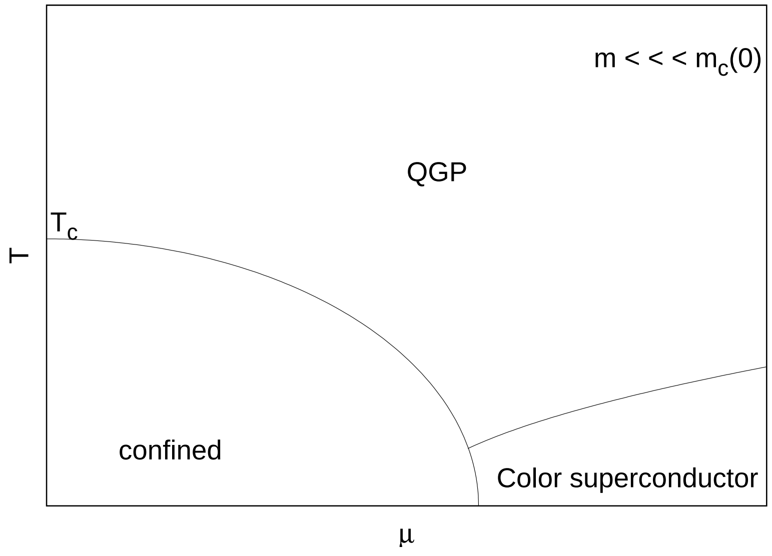

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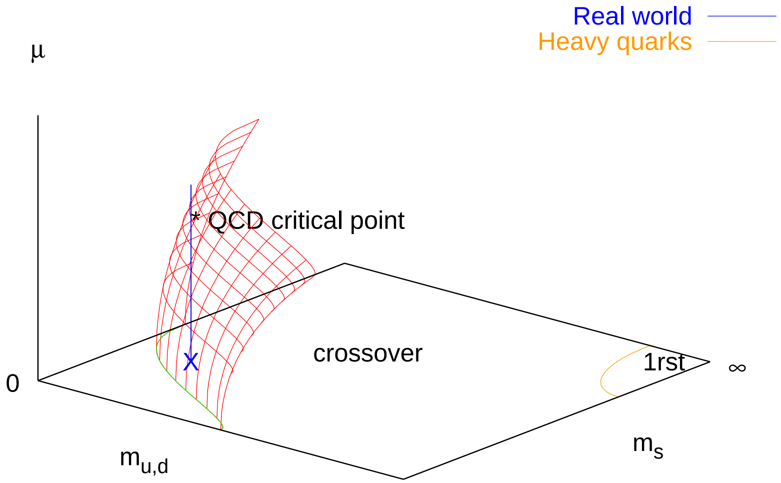

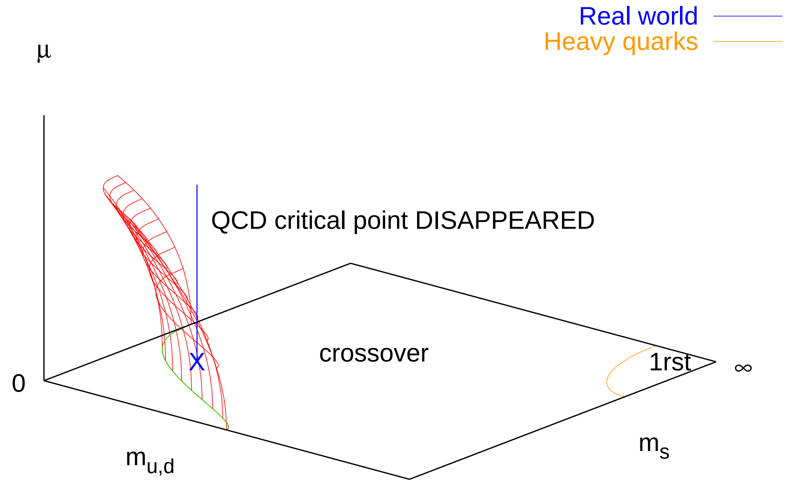

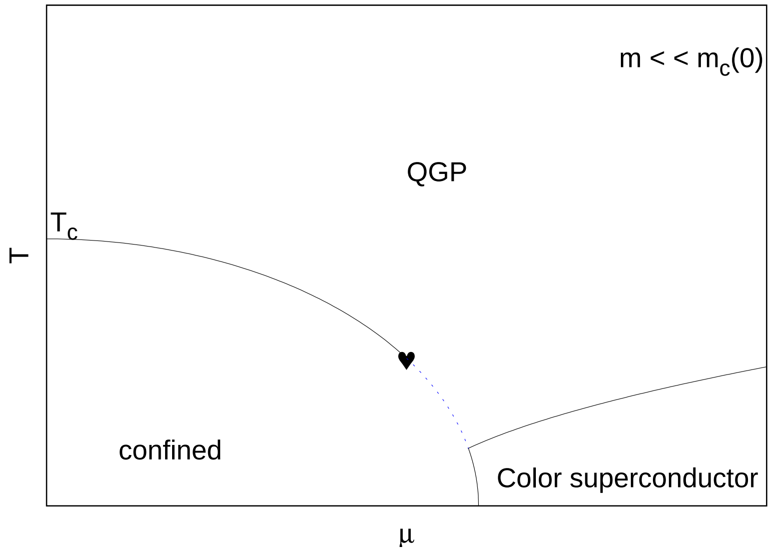

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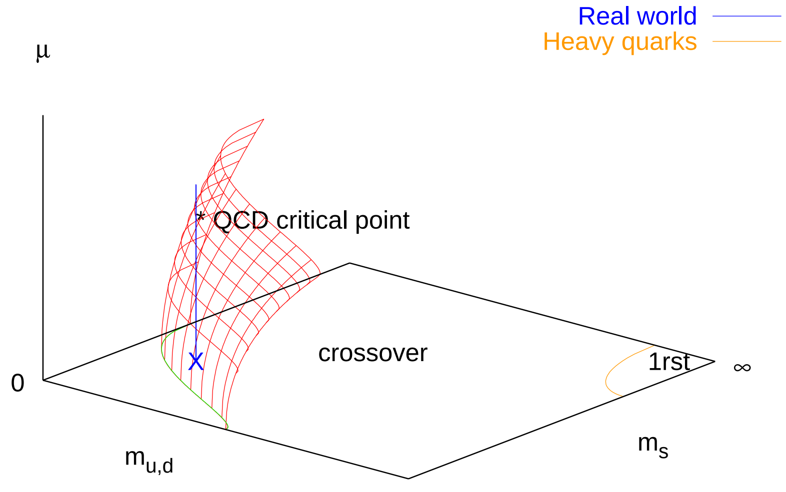
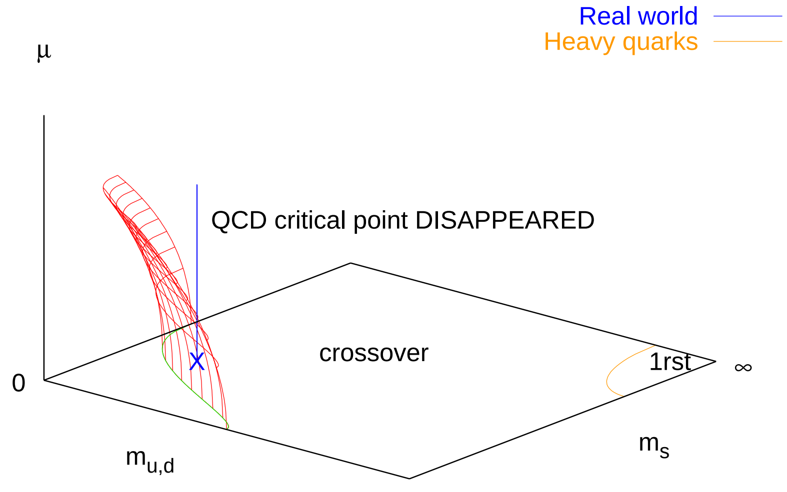
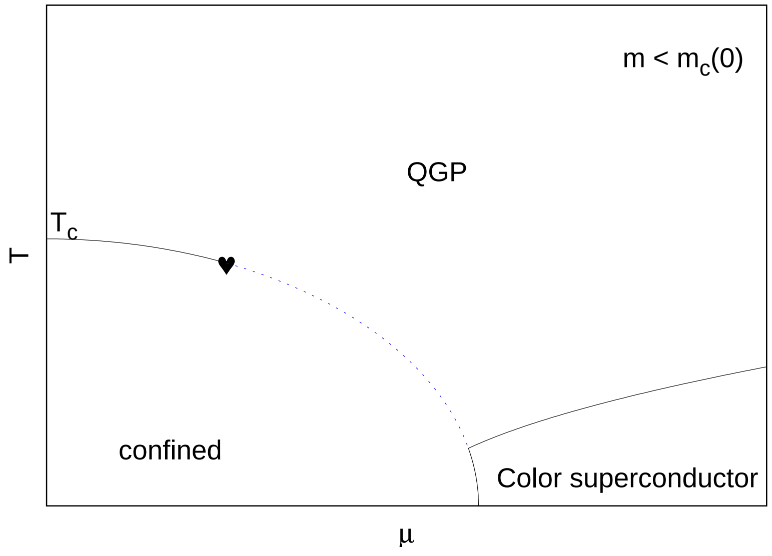
sign of $\frac{dm_c(\mu)}{d\mu^2}\Big|_{\mu=0}$

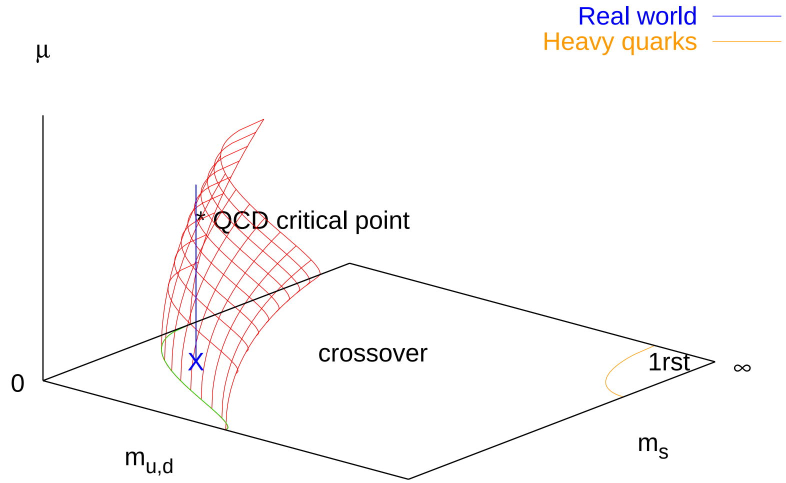


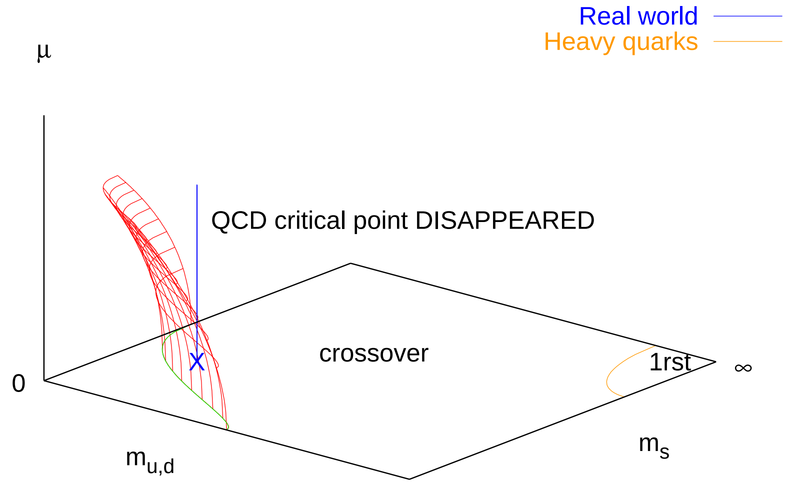


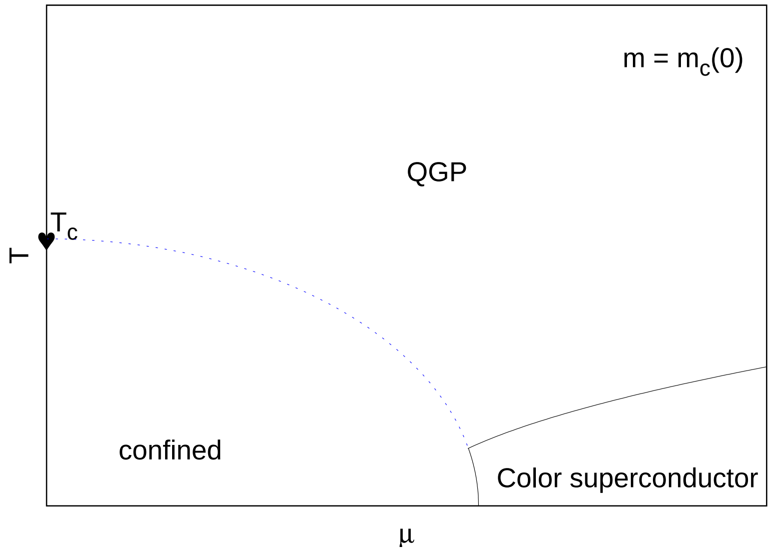


A non-standard scenario: no critical point?

sign of $\frac{dm_c(\mu)}{d\mu^2}\Big|_{\mu=0}$







A non-standard scenario: no critical point?

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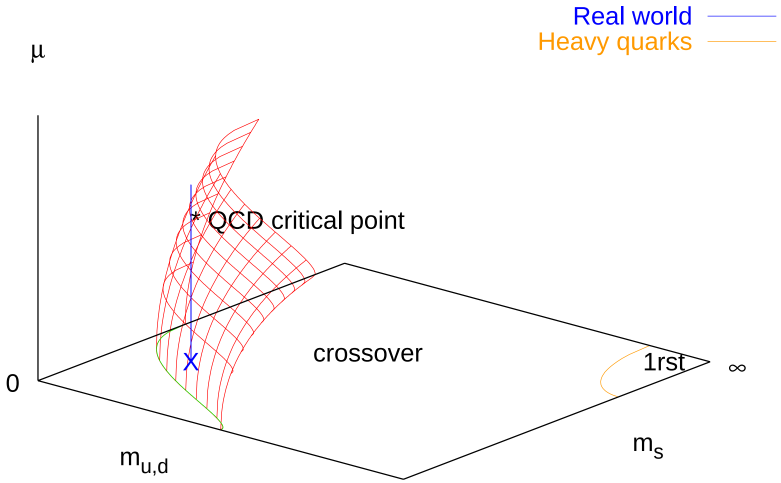
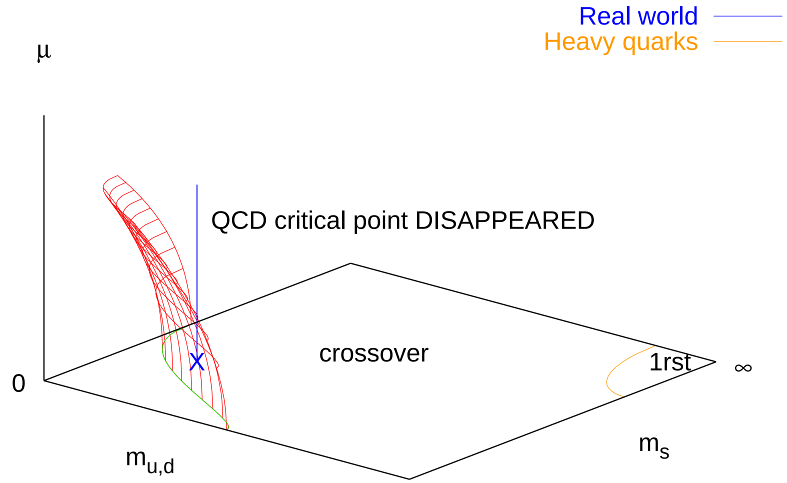
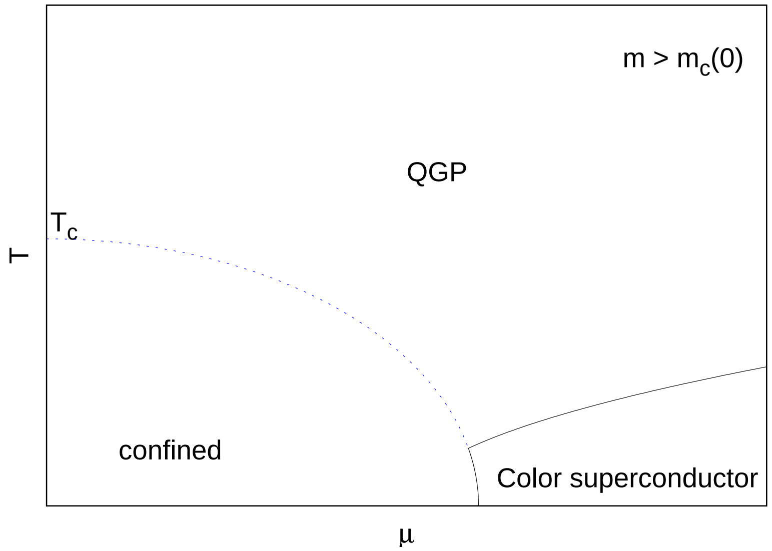






In either case: $m_c(\mu)$ *slowly* varying

smallness of $\frac{dm_c(\mu)}{d\mu^2}|_{\mu=0} \Rightarrow$ **very high quark mass sensitivity of** μ_c

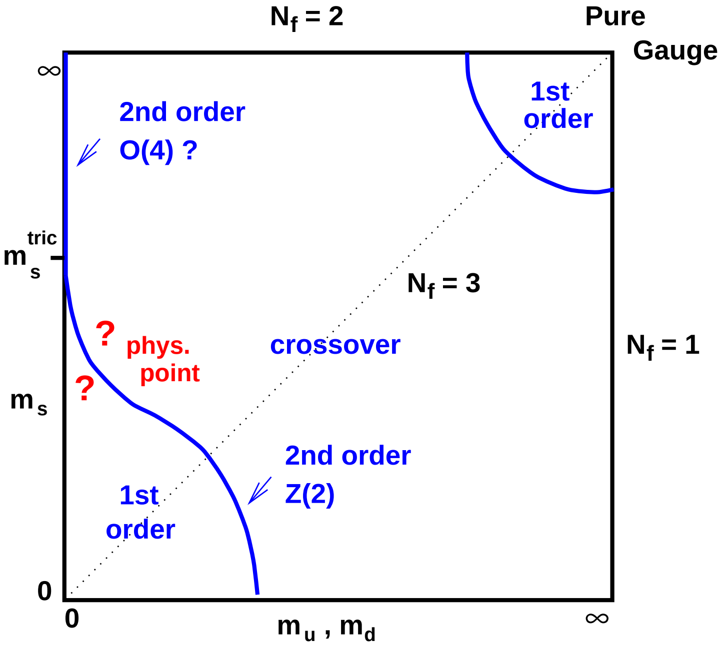
Can one expect the critical point to be at "small" μ?

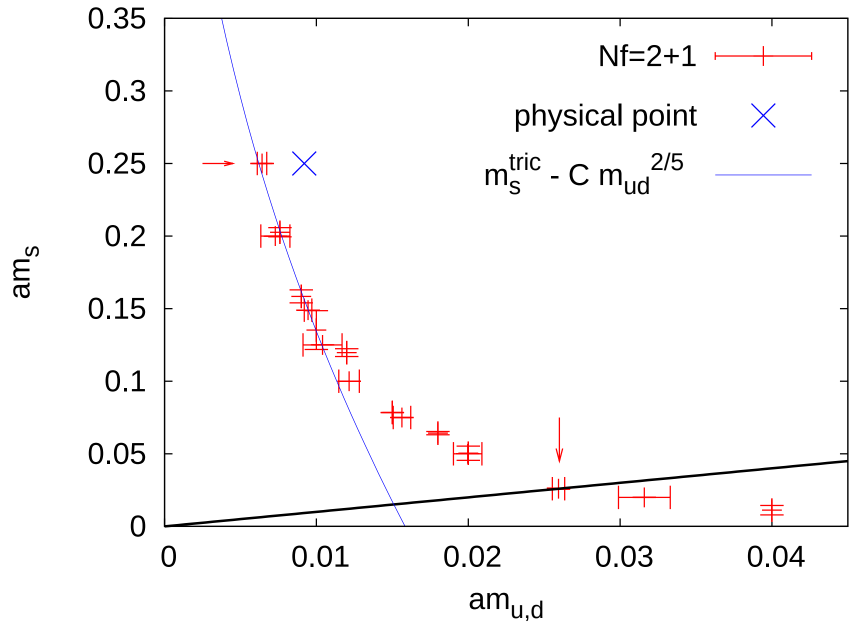
If $\mu_B^c \sim 360$ MeV (FK), then

$$1 < \frac{m}{m_c(\mu = 0)} \lesssim 1.05$$

fine tuning of quark masses!

$N_f = 2+1 : (m_s, m_{u,d})$ phase-diagram at $\mu = 0$





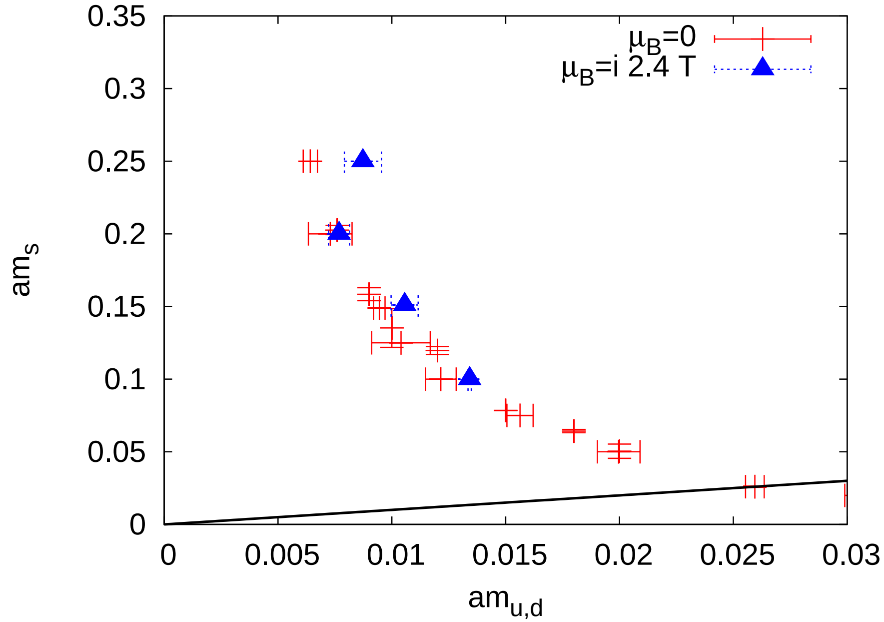
If there is a tricritical point $m_s^{tric} \approx 2.8 T_0 \lesssim 500$ MeV

Setting the scale

(marked by arrows):

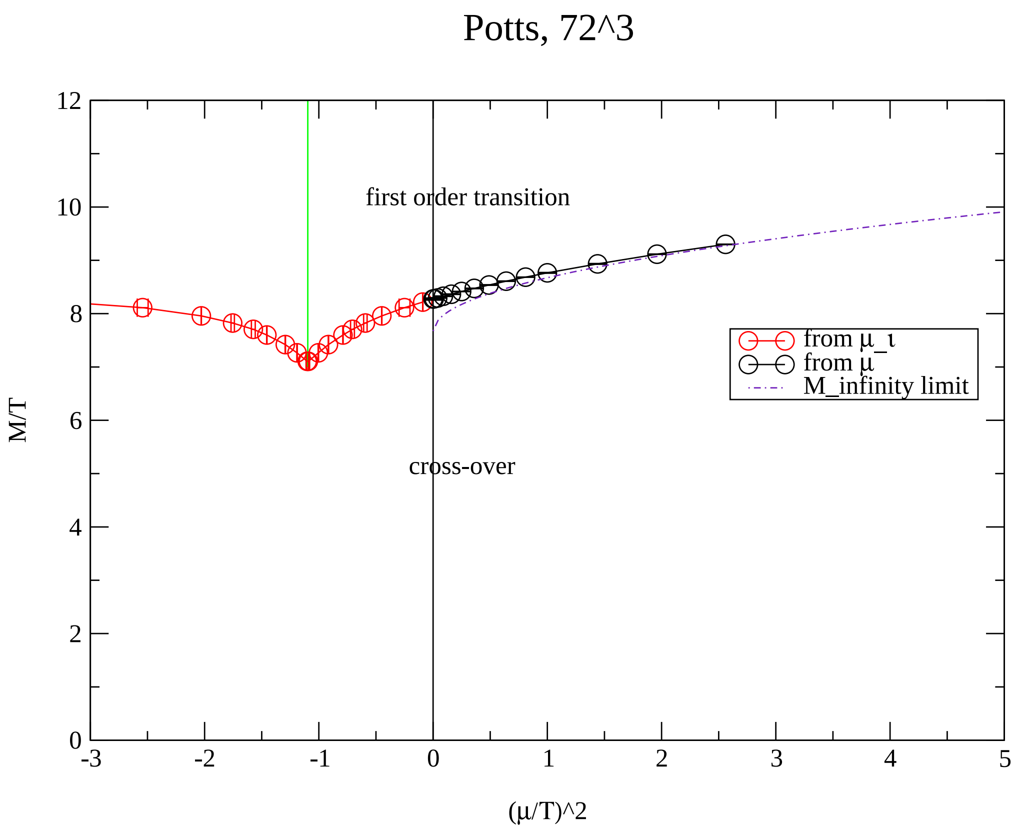
$(am_{u,d}, am_s)$	m_π / m_ρ	m_K / m_ρ
(0.0265,0.0265)	0.304(2)	0.304(2)
(0.005,0.25)	0.148(2)	0.626(9)
physical	0.18	0.6456

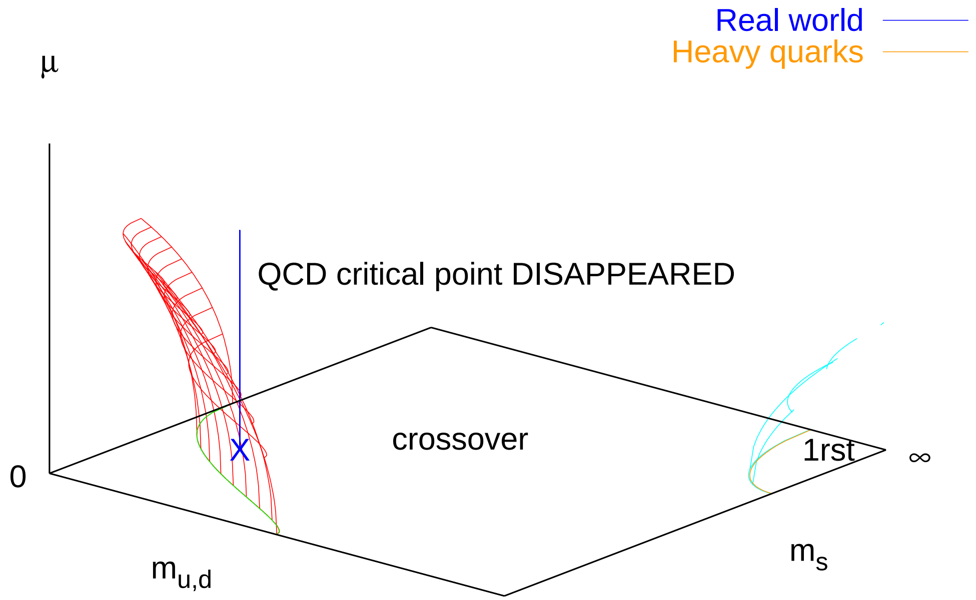
$N_f = 2+1 : (m_s, m_{u,d})$ phase-diagram at $\mu_B = i2.4T$



Real μ: first order region shrinking!

Unusual? ...the same happens for heavy quark masses!





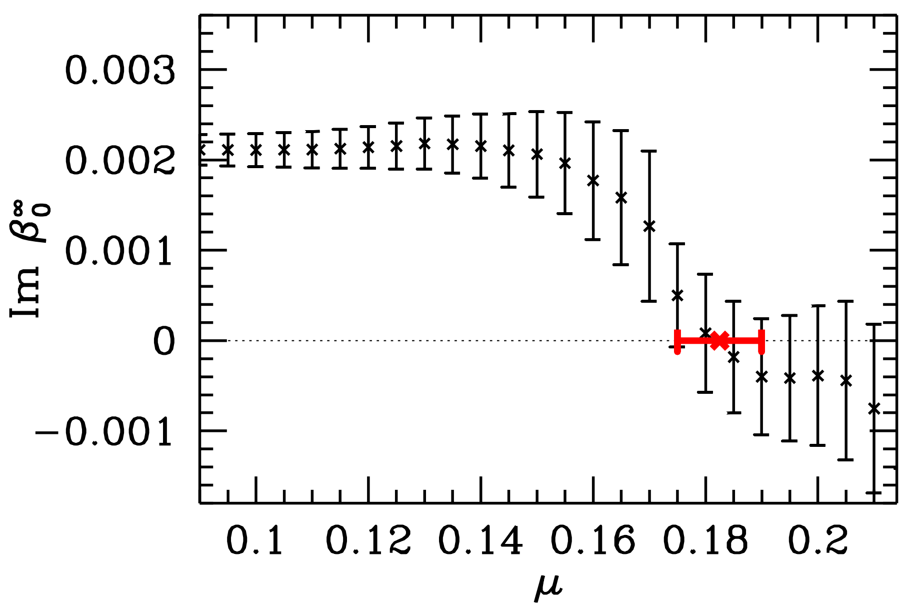
Real μ: first order region shrinking!

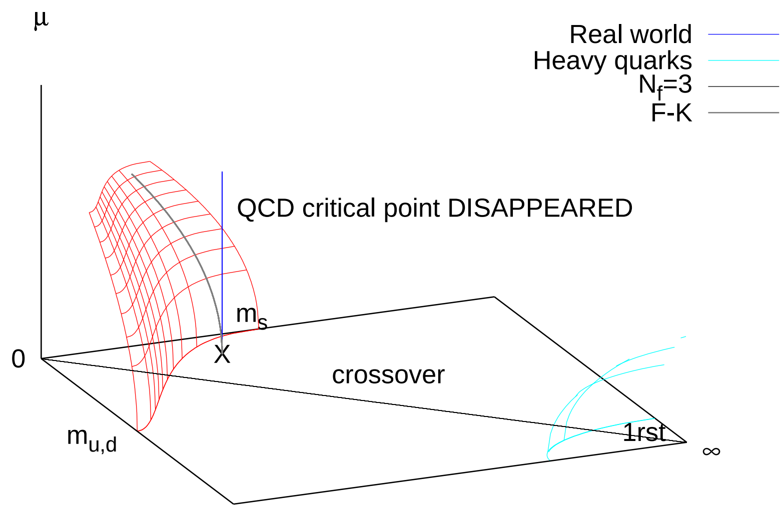
de Forcrand, Kim, Takaishi

Contradiction with other lattice studies? ...not necessarily!

- Fodor & Katz: $\{T_E, \mu_E\} = \{162(2), 120(13)\}$ MeV ?

  different systematics, cutoff effects





F&K keep (am_q) fixed, while $a(T(\mu))$ increases with μ

$\Rightarrow$Lighter quarks at larger μ may cause the phase transition

- Gavai & Gupta: $\mu_E/T_E \lesssim 1$? different theory, $N_f = 2$

Are there other possibilities still....?

- critical point at finite μ not analytically connected to that at $\mu = 0$
- additional critical structure logical possibility

Conclusions

- Mapping of QCD phase diagram possible for $\mu_q/T \lesssim 1$
- Critical endpoint is extremely quark mass sensitive

  $\Rightarrow \mu_c \lesssim 400$ MeV requires nature to fine tune m_q's close to $\mu = 0$ critical line
- existence of QCD critical point as yet inconclusive
- so far $a \sim 0.3$ fm, staggered only
- **Need finer lattices to understand even qualitative features!**