Quantum-classical crossover in electrodynamics

Quantization: CED $\Longrightarrow$ QED

Reality: CED $\Longleftarrow$ QED

Questions:

1. Can we recover
$$S=-\sum_i\left(M_i\int_{x_i}ds+e_i\int_{x_i}A_\mu(x)dx^\mu\right)-\frac{1}{4}\int d^4x(\partial_\mu A_\nu(x)-\partial_\nu A_\mu(x))^2,$$
the usual classical action? [Is there a variational pinciple ? (R. Jordan (1986): not)]
2. Can we separate the one-particle dynamics from the many-particle correlations of the Dirac-see?
3. What kind of corrections appear in the action?
4. Classical or macroscopic theory?
5. How does the environment, treated on the quantum level, lead to $\not{T}$ and define the time arrow?
6. How does decoherence appear?
7. Are macroscopic and microscopic polarisable media similar?
8. What is the role for Quantum Field Theory in the theory of measurement?
9. What happens with the reduced density matrix during blocking? (QRG?)
10. In what kind of models are the mixing effects negligible?

Lesson: QM: Two time axes $\Longrightarrow$ decoherence $\Longrightarrow$ CM: single time, causality

Dynamics of expectation values of local observables

Closed time path method (CTP) of Schwinger

$$\langle\psi(t)|A|\psi(t)\rangle = \langle\psi_i|e^{itH}Ae^{-itH}|\psi_i\rangle = \mathrm{Tr}[A\underbrace{e^{-itH}|\psi_i\rangle\langle\psi_i|e^{itH}}_{\rho(t)}]$$

Generating functional: $H \to H^{\pm}(t) = H \mp \sum_a \int d^3x j_a^{\pm}(t,\boldsymbol{x})O_a(t,\boldsymbol{x})$

$$\begin{aligned}
e^{iW[j^+,j^-]} &= \mathrm{Tr}\bar{T}[e^{i\int_{t_i}^{t_f}dt[H+\sum_a\int d^3x j_a^-(t,\boldsymbol{x})O_a(t,\boldsymbol{x})]}]\rho_f T[e^{i\int_{t_i}^{t_f}dt'[-H(t')+\sum_a\int d^3x j_a^+(t,\boldsymbol{x})O_a(t,\boldsymbol{x})]}]\rho_i \\
&= \mathrm{Tr}T^*[e^{i\int_{t_i}^{t_f}dt'\sum_a j_a^-(t')O_a(t')}\rho_f e^{i\int_{t_i}^{t_f}dt'\sum_a j_a^+(t')O_a(t')}\rho_i] \\
&= \langle\rho_f|T^*[e^{i\sum_a\int_{t_i}^{t_f}dt^- j_a^-(t^-)O_a(t^-)}e^{i\sum_a\int_{t_i}^{t_f}dt^+ j_a^+(t^+)O_a(t^+)}]|\rho_i\rangle \\
&= \mathrm{Tr}\rho_f\rho_i(t_f)
\end{aligned}$$

1. Transition probability rather than amplitude
2. Time symmetric Quantum Mechanics (Aharonov, Bergmann, Lebowitz 1964)
3. Two time axis, doubling of the degrees of freedom to obtain the density matrix (→QRG)
4. Time passes back and forth, retarded and advanced effects separated:

$j^{\pm} = j^a \pm j^r \implies$ j^r: physical, diagonal, retarded,
j^a: non-physical, non-diagonal, advanced

Unitarity: $W[j,-j]=0$, expectation values generated by varying off-diagonal sources

"in-in" (Schwinger) and "in-out" (Feynman) formalisms differ: $|\psi_i\rangle \neq |E\rangle$

$$\langle\psi_i|T[A_n(x_n)\cdots A_1(x_1)]|\psi_i\rangle_H = \langle\psi_i|T^*[\underbrace{e^{-i\int_{x_n^0}^{t_i}dtH(t)}}_{\neq \mathbb{1}} A_n(x_n)e^{-i\int_{t_i}^{x_n^0}dtH(t)}\cdots e^{-i\int_{x_1^0}^{t_i}dtH(t)}A_1(x_1)e^{-i\int_{t_i}^{x_1^0}dtH(t)}]|\psi_i\rangle_S$$

Measurable expectation values:

$$e^{iW[j^+,j^-]} = \mathrm{Tr}\bar{T}[e^{i\int_{t_i}^{t_f}dt[H+\sum_a\int d^3x j_a^-(t,\boldsymbol{x})O_a(t,\boldsymbol{x})]}]\underset{\substack{\uparrow\\ \mathbb{1}}}{\rho_f} T[e^{i\int_{t_i}^{t_f}dt'[-H(t')+\sum_a\int d^3x j_a^+(t,\boldsymbol{x})O_a(t,\boldsymbol{x})]}]\underset{\substack{\uparrow\\ |0\rangle\langle 0|}}{\rho_i}$$

$$\langle O_a(x)\rangle = \frac{\delta W[j^+,j^-]}{\delta j_a^\pm(x)}_{|j^+=-j^-} = \frac{\delta W[j,\bar{j}]}{\delta j_a(x)}_{|j=0} \qquad j^\pm = \frac{j}{2}(1\pm\kappa)\pm\bar{j}$$

j:infinitesimal formal parameter, $\bar{j}$:environment

Problem: $\kappa=0$: $W^*[j^+,j^-]=-W[-j^-,-j^+] \implies W^*[j,\bar{j}]=-W[-j,\bar{j}]$
No $\mathcal{O}\left(j^2\right)$ real part, connected two-point function

Solution: Use $\kappa\neq 0$ mixing diagonal and non-diagonal fluctuations

Linear response: Physical source, $j^a_+(t)=-j^a_-(t)=j^a(t)$, drives the time evolution

$$\langle\mathbb{1}|T^*[F(O_\ell)]|\rho_i\rangle_j = F\left[-i\frac{\delta}{\delta j^+_\ell}\right]\sum_{n_+,n_-}\frac{(-1)^{n_-}}{n_+!n_-!}\left(\sum_a\int dt' j^a(t')\frac{\delta}{\delta j^+_a(t')}\right)^{n_+} \times\left(\sum_b\int dt'' j^b(t'')\frac{\delta}{\delta j^-_b(t'')}\right)^{n_-} e^{iW[j^+,j^-]}{}_{|j^+=-j^-=j}$$

$$\delta\langle\mathbb{1}|O_\ell(t^+)|\rho_i\rangle = i\int_{t_i}^{t_f}dt'\sum_a j_a(t')[\langle\mathbb{1}|T^*[O_a(t^{-'})O_\ell(t^+)]|\rho_i\rangle_{|t^{-'}=t'}-\langle\mathbb{1}|T^*[O_a(t^{+'})O_\ell(t^+)]|\rho_i\rangle_{|t^{+'}=t'}],$$

$\rho_f=\mathbb{1}$ $\implies$ t_f is decreased to t' in the absence of non-physical sources (unitarity) and for $t_f=t'$:

$$\begin{aligned}&\Theta(t'-t)\langle\mathbb{1}|T^*[O_a(t^{-'})O_\ell(t^+)]|\rho_i\rangle\\ &\quad=\mathrm{Tr}\bar{T}[e^{i\int_{t_i}^{t'}dt[H+\sum_a\int d^3x j^-_a(t,\boldsymbol{x})O_a(t,\boldsymbol{x})]}]O_a(t^{-'})T[O_\ell(t^+)e^{i\int_{t_i}^{t'}dt'[-H(t')+\sum_a\int d^3x j^+_a(t,\boldsymbol{x})O_a(t,\boldsymbol{x})]}]\rho_i\\ &\quad=\mathrm{Tr}\bar{T}[e^{i\int_{t_i}^{t'}dt[H+\sum_a\int d^3x j^-_a(t,\boldsymbol{x})O_a(t,\boldsymbol{x})]}]T[O_a(t^{+'})O_\ell(t^+)e^{i\int_{t_i}^{t'}dt'[-H(t')+\sum_a\int d^3x j^+_a(t,\boldsymbol{x})O_a(t,\boldsymbol{x})]}]\rho_i\\ &\quad=\Theta(t'-t)\langle\mathbb{1}|T^*[O_a(t^{+'})O_\ell(t^+)]|\rho_i\rangle_{|t^{+'}=t'}\end{aligned}$$

$\implies$ retarded response in each order due to the interference between the time axis

Free photons

$$e^{iW^\gamma[j^+,j^-]} = \mathrm{Tr}\bar{T}[e^{i\int_{t_i}^{t_f} dt\int_{\boldsymbol{x}}[H(x)+A^-(x)j^-(x)]}]\rho_f T[e^{-i\int_{t_i}^{t_f} dt\int_{\boldsymbol{x}}[H(x)-A^+(x)j^+(x)]}]\rho_i = \int D[\hat{A}]e^{\frac{i}{2}\hat{A}\cdot\hat{D}_0^{-1}\cdot\hat{A}+i\hat{j}\cdot\hat{A}}$$

$$S_M[A] = \int_x\left[-\frac{1}{4}(\partial_\mu A_{\nu,x}-\partial_\nu A_{\mu,x})^2-\frac{\xi}{2}(\partial^\mu A_{\mu,x})^2\right] = \frac{1}{2}A\cdot D_0^{-1}\cdot A$$

Integration/summation = scalar product: $\int dx\phi(x)\chi(x) = \int_x \phi_x\chi_x = \phi\cdot\chi$, $\int dx A_\mu(x)j^\mu(x) = A\cdot j$.
Composite index: $a = (\mu, x)$.

Free propagator:

$$D_0^{-1} = D_T^{-1}+D_L^{-1} \qquad D_T^{-1} = (\Box - i\epsilon)T \qquad D^{-1}L = \xi(\Box - i\epsilon)L \qquad T^{\mu\nu} = g^{\mu\nu}-L^{\mu\nu} \qquad L^{\mu\nu} = \frac{\partial^\mu\partial^\nu}{\Box}$$

$$\hat{A} = \begin{pmatrix} A^+ \\ A^- \end{pmatrix} \qquad \hat{D}_0^{-1} = \begin{pmatrix} D_0^{-1} & 0 \\ 0 & -D_0^{-1*} \end{pmatrix} + \underbrace{\hat{D}_{\mathrm{BC}}^{-1}}_{\text{from op. formalism}}$$

Open boundary condition (OBC): $\rho_i = |0\rangle\langle 0|, \quad \rho_f = \mathbb{1}$

Closed boundary condition (FBC): $\rho_i = \rho_f = |0\rangle\langle 0|$

Gennerating functional for connnected Green functions

$$\text{Propagator :} \qquad \hat{D}_0 = \begin{pmatrix} D_0^n + i\Im D_0 & -\frac{1}{2}D_0^f + i\Im D_0 \\ \frac{1}{2}D_0^f + i\Im D_0 & -D_0^n + i\Im D_0 \end{pmatrix}$$

$$D^{++}_{0\ xy} = D^n_{0\ xy} + i\Im D_{0\ xy} = i\langle 0|T[\phi_x\phi_y|0\rangle = \frac{1}{4\pi}\delta((x-y)^2) - \frac{i}{4\pi^2}P\frac{1}{(x-y)^2}$$

$$\text{retarded} \qquad D^r_{0\ xy} = D^n_{0\ xy} + \frac{1}{2}D^f_{0\ xy} = \frac{1}{2\pi}\Theta(x^0 - y^0)\delta((x-y)^2)$$

$$\text{advanced} \qquad D^a_{0\ xy} = D^n_{0\ xy} - \frac{1}{2}D^f_{0\ xy} = \frac{1}{2\pi}\Theta(y^0 - x^0)\delta((x-y)^2)$$

↑ near-field ↑ far-field

Green functions (Dirac)

$$\text{Gauss integral :} \qquad W^\gamma[\hat{j}] = -\frac{1}{2}\hat{j}\cdot\hat{D}_0\cdot\hat{j}$$

$$W^\gamma_{\rm OBC}[j,\bar{j}] = -\frac{1}{2}^{(\bar{j},j)}\cdot\begin{pmatrix} 0 & D_0^a \\ D_0^r & \kappa D_0^n + i\Im D_0 \end{pmatrix}\cdot\begin{pmatrix}\bar{j}\\ j\end{pmatrix} \qquad \left(j^\pm = \frac{j}{2}(1\pm\kappa)\pm\bar{j}\right)$$

$$W^\gamma_{\rm FBC}[\hat{j}] = -\frac{1}{2}j^+\cdot(D_0^n + i\Im D_0)\cdot j^+ + \frac{1}{2}j^-\cdot(D_0^n - i\Im D_0)\cdot j^-$$

$$= -\frac{1}{2}^{(\bar{j},j)}\cdot\begin{pmatrix} 2i\Im D_0 & D_0^n + \kappa i\Im D_0 \\ D_0^n + \kappa i\Im D_0 & \kappa D_0^n + \frac{1+\kappa^2}{2}i\Im D_0\end{pmatrix}\cdot\begin{pmatrix}\bar{j}\\ j\end{pmatrix}$$

Effective action

Problem: $W[j^+, j^-]$ is complex

Solution: Use the Legendre transform of $\Re W[j^+, j^-]$ ($W[j, -j]$ is real)

Both fields:

$$\Gamma^\gamma[A, A^a] = \Re W^\gamma[j, \bar{j}] - \bar{j} \cdot A^a - j \cdot A \qquad \overbrace{A = \frac{\delta \Re W^\gamma[j, \bar{j}]}{\delta j}}^{\text{physical fields}} \quad \overbrace{A^a = \frac{\delta \Re W^\gamma[j, \bar{j}]}{\delta \bar{j}}}^{\text{auxiliary variable}}$$

$$\underbrace{j = -\frac{\delta \Gamma^\gamma[A, A^a]}{\delta A} \qquad \bar{j} = -\frac{\delta \Gamma^\gamma[A, A^a]}{\delta A^a}}_{\text{Inverse Legendre transformation: E.M.}}$$

$$\Gamma^\gamma_{\rm OBC}[A, A^a] = A^a \cdot D_0^{r-1} \cdot A - \frac{\kappa}{2} A^a \cdot D_0^{r-1} \cdot D_0^n \cdot D_0^{a-1} \cdot A^a \implies A^a = D_0^a j \qquad A = D_0^r \bar{j} + \kappa D_0^n j$$

$$\Gamma^\gamma_{\rm FBC}[A, A^a] = A^a \cdot D_0^{n-1} \cdot A - \frac{\kappa}{2} A^a \cdot D_0^{n-1} \cdot A^a \implies A^a = D_0^n j \qquad A = D_0^n \bar{j} + \kappa D_0^n j$$

Physical fields only:

$$\Gamma^\gamma[A] = \Re W^\gamma[j, \bar{j}] - j \cdot A$$

$$\Gamma^\gamma_{\rm OBC}[A] = \frac{1}{2\kappa} (A + \bar{j} D_0^a) \cdot D_0^{n-1} \cdot (A + D_0^r \bar{j})$$

$$\Gamma^\gamma_{\rm FBC}[A] = \frac{1}{2\kappa} (A + \bar{j} D_0^n) \cdot D_0^{n-1} \cdot (A + D_0^n \bar{j})$$

$$\kappa \neq 0!$$

Effective Quantum Mechanics of non-relativistic charges

N charges with trajectories $\boldsymbol{x}^{(n)}(t)$ and current $j[\boldsymbol{x}]_{\mu x} = \sum_{n=1}^{N} \delta_{x^0,t}\delta_{\boldsymbol{x},\boldsymbol{x}^{(n)}(t)}(1, \dot{\boldsymbol{x}}^{(n)}(t))$ $(\kappa = 0)$:

$$
\begin{aligned}
Z &= \int D[\hat{A}]D[\hat{\boldsymbol{x}}]e^{-\frac{i}{2}\hat{A}\cdot\hat{D}_0^{-1}\cdot\hat{A}+iS_c[\boldsymbol{x}^+]-iS_c^*[\boldsymbol{x}^-]-iej[\boldsymbol{x}^+]\cdot A^+ + iej[\boldsymbol{x}^-]\cdot A^-} \\
&= \int D[\hat{\boldsymbol{x}}]e^{iS_c[\boldsymbol{x}^+]-iS_c^*[\boldsymbol{x}^-]+iW^\gamma\left[-ej[\boldsymbol{x}^+],ej[\boldsymbol{x}^-]\right]} \\
&= \int D[\hat{\boldsymbol{x}}]e^{i[S_c[\boldsymbol{x}^+]-S_c^*[\boldsymbol{x}^-]+\frac{e^2}{2}[-j^+\cdot D_0^n\cdot j^+ + j^-\cdot D_0^n\cdot j^- + j^+\cdot D_0^f\cdot j^-]}\overbrace{e^{-\frac{e^2}{2}(j^+-j^-)\cdot\Im D_0\cdot(j^+-j^-)}}^{\text{decoh.+nat. line w.}}
\end{aligned}
$$

↑	↑	↑	↑
co-moving near field		radiation & mixing	
diagonal		off-diagonal (entenglement)	
independent of BC		testing final BC	
		friction	decoherence

1. Radiation and Abraham-Lorentz force: realized by the coupling of the time axis at $t = t_f$ and require "reflection" from the end of time (ie. dependence on ρ_f)

2. The non-diagonality $|\boldsymbol{x}^+ - \boldsymbol{x}^-| = r$ for time t generates the suppression factor

$$
e^{-\frac{e^2}{2}(j[\boldsymbol{x}^+]-j[\boldsymbol{x}^-])\cdot\Im D_0\cdot(j[\boldsymbol{x}^+]-j[\boldsymbol{x}^-])} \approx e^{-N^2\frac{e^2}{4\pi^2}} \qquad t \gg r
$$

   with the speed of light.

3. Classical limit $e^{iS[\hat{\boldsymbol{x}}]} \approx 1$: "in-out" $\Longrightarrow$ rigid saddle point
   "in-in" $\Longrightarrow$ soft saddle point, strong coupling (degeneracy)

4. Decoherence is off-diagonal effect, appearing indirectly in expectation values of diagonal observables

Free Dirac-see

Problems: 1 One part. connected Green fct.: factorisation by Wick theorem for $n > 2$ (creation, destruction are one particle issues)

Two part. connected Green fct.: no factorisation (polarisation is many particle effect)

2 $J \to a$ is not unique : small vs. large polaron problem

$$
\begin{aligned}
e^{iW^e[\hat{a},\hat{\eta},\hat{\bar{\eta}}]} &= \int D[\hat{\psi}]D[\hat{\bar{\psi}}]e^{i\hat{\bar{\psi}}\cdot[-\hat{G}_0^{-1}+\not{\hat{a}}]\cdot\hat{\psi}+i\hat{\bar{\eta}}\cdot\hat{\psi}+i\hat{\bar{\psi}}\cdot\hat{\eta}+iS^e_{CT}[\hat{a}]} \\
\hat{a} &= \begin{pmatrix} a^+ \\ a^- \end{pmatrix} \qquad \hat{\psi} = \begin{pmatrix} \psi^+ \\ \psi^- \end{pmatrix} \qquad \hat{\bar{\psi}} = \begin{pmatrix} \bar{\psi}^+ \\ \bar{\psi}^- \end{pmatrix} \qquad \hat{\eta} = \begin{pmatrix} \eta^+ \\ \eta^- \end{pmatrix} \qquad \hat{\bar{\eta}} = \begin{pmatrix} \bar{\eta}^+ \\ \bar{\eta}^- \end{pmatrix}
\end{aligned}
$$

$$
\hat{G}_0^{-1} = \begin{pmatrix} G_0^{-1} & \\ 0 & -\gamma^0 G_0^{-1\dagger}\gamma^0 \end{pmatrix} + \hat{G}^{-1}_{\mathrm{BC}} \qquad G_0^{-1} = \not{\partial} + m \qquad S^e_{CT}[\hat{a}] = -\frac{\Delta Z_3 + \beta}{2}\hat{a}\cdot\begin{pmatrix} D_T^{-1} & 0 \\ 0 & -D_T^{-1*} \end{pmatrix}\cdot\hat{a}
$$

Two kinds of charges: - Negative energy states (small polaron) $\hat{\eta} = \hat{\bar{\eta}} = 0$
- Positiv energy states (large polaron)

$$
\begin{aligned}
|\psi_i\rangle &= \prod_{j=1}^{n^-}\left(\int_{\boldsymbol{y}} \bar{\psi}_{t,\boldsymbol{y}}\chi^-_{j,\boldsymbol{y}}\right)\prod_{k=1}^{n^+}\left(\int_{\boldsymbol{y}} \bar{\chi}^+_{k,\boldsymbol{y}}\psi_{t,\boldsymbol{y}}\right)|0\rangle \\
e^{iW^e_c[\hat{a}]} &= \prod_{\sigma=\pm1}\prod_{j=1}^{n^-}\left(\int_{\boldsymbol{x}} \bar{\chi}^-_{j,\boldsymbol{x}}\frac{\delta}{\delta\bar{\eta}^\sigma_{t_f,\boldsymbol{x}}}\int_{\boldsymbol{y}}\chi^-_{j,\boldsymbol{y}}\frac{\delta}{\delta\eta^\sigma_{t_i,\boldsymbol{y}}}\right)\prod_{k=1}^{n^+}\left(\int_{\boldsymbol{x}}\chi^+_{k,\boldsymbol{x}}\frac{\delta}{\delta\eta^\sigma_{t_f,\boldsymbol{x}}}\int_{\boldsymbol{y}}\bar{\chi}^+_{k,\boldsymbol{y}}\frac{\delta}{\delta\bar{\eta}^\sigma_{t_i,\boldsymbol{y}}}\right)e^{iW^e[\hat{a},\hat{\eta},\hat{\bar{\eta}}]}{}_{|\hat{\eta}=\hat{\bar{\eta}}=0}
\end{aligned}
$$

Negative enegy states

$$W_0^e[\hat{a}] \;=\; -i\mathrm{Tr}\ln\overbrace{\hat{G}^{-1}[\hat{a}]}^{\hat{G}_0^{-1}+\hat{\not a}}+S_{CT}[\hat{a}] = -i\mathrm{Tr}\ln\hat{G}_0^{-1}+i\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\mathrm{Tr}(\hat{G}_0\cdot\hat{\not a})^n+S_{CT}[\hat{a}] = -\frac{1}{2}\hat{a}\cdot\hat{\tilde{G}}_R\cdot\hat{a}+\mathcal{O}\left(\hat{a}^4\right)$$

$$\begin{aligned}
i\tilde{G}^{++}_{0a,b} &= \mathrm{Tr}[\hat{G}^{++}_0\cdot\gamma_a\cdot\hat{G}^{++}_0\cdot\gamma_b]+\overbrace{i\Delta Z_3 T(\Box-i\epsilon)}^{\text{photon self en.}/e^2} = \tilde{G}_{0a,b}\\
i\tilde{G}^{-+}_{0a,b} &= \mathrm{Tr}[\hat{G}^{+-}_0\cdot\gamma_a\cdot\hat{G}^{-+}_0\cdot\gamma_b]\\
i\tilde{G}^{--}_{0a,b} &= \mathrm{Tr}[\hat{G}^{--}_0\cdot\gamma_a\cdot\hat{G}^{--}_0\cdot\gamma_b]-i\Delta Z_3 T(\Box+i\epsilon)
\end{aligned}$$

Parametrization:

$$\hat{\tilde{G}} = \begin{pmatrix}\tilde{G}^n+i\Im\tilde{G} & -\frac{1}{2}\tilde{G}^f+i\Im\tilde{G}\\ \frac{1}{2}\tilde{G}^f+i\Im\tilde{G} & -\tilde{G}^n+i\Im\tilde{G}\end{pmatrix}$$

Gradient expansion:

$$\tilde{G}^{\mu\nu}_{0q} = \int_x e^{-iqx}\tilde{G}^{\mu\nu}_{x,0} = T^{\mu\nu}\left[\frac{1}{15\pi}\frac{q^4}{m^2}+\mathcal{O}\left(\frac{q^6}{m^4}\right)\right] \qquad T^{\mu\nu}=g^{\mu\nu}-\frac{\partial^\mu\partial^\nu}{\Box}$$

Renormalization: (Noether theorem is insufficient)

$$\langle 0T[j^\mu_x j^\nu_{x'}]0|\rangle \to \langle 0T[j^\mu_x j^\nu_{x'}]0|\rangle - \beta i\Box\delta_{x,x'}T^{\mu\nu}$$

$$\beta = ?$$

Effective action for negative energy states

$$a^{\pm} = a\frac{1 \pm \kappa}{2} \pm \bar{a}$$

Both fields:

$$\begin{aligned} \Gamma^e[J, J^a] &= \Re W^e[a, \bar{a}] - \bar{a} \cdot J^a - a \cdot J \qquad J = \frac{\delta \Re W^e[a, \bar{a}]}{\delta a} \qquad J^a = \frac{\delta \Re W^e[a, \bar{a}]}{\delta \bar{a}} \\ \Gamma^e_{0\ \mathrm{OBC}}[J, J^a] &= J^a \cdot \tilde{G}_0^{r-1} \cdot J - \frac{\kappa}{2} J^a \cdot \tilde{G}_0^{r-1} \cdot \tilde{G}_0^n \cdot \tilde{G}_0^{a-1} \cdot J^a \\ \Gamma^e_{0\ \mathrm{FBC}}[J, J^a] &= J^a \cdot \tilde{G}_0^{n-1} \cdot J - \frac{\kappa}{2} J^a \cdot \tilde{G}_0^{n-1} \cdot J^a \end{aligned}$$

Physical fields only:

$$\begin{aligned} \Gamma^e[J] &= \Re W^e[a, \bar{a}] - a \cdot J \\ \Gamma^e_{0\ \mathrm{OBC}}[J] &= \frac{1}{\kappa}\left[\frac{1}{2} J \cdot \tilde{G}_0^{n-1} \cdot J + J \cdot \tilde{G}_0^{n-1} \cdot \tilde{G}_0^r \cdot \bar{a}\right] \\ \Gamma^e_{0\ \mathrm{FBC}}[J] &= \frac{1}{\kappa}\left[J \cdot \bar{a} + \frac{1}{2} J \cdot \tilde{G}_0^{n-1} \cdot J\right] \end{aligned}$$

$$\kappa \neq 0$$

Dynamics of world-tubes

Static world-tubes N_j (not necesseraly integer) elementary charges:

$$J^\mu_x = g^{\mu 0}\sum_j N_j\rho(|\boldsymbol{x}-\boldsymbol{x}_j|) \qquad \int_{\boldsymbol{y}}\rho(|\boldsymbol{y}|)=1 \qquad \int_{\boldsymbol{y}} y^2\rho(|\boldsymbol{y}|)\gg\frac{1}{m^2}$$

$$\text{Distance between tubes}:\quad r_{jk} = \int_{\boldsymbol{y},\boldsymbol{z}}\rho(|\boldsymbol{y}-\boldsymbol{x}_j|)|\boldsymbol{y}-\boldsymbol{z}|\rho(|\boldsymbol{z}-\boldsymbol{x}_k|)\gg\frac{1}{m}$$

Effective action:

$$\begin{aligned}
\Gamma^e_0[J] &= -\frac{1}{2\kappa}J\cdot\frac{1}{\frac{1}{15\pi m^2}\Box^2-\beta\Box}\cdot J\\
E_{\rm tot} &= -\lim_{t_f-t_i=\infty}\frac{\Gamma^e_0[J]}{t_f-t_i}=\frac{1}{2\kappa}\sum_{jk}N_jN_kV(|\boldsymbol{x}_j-\boldsymbol{x}_k|)\\
V(|\boldsymbol{z}-\boldsymbol{z}'|) &= \frac{1}{2}\int_{\boldsymbol{x},\boldsymbol{y},\boldsymbol{q}}\rho(|\boldsymbol{x}-\boldsymbol{z}|)\frac{e^{-i(\boldsymbol{x}-\boldsymbol{y})\boldsymbol{q}}}{\beta\boldsymbol{q}^2+\frac{1}{15\pi m^2}\boldsymbol{q}^4}\rho(|\boldsymbol{y}-\boldsymbol{z}'|)\\
&= \frac{1}{4\pi\beta}\int_{\boldsymbol{x},\boldsymbol{y}}\rho(|\boldsymbol{x}-\boldsymbol{z}|)\frac{1-e^{-|\boldsymbol{x}-\boldsymbol{y}|m\sqrt{15\pi\beta}}}{|\boldsymbol{x}-\boldsymbol{y}|}\rho(|\boldsymbol{y}-\boldsymbol{z}'|)\qquad(\beta>0)\\
&= \frac{15m^2}{8}\int_{\boldsymbol{x},\boldsymbol{y}}\rho(|\boldsymbol{x}-\boldsymbol{z}|)(C-|\boldsymbol{x}-\boldsymbol{y}|)\rho(|\boldsymbol{y}-\boldsymbol{z}'|)\qquad(\beta=0)\ \leftarrow\ \text{ez kell!}
\end{aligned}$$

$$E_{\rm tot}=\frac{1}{2\kappa}\left(\sigma CN^2_{\rm tot}+\sigma\sum_{jk}N_jN_k|\boldsymbol{x}_j-\boldsymbol{x}_k|\right)\qquad N_{\rm tot}=\sum_j N_j\qquad \sigma=\frac{15m^2}{8}$$

1. Neutral wave-packets are stable only
2. Forces arising among wave-packets in non-interacting Dirac-see ???

Two non-interacting wave-packets:

a Non-relativistic Quantum Mechanics:

$$\begin{aligned} \psi(\boldsymbol{x}_1, \boldsymbol{x}_2) &= \frac{1}{\sqrt{2N}}[\psi_1(\boldsymbol{x}_1)\psi_2(\boldsymbol{x}_2) - \psi_1(\boldsymbol{x}_2)\psi_2(\boldsymbol{x}_1)] \\ \langle\psi|\Delta|\psi\rangle &= \frac{1}{N}[\langle\psi_1|\Delta|\psi_1\rangle\langle\psi_2|\psi_2\rangle + \langle\psi_2|\Delta|\psi_2\rangle\langle\psi_1|\psi_1\rangle - 2Re\langle\psi_1|\Delta|\psi_2\rangle\langle\psi_2|\psi_1\rangle] \\ \text{as if the potential} \quad U(\boldsymbol{x}_1, \boldsymbol{x}_2) &= -\frac{Re\langle\psi_1|\Delta|\psi_2\rangle\langle\psi_2|\psi_1\rangle}{\langle\psi_1|\psi_1\rangle\langle\psi_2|\psi_2\rangle - |\langle\psi_1|\psi_2\rangle|^2} \quad \text{appeared at the overlap} \end{aligned}$$

b Relativisic Quantum Mechanics: mixing of γ and e^+e^- pairs
propagation of γ on line cone
polarization e^+e^- propagates on the light cone $(\tilde{G} \approx \Box^2)$

Charge confinement in the non-interacting Dirac-see:

Screening by "mesons" due to the minimal coupling for $r > \frac{1}{m}$ $(\sigma = \frac{15m^2}{8})$

Effective action for positive energy states

$$W^e[\hat{a}] = W[\hat{a}]_0^e - i \ln \underbrace{G[\hat{a}]}_{\text{valence prop.}}$$

Well separated wells: $a_\mu = g_{\mu 0} u$ in the rest frame, $m\ell < 1$ justifying the non-relativistic approximation

$$\Gamma^e_{\text{CFBC}}[J] = -(t_f - t_i)(m + E_0) - \int_{t_i}^{t_f} dt \int d^3x u(\boldsymbol{x}) J^0(t, \boldsymbol{x})$$

Bound states are localised $\implies$ charges in the potential wells decouple and

$$M = m + \Delta m_{\text{kin}} + \Delta m_{\text{pol}} + \Delta m_\gamma$$

$$\begin{array}{ccc} \uparrow & \uparrow & \uparrow \\ \text{kin.en.} & \text{polar.} & \text{rad. corr.} \\ 1/(m\ell^2) & m\ell u & e^2/\ell \end{array}$$

$$\text{free particle}: \quad \ell \approx \frac{1}{m} \quad \Longrightarrow \quad M = \mathcal{O}(m) \quad (\text{non} - \text{perturbative renormalization})$$

Interacting electrons and photons

$$\begin{aligned} e^{iW[\hat{a},\hat{j}]} &= \int D[\hat{\psi}]D[\hat{\bar{\psi}}]D[\hat{A}]e^{i\hat{\bar{\psi}}\cdot[\hat{G}_0^{-1}+\hat{\sigma}(\not{a}-e\not{\hat{A}})]\cdot\hat{\psi}+\frac{ie^2}{2e_B^2}\hat{A}\cdot\hat{D}_0^{-1}\cdot\hat{A}+i\hat{j}\hat{\sigma}\cdot\hat{A}+iS_{CT}} \\ \hat{\sigma} &= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \end{aligned}$$

Counterterms:

$$S_{CT} = \underbrace{S_{CT}^{QED}}_{\text{usual}} + S_{CT}^{e\gamma} \qquad S_{CT}^{e\gamma} = -(\Delta Z_3 - \alpha)e\hat{a}\cdot\hat{D}_0^{-1}\cdot\hat{A} + \frac{1}{2}(\Delta Z_3 + \beta)\hat{a}\cdot\hat{D}_0^{-1}\cdot\hat{a}$$

Photon action:

$$S_{CT}^{e\gamma} + \frac{e^2}{2e_B^2}\hat{A}\cdot\hat{D}_0^{-1}\cdot\hat{A} = \frac{1}{2}\hat{A}\cdot\hat{D}_0^{-1}\cdot\hat{A} + \frac{\Delta Z_3}{2}(e\hat{A}-\hat{a})\cdot\hat{D}_0^{-1}\cdot(e\hat{A}-\hat{a}) + \alpha\hat{a}\cdot\hat{D}_0^{-1}\cdot\hat{A} + \frac{\beta}{2}\hat{a}\cdot\hat{D}_0^{-1}\cdot\hat{a},$$

$\Longrightarrow$ $\alpha = \beta = 0$ for the E.M. current

Integration over the electron field:

$$e^{iW[\hat a,\hat j]} = \int D[\hat A] e^{iW^e[\hat a - e\hat\sigma \hat A] + \frac{ie^2}{2e_B^2}\hat A\cdot \hat D_0^{-1}\cdot \hat A + i\hat j\cdot \hat A + iS_{CT}}$$

Two-loop result:

$$\begin{aligned}
W[\hat a,\hat j] &= \frac{1}{2}{}^{(\hat a,\hat j)}\cdot\begin{pmatrix} -\hat{\hat G} & -e\hat{\hat G}\cdot\hat\sigma\hat D_0 \\ -e\hat D_0\hat\sigma\cdot\hat{\hat G} & -\hat D\end{pmatrix}\cdot\begin{pmatrix}\hat a\\ \hat j\end{pmatrix}\\
\hat{\hat G} &= \frac{1}{\hat{\hat G}_0^{-1} - \hat\Sigma^e} \qquad\qquad \hat\Sigma^e \approx e^2\hat\sigma\hat D_0\hat\sigma + e^2\hat{\hat G}_0^{-1}\cdot \hat W_D^{e(4)}\cdot\hat{\hat G}_0^{-1}\\
\hat D &= \frac{1}{\hat D_0^{-1} - \hat\Sigma^\gamma} \qquad\qquad \hat\Sigma^\gamma \approx e^2\hat\sigma(\hat{\hat G}_0 + e^2\hat W_D^{e(4)})\hat\sigma\\
W_{Dab}^{e(4)} &= W_{abcd}^{e(4)}(\hat\sigma i\hat D_0\hat\sigma)_{cd} \qquad\qquad \text{(self energy + vertex correction)}
\end{aligned}$$

Connected Green functions:

$$\Re W[a,\bar a,j,\bar j] = \frac{1}{2}^{(a,j,\bar a,\bar j)}\cdot\begin{pmatrix}\frac{\kappa}{2}(K^{\rm tr}+K) & K\\ K^{\rm tr} & 0\end{pmatrix}\cdot\begin{pmatrix}a\\ j\\ \bar a\\ \bar j\end{pmatrix} + \cdots$$

$$K_{\rm OBC} = \begin{pmatrix} -\tilde G^r & -e\tilde G^r D_0^r\\ -eD_0^r\tilde G^r & -D^r\end{pmatrix} \qquad K_{\rm FBC} = \begin{pmatrix} -\tilde G^n & -e(\tilde G^n D_0^n - \overbrace{\Im\tilde G}^{=0}\,\Im D_0)\\ -e(D_0^n\tilde G^n - \Im D_0\Im\tilde G) & -D^n\end{pmatrix}$$

Effective action I.

($J^{\pm}$ and $A^{\pm}$)

$$\Gamma[\hat{J},\hat{A}] = W[\hat{a},\hat{j}] - \hat{a}\cdot\hat{J} - \hat{j}\cdot\hat{A} \qquad \hat{J} = \frac{\delta W}{\delta \hat{a}} \qquad \hat{A} = \frac{\delta W}{\delta \hat{j}}$$

$$\Gamma[\hat{J},\hat{A}] = \Gamma^{\rm mech}[\hat{J}] + \frac{1}{2}\hat{A}\cdot\hat{D}_0^{-1}\cdot\hat{A} - e\hat{A}\hat{\sigma}\cdot\hat{J}$$

$$\Gamma^{\rm mech}[\hat{J}] = \frac{1}{2}\hat{J}\cdot(\hat{\tilde{G}}_0^{-1} - \hat{\tilde{G}}_0^{-1}\cdot\hat{W}_D^{(4)}\cdot\hat{\tilde{G}}_0^{-1})\cdot\hat{J}$$

Equations of motion:

$$\frac{\delta\Gamma[\hat{J},\hat{A}]}{\delta\hat{J}} = -\hat{a} \qquad \frac{\delta\Gamma[\hat{J},\hat{A}]}{\delta\hat{A}} = -\hat{j}$$

No one-loop corrections left because both variables have been kept

Effective action at a distance: (eliminating fields by their E.M.)

$$\Gamma[\hat{A}] = \frac{1}{2}\hat{A}\cdot\hat{D}^{-1}\cdot\hat{A} \qquad \Gamma[\hat{J}] = \frac{1}{2}\hat{J}\cdot\hat{\tilde{G}}^{-1}\cdot\hat{J} \qquad (\hat{j} = \hat{a} = 0)$$

Effective action II.

$(J,\ J^a,\ A$ and $A^a)$

$$
\begin{aligned}
\Gamma[J,J^a,A,A^a] &= \Re W[a,\bar{a},j,\bar{j}] - J\cdot a - J^a\cdot\bar{a} - A\cdot j - A^a\cdot\bar{j} \\
J &= \frac{\delta\Re W[\hat{a},\hat{j}]}{\delta a}, \qquad J^a = \frac{\delta\Re W[\hat{a},\hat{j}]}{\delta\bar{a}}, \qquad A = \frac{\delta\Re W[\hat{a},\hat{j}]}{\delta j}, \qquad A^a = \frac{\delta\Re W[\hat{a},\hat{j}]}{\delta\bar{j}} \\
\Gamma[J,J^a,A,A^a] &= \frac{1}{2}^{(J,A,J^a,A^a)}\begin{pmatrix} 0 & K^{\mathrm{tr}-1} \\ K^{-1} & -\frac{\kappa}{2}(K^{\mathrm{tr}-1}+K^{-1})\end{pmatrix}\begin{pmatrix} J \\ A \\ J^a \\ A^a\end{pmatrix}
\end{aligned}
$$

OBC:

$$
K^{-1} = \begin{pmatrix}\tilde{G}_0^{r-1} - e^2\tilde{G}_0^{r-1}W_D^{e(4)r}\tilde{G}_0^{r-1} & -e \\ -e & D_0^{r-1}\end{pmatrix} \qquad (W_D^{e(4)r} = \Re W_D^{e(4)++} + \Re W_D^{e(4)-+})
$$

Equations of motion for $J^a = A^a = a = j = 0$:

$$
J = \frac{1}{1 - e^2 W_D^{e(4)r}\tilde{G}_0^{r-1}}\tilde{G}_0^r(eA - \bar{a}) \qquad A = D_0^r(eJ - \bar{j})
$$

FBC: (OBC and $D^r, D^a \to D^n$)

$$
K^{-1} = \begin{pmatrix}\tilde{G}_0^{n-1} - e^2\tilde{G}_0^{n-1}W_D^{e(4)n}\tilde{G}_0^{n-1} & -e \\ -e & D_0^{n-1}\end{pmatrix}
$$

Equations of motion for $J^a = A^a = a = j = 0$:

$$
J = \frac{1}{1 - e^2 W_D^{e(4)n}\tilde{G}_0^{n-1}}\tilde{G}_0^n(eA - \bar{a}) \qquad A = D_0^n(eJ - \bar{j})
$$

Effective action III.

(J and A, treating $\bar{a}$ and $\bar{j}$ as parameters)

$$\begin{aligned}
\Gamma[J,A] &= \Re W[a,\bar{a},j,\bar{j}] - J\cdot a - A\cdot j \qquad J = \frac{\delta \Re W[\hat{a},\hat{j}]}{\delta a} \qquad A = \frac{\delta \Re W[\hat{a},\hat{j}]}{\delta j} \\
\kappa\Gamma[J,A] &= \Gamma^{\mathrm{mech}}[J] + \frac{1}{2} A\cdot D_0^{n-1}\cdot A - eA\cdot J - A\cdot J^{\mathrm{ext}} \\
\Gamma^{\mathrm{mech}}[\hat{J}] &= \frac{1}{2} J\cdot(\tilde{G}_0^{n-1} - \tilde{G}_0^{n-1}\cdot W_D^{e(4)}\cdot \tilde{G}_0^{n-1})\cdot J - J\cdot A^{\mathrm{ext}} \\
J^{\mathrm{ext}} &= -eW_a + D_0^{n-1}\cdot W_j \\
A^{\mathrm{ext}} &= (\tilde{G}_0^{n-1} - \tilde{G}_0^{n-1}\cdot W_D^{e(4)}\cdot \tilde{G}_0^{n-1})\cdot W_a - eW_j
\end{aligned}$$

Equations of motion:

$$\begin{aligned}
J_{\substack{\mathrm{OBC}\\ \mathrm{FBC}}} &= -\tilde{G}^{\overset{r}{n}}(\bar{a} + eD_0^{\overset{r}{n}}\bar{j}) + \frac{e}{\tilde{G}_0^{n-1} - \tilde{G}_0^{n-1} W_D^{(4)n}\tilde{G}_0^{n-1}}[A + D^{\overset{r}{n}}\bar{j} + eD_0^{\overset{r}{n}}\tilde{G}^{\overset{r}{n}}\bar{a})] \\
A_{\mathrm{OBC}} &= eD_0^n J - D_0^r\bar{j} + \frac{e}{2}D_0^f\tilde{G}^r(\bar{a} + eD_0^r\bar{j}) - (e^4 D_0^r W_D^{e(4)r} D_0^r + e^4 D_0^r\tilde{G}_0^r D_0^r\tilde{G}_0^r D_0^r)\bar{j} \\
A_{\mathrm{FBC}} &= eD_0^n J - [D_0^n + e^4 D_0^n W_D^{e(4)n} D_0^n + e^4 D_0^n\tilde{G}_0^n D_0^n\tilde{G}_0^n D_0^n]\bar{j}
\end{aligned}$$

Effective action at a distance: (Schwarzschild-Tetrode-Fokker-Wheeler-Feynman)

$$\kappa\Gamma[J] = \frac{1}{2}J\cdot\tilde{G}^{n-1}\cdot J = \Gamma^{\mathrm{mech}}[J] - \frac{e^2}{2}J\cdot D_0^n\cdot J,$$

$\not{T}$: Completely absorbing Universe (Wheeler-Feynman).

OBC E.M.≈FBC E.M. $\Longrightarrow$ complete absorption on microscopic level

Selection of the time arrow

$$\phi^{\pm} = \underbrace{\frac{\phi^{\pm}+\phi^{\mp}}{2}}_{\phi^r:\ \text{retarded}} + \underbrace{\frac{\phi^{\pm}-\phi^{\mp}}{2}}_{\pm\phi^a:\ \text{advanced}} \qquad j^+\cdot\phi^+ + j^-\cdot\phi^- = j\cdot\phi^r + (j\kappa + 2\bar{j})\cdot\phi^a$$

Boundary condition in time $\implies$ $\not{T}$ on the macroscopic scale only

$$\text{4 fields :} \quad A = \underbrace{eD_0^n J}_{closed:\ \boldsymbol{T}} \underbrace{-D_0^r\bar{j} + \frac{e}{2}D_0^f\tilde{G}^r(\bar{a} + eD_0^r\bar{j}) - (e^4 D_0^r W_D^{e(4)r} D_0^r + e^4 D_0^r\tilde{G}_0^r D_0^r \tilde{G}_0^r D_0^r)\bar{j}}_{open:\ \not{T}}$$

$$\text{2 fields :} \quad A = D_0^r(eJ - \bar{j})$$

variation of $\Gamma[J, J^a, A, A^a]$ $\implies$ retarded and advanced equations for the measurable averages and auxiliary fields (the fixing of the auxiliary fields generates $\not{T}$ equations)

variation of $\Gamma[J, A]$ $\implies$ $\boldsymbol{T}$ for the measurable averages (auxiliary fields are varied, too)

1. Macroscopic, closed system $\implies$ T
2. Quantum-classical crossover, decoherence $\implies$ $\phi^a \approx 0$ $\implies$ $\not{T}$
3. $\not{T}$: IR, eg. spontaneous symmetry breaking?
4. Decoherence is not a solution of the problem, it is only a mechanism for adopting a time arrow

Polarisation of the Dirac-see

Let $\Gamma'[A] = \Gamma[F]$, with $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ the photon effective action and

$$J_\mu = -\frac{1}{e}\frac{\delta\Gamma[A]}{\delta A^\mu}_{|A=0} = \left(\partial^\nu \frac{\delta\Gamma[F]}{\delta F^{\mu\nu}} - \partial^\nu \frac{\delta\Gamma[F]}{\delta F^{\nu\mu}}\right)_{|F=0} \qquad \text{the induced current.}$$

Minimal coupling:

$$-eA \cdot J = \frac{e}{2} F_{\mu\nu} \cdot \Box^{-1} \cdot H^{\mu\nu} \qquad H_{\mu\nu} = \partial_\mu J_\nu - \partial_\nu J_\mu$$

Field strength tensor containing polarisation:

$$G = -2\left(\frac{\delta\Gamma[F]}{\delta F} - \frac{\delta\Gamma[F]}{\delta F}_{|F=0}\right).$$

$\mathcal{O}(J)$ and $\mathcal{O}(A^2)$ are sufficient for E.M.:

$$\Gamma[A] = -\frac{1}{4} F \cdot G - eA \cdot J.$$

Gauge and Lorentz invariant quadratic functional:

$$A \cdot \Gamma^{(2)} \cdot A = -\frac{1}{2} F_{\mu\nu} \cdot \Gamma^{(2)} \cdot \Box^{-1} \cdot F^{\mu\nu}$$

$$\Gamma[A] = -\frac{1}{4} F_{\mu\nu} \cdot \epsilon \cdot F^{\mu\nu} - eA \cdot J.$$

$$\text{dielectric function :} \quad T\epsilon = T(1 + 4\pi\chi) = \frac{\delta^2 \Gamma'[A]}{\delta A \delta A} \cdot \Box^{-1}.$$

Valence nucleus and the electron Dirac-see: $\hat{D}_0^{-1} \to \hat{D}_0^{-1} - \hat{\Sigma}_e^{\gamma}$

$$\kappa\Gamma[J, A] = \Gamma^{\mathrm{mech}}[J] + \frac{1}{2} A \cdot (D_0^{n-1} - \Sigma_e^n) \cdot A - eA \cdot J - A \cdot J^{\mathrm{ext}},$$

$$4\pi\chi = -\Box^{-1} \cdot \Sigma_e^n \qquad \text{Uehling} \quad (1949)$$

Quantum Renormalization Group

Connected Green functions:

$$e^{iW[\hat{j}]} = \int D[\hat{\phi}]e^{iS[\hat{\phi}]-\frac{1}{2}\hat{\phi}\cdot\hat{K}\cdot\hat{\phi}+i\hat{j}\cdot\hat{\phi}}, \qquad \hat{\phi}=(\phi^+,\phi^-), \quad S[\hat{\phi}]=S[\phi^+]-S^*[\phi^-], \quad \hat{K}=\begin{pmatrix} K & 0 \\ 0 & K \end{pmatrix}$$

$$\partial_k W = \frac{1}{2}\mathrm{Tr}\left[\left(\frac{\delta^2 W}{\delta j^+\delta j^+}+\frac{\delta^2 W}{\delta j^-\delta j^-}+i\frac{\delta W}{\delta j^+}\frac{\delta W}{\delta j^+}+i\frac{\delta W}{\delta j^-}\frac{\delta W}{\delta j^-}\right)\cdot\partial_k K\right]$$

Effective action:

$$\Gamma[\phi,\phi^i] = \Gamma[\phi,\phi^a]+i\Im\Gamma[\phi^i,\phi^{ai}], \qquad \Gamma[\phi,\phi^a]=\Re W[j,\bar{j}]-\bar{j}\cdot\phi^a-j\cdot\phi, \quad \Im\Gamma[\phi^i,\phi^{ai}]=\Im W[j,\bar{j}]-\bar{j}\cdot\phi^{ai}-j\cdot\phi^i$$

$$\phi+i\phi^i=\frac{\delta W[j,\bar{j}]}{\delta j}, \qquad \phi^a+i\phi^{ai}=\frac{\delta W[j,\bar{j}]}{\delta\bar{j}}$$

↑ ↑

independent variables

$\hat{\phi}=(\phi,\phi^a)$, $\hat{\phi}^i=(\phi^i,\phi^{ai})$

$$\begin{aligned}\partial_k\Gamma = \mathrm{Tr}\Bigg\{\Bigg[&\left(\frac{\delta^2\Gamma}{\delta\hat{\phi}\delta\hat{\phi}}\right)^{-1}_{\phi\phi}-\kappa\left(\frac{\delta^2\Gamma}{\delta\hat{\phi}\delta\hat{\phi}}\right)^{-1}_{\phi\phi^a}+\frac{1+\kappa^2}{4}\left(\frac{\delta^2\Gamma}{\delta\hat{\phi}\delta\hat{\phi}}\right)^{-1}_{\phi^a\phi^a} \\ &+i\left(\frac{\delta^2\Im\Gamma}{\delta\hat{\phi}^i\delta\hat{\phi}^i}\right)^{-1}_{\phi^i\phi^i}-\kappa i\left(\frac{\delta^2\Im\Gamma}{\delta\hat{\phi}^i\delta\hat{\phi}^i}\right)^{-1}_{\phi^i\phi^{ai}}+i\frac{1+\kappa^2}{4}\left(\frac{\delta^2\Im\Gamma}{\delta\hat{\phi}\delta\hat{\phi}}\right)^{-1}_{\phi^{ai}\phi^{ai}}+\frac{i}{2}(\phi^+\phi^++\phi^-\phi^-)\Bigg]\cdot\partial_k K\Bigg\}\end{aligned}$$

where $\phi^\pm=\phi+i\phi^i-(\kappa\mp1)\frac{\phi^a+i\phi^{ai}}{2}$

Closing: $\hat{\phi}^i[\hat{\phi}]$ is obtained from the E.M:

$$
\begin{aligned}
-\hat{j} &= \frac{\delta \Im \Gamma}{\delta \hat{\phi}^i} = \frac{\delta \Gamma}{\delta \hat{\phi}} \\
\frac{\delta^2 \Im \Gamma}{\delta \hat{\phi}^i \delta \hat{\phi}^i} &= \frac{\delta^2 \Gamma}{\delta \hat{\phi} \delta \hat{\phi}} \cdot \hat{S}, \qquad\qquad \hat{S}_{ab}[\hat{\phi}] = \frac{\delta \hat{\phi}_a}{\delta \hat{\phi}^i_b} \\
\partial_k \hat{S}_{ab} &= \left(\frac{\delta^2 \Gamma}{\delta \hat{\phi} \delta \hat{\phi}} \right)^{-1}_{ac} \left[\left(\frac{\delta^2 \dot{\Gamma}}{\delta \hat{\phi} \delta \hat{\phi}} \right)_{cd} \hat{S}_{db} - \hat{S}_{dc} \left(\frac{\delta^2 \dot{\Gamma}_i}{\delta \hat{\phi} \delta \hat{\phi}} \right)_{de} \hat{S}_{eb} - \frac{\delta \dot{\Gamma}_i}{\delta \hat{\phi}_d} \frac{\delta \hat{S}_{dc}}{\delta \hat{\phi}_e} (S^{-1})_{eb} \right]
\end{aligned}
$$

Separation of the tree-level parts:

$$
\Gamma = \tilde{\Gamma} + i \Im \tilde{\Gamma} + \frac{i}{2} (\phi^+ \cdot \hat{K} \cdot \phi^+ + \phi^- \cdot \hat{K} \cdot \phi^-)
$$

Evolution equations:

$$
\begin{aligned}
\partial_k \tilde{\Gamma} &= \mathrm{Tr} \left\{ \left[\left(\frac{\delta^2 \Re \tilde{\Gamma}}{\delta \hat{\phi} \delta \hat{\phi}} + A \right)^{-1}_{\phi\phi} - \kappa \left(\frac{\delta^2 \Re \tilde{\Gamma}}{\delta \hat{\phi} \delta \hat{\phi}} + A \right)^{-1}_{\phi\phi^a} + \frac{1+\kappa^2}{4} \left(\frac{\delta^2 \Re \tilde{\Gamma}}{\delta \hat{\phi} \delta \hat{\phi}} + A \right)^{-1}_{\phi^a\phi^a} \right] \cdot \partial_k K \right\} \\
\partial_k \tilde{\Gamma}_i &= \mathrm{Tr} \left\{ \left[\left(\frac{\delta^2 \Re \Gamma}{\delta \hat{\phi} \delta \hat{\phi}} \cdot \hat{S} + B \right)^{-1}_{\phi^i\phi^i} - \kappa \left(\frac{\delta^2 \Re \Gamma}{\delta \hat{\phi} \delta \hat{\phi}} \cdot \hat{S} + B \right)^{-1}_{\phi^i\phi^{ai}} + \frac{1+\kappa^2}{4} \left(\frac{\delta^2 \Re \Gamma}{\delta \hat{\phi} \delta \hat{\phi}} \cdot \hat{S} + B \right)^{-1}_{\phi^{ai}\phi^{ai}} \right] \cdot \partial_k K \right\}
\end{aligned}
$$

$$
\begin{aligned}
A_{ab} &= \frac{1}{2} \left(\hat{K}_{ac} \frac{\delta \hat{\phi}^i_c}{\delta \hat{\phi}_b} + \frac{\delta \hat{\phi}^i_c}{\delta \hat{\phi}_a} \hat{K}_{cb} \right) + \hat{\phi}_c \hat{K}_{cd} \frac{\delta^2 \hat{\phi}^i_d}{\delta \hat{\phi}_a \delta \hat{\phi}_b} \\
B_{ab} &= 2\hat{K}_{ab} - 2 \frac{\delta \hat{\phi}_c}{\delta \hat{\phi}^i_a} \hat{K}_{cd} \frac{\delta \hat{\phi}_d}{\delta \hat{\phi}^i_b} - \hat{\phi}_c \hat{K}_{cd} \frac{\delta^2 \hat{\phi}_d}{\delta \hat{\phi}^i_a \delta \hat{\phi}^i_b} - \frac{\delta^2 \hat{\phi}_c}{\delta \hat{\phi}^i_a \delta \hat{\phi}^i_b} \hat{K}_{cd} \hat{\phi}_d
\end{aligned}
$$

Summary

1. Can we recover the classical action? **A: Yes, by mixing the diagonal and off-diagonal fluctuations.**
2. Can we separate the one-particle dynamics from the many-particle correlations of the Dirac-see? **A: Yes, but from the positive energy (large polaron) states only. The overlapping and filled negative energy states (small polarons) remain strongly correlated.**
3. What kind of corrections appear in the action? **A: series truncated in e^2, gradient expansion and number of correlated clusters.**
4. Classical or macroscopic theory? **A: (i) $\Lambda > k_{\mathrm{kv-kl}}$: microscopic, classical (bound states, CDFT); (ii) $\Lambda \approx k_{\mathrm{kv-kl}}$: Classical-quantum crossover, decoherence; (iii) $\Lambda < k_{\mathrm{kv-kl}}$: macroscopic physics.**
5. How does the environment, when treated on the quantum level, lead to $\not T$ and define the time arrow? **A: Decoherence passes the time arrow of the environment in macroscopic physics.**
6. How does decoherence appear? **A: Strong coupling regime and suppression of advances effects.**
7. Are macroscopic and microscopic polarisable media similar? **A: Same treatment for the Dirac-see and the macroscopic polarisation.**
8. Is there a role for Quantum Field Theory in the theory of measurement? **A: defining the Hilbert space as in spontaneous symmetry breaking.**
9. What happens with the reduced density matrix during blocking? **A: Schwinger's CTP method is needed to make it explicite because there are two natural time axis in Quantum Mechanics.**
10. In what kind of models are the mixing effects negligible? **A: In renormalized models only. SM?**