

# Monopole-Antimonopole Correlation Functions in 4D U(1) l.g.t

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based on PLB in prep. [hep-lat/0603022](#)



# Outlook

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- Could be the prototype for confinement.



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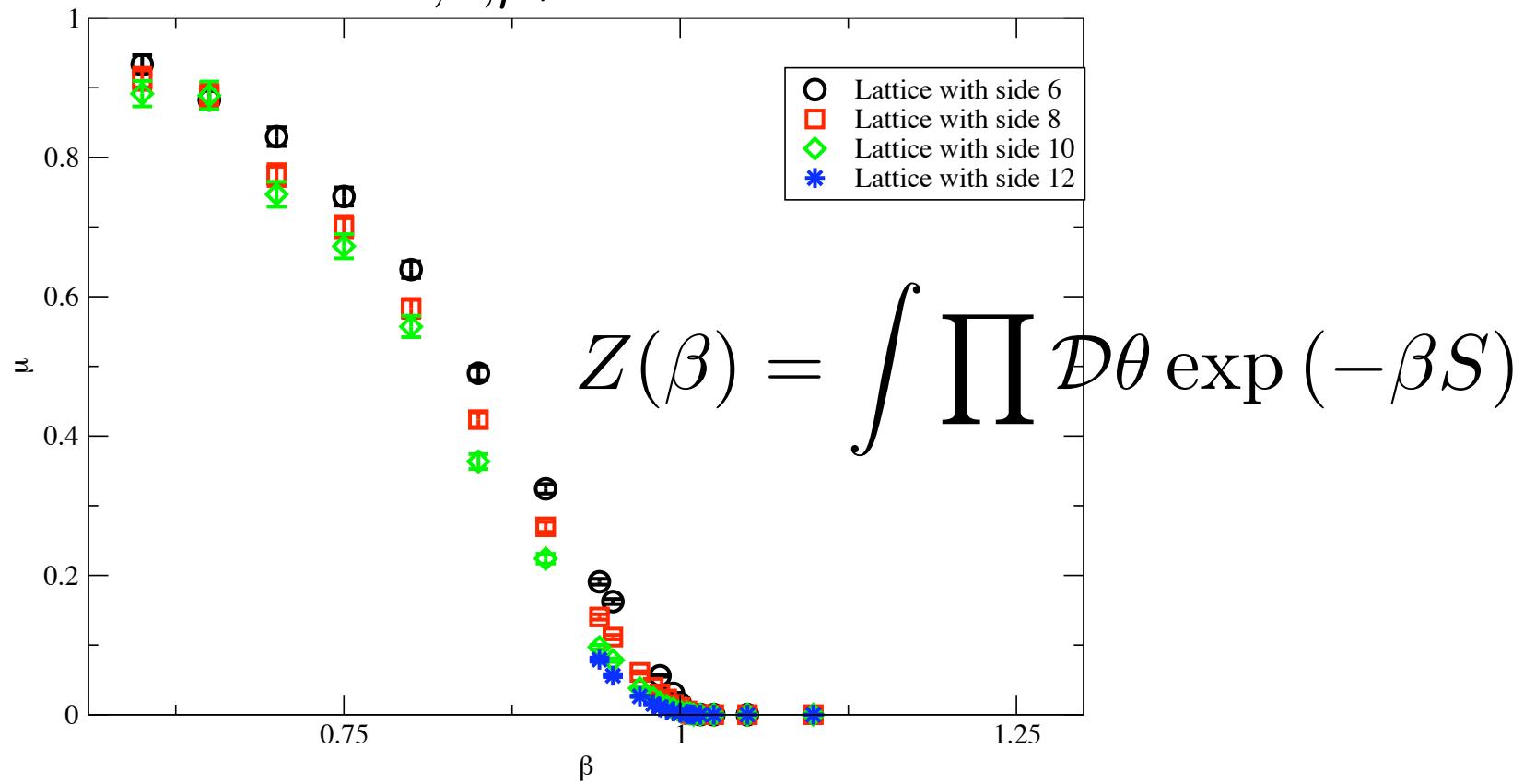
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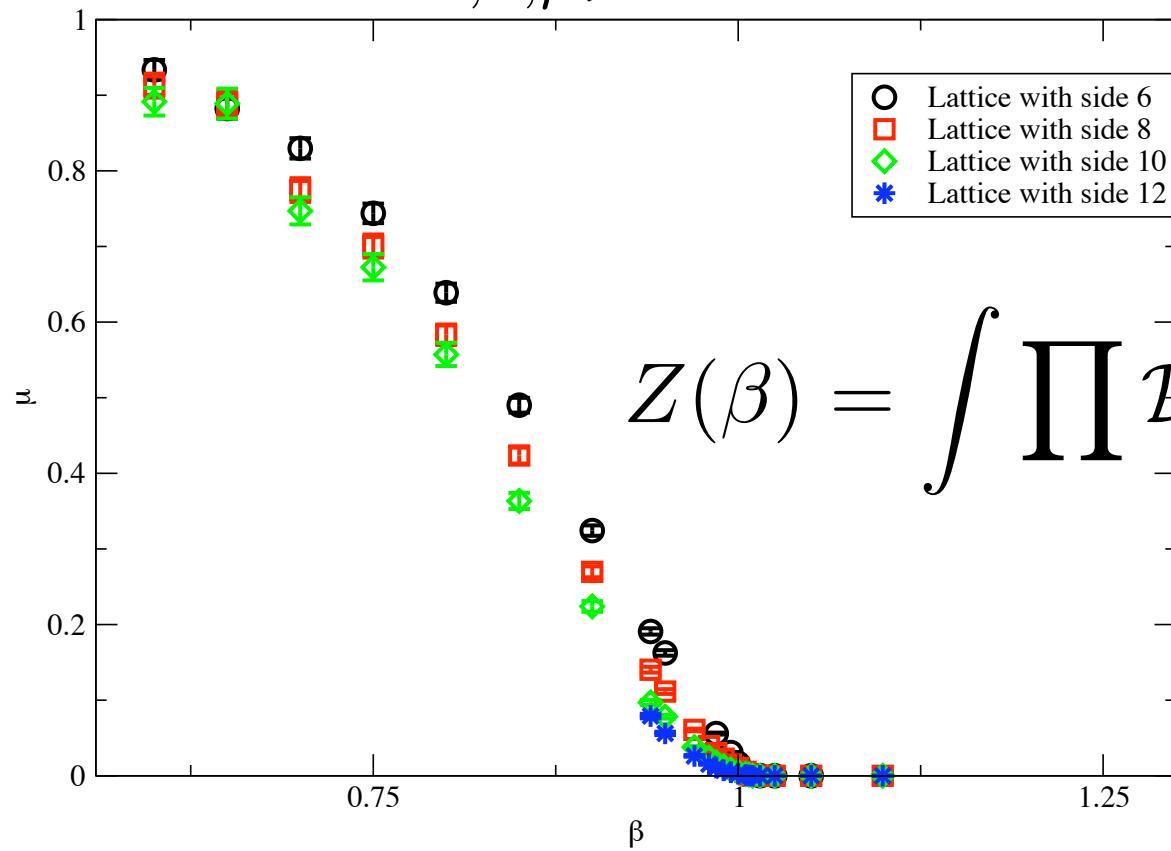
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Weak first order  
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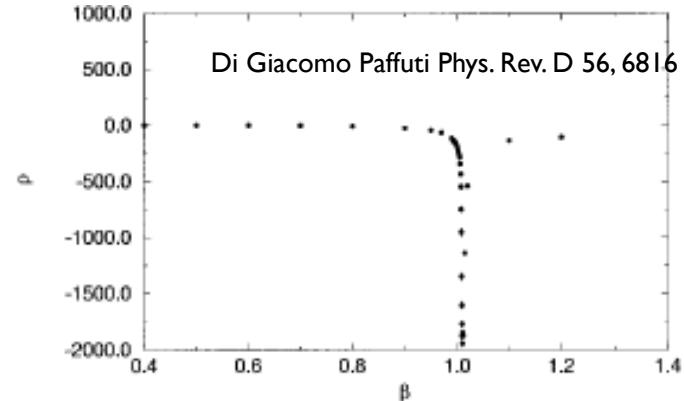


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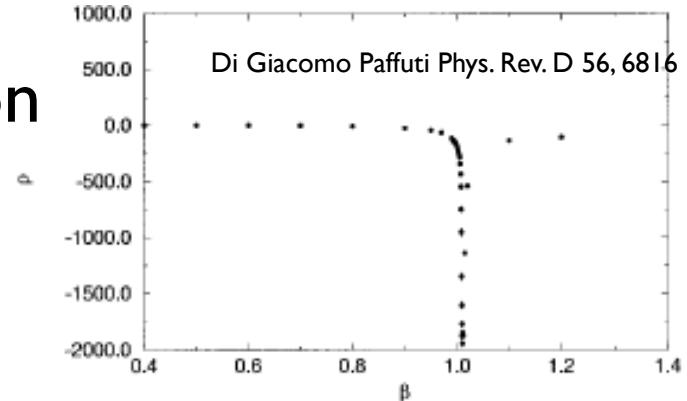


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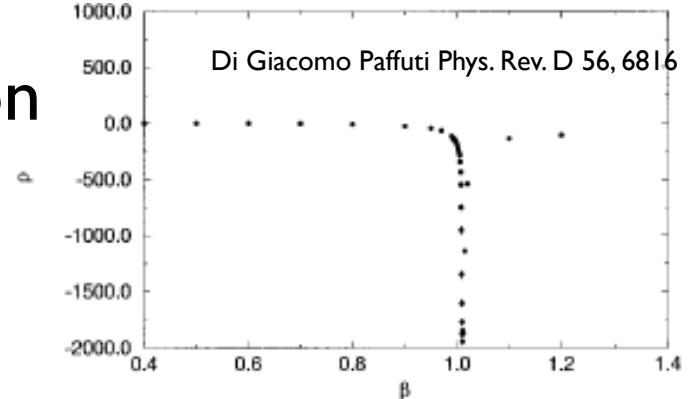


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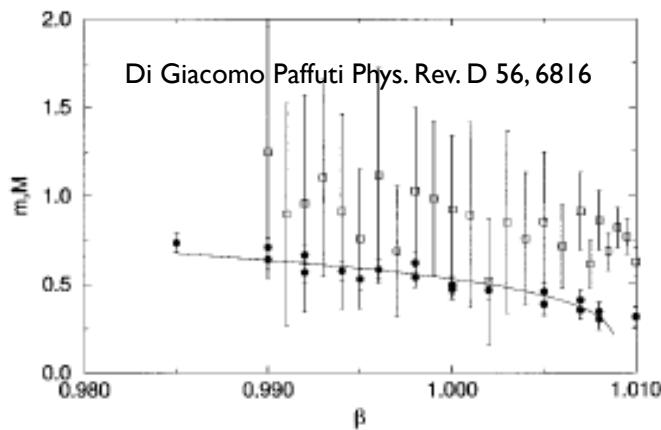


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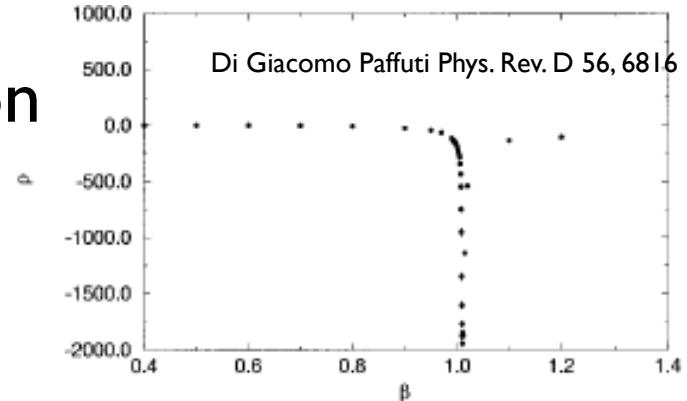


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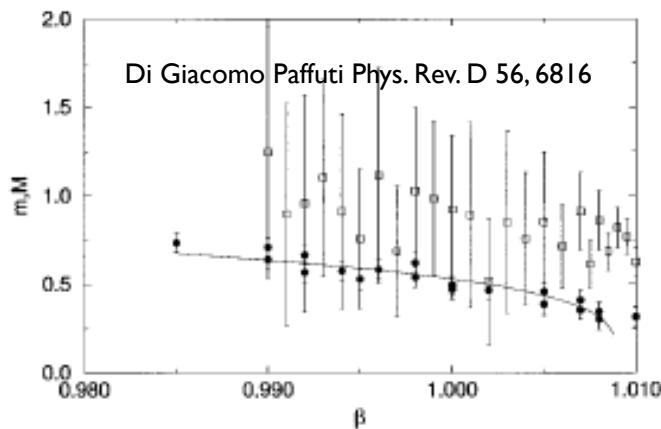


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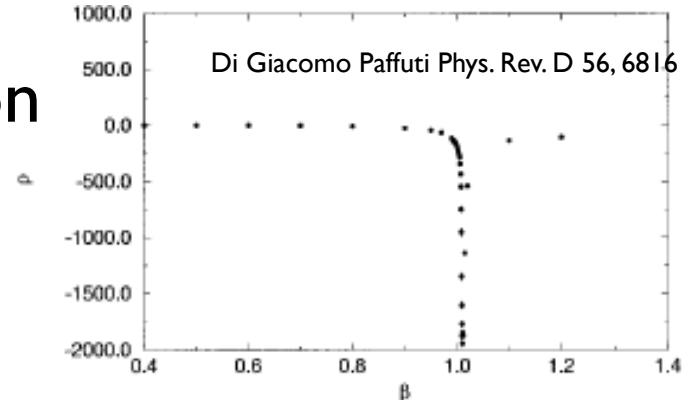


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Anything new from the known spectrum?



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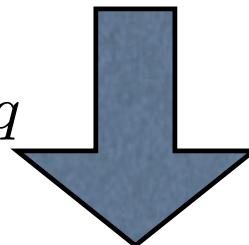
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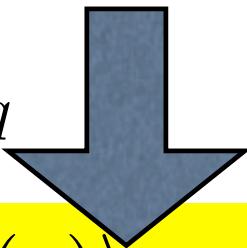
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$$\langle O_0(x)O_1(y) \rangle_c = 0$$



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Frohlich, Marchetti Europhys. Lett. 2, pag 933 1986

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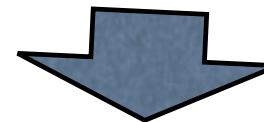
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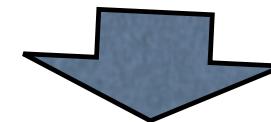
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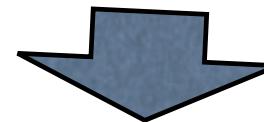
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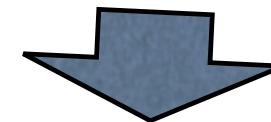
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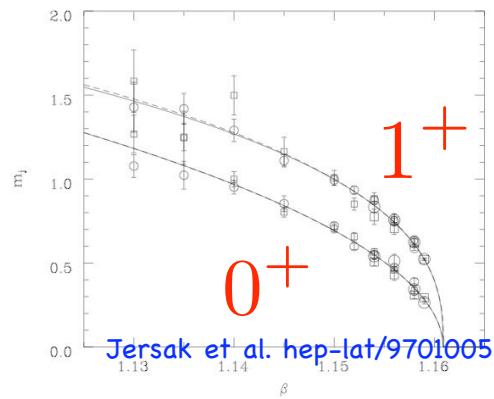
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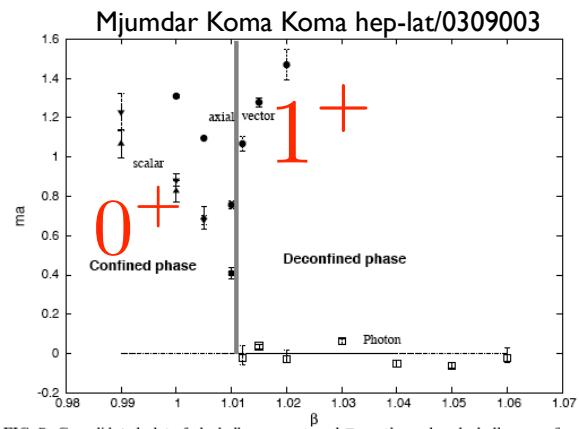
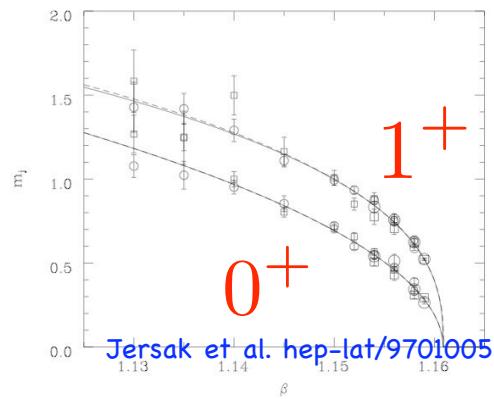


FIG. 7. Consolidated plot of glueball masses.  $\blacktriangle$  and  $\blacktriangledown$  are the scalar glueball masses from the sets  $\partial\partial^*\langle CC\rangle$  and  $\partial^*\partial\langle CC\rangle$  respectively. The  $\bullet$  corresponds to the axial vector mass with zero momentum. The  $\square$  is the photon extracted from the axial vector correlator (See fig. 6).

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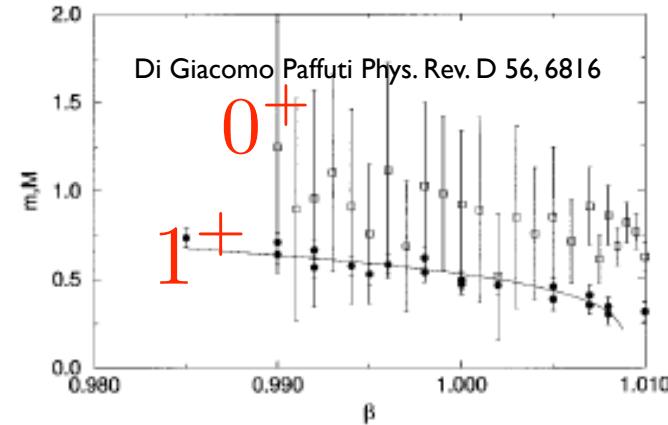
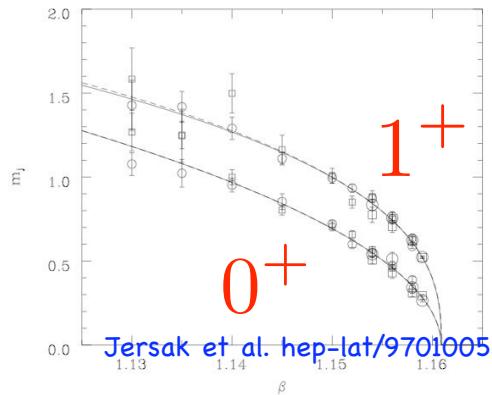


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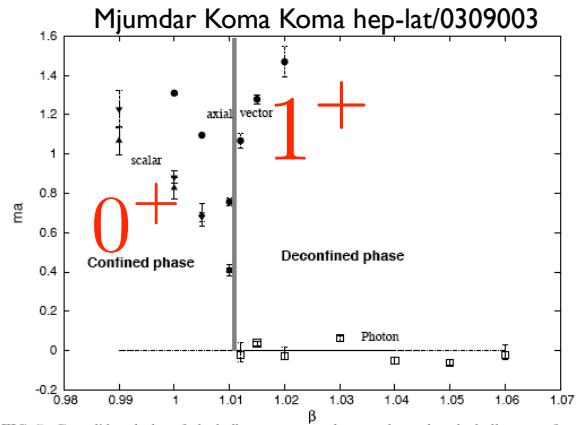


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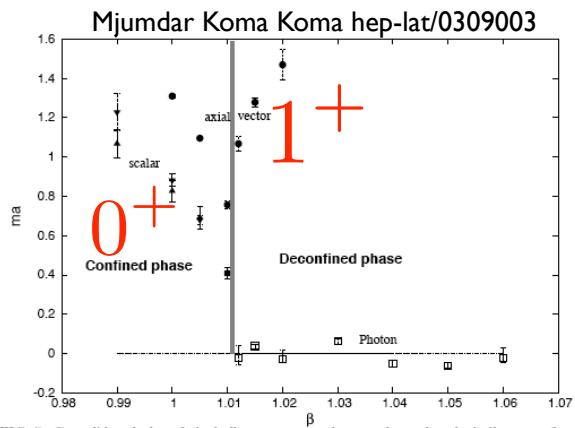
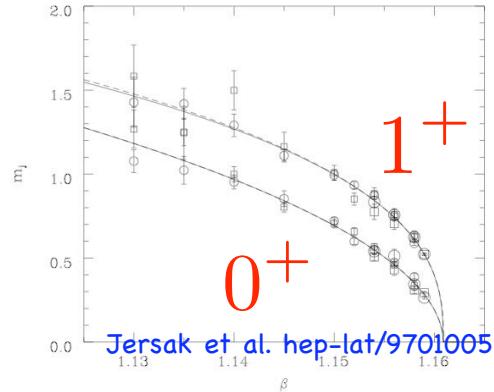


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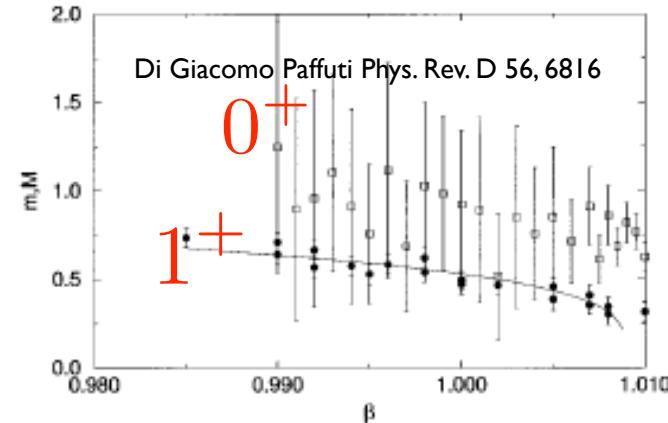


FIG. 6. Mass of the monopole  $M$  (squares) and mass of the dual photon  $m$  (circles) versus  $\beta$ .

Implicitly assume mon operator couples to new particle: magnetic superselection

# Spectrum

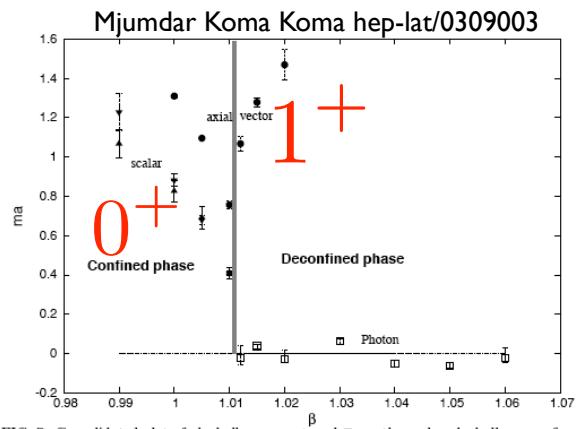
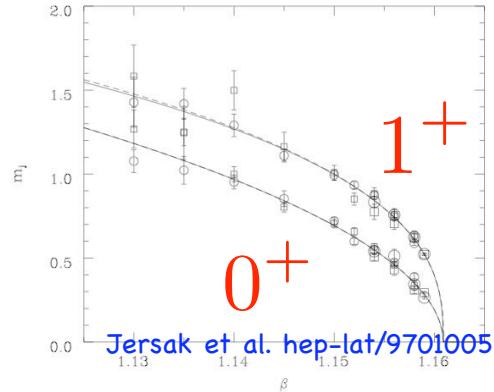


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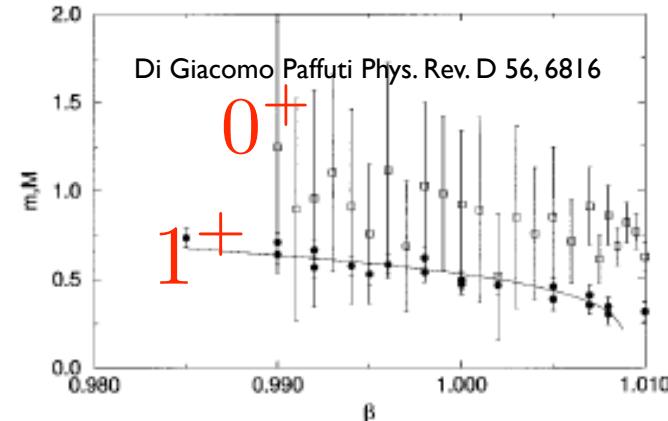


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Implicitly assume mon operator couples to new particle: magnetic superselection contradiction with theorem



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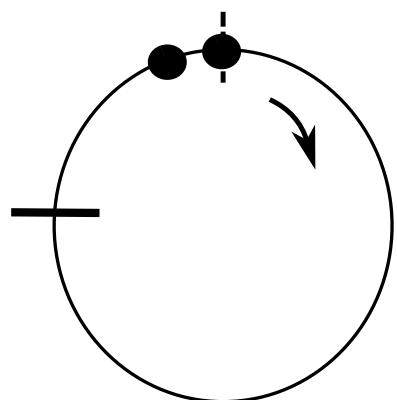
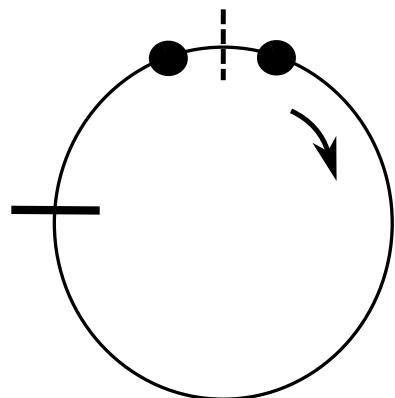


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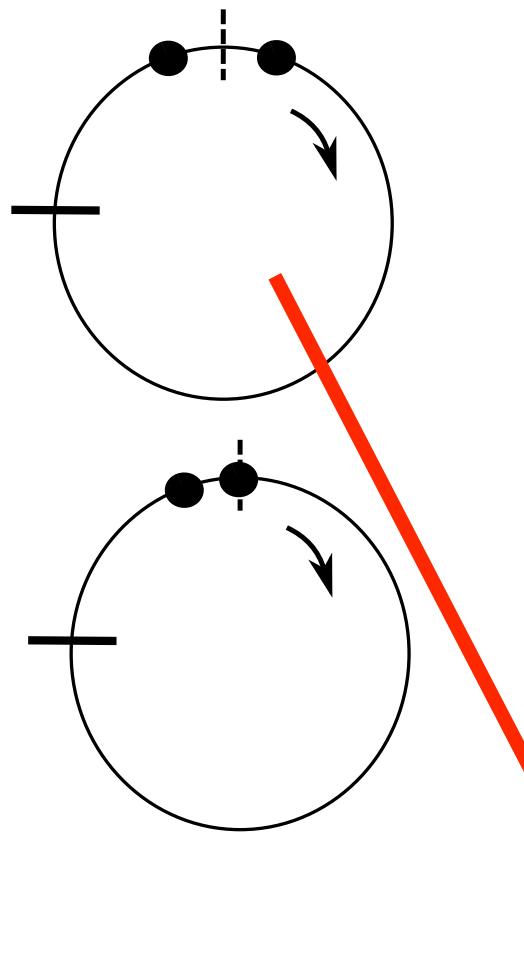


# Symmetric derivatives

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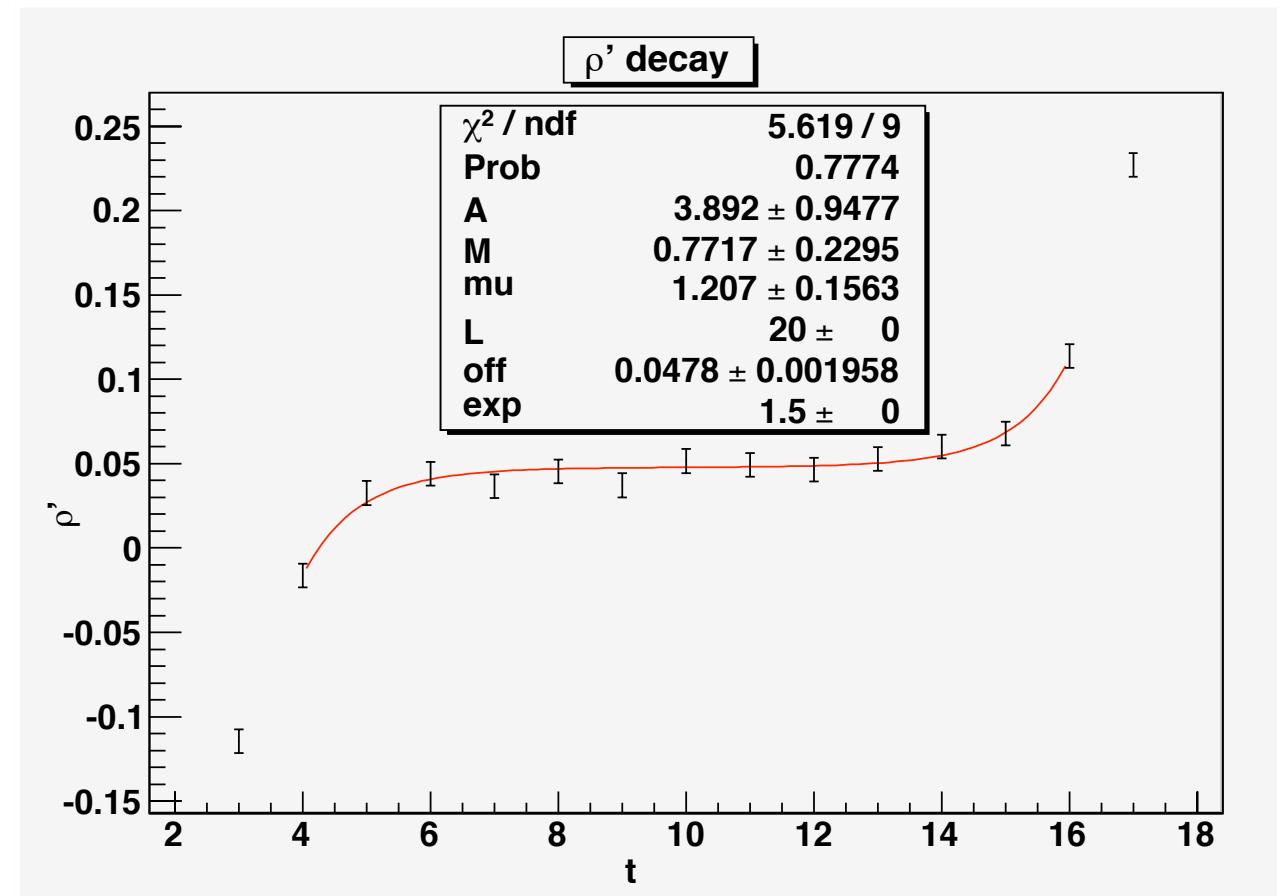
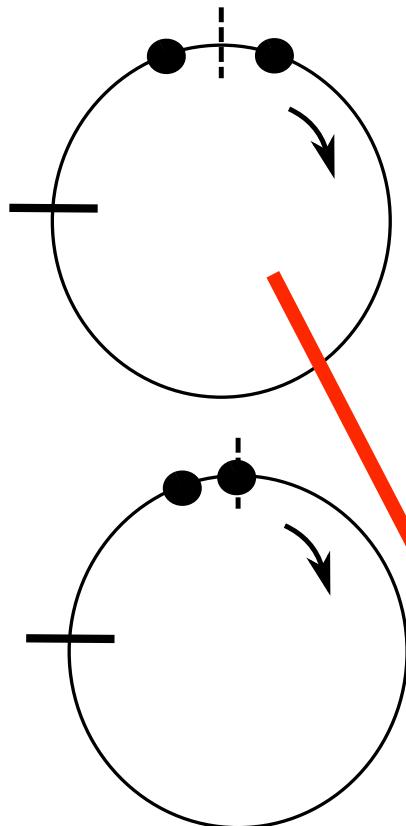


# Symmetric derivatives



More correlated

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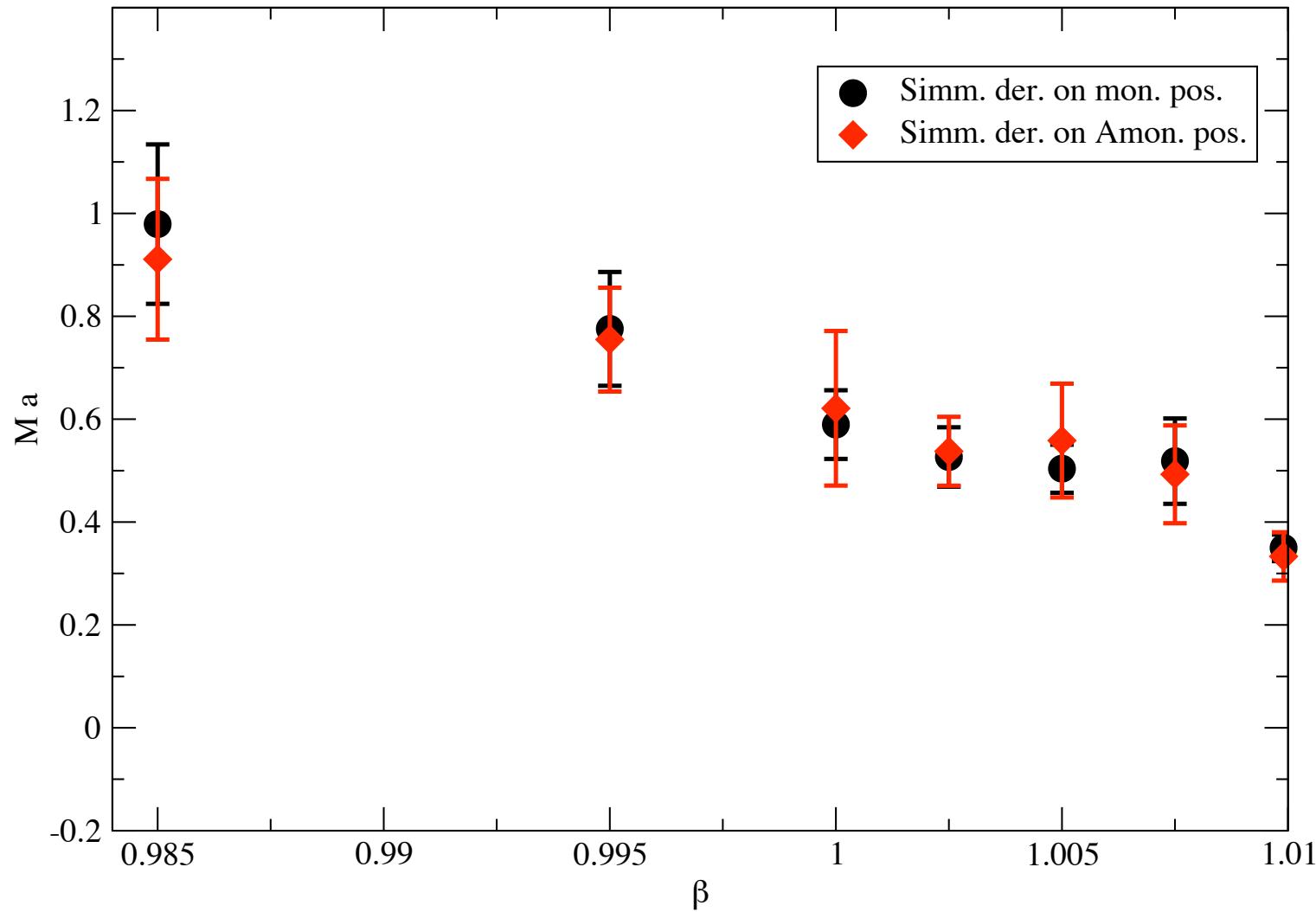


More correlated

# Results

Lightest scalar masses in the confined phase

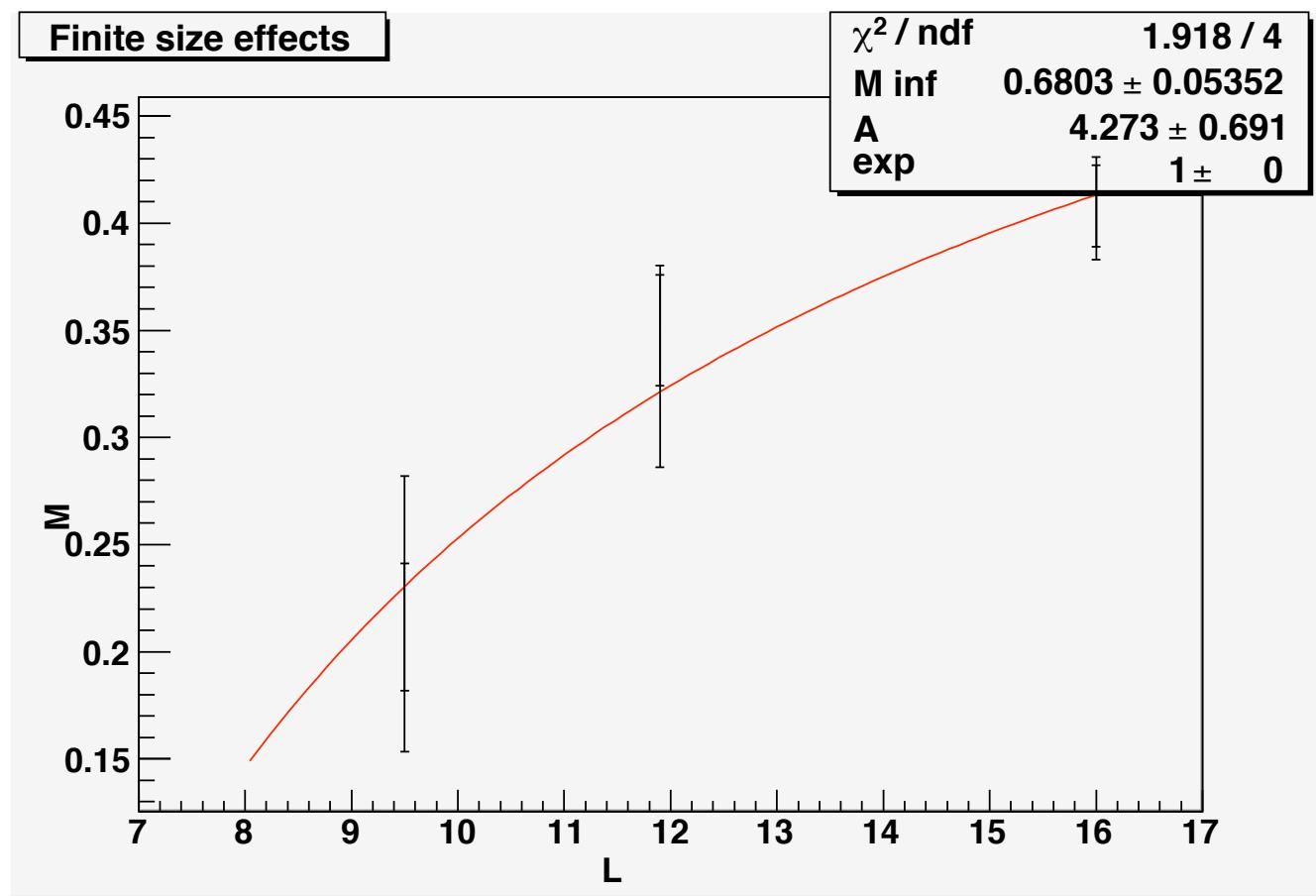
10 x 10 x 10 x 20 lattice and 400000 sweeps



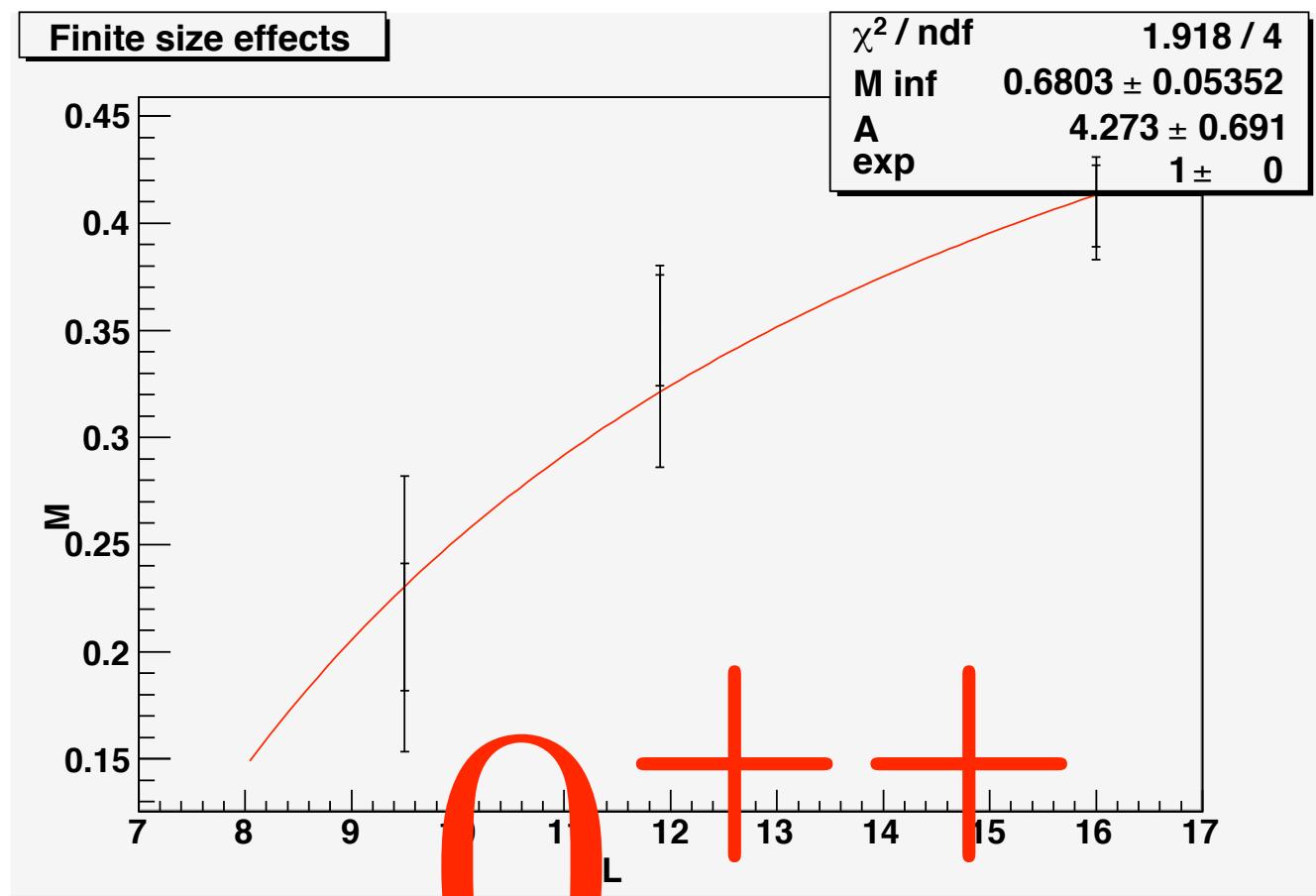


**Compare with g. spect.**

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# Conclusions

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- Introduce a new susceptibility for the monopole creation operator

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