Freezing / Melting of Flexible Membranes

Topological Defects in Planar Membranes

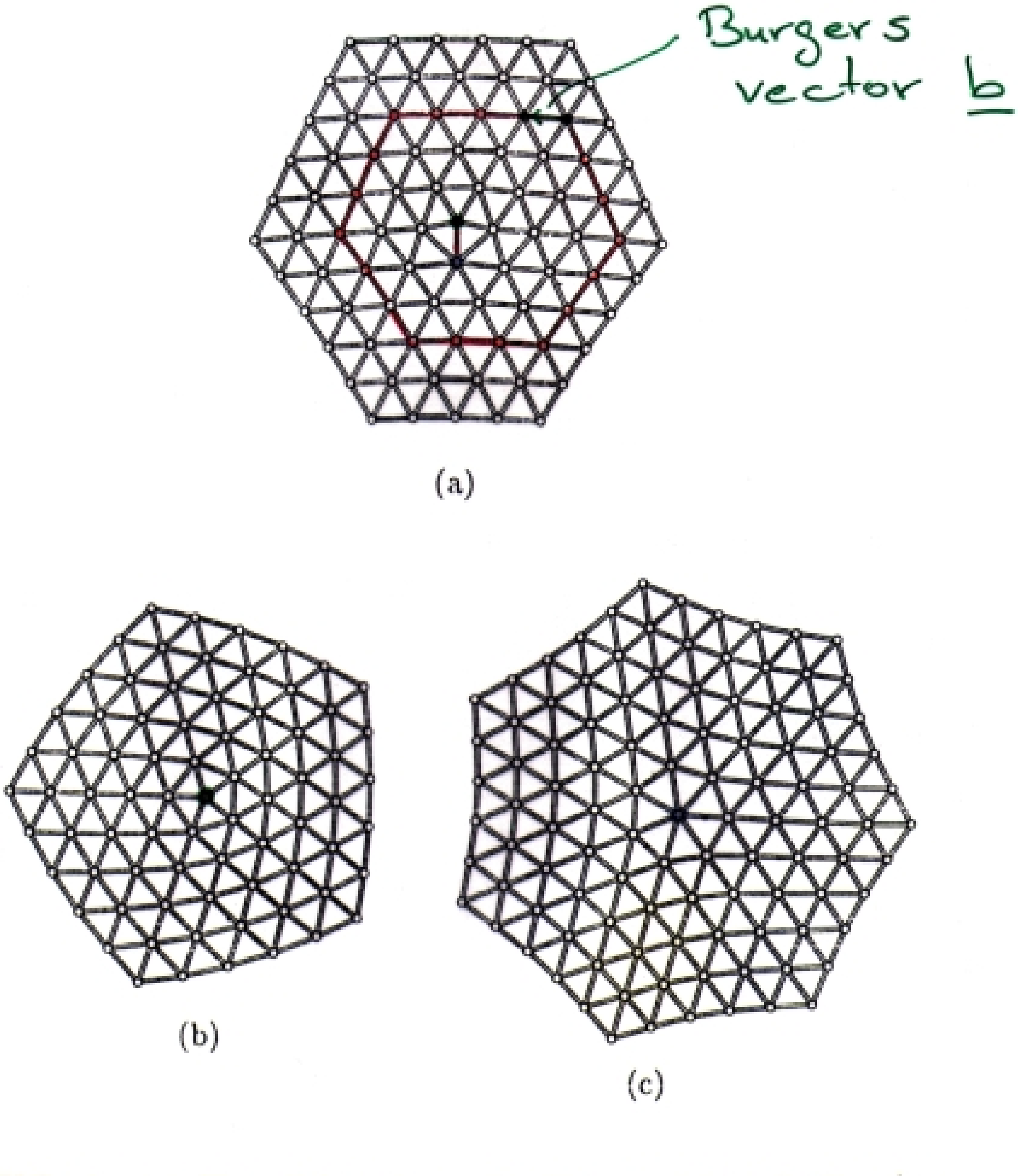


FIG. 1. (a) Flat dislocation. (b) Flat positive disclination ($s = 2\pi/6$). (c) Flat negative disclination ($s = -2\pi/6$).

Defect (Free) Energies:

- Dislocations:

Stretching energy:

$$\boxed{E_s = \frac{K_0 b^2}{8\pi} \ln \frac{R}{a}}$$

with Young modulus $K_0 = \frac{4\mu(\lambda+\mu)}{2\mu+\lambda}$

and Burgers vector $\underline{b}$.

Translational entropy:

$$\boxed{S = k_B \ln \left(\frac{R}{a}\right)^2 = 2 k_B \ln \frac{R}{a}}$$

↷ Free energy:

$$F = E_s - TS = \left(\frac{K_0 b^2}{8\pi} - 2 k_B T\right) \ln \frac{R}{a}$$

Phase transition at $k_B T^* = \frac{K_0 b^2}{16\pi}$

For $T > T^*$:

- Free dislocations destroy (quasi) long-range translational order
- (Quasi) long-range orientational order <u>not</u> affected

↝ hexatic phase

- Disclinations:

Stretching energy:

$$E_s = \frac{K_0 s^2}{32\pi} R^2$$

For $T > T^*$:

Free dislocations screen strain field

↝ hexatic-to-fluid transition at $T^{**} > T^*$

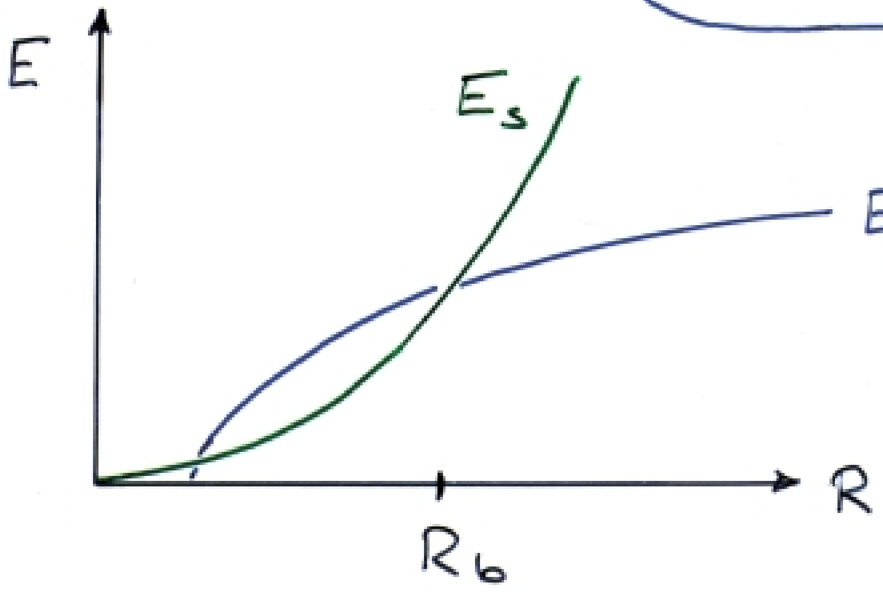
Topological Defects in Flexible Membranes

(Seung & Nelson 1988)

- Disclinations:

Stretching vs. Buckling

$$E_s = \frac{K_0 s^2}{32\pi} R^2 \qquad E_b = s \varkappa \ln \frac{R}{a}$$



↷ Buckling radius

$$\boxed{R_b = 10 \cdot \left(\frac{\varkappa}{K_0 s}\right)^{1/2}}$$

- Dislocations:

Buckling radius

$$\boxed{R_b \simeq 120 \cdot \frac{x}{k_0 b}}$$

Topological Defects: Dislocations

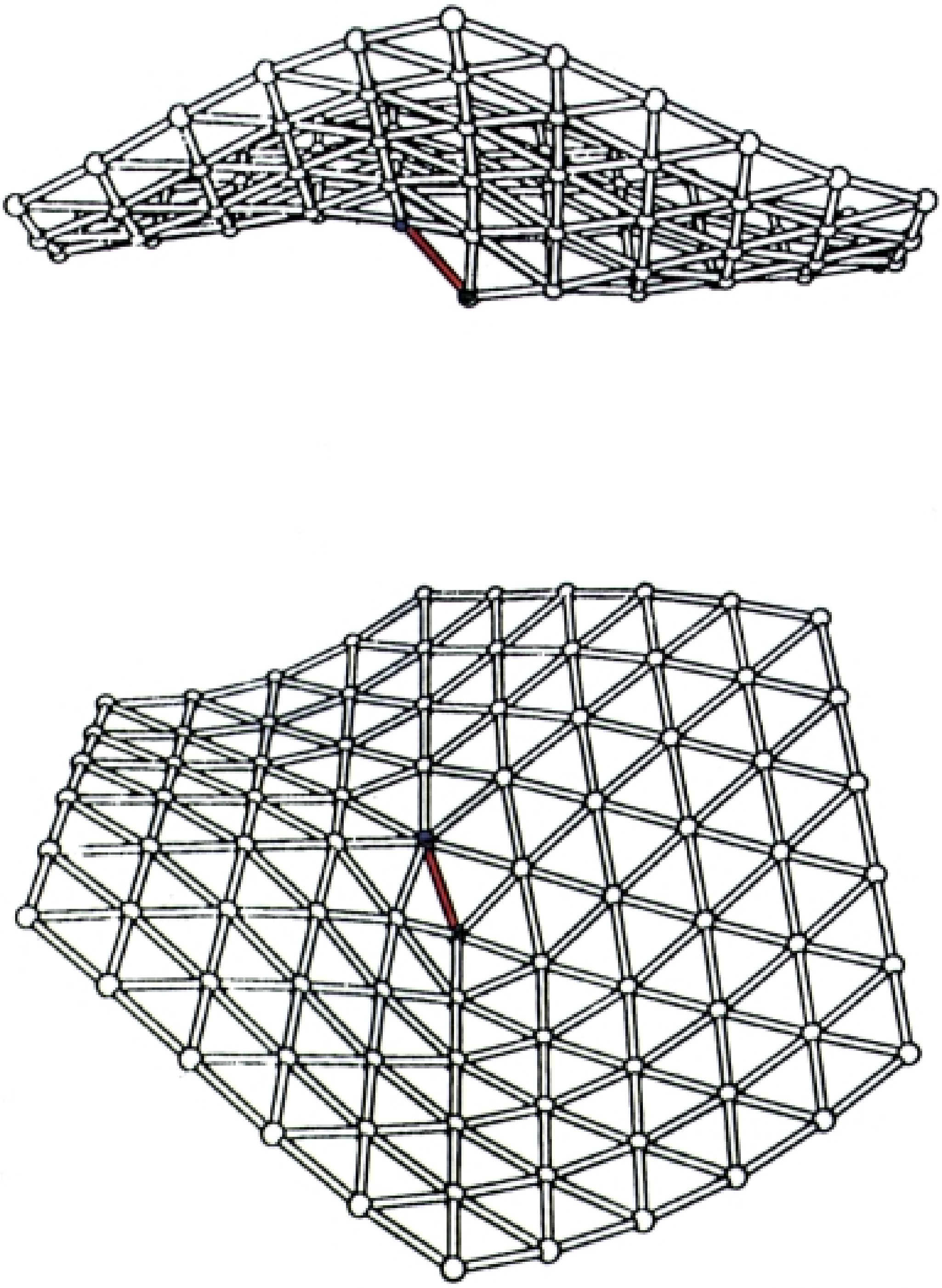
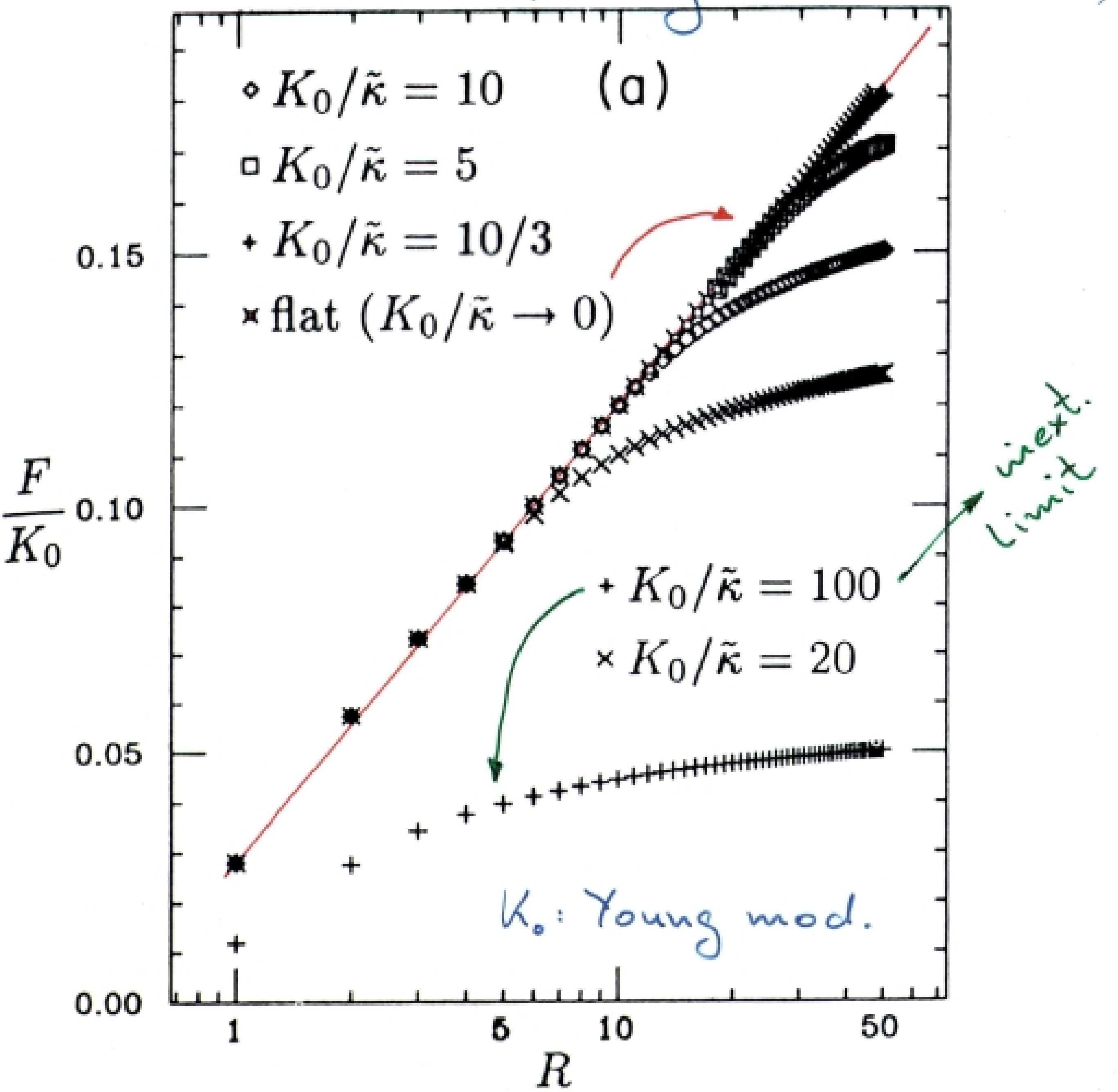


FIG. 10. Two views of a buckled dislocation with $K_0/\bar{\kappa}=100$.

(Seung & Nelson 1988)

Elastic energy of dislocations:

(Seung & Nelson 1988)



K_0: Young mod.

sub-logarithmic growth with system size R

↝ dislocations destroy crystalline order at any T>0

Monte Carlo Simulations

(G.G. & Kroll 1997, 1999)

Parameters:

- Bending rigidity $\varkappa$

  → controls out-of-plane fluctuations

- Tether length ℓ_0

  → controls in-plane density

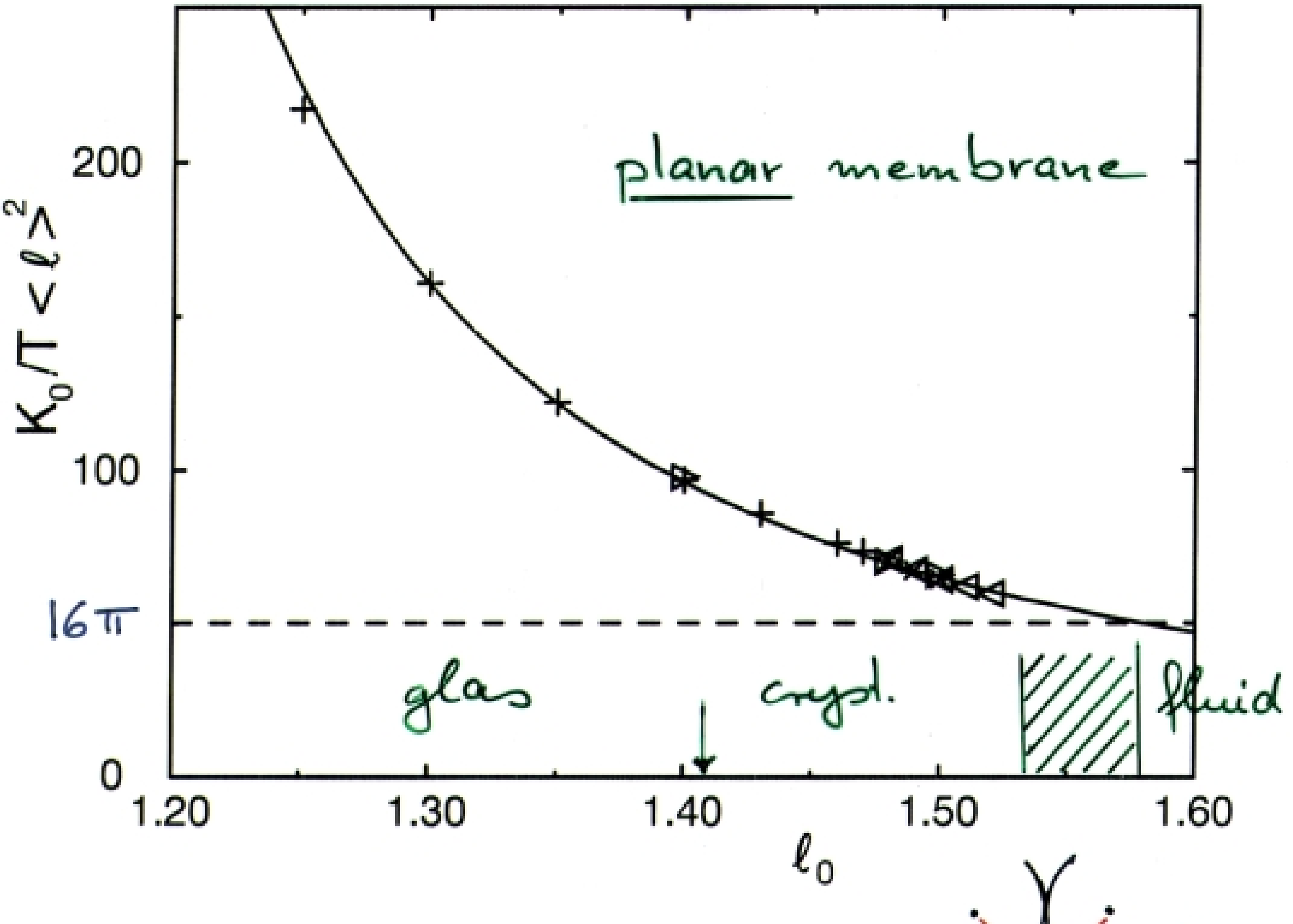
  → Young modulus K_0

Monte Carlo Simulations:

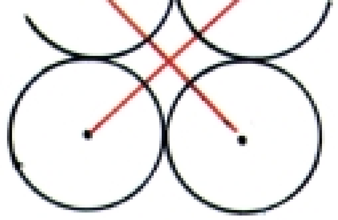
(G.G. & Kroll 1997)

Vary tether length ℓ_0:

- in-plane density
- Young modulus K_0

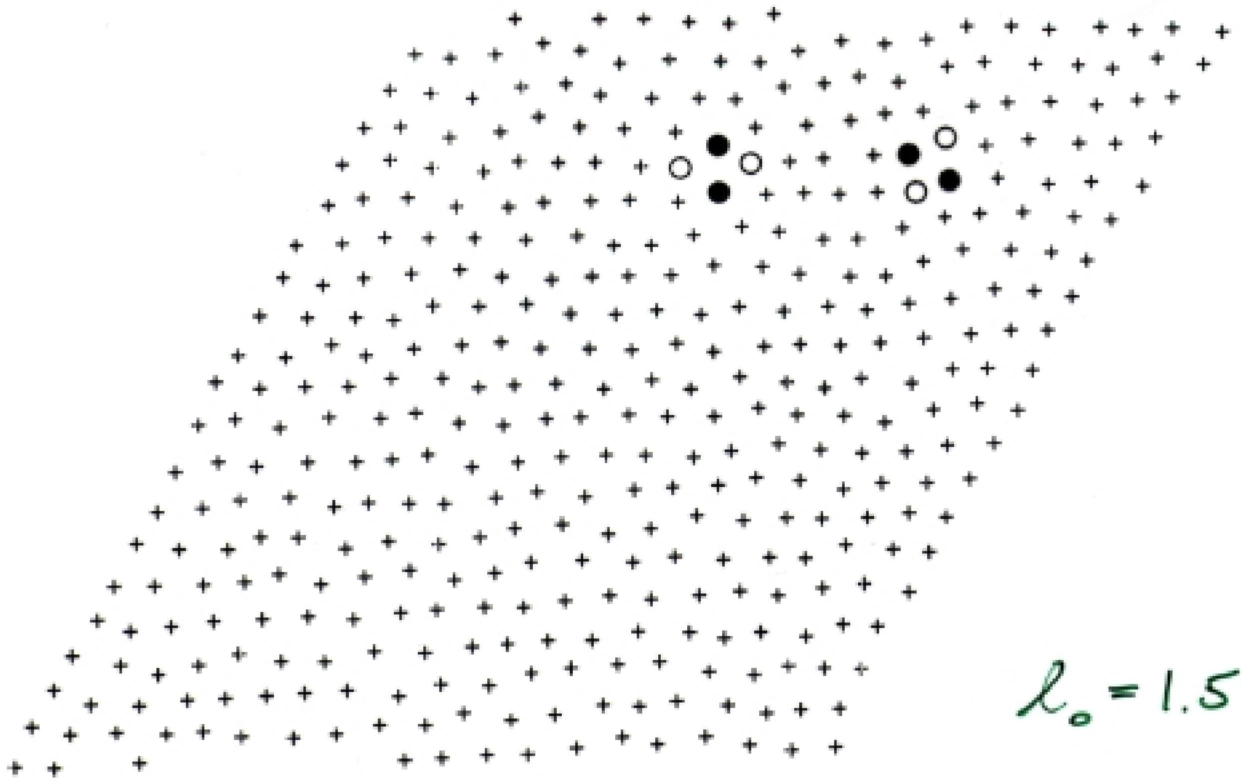


glas: no bond-flips

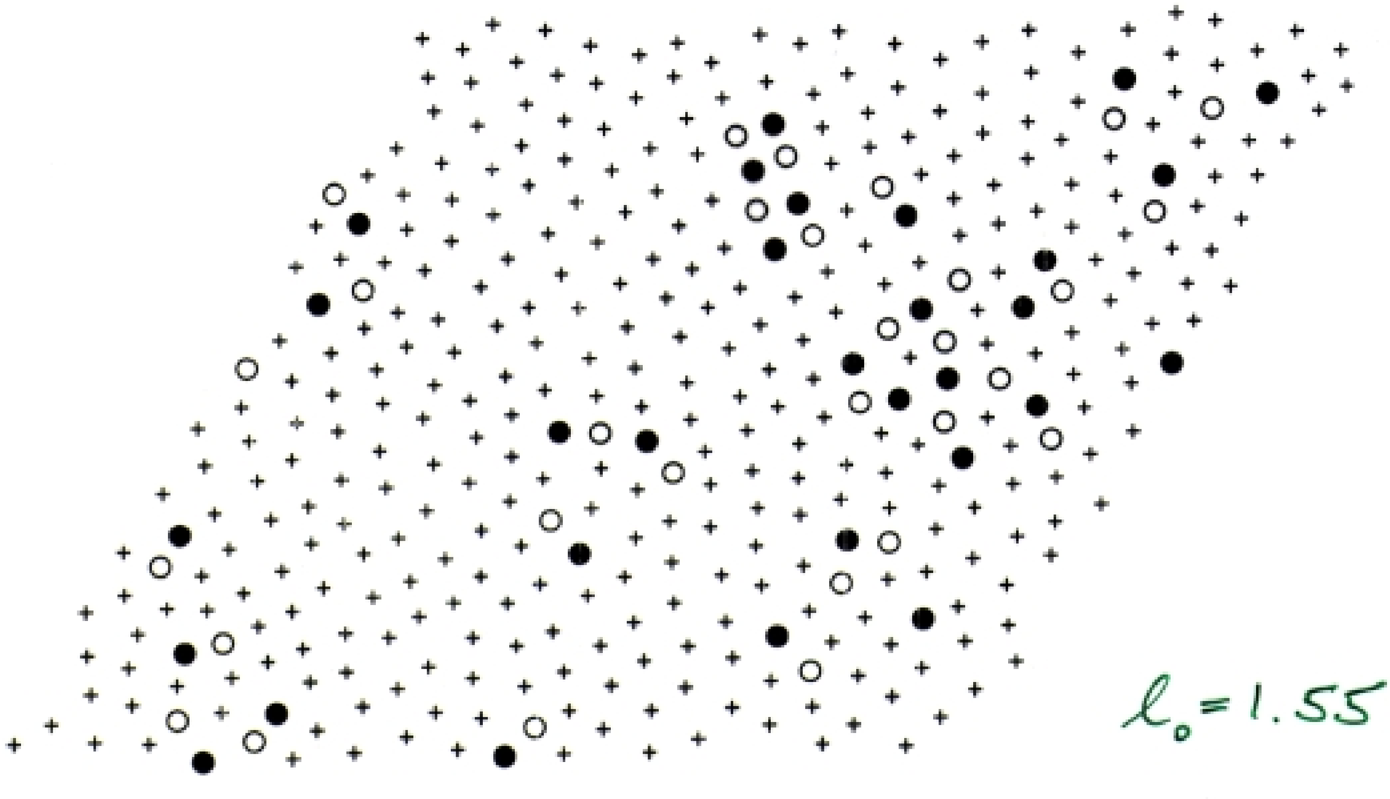


$\ell_0 = \sqrt{2}$

Planar Membranes:



$l_o = 1.53$



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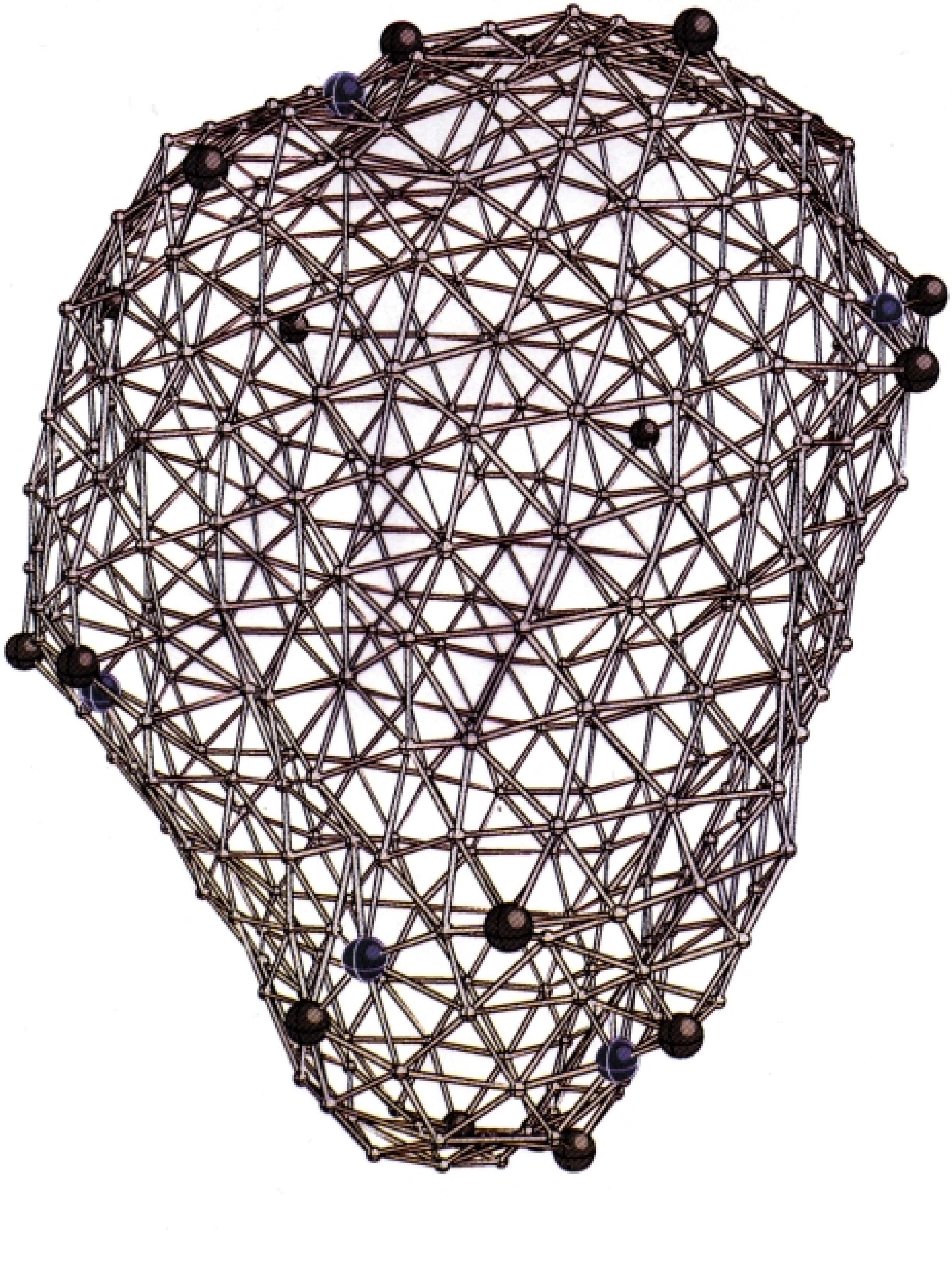
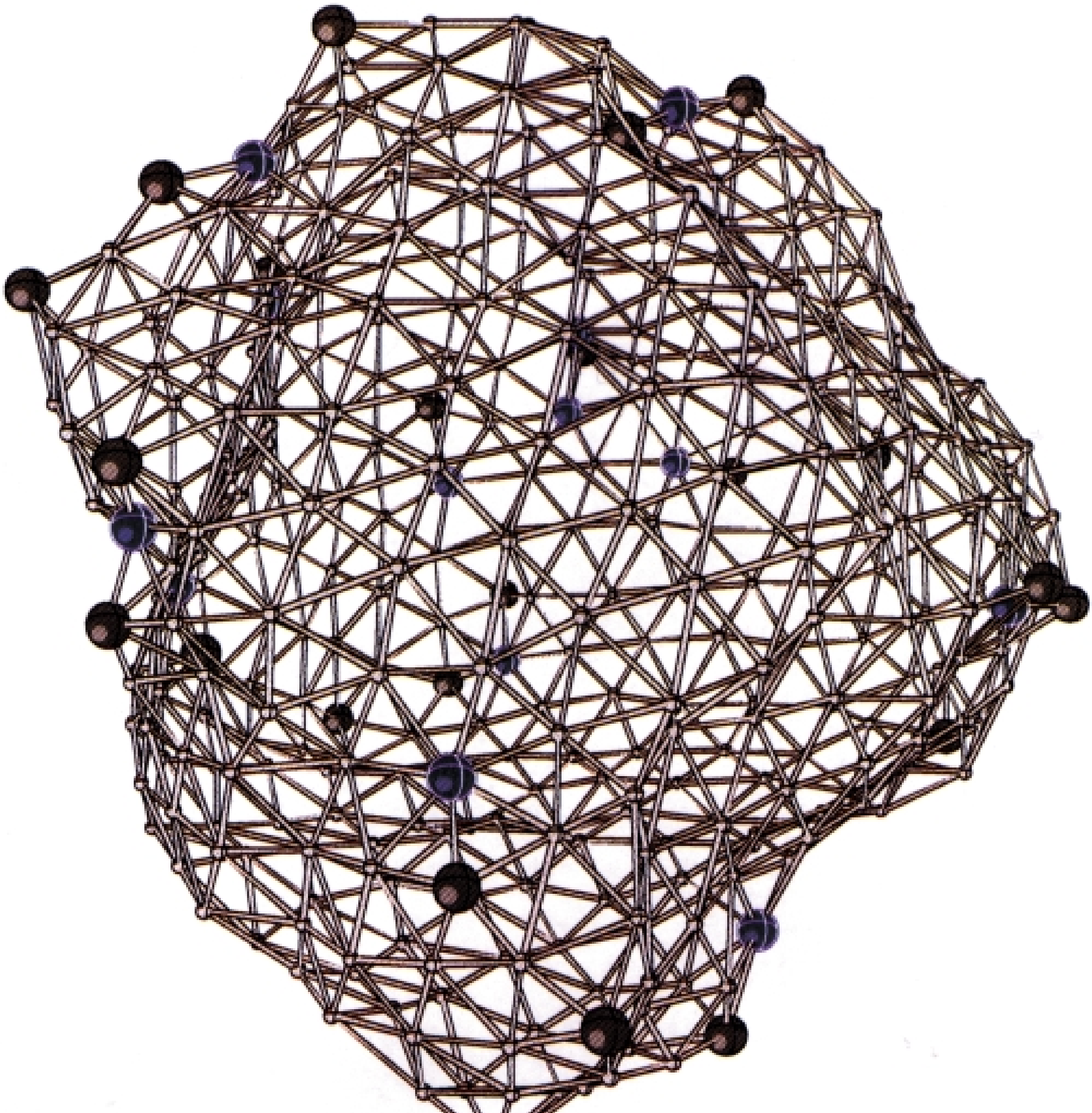
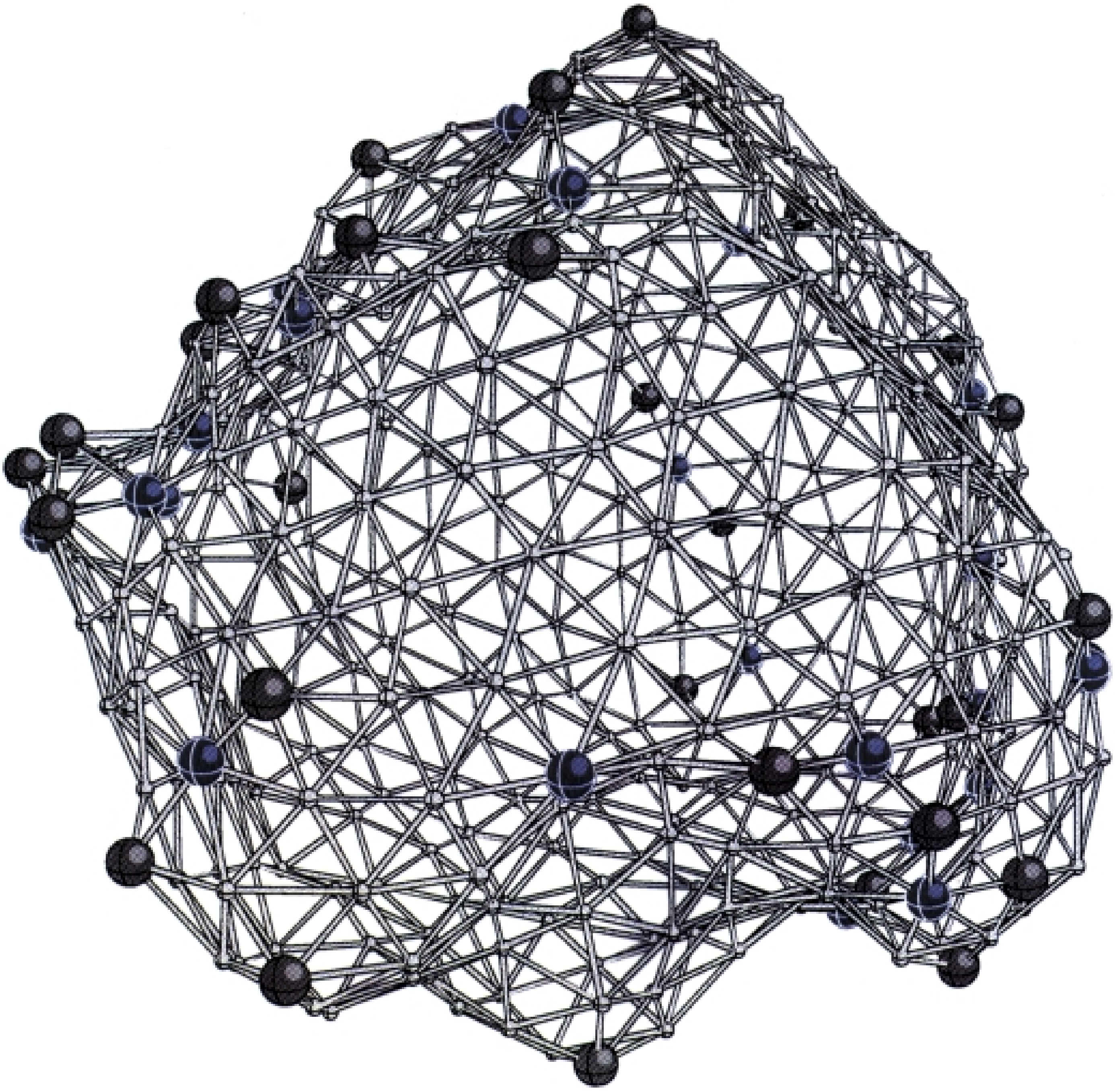






Free Energy of Dislocations

Scaling form:

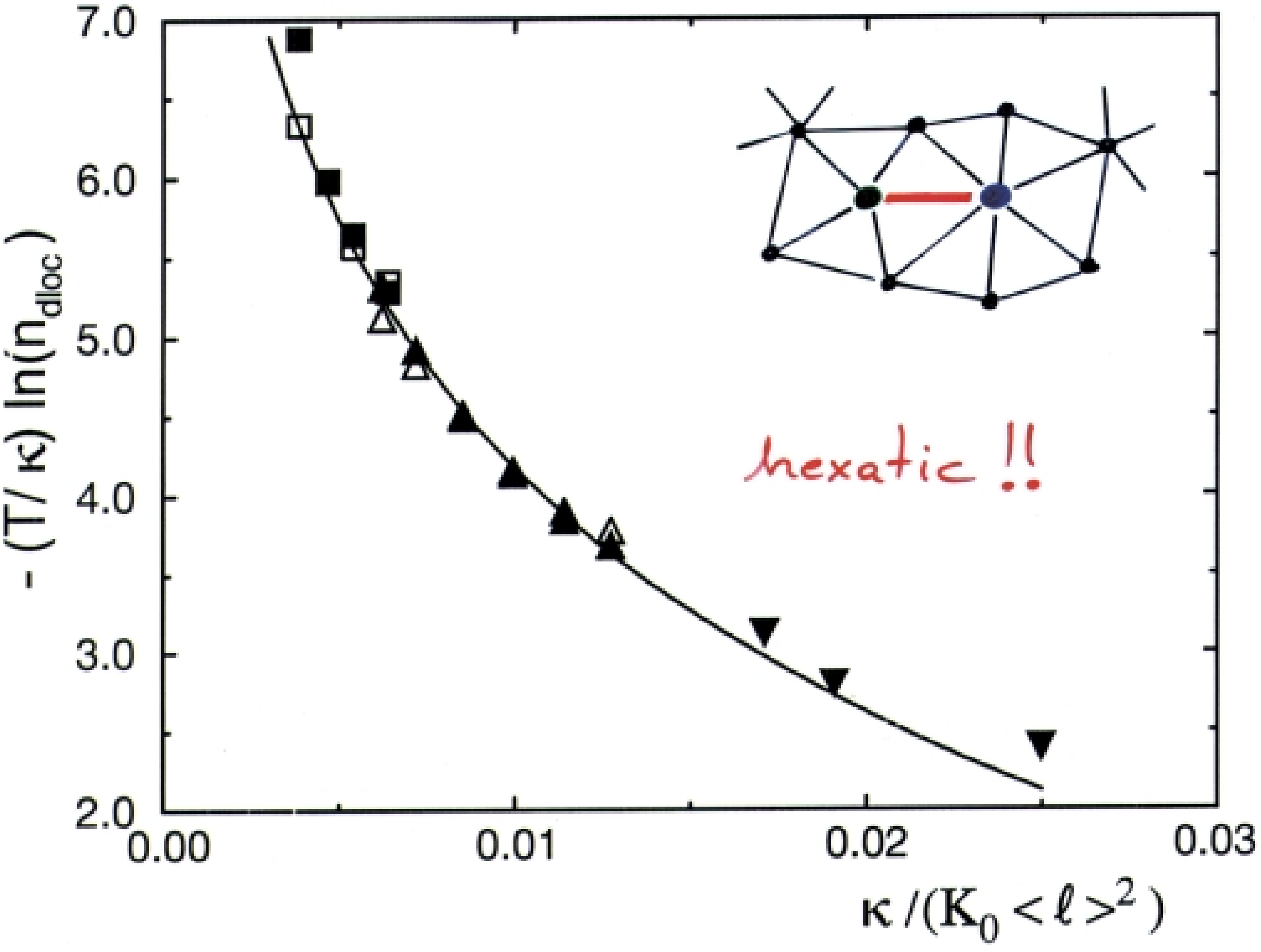
bending

$$\boxed{F_{dloc} = \kappa \cdot \Theta\left(\frac{\kappa}{K_0 \langle \ell \rangle^2}\right)}$$

stretching

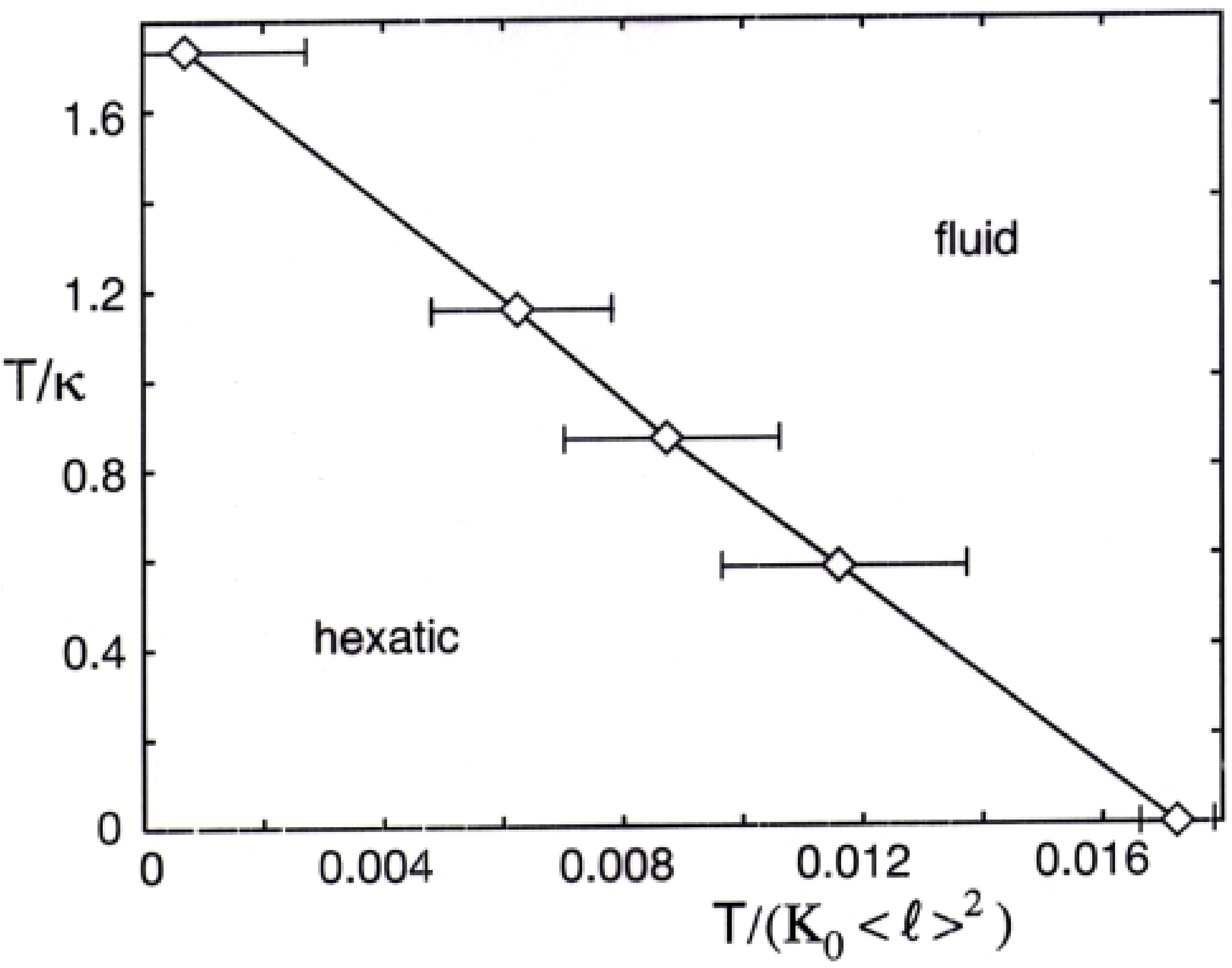
Density: $n_{dloc} \sim \exp(-\beta F_{dloc})$

(G.G. & Kroll, 1997)



hexatic!!

Phase Diagram:



Hexatic-to-fluid transition

- decreasing bending rigidity κ
- decreasing in-plane density

Experiments

- Hexatic phase :

  Best candidate :

  Crystallization of membrane proteins (bacteriorhodopsin) in lipid bilayers

  (Koltover et al. 1999)

- Topological defects :

  Vesiculation driven by clathrin coat

  (Mashl & Bruinsma 1998)

Clathrin-Coated Pits & Vesicles

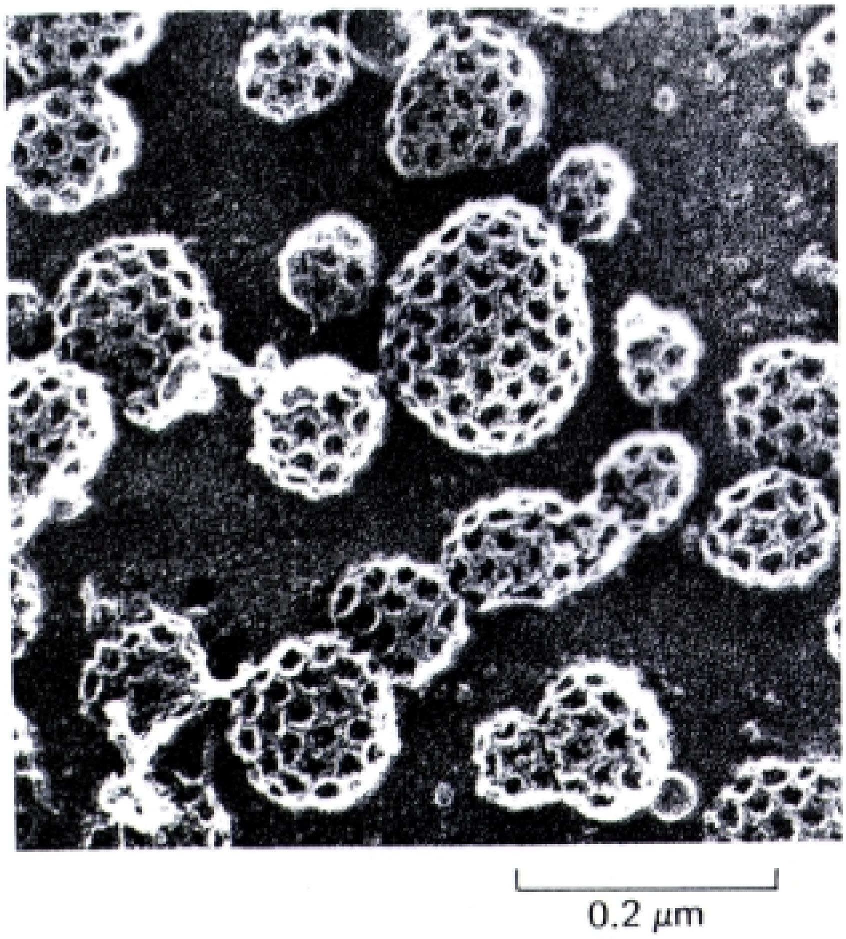


Figure 6–73 Electron micrograph of numerous coated pits and vesicles on the inner surface of the plasma membrane of cultured fibroblasts. The cells were rapidly frozen in liquid helium, fractured, and deep-etched to expose the cytoplasmic surface of the plasma membrane. (Reproduced from J. Heuser, *J. Cell Biol.* 84:560–583, 1980. By copyright permission of the Rockefeller University Press.)

(Alberts et al., "The Cell")