CONFINEMENT AND CHROMOMAGNETIC FIELDS

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Introduction

To study the vacuum structure of the lattice gauge theories we introduced a gauge invariant effective action, defined by using the lattice Schrödinger functional

$$\mathcal{Z}[U^{\rm ext}_\mu] = \int \mathcal{D}U \; e^{-S_W} \,, \tag{1}$$

where S_W is the Wilson action.

The functional integration is extended over links on a lattice with the hypertorus geometry and satisfying the constraints

$$U_k(x)|_{x_4=0} = U^{\rm ext}_k(x) \,. \tag{2}$$

$U^{\rm ext}_k(x)$ is the lattice version of the external continuum gauge field $\vec{A}^{\rm ext}(x) = \vec{A}^{\rm ext}_a(x)\lambda_a/2$:

$$U^{\rm ext}_\mu(x) = {\rm P} \exp\left\{ ig \int_0^1 dt \, A^{\rm ext}_{a,\mu}(x + t\hat{\mu}) \frac{\lambda_a}{2} \right\} \,, \tag{3}$$

where $\rm P$ is the path-ordering operator and g the gauge coupling constant.

The lattice effective action for the external static background field $\vec{A}^{\rm ext}(x)$ is

$$\Gamma[\vec{A}^{\rm ext}] = -\frac{1}{L_4} \ln \left\{ \frac{\mathcal{Z}[\vec{A}^{\rm ext}]}{\mathcal{Z}[0]} \right\} \,, \tag{4}$$

L_4 is the extension in Euclidean time, $\mathcal{Z}(0)$ is the lattice Schrödinger functional without the external background field ($U^{\rm ext}_k = \mathbf{1}$).

- $\Gamma[\vec{A}^{\rm ext}]$ is invariant for lattice gauge transformations of the external links $U_k^{\rm ext}$;
- $\lim_{T\to\infty}\Gamma[\vec{A}^{\rm ext}] = E_0[\vec{A}^{\rm ext}] - E_0[0]$, $E_0[\vec{A}^{\rm ext}]$ vacuum energy in presence of the external background field.

At FINITE TEMPERATURE, we consider the thermal partition function in presence of a given static background field:

$$\mathcal{Z}_T\left[\vec{A}^{\rm ext}\right] = \int_{U_k(\beta_T,\vec{x})=U_k(0,\vec{x})=U_k^{\rm ext}(\vec{x})} \mathcal{D}U\, e^{-S_W}\,, \qquad (5)$$

- the integrations are over the dynamical links with p.b.c.'s in the time direction.
- at the spatial boundaries we keep the boundary conditions $U_k(\vec{x},x_4) = U_k^{\rm ext}(\vec{x})$ used in the Schrödinger functional.

If we send the physical temperature to zero: the thermal functional Eq. (5) reduces to the zero-temperature Schrödinger functional with the constraints $U_k(x)|_{x_4=0} = U_k^{\rm ext}(\vec{x})$.

We used the above defined lattice effective action

to investigate

$$\boxed{\text{the response of the vacuum to an external constant Abelian chromomagnetic field.}}$$

Abelian ChromoMagnetic Background Field: $T = 0$

Let us consider a static external constant Abelian chromomagnetic field

$$\vec{A}_a^{\rm ext}(\vec{x}) = \vec{A}^{\rm ext}(\vec{x})\delta_{a,3}\,, \quad \vec{A}_k^{\rm ext}(\vec{x}) = \delta_{k,2}x_1 H\,. \tag{6}$$

On the lattice

$$U_1^{\rm ext}(\vec{x}) \quad = \quad U_3^{\rm ext}(\vec{x}) = \mathbf{1}$$

$$U_2^{\rm ext}(\vec{x}) = \begin{bmatrix} e^{i\frac{gHx_1}{2}} & 0 & 0 \\ 0 & e^{-i\frac{gHx_1}{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix}\,. \tag{7}$$

To be consistent with the hypertorus geometry we impose that the magnetic field H is quantized

$$\frac{gH}{2} = \frac{2\pi}{L_1} n_{\rm ext}\,, \tag{8}$$

with $n_{\rm ext}$ integer.

Due to the gauge invariance of our effective action for static gauge transformations of the background field, it turns out that $\Gamma[\vec{A}^{\rm ext}]$ is proportional to the spatial volume V.

Therefore the relevant quantity is the density of the effective action

$$\varepsilon[\vec{A}^{\rm ext}] = -\frac{1}{\Omega}\ln\left[\frac{\mathcal{Z}[A^{\rm ext}]}{\mathcal{Z}[0]}\right]\,, \tag{9}$$

where $\Omega = V \cdot T$.

To avoid the problem of computing a partition function which is the exponential of an extensive quantity we consider the derivative of

$\varepsilon[\vec{A}^{\text{ext}}]$ with respect to β by taking n_{ext} (i.e. gH) fixed

$$\varepsilon'\left[\vec{A}^{\text{ext}}\right] = \frac{\partial \varepsilon\left[\vec{A}^{\text{ext}}\right]}{\partial \beta} = -\frac{1}{\Omega}\left[\frac{1}{\mathcal{Z}[U^{\text{ext}}]}\frac{\partial \mathcal{Z}[U^{\text{ext}}]}{\partial \beta} - \frac{1}{\mathcal{Z}[0]}\frac{\partial \mathcal{Z}[U^{\text{ext}}]}{\partial \beta}\right]$$
$$= \left\langle \frac{1}{\Omega}\sum_{x,\mu>\nu}\frac{1}{3}\text{ReTr}U_{\mu\nu}(x)\right\rangle_0 - \left\langle \frac{1}{\Omega}\sum_{x,\mu>\nu}\frac{1}{3}\text{ReTr}U_{\mu\nu}(x)\right\rangle_{\vec{A}^{\text{ext}}} \tag{10}$$

$U_{\mu\nu}(x)$'s are the plaquettes in the (μ,ν) plane.

If we consider the contribution due to the dynamical links (i.e. the links that are not constrained in the functional integration)

$$\begin{aligned}\varepsilon'_{\text{int}}[\vec{A}^{\text{ext}}] = &\left\langle \frac{1}{\Omega_{\text{int}}}\sum_{x\in\tilde{\Lambda},\mu>\lambda}\frac{1}{3}\text{ReTr}U_{\mu\nu}(x)\right\rangle_0 \\ &- \left\langle \frac{1}{\Omega_{\text{int}}}\sum_{x\in\tilde{\Lambda},\mu>\lambda}\frac{1}{2}\text{ReTr}U_{\mu\nu}(x)\right\rangle_{\vec{A}^{\text{ext}}},\end{aligned} \tag{11}$$

where $\tilde{\Lambda}$ is the ensemble of the internal lattice sites.

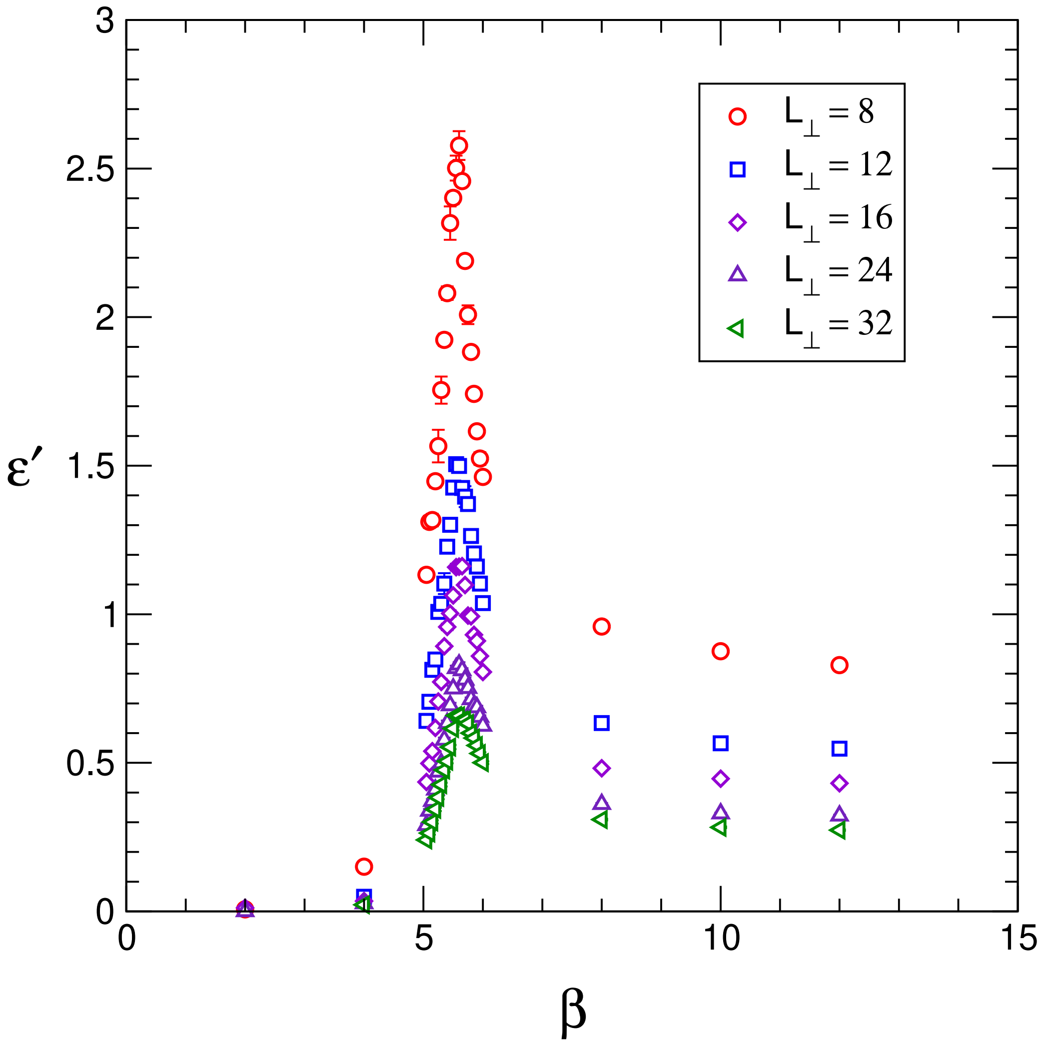
In order to extract $\varepsilon_{\text{int}}(\beta, n_{\text{ext}})$ we can numerically integrate the data for $\varepsilon'_{\text{int}}(\beta, n_{\text{ext}})/\varepsilon'_{\text{ext}}$ using the trapezoidal rule

$$\varepsilon_{\text{int}}(\beta, n_{\text{ext}}) = \varepsilon'_{\text{ext}}\int_0^\beta \frac{\varepsilon'_{\text{int}}(\beta, n_{\text{ext}})}{\varepsilon'_{\text{ext}}}\,d\beta', \tag{12}$$

with $\varepsilon'_{\text{ext}}$ the derivative of the energy density due to the external background field:

$$\varepsilon'_{\text{ext}} = \frac{2}{3}\left[1-\cos\left(\frac{gH}{2}\right)\right] = \frac{2}{3}\left[1-\cos\left(\frac{2\pi}{L_1}n_{\text{ext}}\right)\right]. \tag{13}$$

$$L_4 = L_1 = 32 \qquad L_2 = L_3 = L_\perp \tag{14}$$



In the weak coupling region we find that

$$\varepsilon_{\rm int}(\beta, n_{\rm ext}) \simeq a(n_{\rm ext})\frac{1}{2}H^2\,, \quad \beta \gg 1 \tag{15}$$

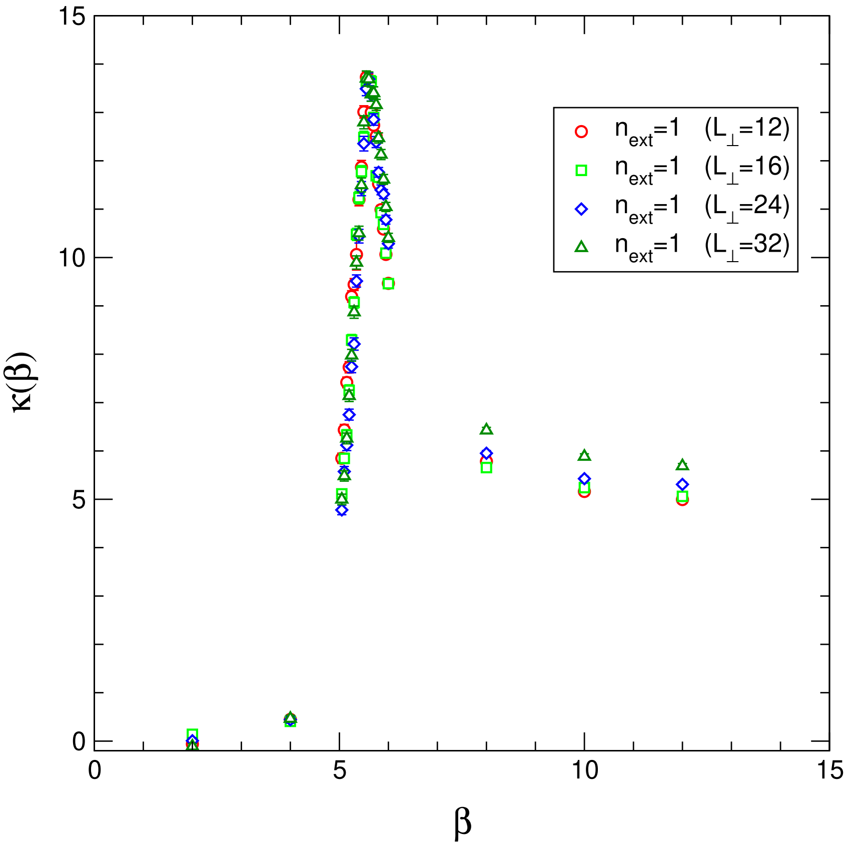
where $a(n_{\rm ext})$ decreases by increasing $L_\perp$ or the external background field. This peculiar behavior is very different from the Abelian case where we found that $a(n_{\rm ext}) \simeq 1$ independently on $L_\perp$ and $n_{\rm ext}$.

In order to perform the thermodynamic limit we introduce the scaling variable:

$$x = \frac{a_H}{L_{\rm eff}} \,, \tag{16}$$

where $a_H = \sqrt{\frac{2\pi}{gH}}$ is the magnetic length, and $L_{\rm eff} = \Omega_{\rm int}^{1/4}$ is the lattice effective linear size. We try the scaling law:

$$x^{-\alpha} \frac{\varepsilon'_{\rm int}(\beta, n_{\rm ext}, L_{\rm eff})}{\varepsilon'_{\rm ext}} = \kappa(\beta) \,. \tag{17}$$



We see that our numerical data can be arranged on the scaling curve $\kappa(\beta)$. It is remarkable that the value of the exponent $\alpha = 1.5$ in Eq. (17) agrees with the one we found for the SU(2) gauge theory.

From Eq. (17) we can determine the infinite volume limit of the vacuum energy density $\varepsilon_{\rm int}$. We have:

$$\lim_{L_{\rm eff} \to \infty} \varepsilon_{\rm int}(\beta, n_{\rm ext}, L_{\rm eff}) = \varepsilon'_{\rm ext} \int_0^\beta d\beta' \, \kappa(\beta') \lim_{L_{\rm eff} \to \infty} \left(\frac{a_H}{L_{\rm eff}} \right)^\alpha = 0 \tag{18}$$

in the whole range of β. As a consequence, in the continuum limit $(L_{\rm eff} \to \infty, \beta \to \infty)$ the SU(3) vacuum screens the external chromomagnetic Abelian field.

Abelian ChromoMagnetic Background Field: $T \neq 0$

We can extend the study of the SU(3) gauge system in an external chromomagnetic Abelian field to finite temperature.
On the lattice the physical temperature $T_{\rm phys}$ is introduced by (in units of $\kappa_B = 1$)

$$1/T_{\rm phys} = L_t \cdot a \,, \tag{19}$$

where L_t is the linear extension in the time direction $L_t = L_4$, while the extension on the spatial directions should be infinite. In numerical simulations, however, the spatial extension would of course be finite. In order to approximate the thermodynamic limit $L_s \gg L_t$.

We perform our numerical simulations on $64^3 \times L_t$, $128 \times 64^2 \times L_t$ lattices by imposing $L_t/L_s \leq 4$, in order to avoid finite volume effects.

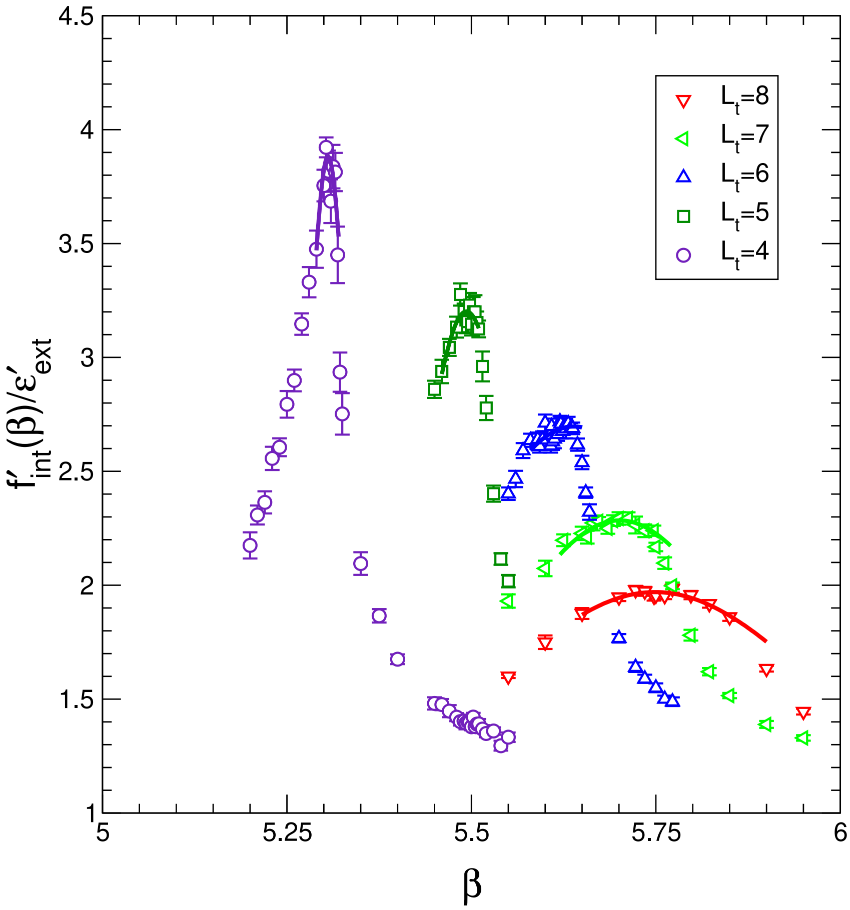
In the case of constant external chromomagnetic field the relevant quantity is the density of the free energy

$$f[\vec{A}^{\rm ext}] = -\frac{1}{V L_t} \ln \frac{\mathcal{Z}_T[\vec{A}^{\rm ext}]}{\mathcal{Z}_T[0]} \,, \quad V = L_s^3 \,. \tag{20}$$

We found that, if we start with the SU(3) gauge system at zero temperature in a constant Abelian chromomagnetic background field of fixed strength ($n_{\rm ext} = 1$) and increase the temperature, then the perturbative tail of the β-derivative of the free energy density $f'_{\rm int}(\beta, n_{\rm ext})/\varepsilon'_{\rm ext}$ increases with $1/L_t$ and tends towards the "classical" value $f'_{\rm int}(\beta, n_{\rm ext})/\varepsilon'_{\rm ext} \simeq 1$.

We may conclude that by increasing the temperature there is no screening effect in the free energy density confirming that the zero-temperature screening of the external field is related to the confinement.

The information of $f'_{\rm int}(\beta, n_{\rm ext})/\varepsilon'_{\rm ext}$ at finite temperature can be used to get an estimate of the deconfinement temperature T_c.



We see clearly that the pseudocritical coupling $\beta^*(L_t)$ depends on L_t. To determine the pseudocritical couplings we parameterize $f'_{\rm int}(\beta, L_t)$ near the peak as

$$\frac{f'_{\rm int}(\beta, L_t)}{\varepsilon'_{\rm ext}} = \frac{a_1(L_t)}{a_2(L_t)[\beta - \beta^*(L_t)]^2 + 1} \,. \tag{21}$$

Having determined $\beta^*(L_t)$ we estimate the deconfinement temperature as

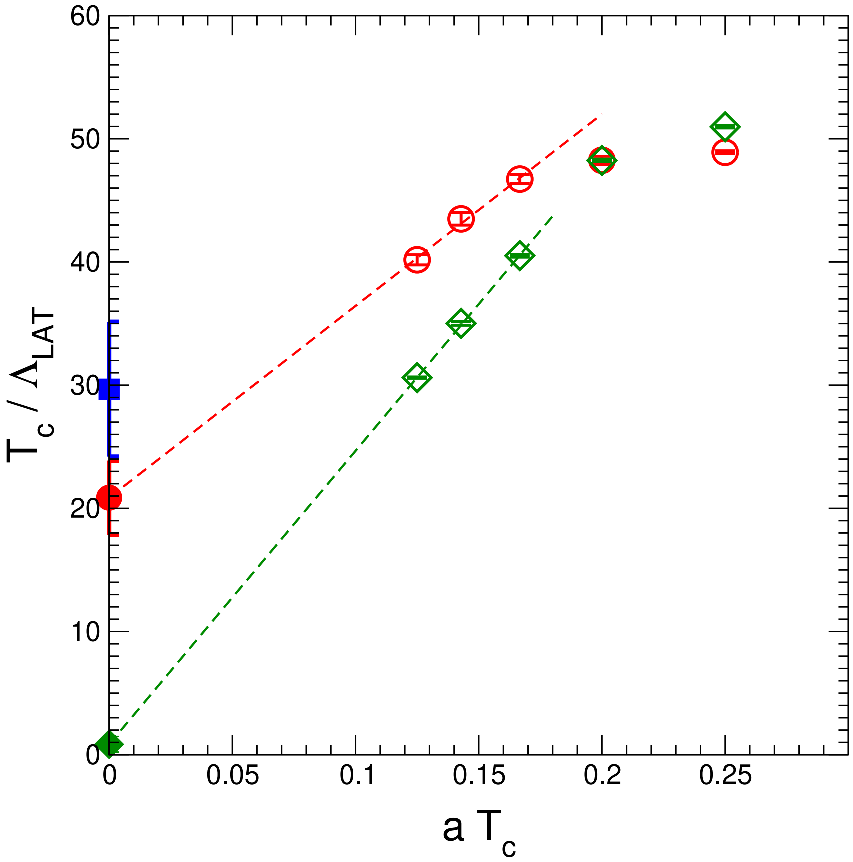
$$\frac{T_c}{\Lambda_{\rm latt}} = \frac{1}{L_t} \frac{1}{f_{SU(3)}(\beta^*(L_t))} \,, \tag{22}$$

where

$$f_{SU(N)}(\beta) = \left(\frac{\beta}{2Nb_0}\right)^{51/121} \exp\left(-\beta\frac{1}{4Nb_0}\right) \,, \tag{23}$$

where $b_0 = (11N)/(48\pi^2)$ and N is the color number.

To obtain the continuum limit critical temperature we have to extrapolate $\frac{T_c}{\Lambda_{\rm latt}}$ to the continuum. We perform a linear extrapolation to the continuum of our data for $T_c/\Lambda_{\rm latt}$.



For comparison we also display the continuum limit of the critical temperature without background field

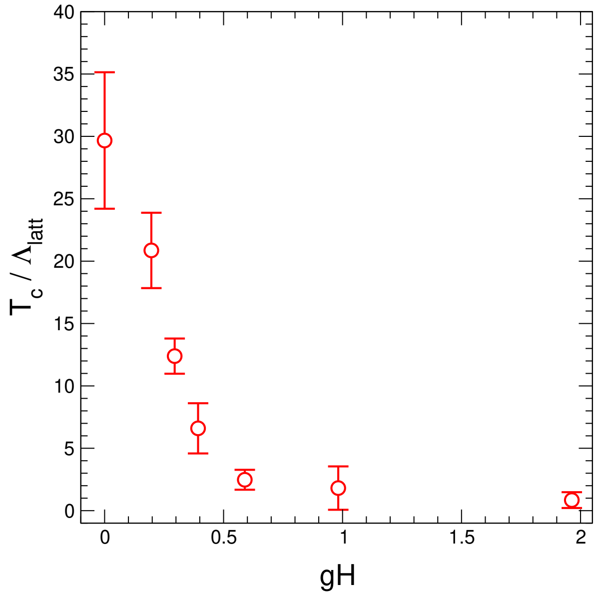
$$\frac{T_c}{\Lambda_{\rm latt}} = 29.67 \pm 5.47 \,. \tag{24}$$

T_c versus gH

Our derivation of $\frac{T_c}{\Lambda_{\mathrm{latt}}}$ shows that the continuum limit deconfinement critical temperature does depend on the applied Abelian chromomagnetic field. Therefore we decided to vary the strength of the applied external Abelian chromomagnetic background field to study quantitatively the dependence of T_c on gH.

We perform numerical simulations on $64^3 \times L_t$ lattices with $n_{\text{ext}} = 1, 2, 3, 5, 10$. As a check of possible finite volume effects we also simulated our gauge system on $128 \times 64^2 \times L_t$ lattice with $n_{\text{ext}} = 3$. We found that our numerical data do not display appreciable finite volume effects.

Following previous steps we are able to determine the critical temperature $\frac{T_c}{\Lambda_{\mathrm{latt}}}$ as a function of the external chromomagnetic field gH.



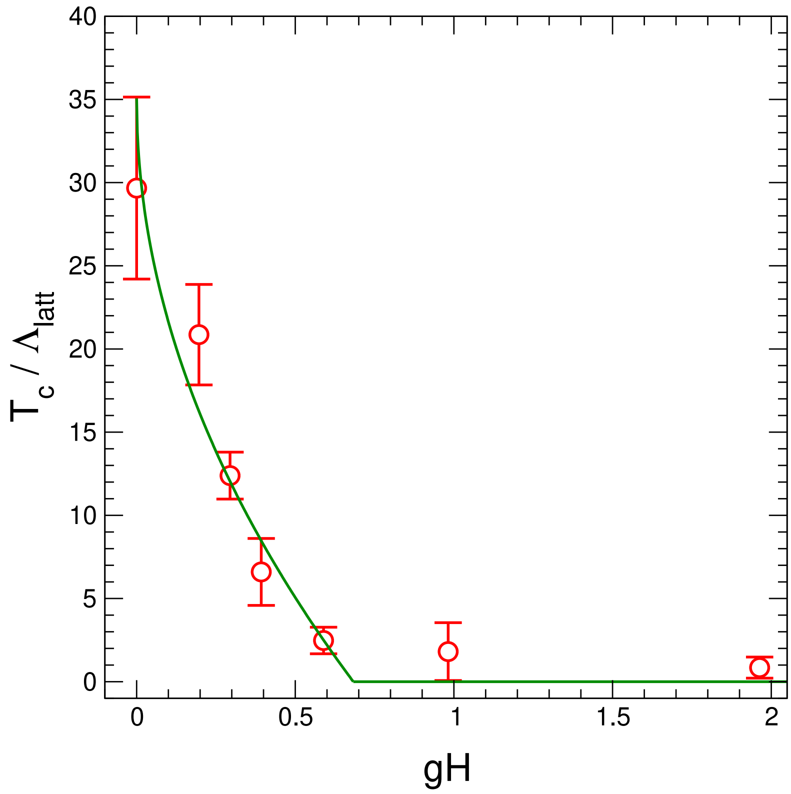
We see that the critical temperature decreases by increasing the external Abelian chromomagnetic field.

If the magnetic length $a_H \sim 1/\sqrt{gH}$ is the only relevant scale of the problem, then for dimensional reasons one expects that:

$$T_c^2 \;\sim\; gH\,. \tag{25}$$

Indeed we try to fit our data with

$$\frac{T_c(gH)}{\Lambda_{\text{latt}}} = \frac{T_c(0)}{\Lambda_{\text{latt}}} + \alpha\sqrt{gH}\,. \tag{26}$$



We find a satisfying fit with $\alpha = -42.4 \pm 7.4$ and $T_c(0)/\Lambda_{\text{latt}}$ that agrees within errors with Eq. (24).

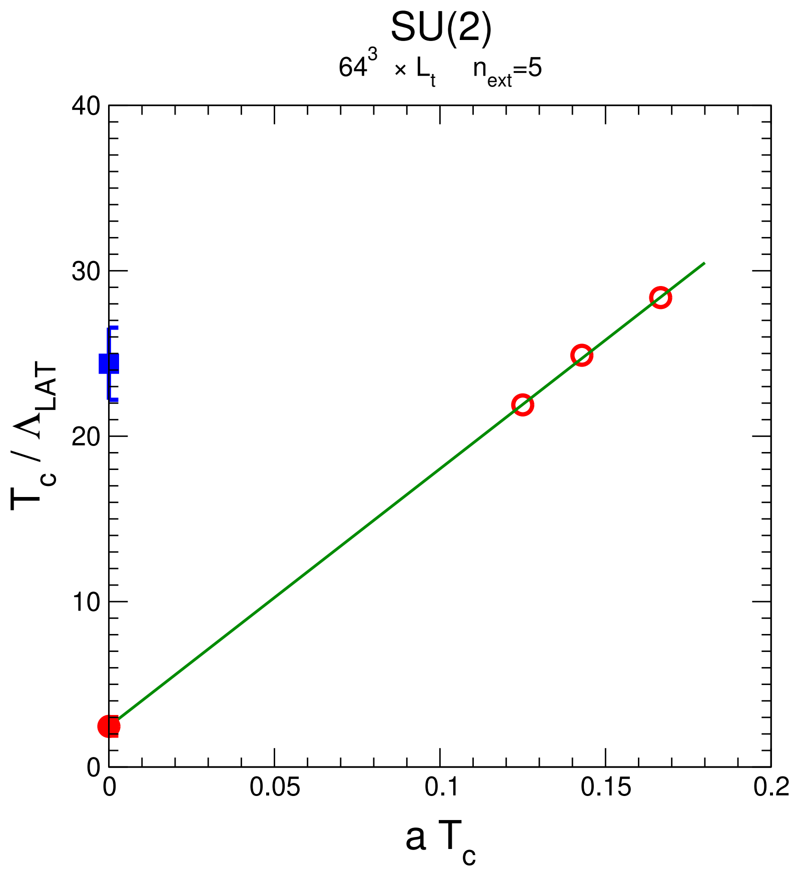
We see that there exists a critical field

$$gH_c \simeq 0.68 \tag{27}$$

such that $T_c = 0$ for $gH > gH_c$. In physical units the critical field turns out to be:

$$\frac{\sqrt{gH_c}}{T_c} = 8.5 \pm 1.4 \,. \tag{28}$$

Our preliminary data suggest that the same results hold also for $SU(2)$



where the continuum limit of the critical temperature without background field is:

$$\frac{T_c}{\Lambda_{\rm latt}} = 24.38 \pm 2.18 \,. \tag{29}$$

P-Stars

One of the most intriguing aspect of modern astrophysics is the possible existence of Stange Stars which are made entirely of deconfined u, d, s quarks. The possible existence of strange stars is a direct consequence of the so called strange matter hypothesis, namely that u, d, s quarks in equilibrium with respect to weak interactions could be the true ground state of hadronic matter.

Our lattice results (without dynamical quarks) shows that the continuum limit deconfinement critical temperature goes to zero in a strong enough chromomagnetic field. Therefore, it is conceivable that the QCD ground state in presence of a strong Abelian chromomagnetic background field is actually made of deconfined quarks.

Thus, our results implies the exciting possibility of a new class of compact quark stars (P-STARS) made of deconfined up and down quarks immersed in a chromomagnetic field and in β-equilibrium.

We needs the equation of state appropriate for the description of deconfined quarks and gluons in an Abelian chromomagnetic field.

From Eq. (28) we estimate $\sqrt{gH_c} \sim 1\, GeV$ corrisponding to $H_c \sim 10^{19}\, Gauss$. We see that up and down quarks are in the lowest Landau level.

We need the thermodynamical potential (at $T = 0$):

QUARKS

$$\varepsilon_q = \frac{3}{4\pi^2} gH \left(\mu_u^2 + \mu_d^2\right), \tag{30}$$

ELECTRONS

$$\varepsilon_e = \frac{\mu_e^4}{4\pi^2}, \tag{31}$$

GLUONS

$$\varepsilon_g = \frac{11}{32\pi^2} (gH)^2 . \tag{32}$$

The pressure P is given by:

$$P = \frac{3}{4\pi^2} gH \left(\mu_u^2 + \mu_d^2\right) + \frac{\mu_e^4}{4\pi^2} - \frac{11}{32\pi^2} (gH)^2 . \tag{33}$$

We have the following constraints:

$$\mu_e + \mu_u = \mu_d \qquad \beta - equilibrium \tag{34}$$

$$\frac{2}{3} n_u - \frac{1}{3} n_d = n_e \qquad charge\ neutrality \tag{35}$$

where:

$$n_u = \frac{3}{2\pi^2} gH \, \mu_u \ , \ n_d = \frac{3}{2\pi^2} gH \, \mu_d \ , \ n_e = \frac{\mu_e^3}{3\pi^2} . \tag{36}$$

We obtain the following Equation of State :

$$P = \epsilon - \frac{\mu_e^4}{6\pi^2} - \frac{11}{16\pi^2}(gH)^2 \, . \tag{37}$$

To study some properties of a star we consider it to be a spherically symmetric object, corresponding to a non rotating star.
The stability of the star is governed by the general-relativistic equation of hydrostatic equilibrium for a spherical configuration of quark matter, which is the Tolman-Oppenheimer-Volkov equation:

$$\frac{dP}{dr} = -\frac{GM(r)\varepsilon(r)}{r^2}\left[1 + \frac{4\pi r^3 P(r)}{M(r)}\right] \frac{1 + \frac{P(r)}{\varepsilon(r)}}{1 - \frac{2GM(r)}{r}} \, , \tag{38}$$

$$\frac{dM}{dr} = 4\,\pi\, r^2\, \varepsilon(r) \, . \tag{39}$$

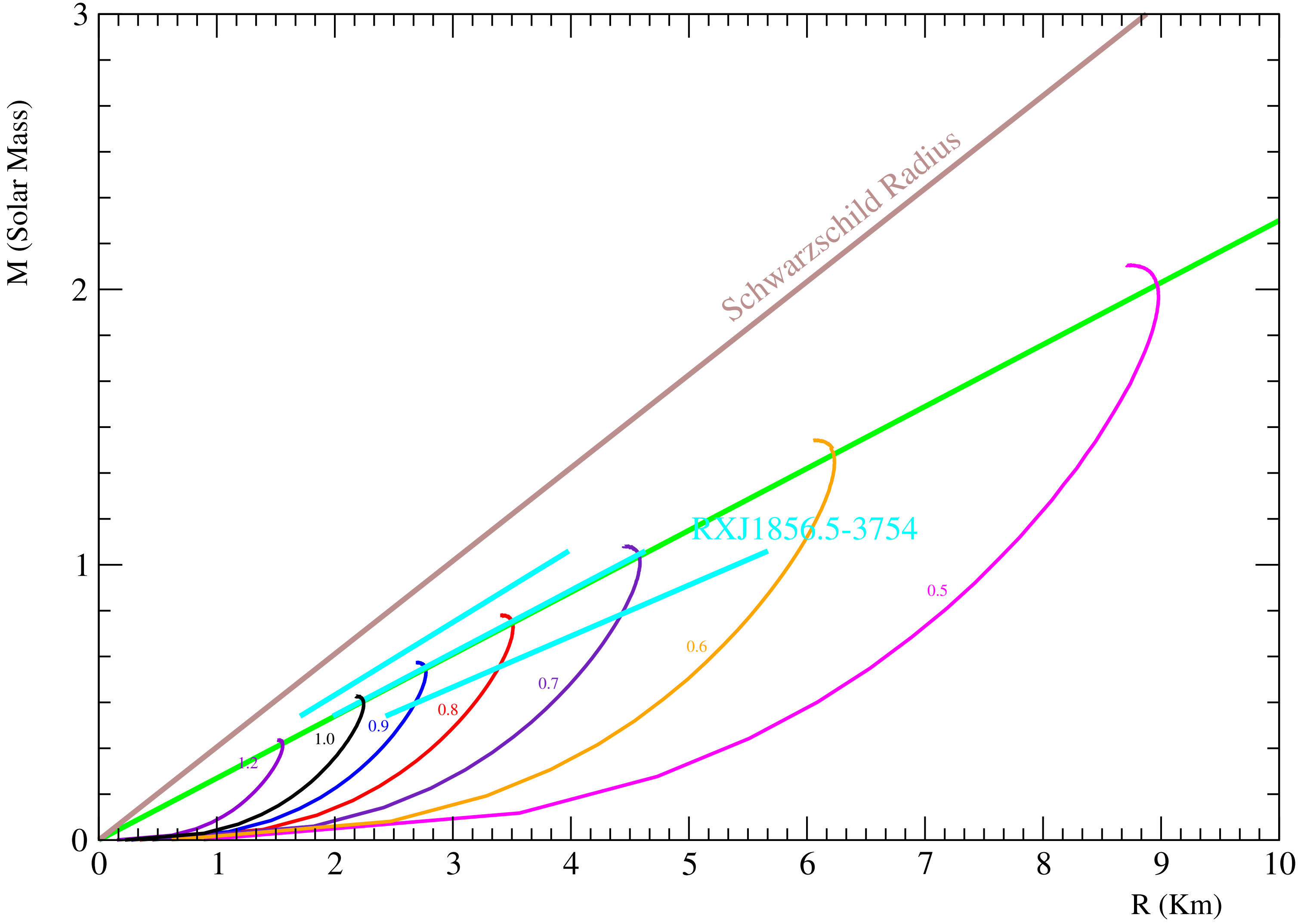
These equations can be solved numerically for a given central density ε_c and obtain $\epsilon(r)$ and $P(r)$.

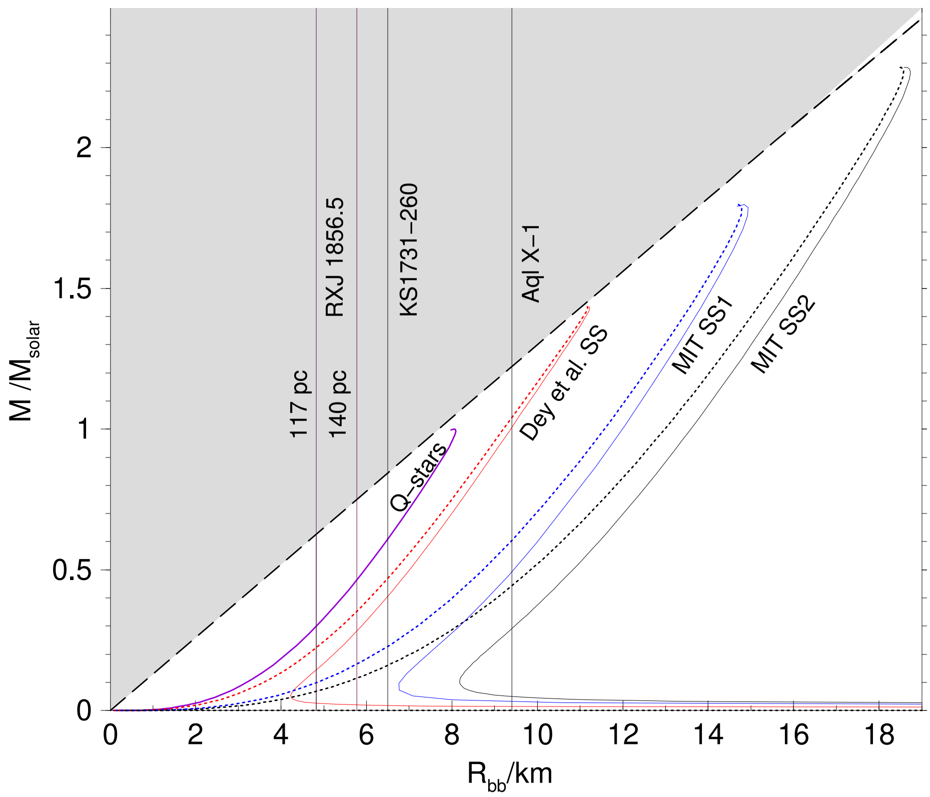
The radius of the star is determined by :

$$P(r = R) = 0 \, , \tag{40}$$

and the total mass by:

$$M = M(r = R) \, . \tag{41}$$


M (Solar Mass)
R (Km)
Schwarzschild Radius
RXJ1856.5-3754
1.2
1.0
0.9
0.8
0.7
0.6
0.5
0
1
2
3
4
5
6
7
8
9
10


117 pc
140 pc
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Dey et al. SS
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M /M_solar
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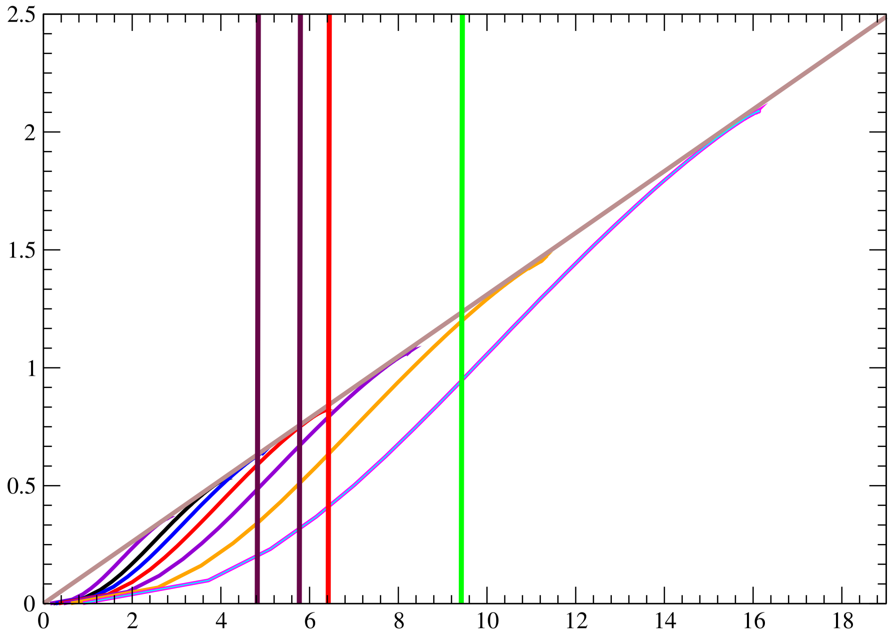

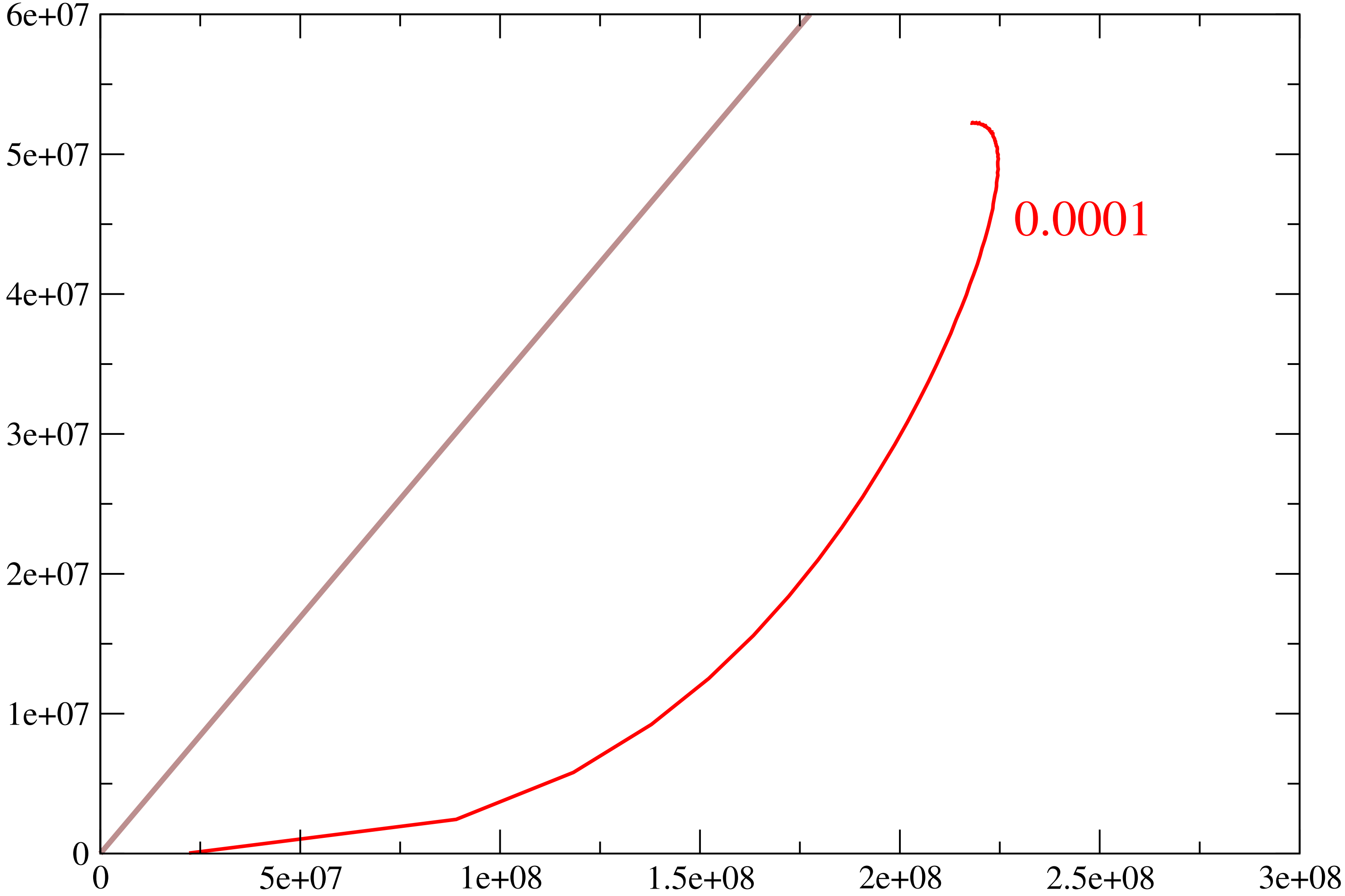

6e+07
5e+07
4e+07
3e+07
2e+07
1e+07
0
0
5e+07
1e+08
1.5e+08
2e+08
2.5e+08
3e+08
0.0001

Conclusions

There is an intimate connection between Abelian chromomagnetic fields and color confinement.

The existence of a critical chromomagnetic field is naturally explained if the vacuum behaves as an ordinary relativistic color superconductor, a condensate of a color charged scalar field whose mass is proportional to the inverse of the magnetic length.

A natural candidate for such tachyonic color charged scalar field is the famous Nielsen-Olesen unstable mode.

The chromomagnetic condensate cannot be uniform due to gauge invariance of the vacuum, which disorders the system in such a way that there are not long range color correlations.

Another important aspect of this work is related to astrophysics applications. Our results implies the exciting possibility of a new class of compact quark stars.

We expect that our proposal for P-Star, the new class of quark star, will be corroborated by new data in the near future.