CRYSTALLINE COLOR SUPERCONDUCTIVITY

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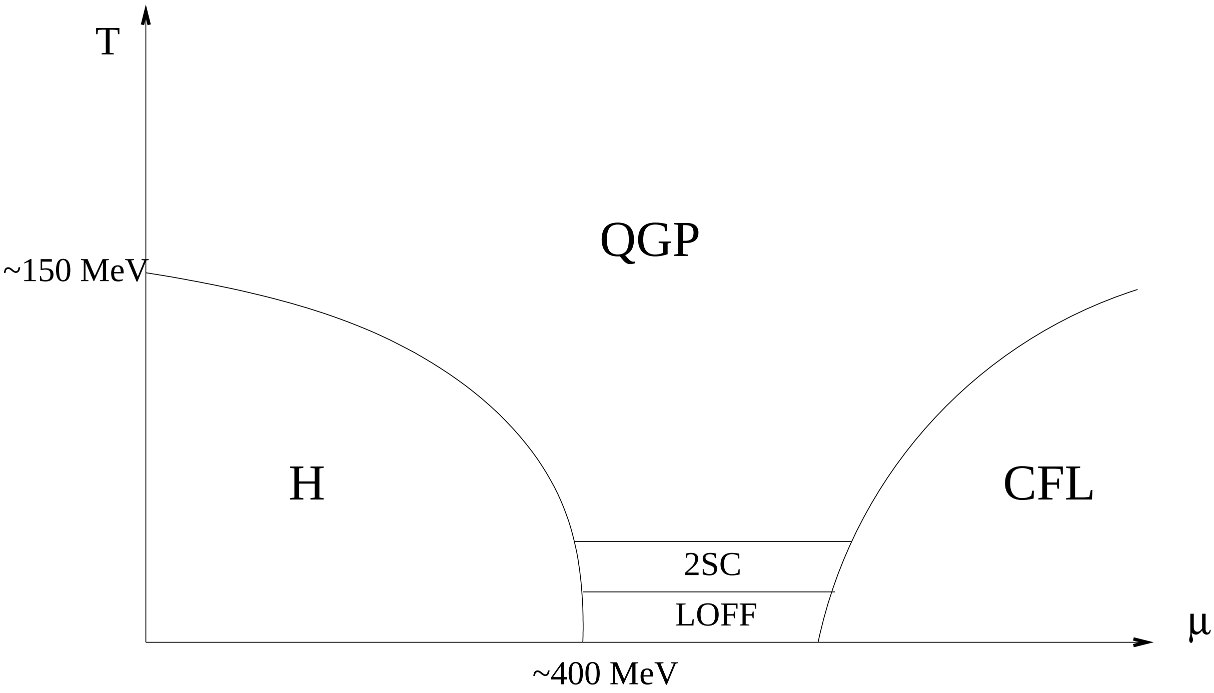
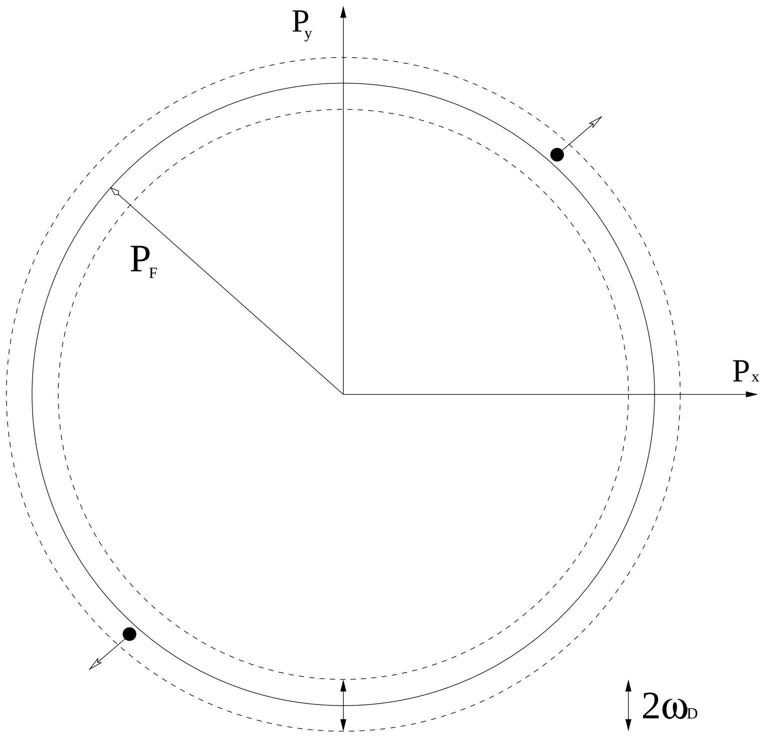
QCD phase diagram



Phase	Condensate
QGP	Free quarks
H	$< \bar{\psi}\psi >$ $< \psi\psi\psi >$
CSC	$< \psi\psi >$

$$\begin{aligned} 2SC &\rightarrow LOFF \\ O(3) &\rightarrow O_h \end{aligned} \tag{1}$$

SUPERCONDUCTIVITY



- Surface effect for arbitrary weak fermion attraction.
- Cooper pairs have opposite momentum and spin.
- Dispersion law $\omega(p) \simeq \sqrt{p^2 + \Delta^2}$.
- $\Delta \simeq 2\omega_D e^{-2/(g\nu_F)}$.

ω_D Debye frequency, g four fermion coupling, ν_F density of states on the Fermi surface.

SUPERCONDUCTING ALLOYS

A.I.**L**arkin N.**O**vchinnikov P.**F**ulde P.A.**F**errel

- The system

Type II superconductor containing paramagnetic impurities.

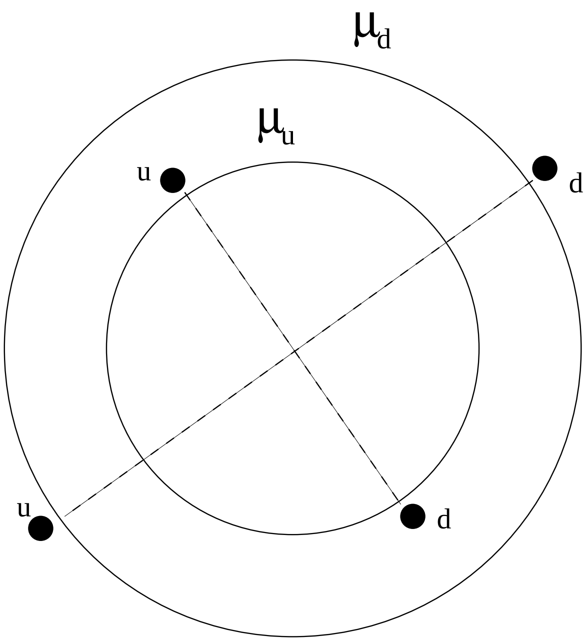
The control parameter is

$$I = N_i S a \tag{2}$$

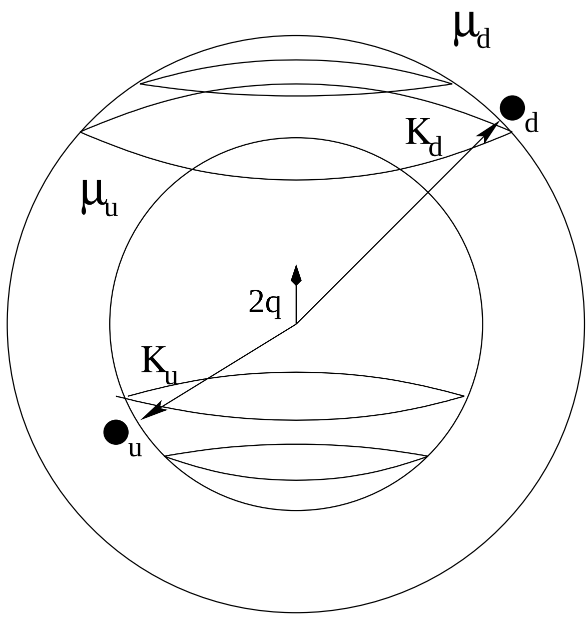
N_i concentration of impurities

S spin of impurities

a integral of the exchange potential



BCS



LOFF

$$\begin{array}{ccc} BCS \rightarrow & LOFF & \rightarrow NORM \\ \frac{\Delta_0}{\sqrt{2}} < & I & < K\Delta_0 \end{array} \tag{3}$$

BCS pairs pay free energy price.

LOFF pairs are near their own Fermi surface.

K depends on the crystalline structure. For the plane wave $K = .75$.

LOFF PHASE

- The momentum of a pair is non zero: $\vec{K}_u + \vec{K}_d = 2\vec{q}$.

- The phase space consists of *pairing regions* and *blocking regions*.

- The gap is $\Delta_{\text{LOFF}} \equiv \Delta_{\text{LOFF}}(\vec{r}) \ll \Delta_0$.

- Weak superconductor: existence of a mixed phase (normal/superconductor). The Meissner effect does not hold.

- To have the LOFF phase we need Fermi surfaces far apart.

- Difficult to observe LOFF phase in alloys: random scattering of electrons on impurities destroys coherence.

LOFF PHASE IN QCD

Quark matter at asymptotic densities and at low temperature.

- Weak equilibrium
  $d \leftrightarrow u + e$

$$\begin{aligned}\mu_u &= \mu - \tfrac{2}{3}\mu_e\\ \mu_{d,s} &= \mu + \tfrac{1}{3}\mu_e\end{aligned}$$

- Electrical neutrality $\frac{2}{3}n_u - \frac{1}{3}n_d - \frac{1}{3}n_s - n_e = 0$

$$\begin{aligned}n_{u,d} &= \frac{1}{\pi^2}\mu_{u,d}^3\\ n_s &= \frac{1}{\pi^2}(\mu_s^2 - m_s^2)^{3/2}\\ n_e &= \frac{1}{3\pi^2}\mu_e^3\end{aligned} \tag{4}$$

For $m_s = 0$ (CFL) we get $n_u = n_d = n_s$, $n_e = 0$.

For $m_s \sim \mu_s$ (2SC/LOFF)
$$\begin{aligned}\mu_u &= \bar{\mu} - \delta\mu\\ \mu_d &= \bar{\mu} + \delta\mu\end{aligned}$$

EFFECTIVE LAGRANGIAN APPROACH

For us is a tool to perform calculations.

We need to find:

1. The effective degrees of freedom.

2. The actual ground state and the available phase space.

3. Relevant and marginal operators under renormalization group.

1) Decoupling of negative energy states.

Our starting point is the lagrangian

$$\mathcal{L} = \bar{\Psi}(i\not{D} + \mu\gamma_0)\Psi - \frac{1}{4}F^a_{\mu\nu}F^{\mu\nu,a} \ . \qquad (5)$$

Only fermions near the Fermi surface take part in Cooper pairing. Their momentum is large $|\vec{p}| \sim \mu$ thus $\not{D} \to \not{\partial}$ and the Dirac equation is

$$(\not{p} + \mu\gamma_0)\Psi = 0. \qquad (6)$$

The eigenvalues of $H_D = p_0$ are $E_\pm = -\mu \pm |\vec{p}|$. Therefore negative energy states are deep inside the Fermi sphere and decouple.

2) Momentum decomposition.

$$p^\mu = \mu v^\mu + \ell^\mu \qquad v^\mu = (0, \vec{v}_F) \tag{7}$$

ℓ is the residual momentum of the quasi-particles which live in a narrow band close to the Fermi surface. We require $|\ell| < \delta$ with δ an ultraviolet cutoff (in normal superconductors $\delta = \omega_D$).

$$\Psi(x) = \sum_{\vec{v}} e^{-i\mu v\cdot x}\psi \ . \tag{8}$$

with $\psi \equiv \psi_{\vec{v}}(x)$. The lagrangian for quarks is

$$\mathcal{L}_{\text{eff}} = \sum_{\vec{v}} \left[\psi^\dagger iV\cdot D\psi - P^{\mu\nu}\psi^\dagger \frac{1}{2\mu + i\tilde{V}\cdot D} D_\mu D_\nu \psi\right] \tag{9}$$

$V^\mu = (1, \vec{v}_F)$, $\tilde{V}^\mu = (1, -\vec{v}_F)$,

$P^{\mu\nu} = g^{\mu\nu} - \frac{1}{2}\left[V^\mu \tilde{V}^\nu + V^\nu \tilde{V}^\mu\right]$

3) Interactions.

As in the Landau theory of the Fermi liquid the only non irrelevant operator is

$$\mathcal{L}_I = F \int dx \ \left(\bar{\psi}\gamma^\mu T^A \psi\right)\left(\bar{\psi}\gamma_\mu T^A \psi\right) \qquad (10)$$

4 fermi interaction with quantum numbers of 1 gluon exchange. No need to resort to phonons. QCD superconductivity more robust than in metals (higher T_C/E_F).

We make a mean field ansatz

$$\psi\psi\psi\psi \to \langle\psi\psi\rangle\psi\psi \qquad (11)$$

In general the condensate is

$$\langle \psi^T_{\alpha i} C \psi_{\beta j} \rangle = \frac{\Delta}{2} \epsilon_{\alpha\beta I} \epsilon_{ijK} \Omega_{IK} \qquad (12)$$

- $\Omega_{IK} = \delta_{IK} \qquad (CFL)$

$SU(3)_c \times SU(3)_L \times SU(3)_R \times U(1)_B \to SU(3)_{cLR} \times Z_2$

- $\Omega_{IK} = \delta_{I3}\delta_{K3} \qquad (2SC/LOFF)$

$SU(3)_c \to SU(2)_c$ chiral symmetry is unbroken (it is not a superfluid)

Ω_{IK} is determined by free energy analysis.

- Moreover in the LOFF phase the gap is spatial dependent

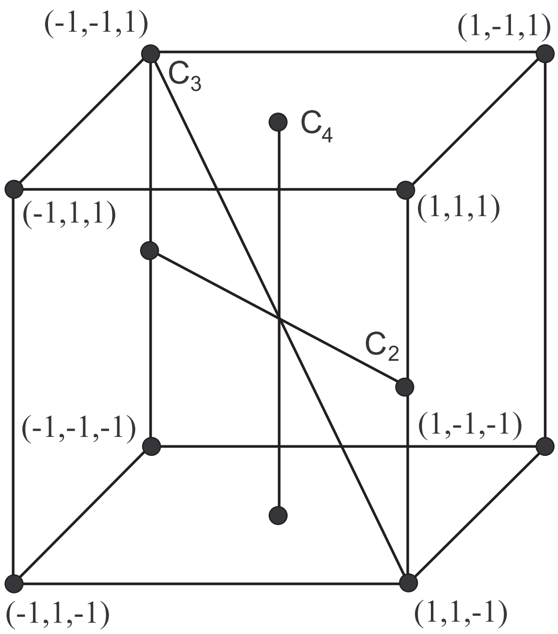
$\Delta_{\mathrm{LOFF}} = \Delta e^{i2\vec{q}\cdot\vec{x}} \Rightarrow$ Breaking of spatial invariance.

More than one direction.

$|\vec{q}|$ is determined by minimizing the free energy, $\hat{q}$ spontaneously. Why only one direction?

$$\Delta_{\mathrm{LOFF}} = \sum_m \Delta_m e^{i\vec{q}_m\cdot\vec{r}} \qquad (13)$$

defines a lattice structure in momentum space.



The favored one seems to be the cubic lattice (Bowers,Rajagopal).

Effective lagrangian for phonons

Breaking of spatial symmetries $\to$ 3 NGBs: phonons.

Phonons describe the fluctuations of the crystal planes.

$$2qx_i \to 2qx_{k_i} + i\frac{\phi^{(i)}}{f} \tag{14}$$

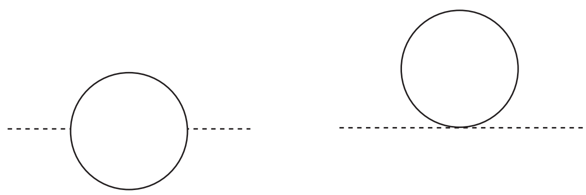
with $x_{k_i} = \frac{k_i \pi}{q}$ and

$$\int d\vec{x} f(\vec{x}) \to \sum_{k_i, k_2, k_3} f(x_{k_i}) \tag{15}$$

The interaction part of the action is

$$S_{\text{int}} \propto \int dt \sum_{k_i} \sum_{\vec{v}} \sum_{m} \Delta e^{i \sum_i \epsilon_i^{(m)} \phi^{(i)}} \psi^T C \psi \quad (16)$$

In this way we get $\mathcal{L}_{\phi\psi\psi}$ and $\mathcal{L}_{\phi\phi\psi\psi}$ which allow the evaluation of the self energy diagrams



Finally we get the 1-loop lagrangian

$$\begin{aligned}\mathcal{L}_{\text{eff}}(\phi) &= \frac{1}{2}\sum_i (\dot{\phi}^{(i)})^2 - \frac{a}{2}\sum_i (\vec{\nabla}\phi^{(i)})^2 \\ &\quad - \frac{b}{2}\sum_i (\partial_i \phi^{(i)})^2 - \frac{c}{2}\sum_{i<j} (\partial_i \phi^{(i)} \partial_j \phi^{(j)})\end{aligned}$$

and we find $a = \frac{1}{12}$, $b = 0$, $c = \frac{3(\delta\mu/q)^2 - 1}{12}$

Gluon Dynamics

$SU(2)_c$ gluons: free lagrangian + 1-loop corrections. Medium as a dielectric.

1) 2SC

$$\mathcal{L} = \frac{1}{2}(E_i^a E_i^a - B_i^a B_i^a) + \frac{k}{2} E_i^a E_i^a \qquad (17)$$

$\epsilon = 1 + k$, $\lambda = 1$; $k \propto (g\mu/\Delta)^2$.

2) LOFF (z symmetry axis)

$$\mathcal{L} = \frac{1}{2}(\epsilon_{ij} E_i^a E_j^a - B_i^a B_i^a) + \frac{1}{2} m_D^2 A_0^a A_0^a \qquad (18)$$

$\epsilon_{ij} = diag(1 + k_1, 1 + k_1, 1 + k_2)$, $m_D \propto g\mu$.

3) LOFF(CUBE)

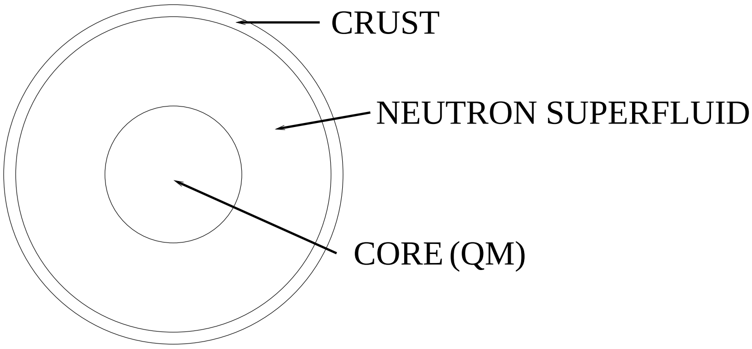
$$\mathcal{L} = \frac{1}{2}(\tilde{\epsilon}_{ij} E_i^a E_j^a - B_i^a B_i^a) + \frac{1}{2} M_D^2 A_0^a A_0^a \qquad (19)$$

$\tilde{\epsilon}_{ij} = \delta_{ij} k_3$. The gluon velocity is $v = 1/\sqrt{\epsilon}$.

In 1) and 3) the gluon velocity is isotropic.

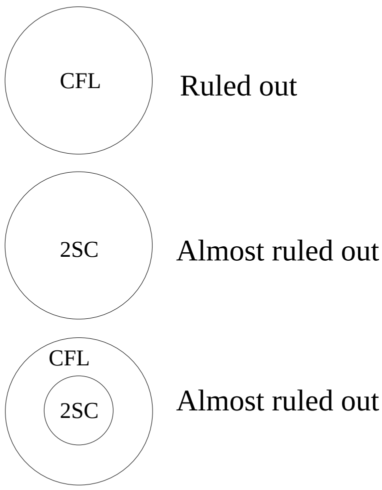
Astrophysical implications

Neutron star

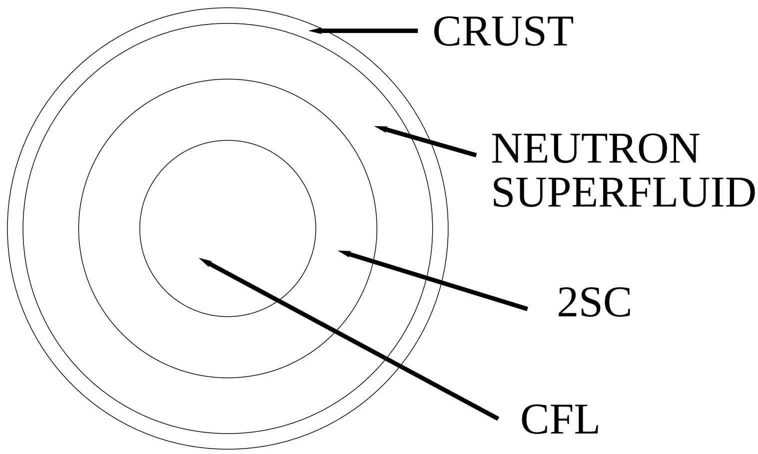


Strange matter hypothesis: uds matter more stable than hadronic matter. $r-$ mode instability: spin-down of all rotating objects in absence of viscous forces, regards the outer part of the star.

"Pure" Strange stars



Hybrid star

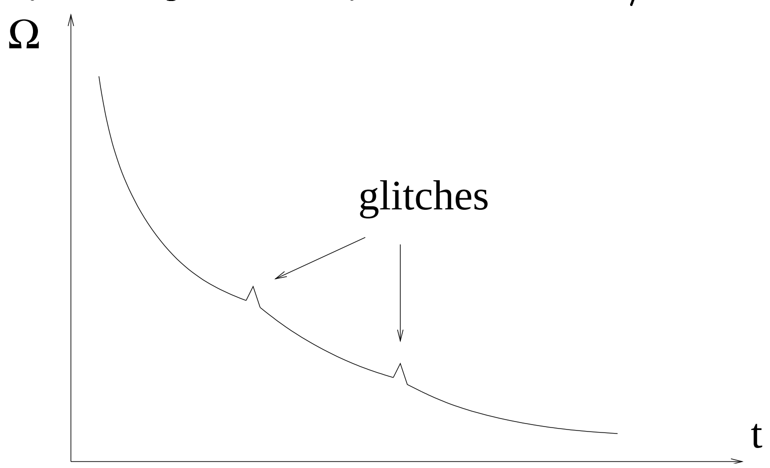


Pulsar Glitches

- Pulsars: rotating compact objects.

$T \sim 1.5\text{ms} - 4.3\text{s}$ $\;M \sim 1.5 M_{\odot}$ $R \sim 10\text{Km}$

- Glitches: Sudden jumps in the rotation frequency of a pulsar $\Delta\Omega/\Omega \sim 10^{-6} - 10^{-9}$.



Accepted mechanism: starquake + angular momentum transfer from the neutron superfluid to the crust.

Strange star mechanism : starquake(?) + unpinning of vortices in the LOFF phase.

References

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