DECONFINEMENT TRANSITION IN LATTICE GAUGE THEORIES: CRITICAL INDICES AND SCREENING MASSES NEAR CRITICALITY

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Plan of the talk

- Universality and Svetitsky-Yaffe conjecture

- Finite-size behavior at criticality and the critical index ν of the correlation length
  - the case of (2+1)D SU(3) gauge theory
  - the case of (3+1)D SU(2) gauge theory

- 3D Ising universality class
  - spectrum of massive excitations in 3D Ising model and 3D lattice regularized ϕ^4 theory in the broken symmetry phase
  - predictions, through universality, of the spectrum of screening masses in the deconfined phase of (3+1)D SU(2) gauge theory at finite temperature

- Numerical determination of the screening masses in the deconfined phase of in (3+1)D SU(2) gauge theory near criticality

- Conclusions

Universality and Svetitsky-Yaffe conjecture

- $(d+1)$-dimensional SU(N) pure gauge theory at finite temperature undergoing a *continuous* deconfinement transition at T_c

  order parameter: Polyakov loop $\boxed{P(x,y,z) \equiv \mathbf{Tr} \prod_{n_4=1}^{N_t} U_4(x,y,z,an_4)}$

  $$\langle P \rangle = 0 \quad (T < T_c) \qquad\qquad \langle P \rangle \neq 0 \quad (T > T_c)$$

  breaking of the center symmetry

- integrating out all degrees of freedom except the order parameter $\rightarrow$ d-dimensional effective model
  - globally invariant under the center of the gauge group
  - with only short-range interactions
  - with long-range fluctuations in the order parameter dominating at criticality

  [B. Svetitsky and L.G. Yaffe (1982)]

- Svetitsky-Yaffe conjecture: the $(d+1)$-dimensional SU(N) gauge theory and the d-dimensional effective statistical model belong to the same universality class

  Critical properties (such as critical indices, finite-size scaling, correlation functions at criticality, ...) are predicted to coincide.

  [J. Engels et al. (1990, 1992)]
  [M. Teper (1993)]
  [M. Caselle and M. Hasenbusch (1996)]
  [K. Pinn and S. Vinti (1998)]

	$(d+1)$-dimensional pure gauge theory at $T \neq 0$	d-dimensional effective model
$d=2$	SU(2)	Z(2)
$d=2$	SU(3)	Z(3)
$d=3$	SU(2)	Z(2)

Finite-size behavior at criticality and the critical index ν of the correlation length

d-dimensional statistical model:
critical indices from *finite-size behavior* of suitable operators

In particular, finite size behavior of the (lattice) energy operator E:

$$\langle E \rangle_L \sim \langle E \rangle_\infty + k L^{\frac{1}{\nu}-d}$$

$L =$ "(large) lattice size", $\quad k =$ "non-universal constant"

[M. Hasenbusch and K. Pinn (1997)]

$(d+1)$-dimensional pure gauge theory

$\hat{O}$: operator invariant under gauge and center symmetry

Examples: Wilson loop $\quad P(\vec{x})P^\dagger(\vec{y}) \quad \ldots\ldots$

$\vec{x}$, $\vec{y}$: different sites of the d-dimensional space.

Ansatz: $\quad \hat{O} = c_I \ I + c_\epsilon \ \epsilon +$ irrelevants

I: identity operator $\quad \epsilon$: (scaling) energy operator of the statistical model

[F. Gliozzi and P. Provero (1997)]
[R. Fiore, F. Gliozzi and P. Provero (1998)]
for the case of SU(2) in (2+1)-dimension.

$$\longrightarrow \quad \langle \hat{O} \rangle_L \sim \langle \hat{O} \rangle_\infty + c L^{\frac{1}{\nu}-d}$$

Numerical results

Lattice operators

1) electric plaquette 2) magnetic plaquette 3) $P(\vec{x})P^\dagger(\vec{y})$

($\vec{x}$, $\vec{y}$ are neighbor sites in the spatial lattice)

The case of (2+1)D SU(3)

[R. Fiore, A. P. and P. Provero (2001)]

$N_t = 2$ $L = N_x = N_y$ from 7 to 30 $\beta_c(N_t = 2) = 8.155$

[β_c from J. Engels et al. (1997)]

$$\boxed{\nu = 0.827(22)} \qquad \chi^2_{\rm red} = 0.84\ .$$

Exact value in the 2-dimensional 3-state Potts model: $\boxed{\nu = \frac{5}{6} = 0.833\cdots}$.

Monte Carlo determination (χ^2 analysis of the Polyakov loop susceptibility):

$$\boxed{\nu_{MC} = 0.90(20)}$$

[J. Engels et al. (1997)]

The case of (3+1)D SU(2)

[R. Fiore, A. P., P. Provero and C. Vena (2001)]

$N_t = 2$ $L = N_x = N_y = N_z$ from 5 to 26 $\beta_c(N_t = 2) = 1.8735$

[β_c from S. Fortunato et al. (2001)]

$$\boxed{\nu = 0.6298(28)} \qquad \chi^2_{\rm red} = 0.96\ .$$

Result in the 3-dimensional Ising model by the high-T expansion:

$$\boxed{\nu = 0.63012(16)}$$

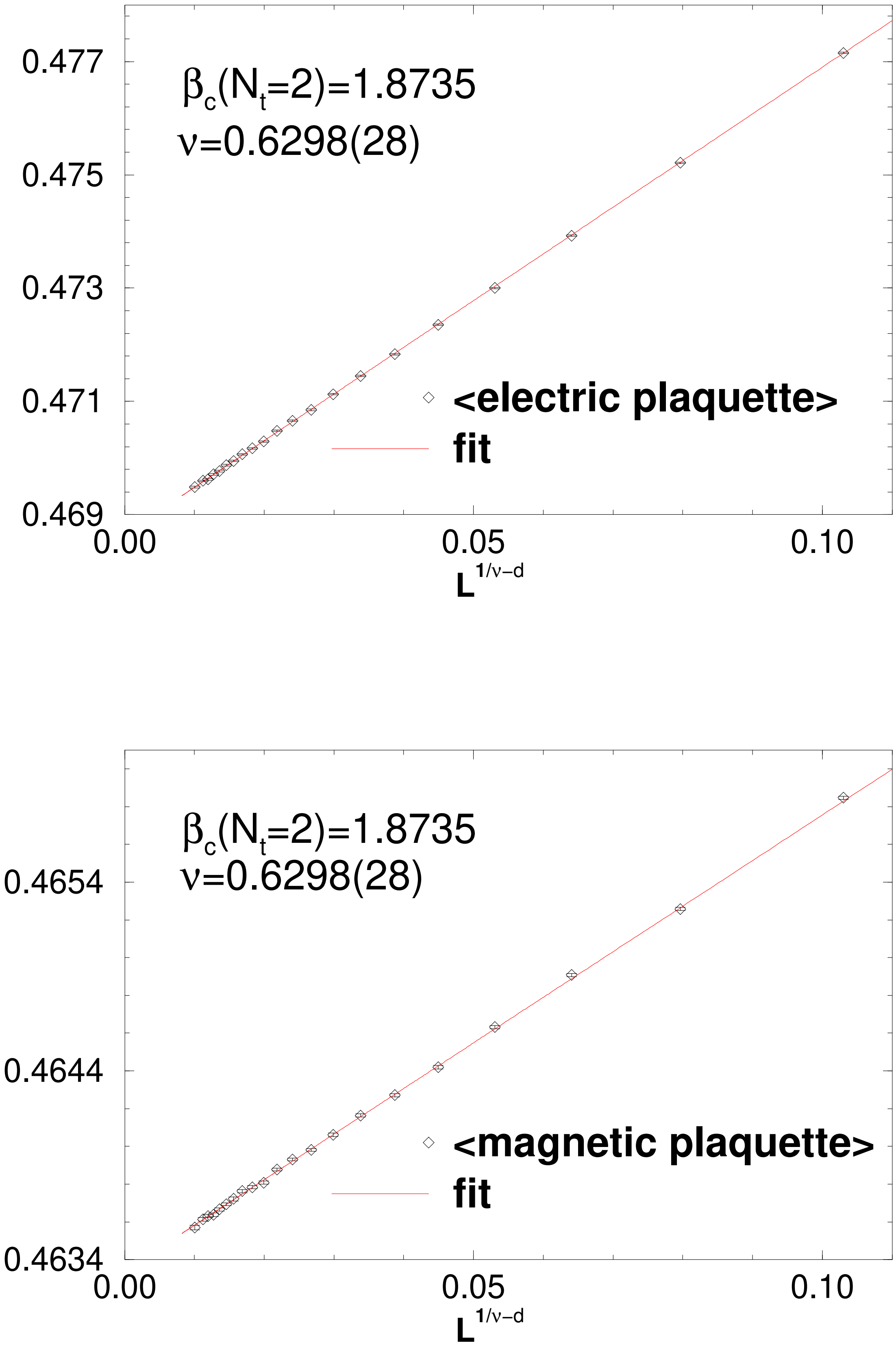
[M. Campostrini et al. (2002)]

Monte Carlo determination (χ^2 analysis of the Polyakov loop susceptibility):

$$\boxed{\nu_{MC} = 0.630(9)}$$

[S. Fortunato et al. (2001)]

SU(2), $d = 3$



0.477
0.475
0.473
0.471
0.469
0.00
0.05
0.10
$L^{1/\nu-d}$
$\beta_c(N_t=2)=1.8735$
$\nu=0.6298(28)$
<electric plaquette>
fit
0.4654
0.4644
0.4634
0.00
0.05
0.10
$L^{1/\nu-d}$
$\beta_c(N_t=2)=1.8735$
$\nu=0.6298(28)$
<magnetic plaquette>
fit

Spectrum of massive excitations in 3D Ising model and 3D lattice regularized ϕ^4 theory in the broken symmetry phase

[M. Caselle, M. Hasenbusch and P. Provero (1999)]

- **3D Ising model**

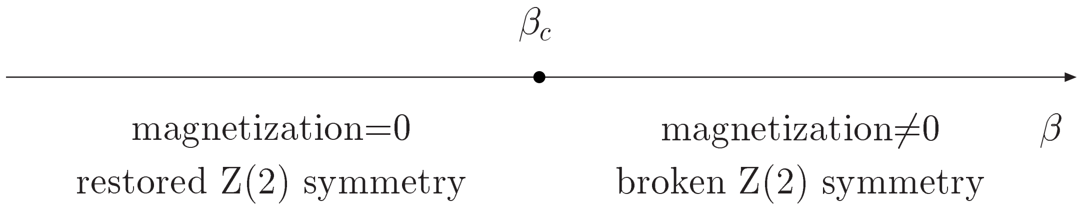
$$S = -\beta \sum_{<n,m>} s_n s_m \,, \qquad\qquad \beta_c = 0.2216543(4)$$

Simulations at $\beta = 0.23142$ and $\beta = 0.2275$

- **3D lattice ϕ^4 theory**

$$S = -\beta \sum_{<n,m>} \phi_n \phi_m + \sum_n \phi_n^2 + \lambda \sum_n (\phi_n^2 - 1)^2 \,, \qquad\qquad \beta_c(\lambda = 1.1) = 0.3750966(4)$$

Simulations at $\beta = 0.405$, $\beta = 0.385$ and $\beta = 0.3798$



The spectrum has been determined from the analysis by the so-called "variational" method (see below) of wall-wall correlations of spin operators.

Three 0^+ states detected; results for their mass ratios agree in the two models:

$$\frac{m_2}{m_1} = 1.83(3) \qquad\qquad \frac{m_3}{m_1} = 2.45(10)$$

Universality is valid far beyond critical indices.

The first excitation is below the two particle production threshold $\rightarrow$ non-perturbative state, identified as a bound state of the fundamental excitation.

[M. Caselle, M. Hasenbusch, P. Provero, K. Zarembo (2001)]

The second mass state could be a fictitious one which mimics the two-particle cut.

- **Predictions from universality**

3D Ising	$\longleftrightarrow$	**(3+1)D SU(2) at $T \neq 0$**
broken phase		**deconfined phase**
magnetization		**Polyakov loop**

$$\beta_c(N_t = 4) = 2.29895(10)$$

$\langle P \rangle = 0$	$\langle P \rangle \neq 0$	β
restored Z(2) symmetry	broken Z(2) symmetry	

Analysis of wall-wall correlations of Polyakov loop operators in the deconfined phase near the critical temperature.

3D Ising	$\longleftrightarrow$	**(3+1)D SU(2) at $T \neq 0$**
0^+ **states**		0^+ **screening masses**

- **Predictions from duality**

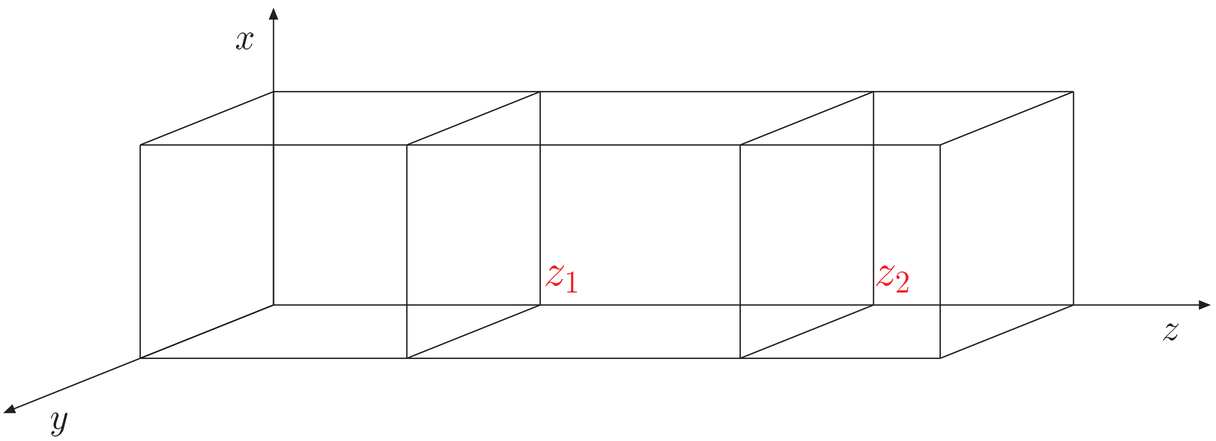
3D Ising model	$\longleftrightarrow$	**3D Z(2) gauge theory**
broken phase		**confined phase**
0^+ **states**		0^+ **glueball spectrum**

$$\frac{m_2}{m_1} = 1.88(2) \qquad \frac{m_3}{m_1} = 2.59(4)$$

[M. Caselle, M. Hasenbusch, P. Provero, K. Zarembo (2001)]

Monte Carlo determination of the spectrum

0^+ **channel, lattice** $N_x \times N_y \times N_z \times N_t$



Polyakov loop $\boxed{P(x,y,z) \equiv \mathbf{Tr} \prod_{n_4=1}^{N_t} U_4(x,y,z,an_4)}$

Wall-average (zero momentum projection):

$$\overline{P}(z) \equiv \frac{1}{N_x N_y} \sum_{x,y} P(x,y,z)$$

Connected wall-wall correlation function:

$$\begin{aligned} G(|z_1 - z_2|) &\equiv \langle \overline{P}(z_1)\overline{P}(z_2)\rangle - \langle \overline{P}(z_1)\rangle\langle \overline{P}(z_2)\rangle \\ &= A\Big(\exp[-m|z_1 - z_2|] + \exp[-m(N_z - |z_1 - z_2|)]\Big) \\ &+ B\Big(\exp[-m^*|z_1 - z_2|] + \exp[-m^*(N_z - |z_1 - z_2|)]\Big) + \dots \end{aligned}$$

m = **"lowest mass in the** 0^+ **channel"**

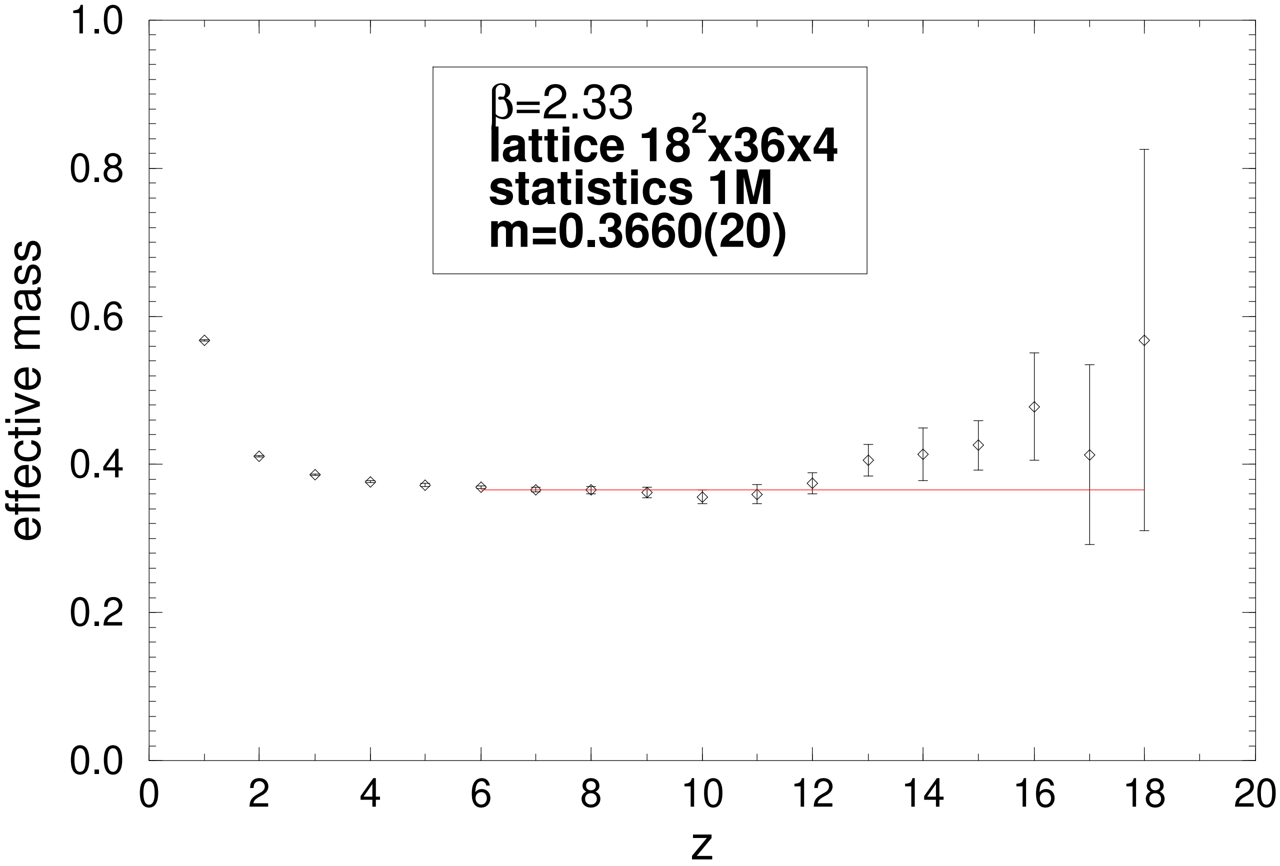
m^* = **"1st excited level in the** 0^+ **channel"**

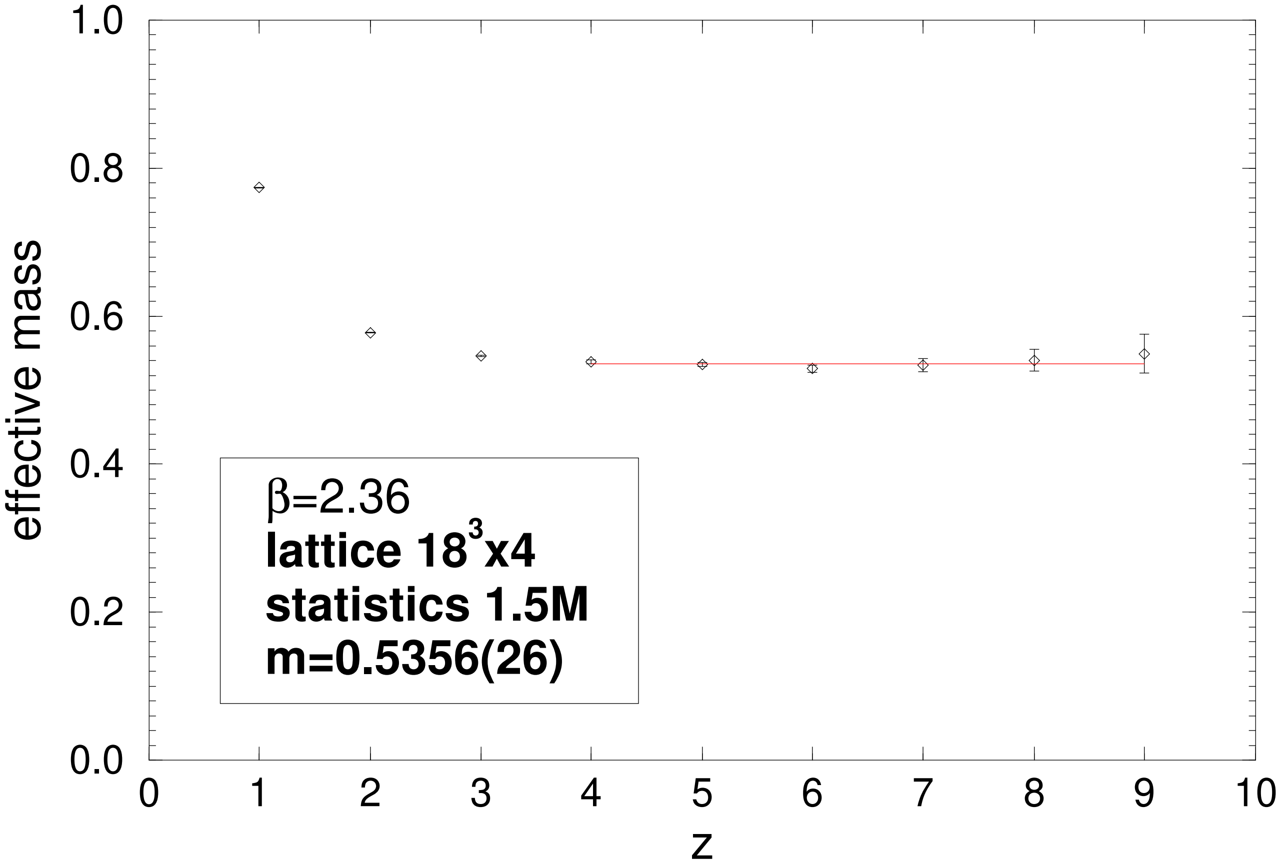
$\dots$

Nota bene: $\langle \overline{P}(z)\rangle \neq 0$ **in the high temperature phase**
$\rightarrow$ z**-independent term in** $G(|z_1 - z_2|)$ **of the form** $C_0 \exp[-mN_z]$**?**

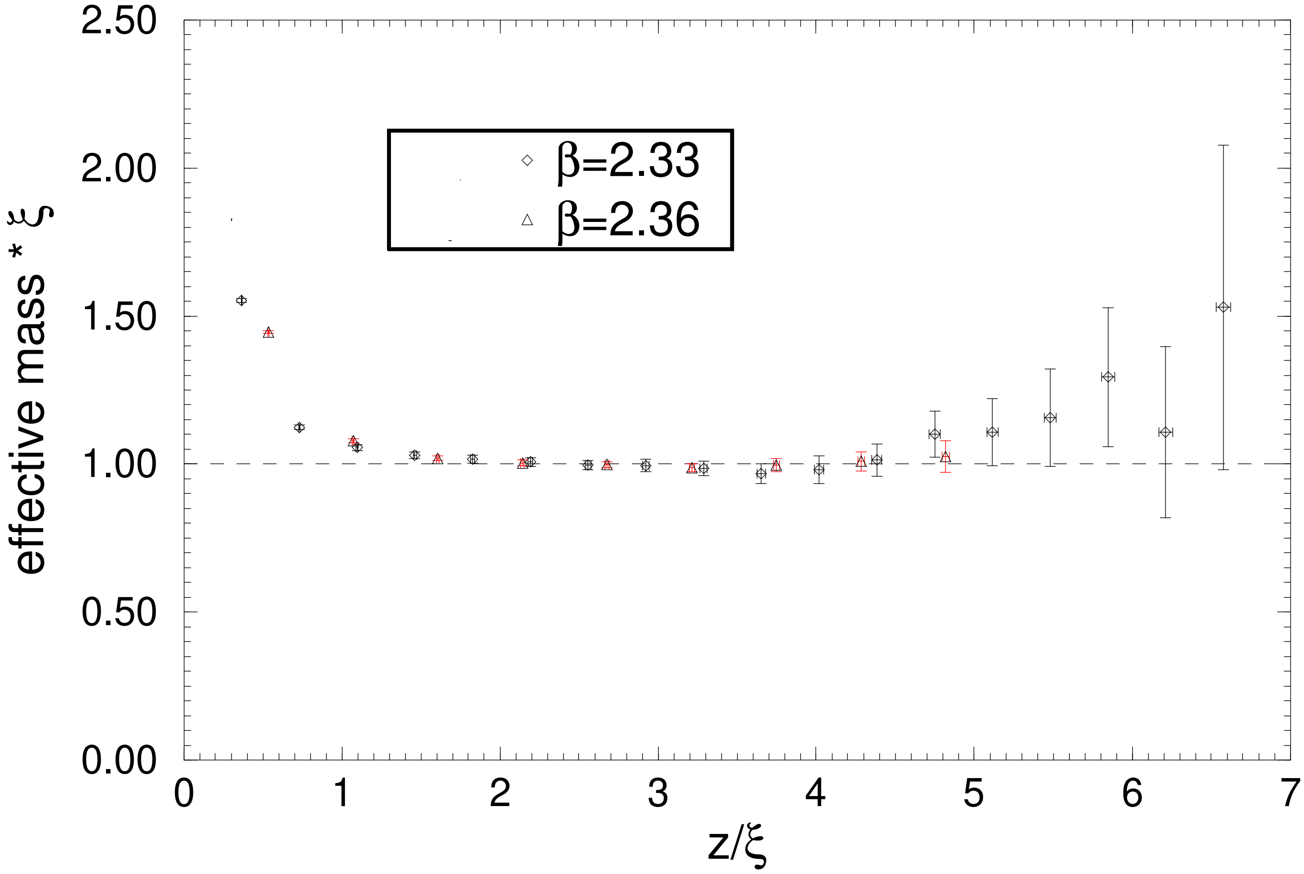
[I. Montvay, P. Weisz (1987)]

No, since such a term is sub-dominant for $m^* < 2m$**, as we expect to be the case here.**





$$G(z) = A\Big(\exp[-m_{eff} z] + \exp[-m_{eff}(N_z - z)]\Big)$$



Data from the two different β's fall on the same curve before reaching the plateau: there must exist physical excited states.

“Variational” method

[A.S. Kronfeld (1990); M. Lüscher, U. Wolff (1990)]

- **Define a basis of (wall-averaged) operators, $\{O_\alpha\}$;**
- **compute the cross-correlation matrix among these operators**

$$C_{\alpha\beta}(z) = \langle O_\alpha(z) O_\beta(0) \rangle - \langle O_\alpha(z) \rangle \langle O_\beta(0) \rangle \ ;$$

- **diagonalize $C_{\alpha\beta}(z)$ to obtain the eigenvalues**

$$\lambda_i(z) \propto \exp[-m_i z] + \exp[-m_i (N_z - z)] \ ;$$

- **extract the masses m_i.**

- **The effectiveness of the method relies on the choice of a “good” set of operators;**
- **it is convenient to define operators living on different length scales (for instance, by use of recursive definitions);**
- **by defining operators with non-trivial transformation under spatial rotation, it is possible to look for states with non-zero angular momentum.**

First set of operators (channel 0^+)

[M. Caselle, M. Hasenbusch, P. Provero (1999)

for 3D Ising model and 3D lattice regularized ϕ^4 theory]

Recursive definition

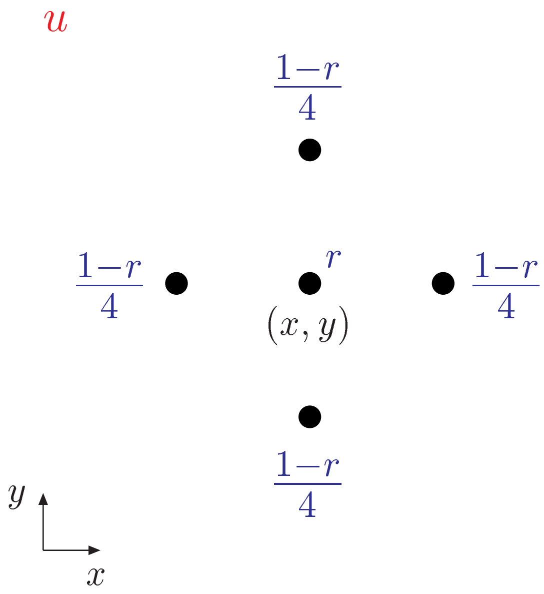
Polyakov loop $\boxed{P(x,y,z) \equiv \mathbf{Tr} \prod_{n_4=1}^{N_t} U_4(x,y,z,an_4)}$

$$P^{(0)}(x,y,z) = P(x,y,z)$$

$$P^{(n+1)}(x,y,z) = \mathbf{sign}(u)\Big[(1-w)|u| + wy\Big]$$

$$u = rP^{(n)}(x,y,z)$$

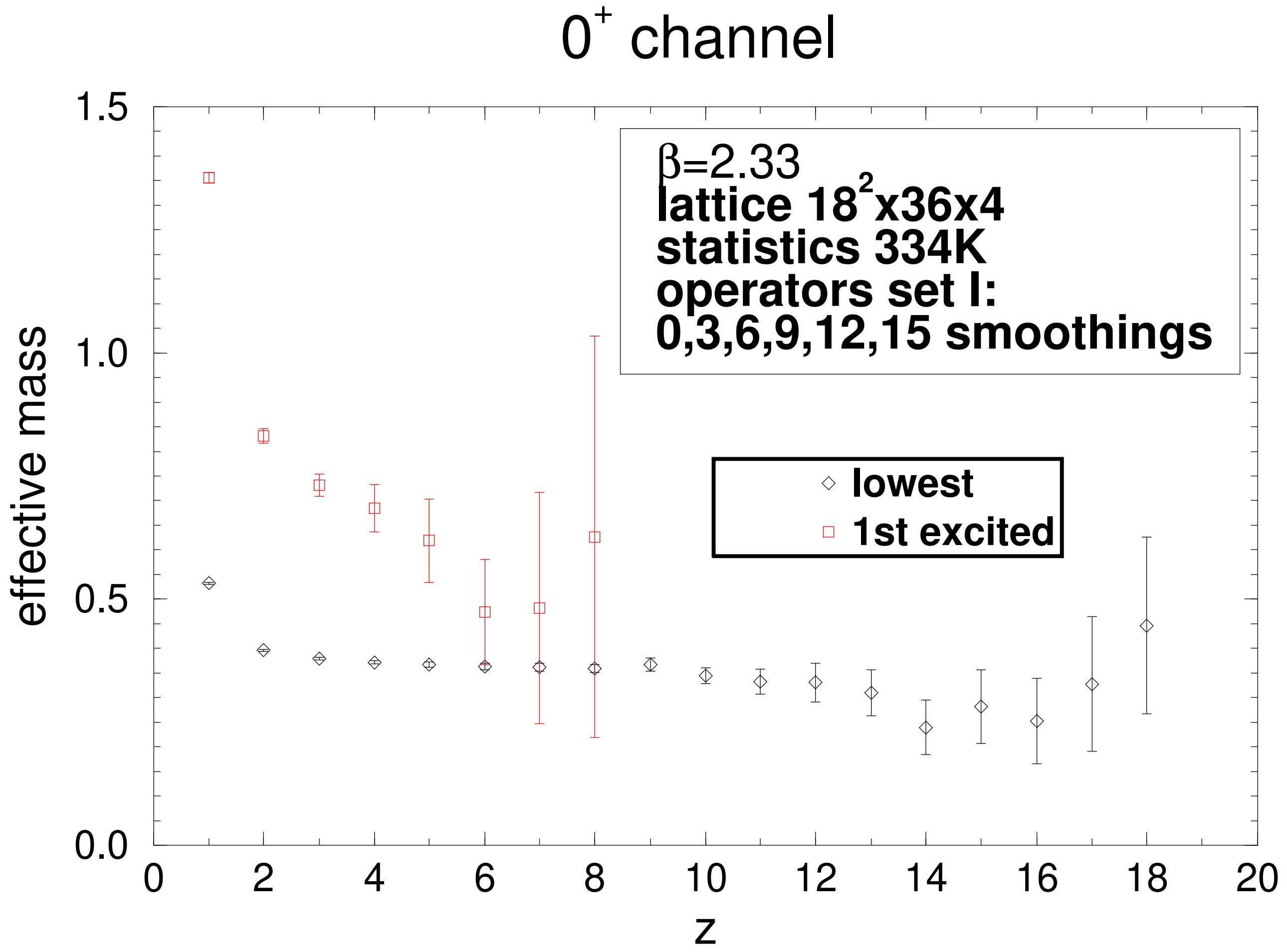
$$+\frac{1-r}{4}\Big(P^{(n)}(x-a,y,z) + P^{(n)}(x,y-a,z) + P^{(n)}(x+a,y,z) + P^{(n)}(x,y+a,z)\Big)$$



$$\overline{P}^{(n)}(z) = \frac{1}{N_x N_y} \sum_{x,y} P^{(n)}(x,y,z)$$

Choice of parameters:

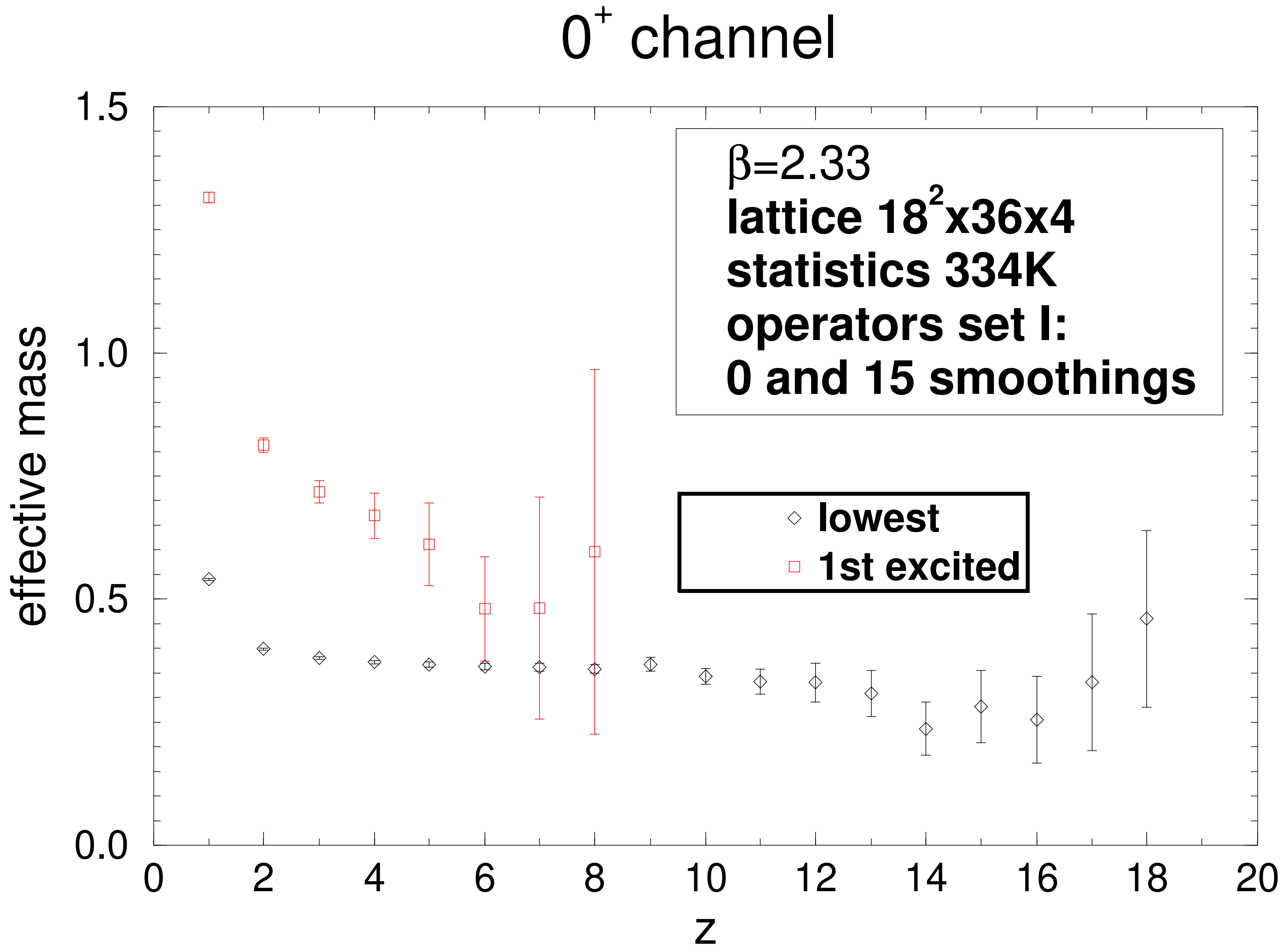
$r = 0$ $\qquad$ $w = 0.1$ $\qquad$ $y = \langle P(x,y,z) \rangle$



$$m(0^+) = 0.3629(50) \qquad \frac{m^*(0^+)}{m(0^+)} = 1.89(16)$$
$$m^*(0^+) = 0.684(48)$$

3D Ising and (lattice) ϕ^4:

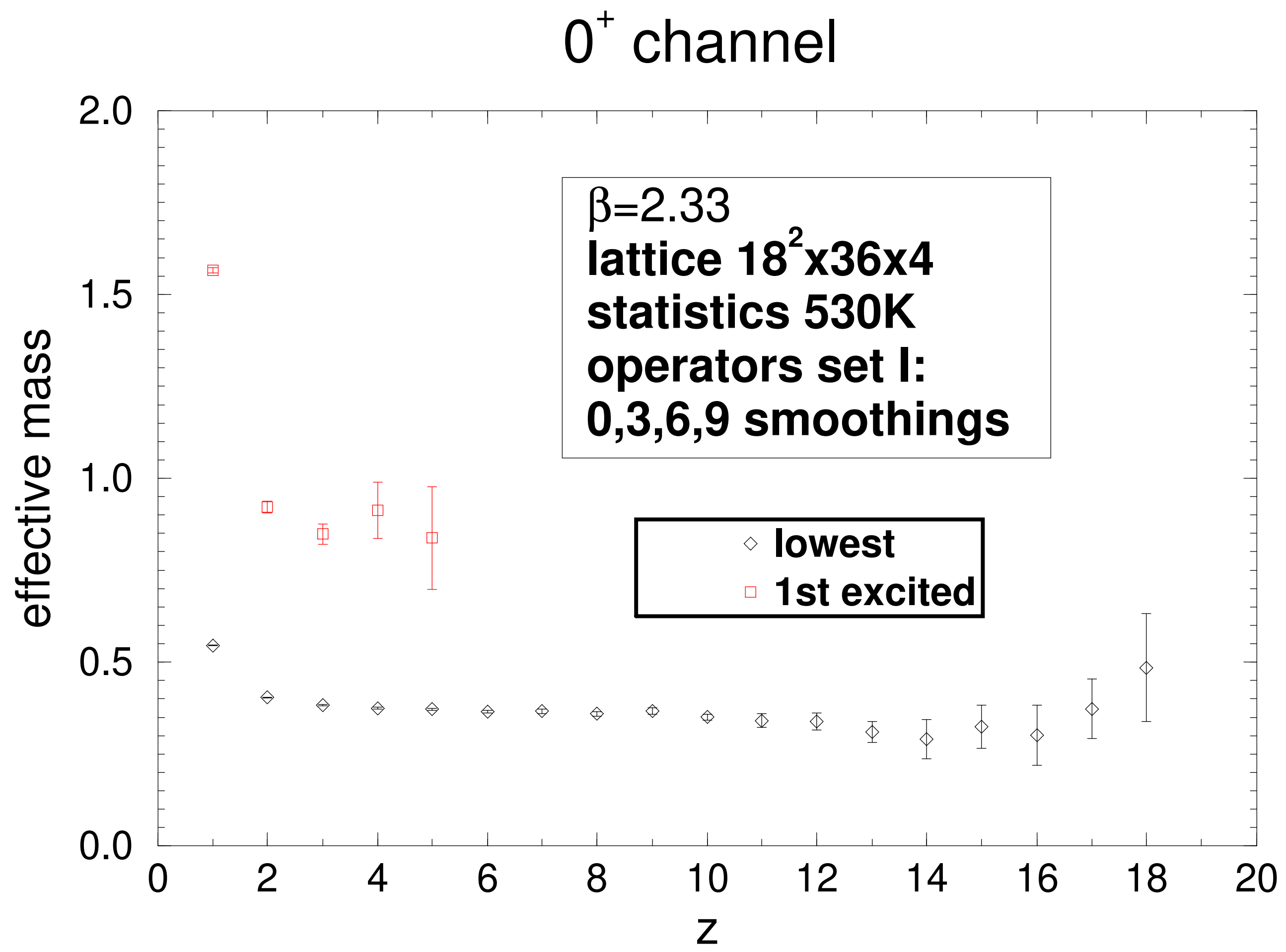
$$\frac{m_2}{m_1} = 1.83(3)$$



$$\begin{aligned} m(0^+) &= 0.3620(51) \\ m^*(0^+) &= 0.670(46) \end{aligned} \qquad \frac{m^*(0^+)}{m(0^+)} = 1.85(15)$$

3D Ising and (lattice) ϕ^4:

$$\frac{m_2}{m_1} = 1.83(3)$$



$$
\begin{aligned}
m(0^+) &= 0.3615(40) \\
m^*(0^+) &= 0.848(28)
\end{aligned}
\qquad
\frac{m^*(0^+)}{m(0^+)} = 2.35(10)
$$

3D Ising and (lattice) ϕ^4:

$$
\frac{m_3}{m_1} = 2.59(4)
$$

Here the length scale of the operators is not large enough for the bound state to be detected; the state seen here could be the analog of the two-particle cut in the Ising model.

Second set of operators (channels 0^+ and 2^+)

[V. Agostini, G. Carlino, M. Caselle, M. Hasenbusch (1996) for 3D gauge Ising model]

Polyakov loop $\boxed{P(x,y,z) \equiv \mathbf{Tr} \prod_{n_4=1}^{N_t} U_4(x,y,z,an_4)}$

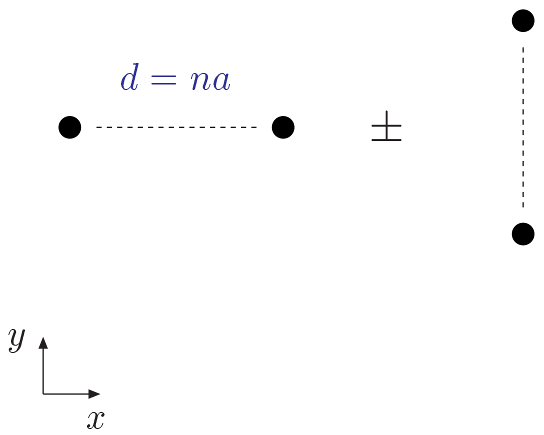
Channel $0^+(2^+)$

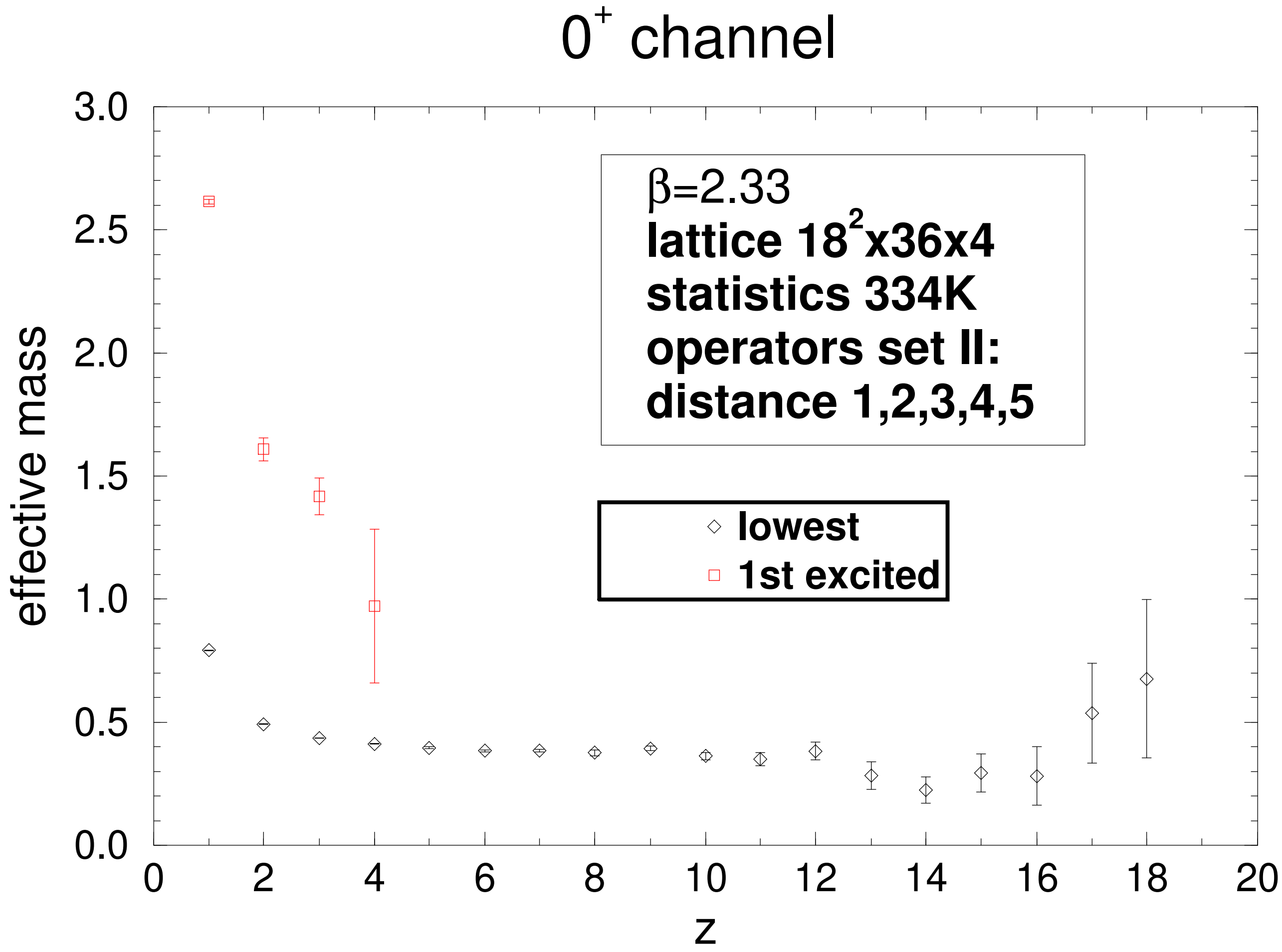
$$\overline{P}^{(n)}(z) = \frac{1}{N_x N_y} \sum_{x,y} \Big[P(x,y,z)P(x+na,y,z) \pm P(x,y,z)P(x,y+na,z) \Big]$$

$$n = 1, \ldots, 5$$

$0^+(2^+)$

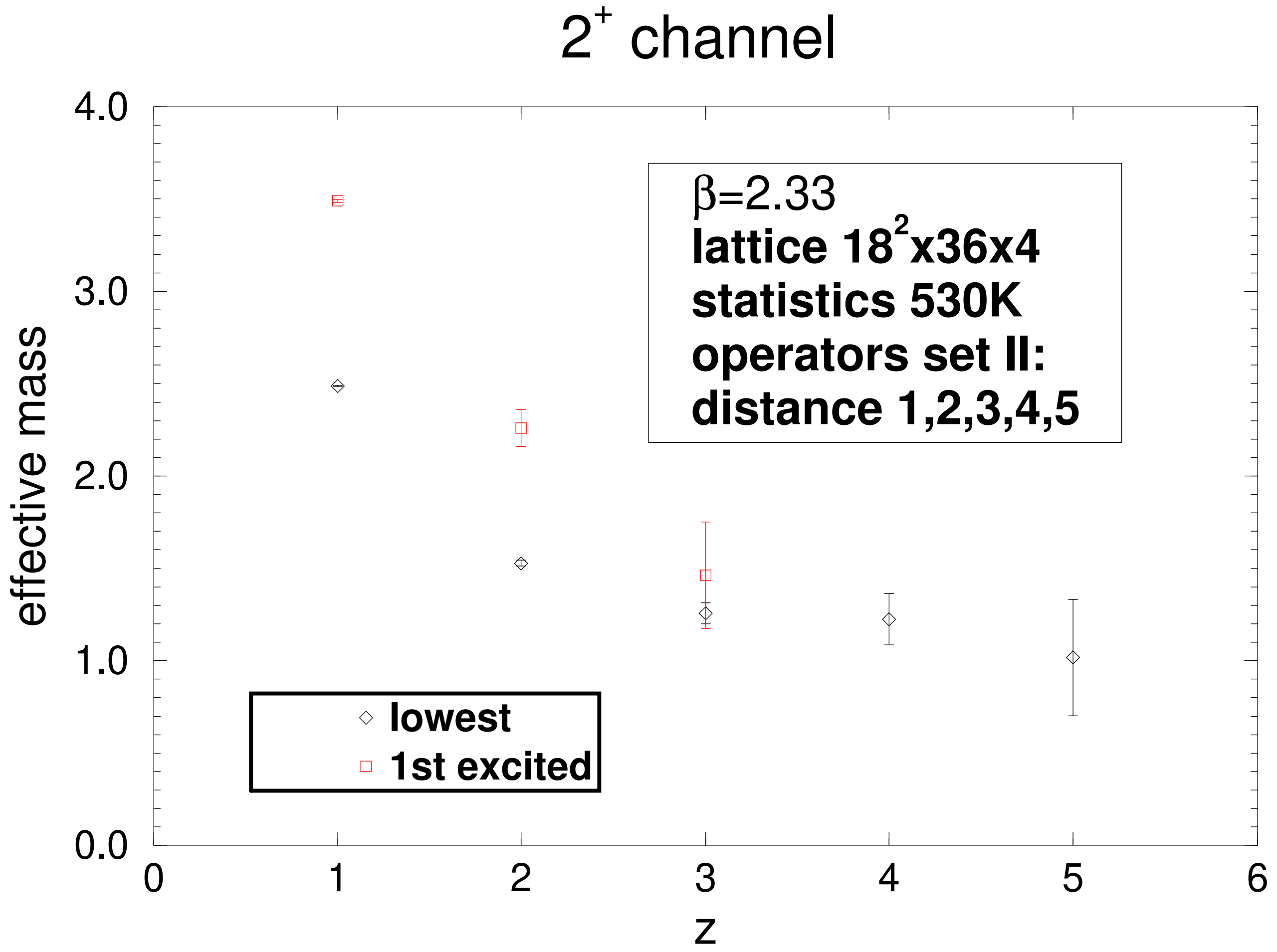
$n = 1, \ldots, 5$





$$m(0^+) = 0.3828(40)$$

No bound state detected in the 0^+ channel by these operators. The lowest mass turns out to be larger than previous determinations.



$$m(2^+) = 1.23(14) \qquad \frac{m(2^+)}{m(0^+)} = 3.36(40)$$

$[m(0^+) = 0.3660(20)$ **from the naive wall-wall correlation**$]$

3D gauge Ising (glueball spectrum):

$$\frac{m(2^+)}{m(0^+)} = 2.59(4) \qquad \frac{m^*(2^+)}{m(0^+)} = 3.23(7)$$

[V. Agostini, G. Carlino, M. Caselle, M. Hasenbusch (1996)]

The result found seems to be in agreement with the $m^*(2^+)$**, more than with** $m(2^+)$**.**

Conclusions

- **The determination of the critical index ν of (2+1)D SU(3) and (3+1)D SU(2) pure gauge theories by finite size scaling analysis of simple lattice operators, such as the plaquette, is a further confirmation of the predictive power of the Svetitsky-Yaffe conjecture.**

- **There is evidence of the existence of excited states in the 0^+ channel of the spectrum of screening masses in the deconfined phase of (3+1)D SU(2) gauge theory near the critical temperature.**

- **The first excited state in the (3+1)D SU(2) gauge theory has been detected applying the so-called "variational" method to a set of (wall-averaged) operators with large scale.**

- **The mass of this state is compatible with the mass of the bound state $(0^+)^*$ in the broken phase of 3D Ising and (lattice) ϕ^4, in agreement with universality.**

- **A similar analysis in the 2^+ channel with a different set of operators led to results which demand further investigation.**