The Higgs condensate as physical medium: observable consequences and lattice tests

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SUMMARY

1) The physical meaning of “triviality”
2) Spontaneous symmetry breaking in $\lambda\Phi^4$ theory as a condensation phenomenon
3) The RG-description of the condensed phase
4) Phenomenology of the Higgs particle: differences with respect to perturbation theory
5) Check with lattice simulations
6) Conclusions

The Standard Model vacuum is based on the simple concept of Spontaneous Symmetry Breaking (SSB) induced by a $\lambda\Phi^4$ theory. The idea is very simple : a non vanishing expectation value

$$<\Phi> = v \neq 0$$

of a scalar field Φ represents the simplest ingredient to generate the particle masses (g_i are dimensionless couplings)

$$m_i = g_i \, v$$

At the quantum level, where the value $|\phi| = v$ denote the degenerate absolute minima of the quantum effective potential $V_{eff}(\phi)$, SSB amounts to a Bose-condensation phenomenon of spontaneously created scalar quanta in the $\vec{k} = 0$ mode (see `t Hooft).

Trying to reconcile this intuitive picture with the generally accepted "triviality" of $\lambda\Phi^4$ theories in 3+1 space-time dimensions leads to a deeper understanding of the underlying physical system.

Following this way, one discovers the inadequacy of the standard perturbative approach with sizeable phenomenological differences.

In search of the ultimate building blocks

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Bose condense. The consequence is that this liquid can stream through the tiniest holes without the least amount of resistance.†

Because separate electrons have spin $\frac{1}{2}$, they themselves cannot Bose condense. Particles whose spin is an integer plus one-half (fermions) must all be in *different* quantum states, because of Pauli's exclusion principle. This is why superconductivity can only occur if pair formation has taken place. Yes, I realize that this will raise a few questions, and I apologize beforehand. I have again tried to translate formulae into words, which implies that the reasonings may sound very unsatisafactory. Just look at this as a somewhat unwieldy 'quantum logic'!

That superconductivity could be of importance for elementary particles was discovered by the Belgian François Englert, the American Robert Brout and the Scotsman Peter Higgs. They proposed a model for elementary particles in which electrically charged particles without spin undergo Bose condensation. This time, however, the condensation takes place not inside some material, but in empty space (the 'vacuum') itself. The forces among these particles have then been chosen in such a special way that it saves energy to fill the vacuum with particles rather than keeping it empty. These particles are not directly observable. We would experience this state, in which space and time are sizzling with Higgs particles (as they are now called), but in which the energy is as low as it can ever be, as if space-time were completely empty.

The Higgs particles are the quanta of the 'Higgs field'. A characteristic of the Higgs field is that the energy in it is lowest when the field has a certain strength, and not when it is zero. What we experience as empty space is nothing but the field configuration that has the lowest possible energy. If we move from field jargon to particle jargon, this means that empty space is actually filled with Higgs particles. They have 'Bose condensed'.

This empty space has lots of properties in common with the interior of a superconductor. The electromagnetic field here also has a short range. This is directly related to the fact that in such a world the photon has a certain amount of rest mass.

And yet we have a complete gauge symmetry; gauge invariance is not violated anywhere. And thus we have learned how to turn a photon into a 'massive' particle *without* violating gauge invariance. All we had to do was to add these

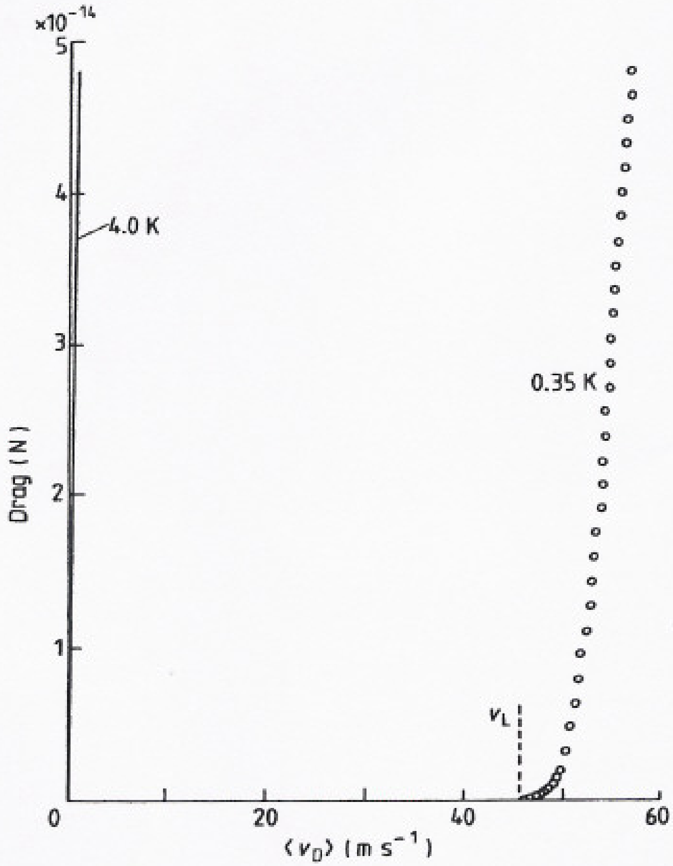

Figure 2.9 Drag on a negative ion moving through liquid He II at 0.35 K and 25 atm, as a function of the average ionic drift velocity $\langle v_D \rangle$. Drag sets in when $\langle v_D \rangle$ reaches critical value which is close to roton v_L. In contrast, drag on ion in liquid He I at 4.0 K sets in as soon as it starts to move, showing that superfluidity is absent. (After Allum *et al* 1977.)

with those calculated from currently accepted values of the Landau parameters (see §2.5) to within about 1 %. If those parameters were to be measured directly by neutron scattering in this region of the He II phase diagram, even more valuable comparisons could then be made.

As we shall describe in the next chapter, many superflow experiments involve the flow of He II through channels and past obstacles. Usually it is found that there is a non-dissipative flow up to a critical value of superfluid velocity, above which dissipation sets in, a situation for which figure 2.8 could be used as an illustration. It might then seem feasible to use the same argument

For instance, introducing the "physical" definition of the vacuum expectation value v_R (in the Standard Model $v_R \approx 246$ GeV) and the Higgs boson mass M_H, perturbation theory predicts

$$\left[\frac{M_H^2}{v_R^2}\right]_{PT} \approx \frac{1}{\ln \Lambda}$$

Λ being the ultraviolet cutoff. Thus, if one would measure a value $M_H = 750$ GeV, the cutoff Λ is predicted to be below 2 TeV thus implying the existence of "new physics" at that energy scale.

Instead, one finds evidence for the alternative result

$$\frac{M_H^2}{v_R^2} = \Lambda\text{- independent}$$

This has been discussed in a series of papers (M.C. and P.M.Stevenson 1994-2000) after some early attempts (V. Branchina, P.Castorina, M.C. , D. Zappala' 1990-1992).

As phenomenological consequences are substantial, a (successful) check of the picture with lattice simulations has been performed (P.Cea, M.C., L.Cosmai 1998-2004).

The meaning of "triviality"

What does "triviality" exactly mean ? It is a statement dictating zero scattering in the continuum limit of 3+1 dimensional QFT (random walks in 3 space dimensions have zero probability of intersecting).

The standard perturbative interpretation of this result is the following : without an ultraviolet cutoff Λ there would be no scalar self-interactions and without them no symmetry breaking. Therefore, the theory must have an ultraviolet cutoff whose magnitude is set by the (one-loop) renormalized self coupling

$$\lambda_R \approx \frac{1}{\ln \Lambda}$$

Finally, using the perturbative relation

$$\left[\frac{M_H^2}{v_R^2}\right]_{PT} \approx \lambda_R$$

one obtains the previously quoted result.

This perturbative interpretation of the theory can be seriously questioned. In fact, the perturbative β-function of $\lambda\Phi^4$ has an alternating-sign structure and predicts either Landau poles (odd orders) or spurious ultraviolet fixed points (even orders). In the latter case, there would be no reasons for a vanishing λ_R.

An alternative interpretation of "triviality" is obtained starting from the remark that, as known from solid state physics, a vanishingly small two-body coupling can coexist with a non trivial ground state.

Example:

Hard-Sphere Bose Gas (Lee, Huang, Yang 1957)

Consider a system of

N atoms in a volume V

with mass m and scattering length a

with number density

$$n = \frac{N}{V}$$

Assume

$na^3 << 1$	**diluteness**
$ka << 1$	**low-energy**

Low-lying excitations are phonons

atoms $\left(a^{+}(k), a(k)\right) \Rightarrow \left(b^{+}(k), b(k)\right)$ **phonons**

Bogolubov transform

$$H_{eff} = N\frac{2\pi na}{m} + \sum_{k\neq 0}\frac{k}{2m}\sqrt{k^2 + 16\pi na}\; b^{+}(k)b(k)$$

Consider the case where the phonon spectrum becomes "exact". Since phonon-phonon interaction processes become important at higher momenta, one can take the limit

$$a \to 0$$

(so that the condition ka<< 1 is valid at any finite momentum) in such that the product na= fixed. In this case, one generates a hierarchy of systems with the same sound velocity

$$c_s^{\;2} = \frac{4\pi na}{m^2}$$

but smaller and smaller phonon-phonon interactions.

This type of system is "trivial" (in a technical sense) but not entirely trivial (in the physical sense). Namely, the excitations are free-field like but are not the non-interacting atoms with spectrum $k^2/(2m)$ as if the limit were taken at a fixed density n=constant.

Notice that the density $n \approx 1/a$ diverges when $a \to 0$. However, the system becomes infinitely dilute since, for na=constant, the limit $a \to 0$ gives

$$\varepsilon = na^3 \to 0$$

Therefore, the average spacing among the atoms $d \approx \frac{1}{\sqrt[3]{n}}$ vanishes in physical units but it diverges in units of the scattering length. In this sense, the "triviality" condition establishes a hierarchy of length scales that decouple in the limit $a \to 0$ being connected by inverse powers of ε

$$a \ll \frac{1}{\sqrt[3]{n}} \ll \frac{1}{\sqrt{na}} \ll \frac{1}{na^2}$$

In QFT the non-zero density of scalar quanta originates from the spontaneous decay of the empty vacuum state $|0\rangle$ of perturbation theory.

The second-quantized Hamiltonian is

$$H = \frac{1}{2} : \Pi^2 + (\nabla\Phi)^2 + m^2\Phi^2 : + \frac{\lambda}{4!} :\Phi^4:$$

where normal ordering is defined with respect to $|0\rangle$ and the field operator is

$$\Phi(x) = \frac{1}{\sqrt{V}} \sum_k \frac{1}{\sqrt{2E(k)}} \left(a(k) e^{ikx} + a^+(k) e^{-ikx} \right)$$

with $a(k)|0\rangle = 0$ and $E(k) = \sqrt{k^2 + m^2}$.

The operators $a(k)$ and $a^+(k)$ annihilate and create the elementary scalar quanta (with mass m) of the field $\Phi(x)$, the "phions".

The possibility of SSB (and Bose condensation) can be explored by considering the simple class of states

$$|\psi\rangle = e^{\varphi a_0^+}|0\rangle$$

for which

$$\phi = \langle\Phi\rangle_\psi = \frac{1}{\sqrt{2Vm}}2\varphi$$

and

$$n = \left\langle \frac{\sum_k a^+(k)a(k)}{V} \right\rangle_\psi = \varphi^2$$

so that

$$n = \frac{1}{2}m\phi^2$$

Therefore, one can interchange $n \Leftrightarrow \phi$ and SSB, as the condition for non trivial absolute minima of the effective potential $V_{eff}(\phi)$, becomes equivalent to Bose condensation.

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An obvious question immediately arises: how can a $\lambda\Phi^4$ theory with both a repulsive contact interaction $(\lambda > 0)$ and physical mass $(m > 0)$ exhibit a condensed vacuum ?

Namely, in these conditions, how can a system with an average number N of quanta have lower energy than the trivial empty vacuum state $|0\rangle$?

Consider the case where all quanta sit in the k=0 state. There are :

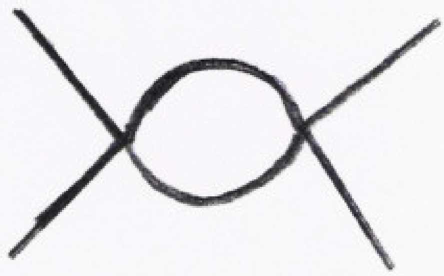
i) the positive contribution from the rest mass

$$N\,m$$

ii) the positive repulsive hard-sphere repulsion

$$N\,\frac{2\pi n a}{m} \qquad \text{where } a=\frac{\lambda}{8\pi m}$$

The point is that the interparticle potential is not always repulsive. The t- and u-channel one-loop contributions



provide an ultraviolet-finite part that gives rise to an attractive $\approx -1/r^3$ potential.

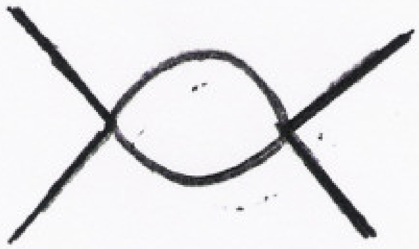
In our case, the interparticle potential (see Feinberg, Sucher, Au 1989) can be derived by considering the elastic collision of two scalar particles in the c.m. frame. The scattering matrix element $M(q)$ depends on the 3-momentum transfer q and, parametrically, on the c.m. energy E. One gets

$$U_{ip}(r) = \int \frac{d^3q}{(2\pi)^3}\, e^{-iqr}\,|M(q)|$$

Thus, in the tree approximation one gets

$$|M| = \lambda \qquad \Rightarrow \quad U_{ip}(r) = \frac{\lambda}{4m^2}\,\delta(r)$$

At one loop and for zero-momentum particles (E=m)



$$\Rightarrow \quad -\frac{\lambda^2}{256\pi^3 m^2}\,\frac{F(2mr)}{r^3}$$

with

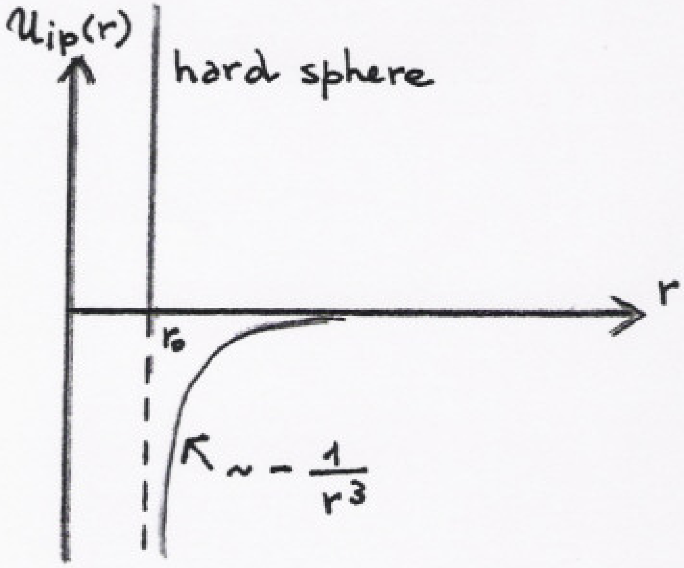
$$F(2mr) = 2mr\,K_1(2mr) \;\rightarrow\; \begin{cases} 1 & \text{(for } m \to 0) \\ \sqrt{\pi m r}\; e^{-2mr} & \text{(for } r \to \infty) \end{cases}$$

A qualitative estimate of the energy density for a dilute system (only two-body contributions) for an N-particle system can be obtained from the average interaction

$$\overline{U} = \frac{1}{V}\int U_{ip}(r)\, d^3r$$

using the estimate $\frac{1}{2}N(N-1)\,\overline{U} \approx \frac{N^2}{2}\,\overline{U}$.

For an interaction of the type

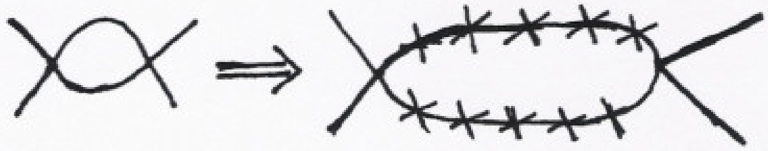


one gets an energy density

$$E(n) = \quad n\,m \quad + \quad \frac{\lambda n^2}{8m^2} \quad - \quad \frac{\lambda^2 n^2}{64\pi^2 m^2} \int_{r_0}^{r_{max}} \frac{dr}{r}$$

rest mass — short-range repulsion — long-range attraction

If one sets $r_{max} = r_{max}(n) \Rightarrow$ the interaction gets screened since the propagation takes place in a background



and transforms $n = \frac{1}{2} m\phi^2$, one reproduces the well known structure of the one-loop effective potential

$$V_{eff}(\phi) = \frac{1}{2} m^2\phi^2 + \frac{\lambda}{4!}\phi^4 + \frac{\lambda^2\phi^4}{256\pi^2} \ln\frac{\lambda\phi^2}{2\Lambda^2}$$

$(r_0 = 1/\Lambda)$

The phase transition associated with SSB is "weakly first order", namely, it occurs for a positive value $m = m_c > 0$. However, approaching the continuum limit m_c vanishes in units of the physical scale of the broken phase.

This can easily be checked computing the effective potential for m=0 (the "Coleman-Weinberg regime") and checking that there is a non-trivial minimum for $\phi \neq 0$.

$$V^{1-\text{loop}}(\phi) = \underbrace{\frac{\lambda}{4!}\phi^4}_{\text{classical energy}} + \underbrace{\frac{\lambda^2\phi^4}{256\pi^2}\left(\ln\frac{\lambda\phi^2}{2\Lambda^2} - \frac{1}{2}\right)}_{\text{zero-point energy of a field with mass } M^2(\phi) = (\lambda\phi^2/2)}$$

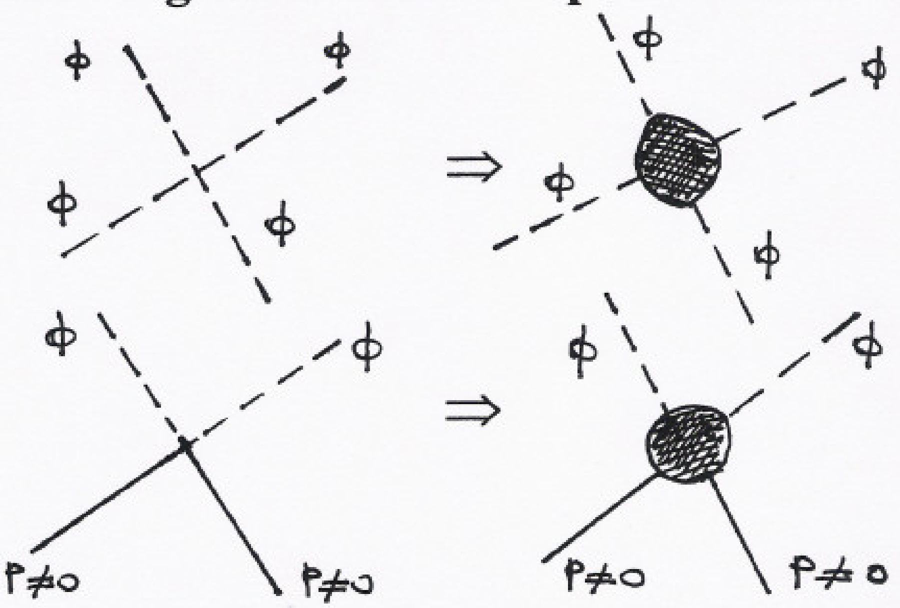
This exhibits non-trivial absolute minima $\phi = \pm v$ for

$$M^2(v) = \frac{\lambda v^2}{2} = \Lambda^2 e^{-32\pi^2/(3\lambda)} \equiv M^2{}_H$$

or

$$\lambda = \frac{16\pi^2}{3\ln(\Lambda / M_H)}$$

The one-loop structure can be given a non-perturbative meaning. Indeed, it reproduces itself in those approximations where the shifted fluctuation field is governed by a quadratic Hamiltonian (one-loop, Gaussian, post-gaussian). These approximations exhibit the following simultaneous replacements



Therefore, the "triviality-compatible" approximations to the effective potential, namely

$$\left\{ V^{1-\mathrm{loop}}, V^{\mathrm{gauss}}, V^{\mathrm{post-gauss}} \right\} \equiv V^{\mathrm{triv}}$$

display the same structure

$$V^{\mathrm{triv}}(\phi) = \frac{\tilde{\lambda}(\phi)}{4!}\phi^4 \quad + \quad \sum_k \sqrt{k^2 + \tilde{M}^2(\phi)}$$

with $$\tilde{M}^2(\phi) = \frac{\tilde{\lambda}}{2}\phi^2$$

Therefore, in leading order, one finds the same type of structure

$$\tilde{\lambda} = \frac{c}{\ln(\Lambda / M_H)} \approx \frac{M_H{}^2}{v^2}$$

where

$$M_H = \tilde{M}(\phi = v)$$

The "triviality-compatible" approximations exhibit a characteristic feature : a large logarithmic rescaling of the bare vacuum field. To this end, let us introduce the "physical" normalization of ϕ, say ϕ_R. This is defined from the condition

$$\left.\frac{d^2V_{eff}}{d\phi^2{}_R}\right|_{\phi_R = v_R} = M^2{}_H$$

Therefore, if we set

$$\phi^2{}_R = \frac{\phi^2}{Z_\phi}$$

we obtain

$$Z_\phi \approx \frac{v^2}{M^2{}_H} \approx \ln\frac{\Lambda}{M_H}$$

In terms of ϕ_R, the ("trivial") effective potential has the approximation-independent form

$$V^{triv}(\phi_R) = \pi^2\phi^4{}_R(\ln\frac{\phi^2{}_R}{v_R{}^2} - \frac{1}{2})$$

and one finds

$$M^2{}_H = 8\pi^2 v^2{}_R$$

in all approximations (one-loop, gaussian, post-gaussian).

Outside of the Coleman-Weinberg regime, one finds the more general expressions $(0 < \varsigma \leq 2)$

$$V^{triv}(\phi_R) = 2\pi^2\varsigma(\varsigma - 1)\phi^2{}_R(v^2{}_R - \phi^2{}_R) +$$

$$\pi^2\varsigma^2\phi^4{}_R(\ln\frac{\phi^2{}_R}{v^2{}_R} - \frac{1}{2})$$

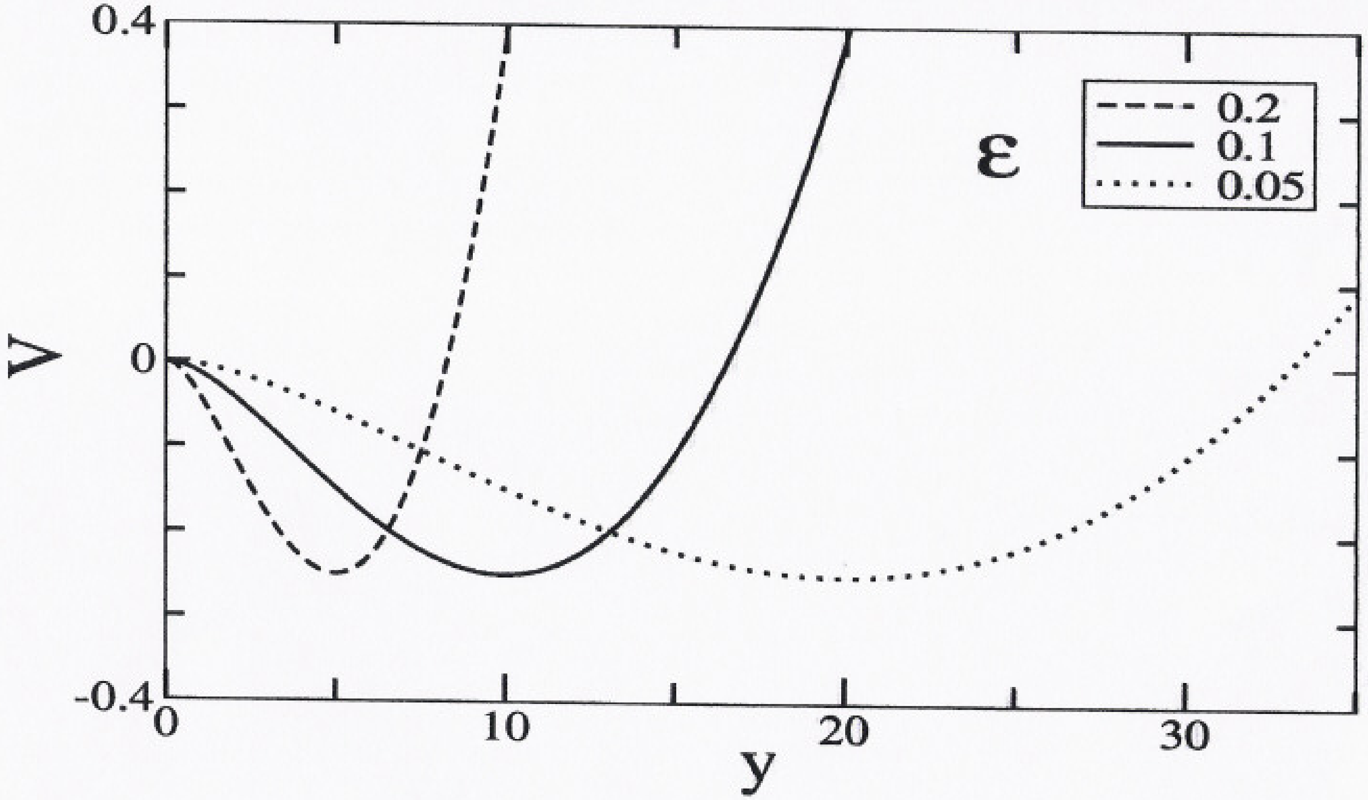
and

$$M^2{}_H = 8\pi^2\varsigma v^2{}_R$$

that reduce to the previous ones for $\varsigma = 1$**.**

M.C.
D. Zappalà
2004

$$V = \frac{\varepsilon^2 y^2}{2}\left[\ \ln \varepsilon y - 1/2\ \right]$$



The rescaling of the vacuum field, $Z = Z_\phi$, as defined above by matching the quadratic shape of the effective potential with the physical Higgs boson mass, should not be confused with the more conventional quantity, say $Z = Z_{prop}$, as defined by the residue of the shifted field propagator.

$Z = Z_{prop}$ can be related, through the Kallen-Lehmann representation to the normalization of the one-particle states. This representation can only exist for a field that admits an asymptotic Fock representation (as in QED).

In this case, one defines $\Phi(x,t) = \langle \Phi \rangle + h(x,t)$ so that

$$\langle \Phi(x,t)\Phi(0,0) \rangle_{conn.} = \langle h(x,t)h(0,0) \rangle = C(x,t)$$

Taking the Fourier transform, one finds

$$C(k,t) = \sum_\alpha | A_\alpha |^2 e^{-E_\alpha(k)t}$$

where $E_\alpha(k)$ are the α-particle energy eigenvalues for a given 3-momentum k.

In this framework

$$\mathbf{Z}_{\mathbf{prop}} = \frac{|\mathbf{A}_1|^2}{\sum_{\alpha} |\mathbf{A}_{\alpha}|^2} \leq 1$$

In the presence of SSB, the Fock representation exists only for the shifted fluctuation field (that has a vanishing expectation value) and is defined "after' fixing the vacuum state. For this reason, beyond perturbation theory, the field rescaling cannot be given as an overall "operatorial" condition of the type $\Phi_B = \sqrt{Z}\Phi_R$.

Physical interpretation of Z_ϕ.

$$M^2_H = \frac{\lambda v^2}{2} = 8\pi(na)$$

The "triviality" of the theory implies a continuum limit where the scattering length $a \rightarrow 0$. As in the non-relativistic example of the hard-sphere Bose gas, the mass scale

$$M^2_H \approx v^2_R$$

can remain fixed only if the particle density $n \rightarrow \infty$ in such a way that the product (na) remains fixed.

In the conventional perturbative picture, where vacuum field and fluctuations undergo the same unit rescaling, one identifies bare and physical vacuum fields so that

$$M^2_H \approx \frac{v^2}{\ln \Lambda} \approx \frac{(v^2_R)_{pert}}{\ln \Lambda}$$

Thus if $v^2 \approx (v^2_R)_{pert}$ is identified with the physical scale (say 246 GeV in the Standard Model) one predicts $M_H \rightarrow 0$ when $\Lambda \rightarrow \infty$. In particle language, this means to take the limit $a \rightarrow 0$ for n=constant. Therefore, the behaviour $Z_\phi \approx \ln \Lambda$, that provides a finite M_H in the continuum limit, is equivalent to a denser and denser scalar condensate.

Check of the picture with lattice simulations.

Define the theory on the lattice in terms of a lattice field

$$\langle \Phi_{latt} \rangle = v_B$$

(B="Bare") at a locality level $\Lambda \approx \pi / a$. Compute

$$\left.\frac{d^2 V_{eff}(\phi_B)}{d\phi^2_B}\right|_{\phi_B = v_B} = \frac{M^2_H}{Z_\phi} = \frac{1}{\chi_2(0)}$$

where $\chi_2(0)$ indicates the bare zero-momentum susceptibility. A lattice simulations where one compares the scaling properties of M_H and $\chi_2(0)$ can easily resolve the issue. If perturbation theory is right, assuming a unique trivial rescaling for vacuum field and fluctuations, a lattice calculation of

$$Z_\phi \equiv M^2_{latt} \chi_{latt}$$

should unambiguously approach unity in the continuum limit.

First method

Compute the propagator on the lattice and compare the lattice data to the (lattice version of the) two-parameter form

$$G(p) = \frac{Z_{prop}}{p^2 + m^2_{latt}}$$

Then compute χ_{latt} and compare Z_{prop} with $Z_\phi = m^2_{latt}\chi_{latt}$. Perturbation theory predicts $Z_\phi \approx Z_{prop}$ while the alternative interpretation of "triviality" predicts $Z_{prop} \approx 1$ and $Z_\phi \approx \ln\Lambda$.

Check these predictions with lattice data taken in the 4D Ising limit of $\lambda\Phi^4$ theory.

T. Lee
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2000

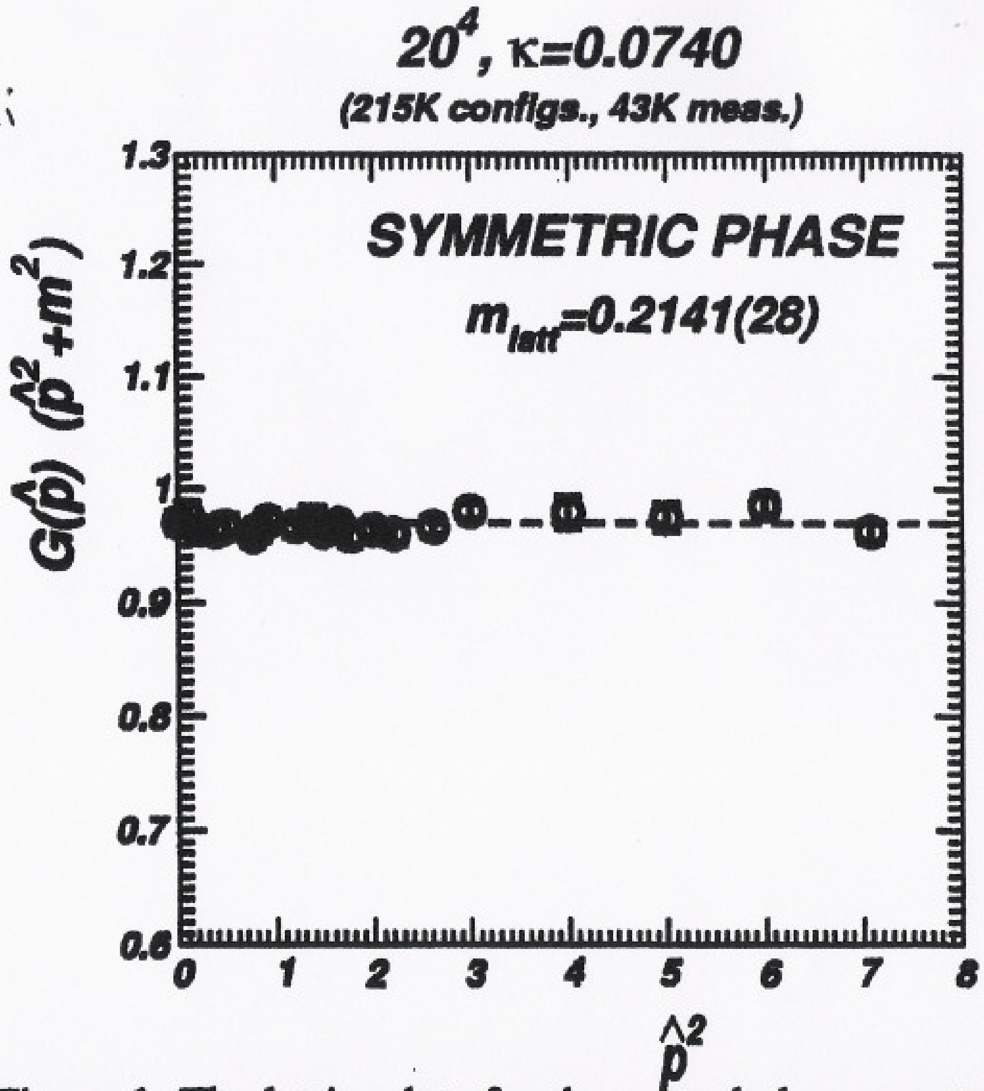

Figure 1. The lattice data for the re-scaled propagator at $\kappa = 0.0740$ in the symmetric phase. The zero-momentum full point is defined as $Z_\varphi = m^2_{\rm latt}\chi$. The dashed line indicates the value of $Z_{\rm prop}$.

spacing. To be confident that finite-size effects are

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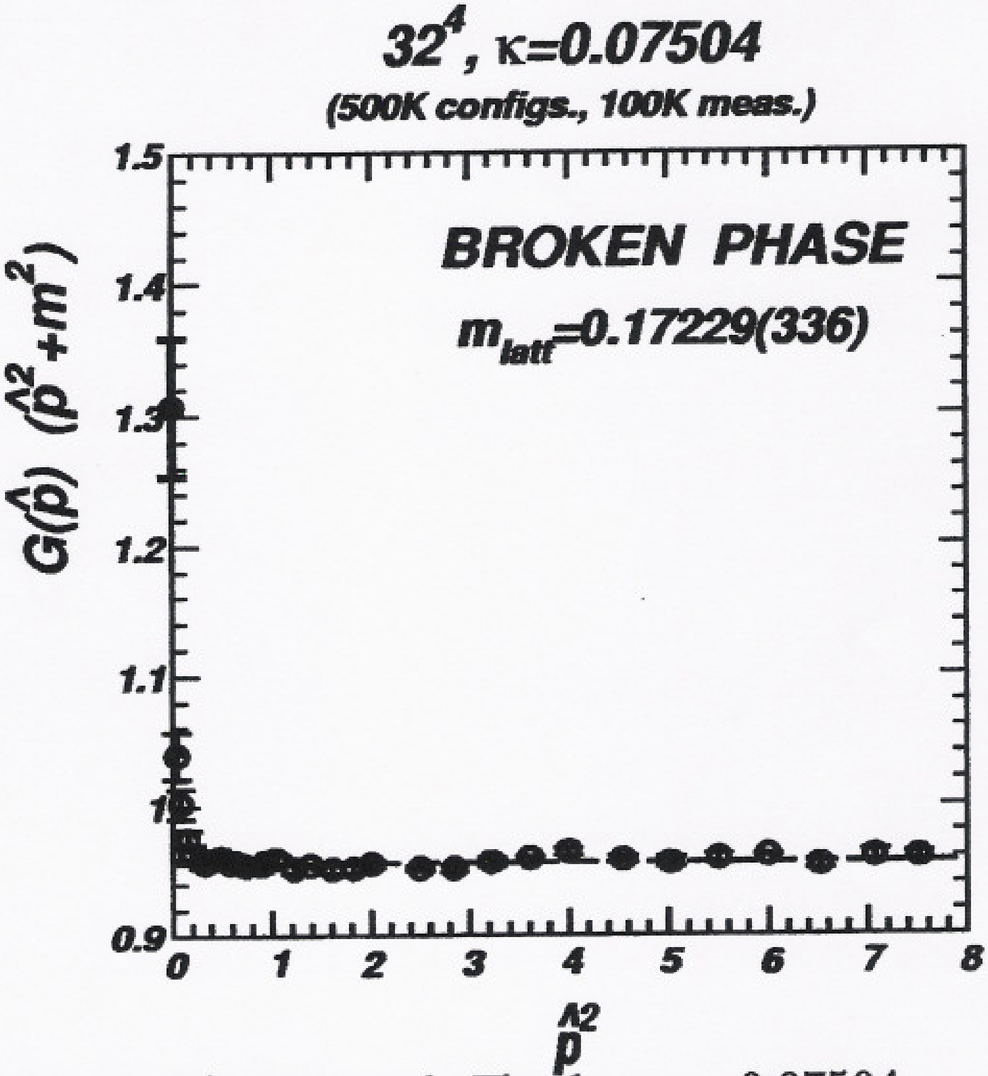

Figure 2. The same as in Fig. 1 at $\kappa = 0.07504$.

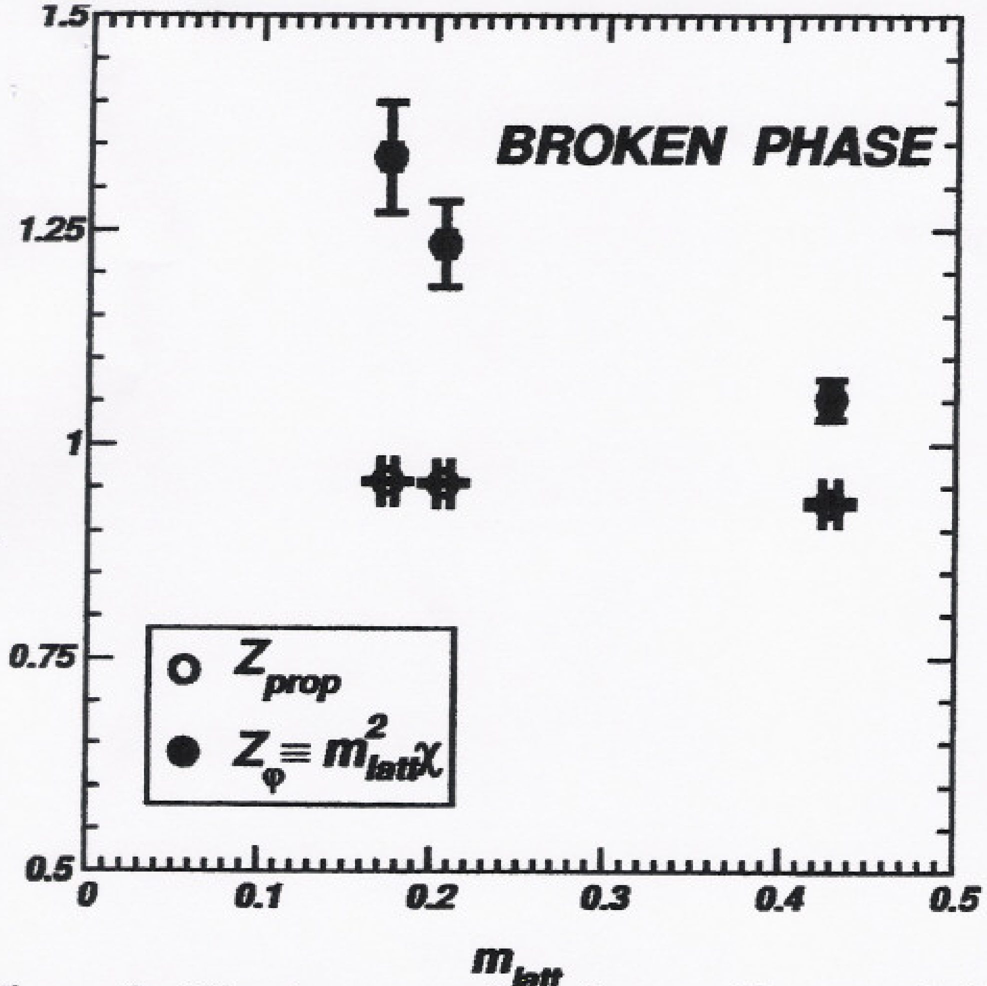

Figure 3. The two re-scaling factors $Z_{\rm prop}$ and Z_φ as a function of $m_{\rm latt}$. (The continuum limit corresponds to $m_{\rm latt} \to 0$.)

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2000

Second method

Again in the Ising limit, for any value of the hopping parameter κ in the broken phase, take the lattice mass from perturbation theory as an external input

$$m_{pert}(\kappa) = m_{input}(\kappa)$$

and just compute the lattice zero-momentum susceptibility $\chi_{latt}(\kappa)$. Using the central values reported in the Luescher-Weisz tables, one can check the consistency of the perturbative trend and compare at each κ the quantity

$$Z_{\phi} = 2\kappa m^2{}_{input}\, \chi_{latt}$$

with the perturbative estimate

$$Z_{LW} = 2\kappa Z_R$$

where Z_R is also given in the Luescher-Weisz tables.

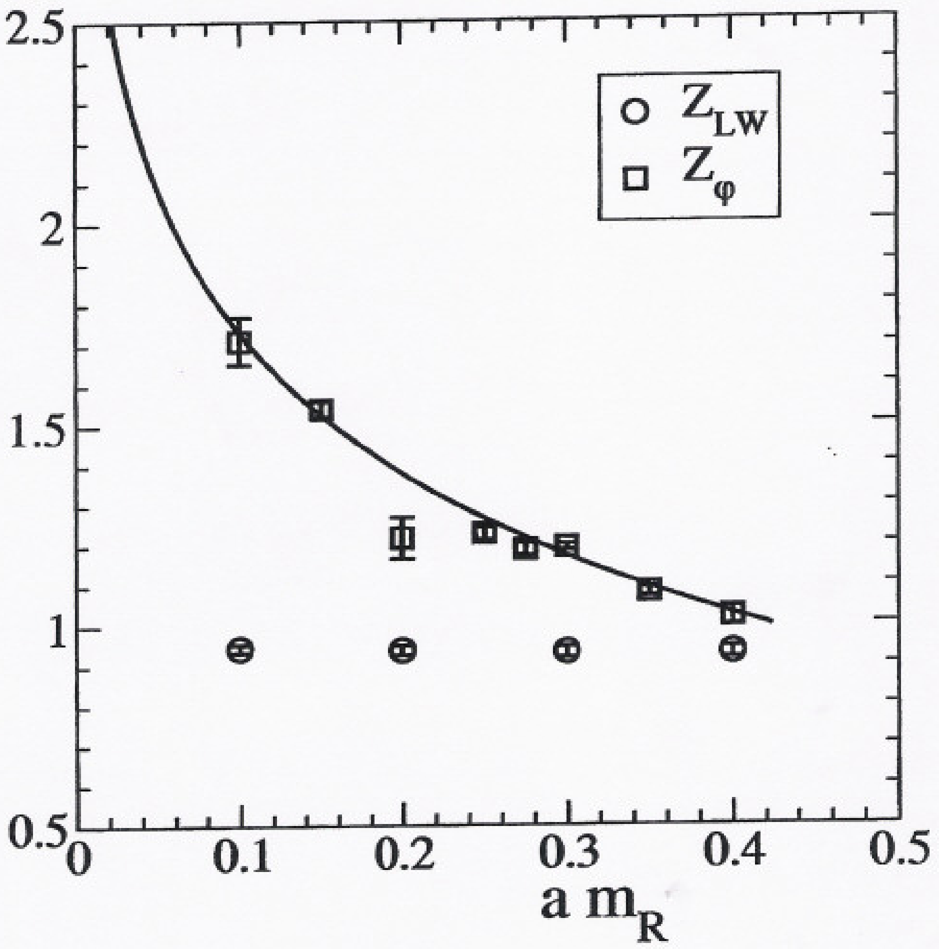

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Figure 1: The lattice data for Z_φ, as defined in Eq. (2.4), and its perturbative prediction Z_{LW} versus $m_{input} = am_R$. The solid line is a fit to the form Eq. (2.6) with $B = 0.50$.

Third method

With the first two methods one has to determine the value of the physical Higgs mass $m_R(\kappa)$ at each given value of the hopping parameter κ in the broken-symmetry phase. There is a third method, however, where this is not needed and one can get a precise lattice test just using the zero-momentum susceptibility and the bare vacuum field.

To this end, let us observe that both in perturbation theory and in the alternative picture of "triviality", one predicts

$$\frac{v^2_B}{m^2_R} \approx \ln(\Lambda / m_R)$$

Therefore in perturbation theory, where one predicts $\chi_2(0)m^2_R \approx 1$, one expects (PT=Perturbation-Theory)

$$\left[v^2_B \chi_2(0)\right]_{PT} \approx \ln(\Lambda / m_R)$$

On the other hand, in the alternative picture, where one predicts $\chi_2(0)m^2_R \approx \ln(\Lambda / m_R)$ one would rather expect (CS=Consoli-Stevenson)

$$\left[v^2_B \chi_2(0)\right]_{CS} \approx \ln^2(\Lambda / m_R)$$

These two predictions can be compared with the lattice data for the product $v^2{}_B\chi_{latt}$. The data can be fitted to the three-parameter form

$$\alpha\,|\,\ln|\,\kappa-\kappa_c\,||^\gamma$$

where α is a normalization constant and one has to set

$\gamma=1$	**Perturbation-Theory**
$\gamma=2$	**Consoli-Stevenson**

As one can check, the lattice data prefer unambiguously the fit with $\gamma=2$. In this case, one obtains a very precise determinations of the critical point $\kappa_c=0.074819(11)$ to compare with the value $\kappa_c=0.074834(15)$ obtained from the symmetric phase (Gaunt et al. 1979).

P. Cea
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2004

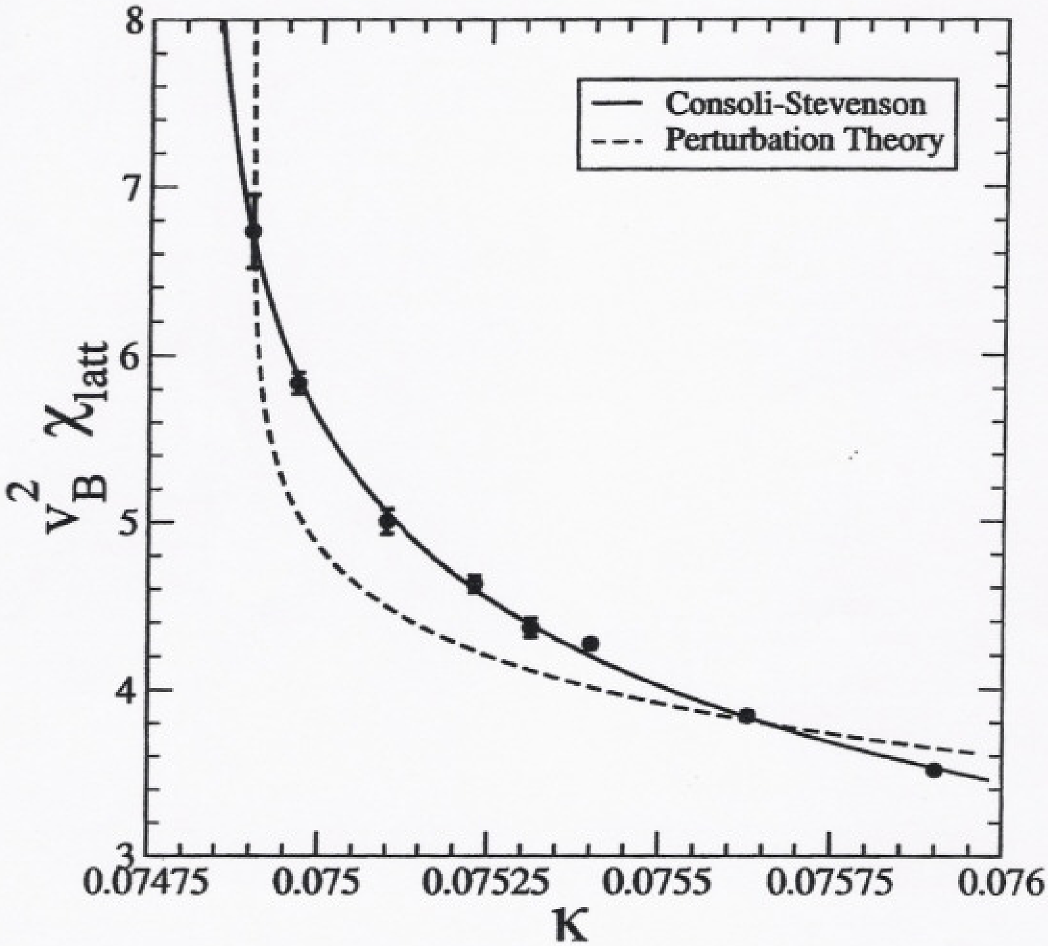

Figure 2: We show the lattice data for $v_B^2 \chi_{latt}$ of Table 3 together with the corresponding fit Eq. (3.4) for $\gamma = 2$ (Consoli-Stevenson) and $\gamma = 1$ (Perturbation Theory).

Conclusions

1) In QFT, SSB is described as a condensation phenomenon. This result is naturally recovered in the class of "triviality-compatible" approximations to the effective potential, $V_{eff} \equiv V_{triv}$, where the phase transition is (weakly) first-order.

2) The structure of V_{triv} is such that one needs a logarithmically divergent rescaling $Z_\phi \approx \ln \Lambda$, from the bare vacuum field v_B to the physical vacuum field v_R, in order the quadratic shape of V_{triv} to match the physical Higgs boson mass M_H.

3) As a consequence, M_H and v_R scale uniformly with Λ, differently from the perturbative predictions, and the ultraviolet cutoff becomes invisible to low-energy physics.

4) The large logarithmic rescaling Z_ϕ admits a physical interpretation in terms of a denser and denser condensate that compensates for the vanishingly small strength of the two-body coupling in the continuum limit ("triviality").

5) This type of picture is supported by the results of lattice simulations performed in the 4D Ising limit of $\lambda\Phi^4$ theory.