$N_F = 2$ QCD AS A PROBE OF CONFINEMENT

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SB.P

I. INTRODUCTION & MOTIVATION

- $N_F = 2$ QCD RELEVANT TO UNDERSTAND CONFINEMENT

- PHASE DIAGRAM [FIG 1]

  - TRANSITION LINE : PEAKS OF SUSCEPTIBILITIES ($C_V, \chi_{\langle\bar\psi\psi\rangle}, \chi_{\langle L\rangle} \ldots$)

  - $m = \infty$ ($m \gtrsim 3$ GeV) QUENCHED $\langle L \rangle$ Z_3

  - $m \simeq 0$ CHIRAL TRANSITION $\langle\bar\psi\psi\rangle$

  3 TRANSITIONS : - CHIRAL RESTORATION
  - $U_A(1)$ RESTORATION
  - DECONFINEMENT

  - INTERMEDIATE m ?

  Z_3 BROKEN ; CHIRAL SYMMETRY BROKEN BY MASS.

HOW TO DEFINE CONFINEMENT & DECONFINEMENT ?

$m_q = m_u = m_d \qquad N_F = 2$

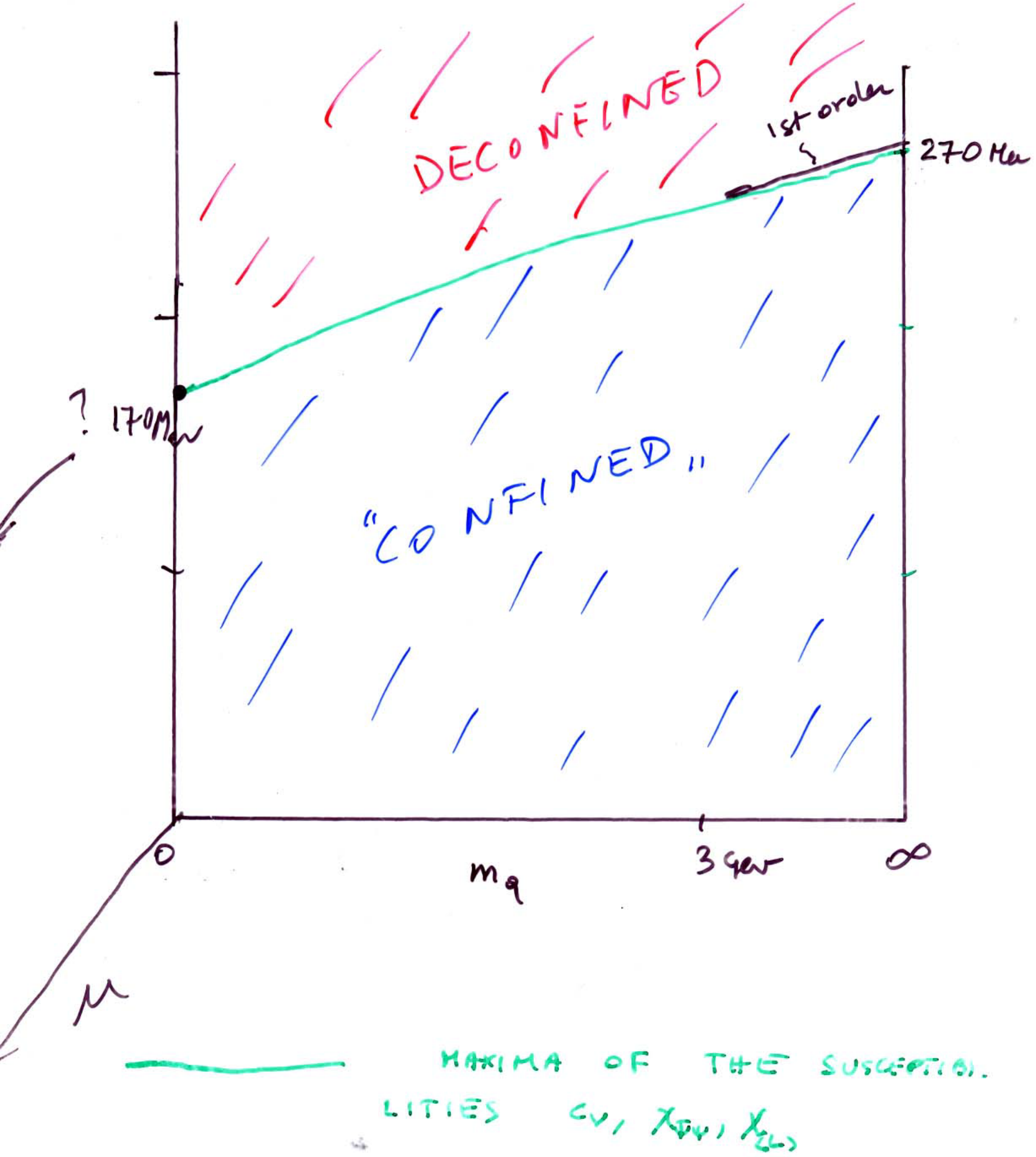
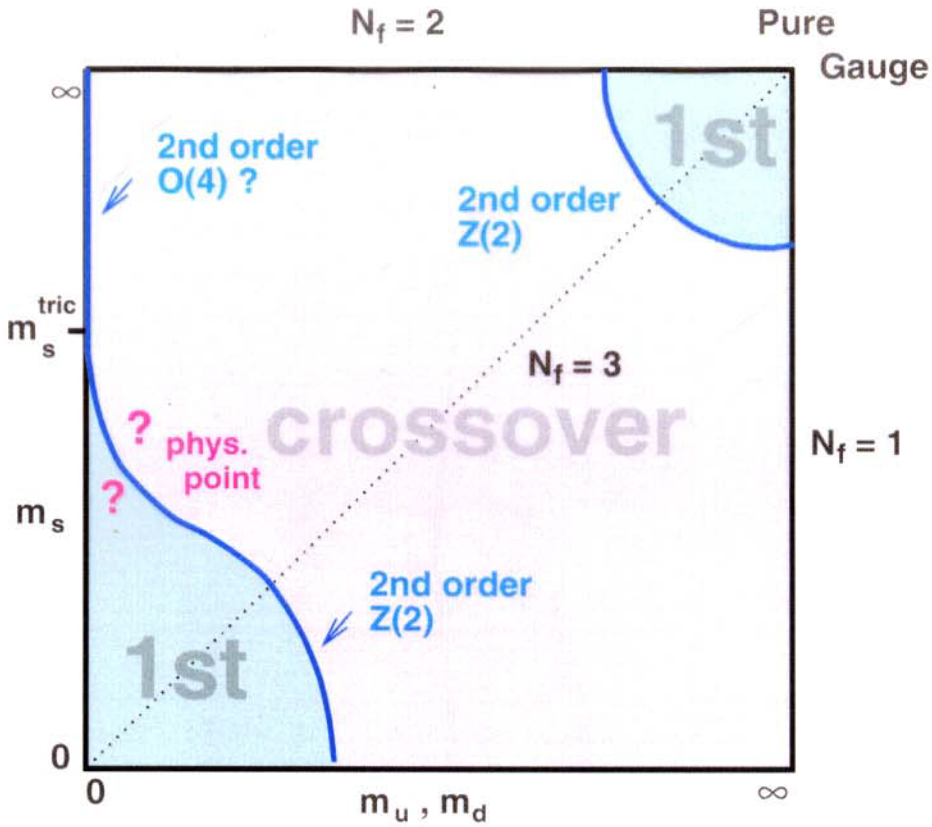


FIG 1



$N_f = 2$
Pure Gauge
∞
2nd order O(4) ?
1st
2nd order Z(2)
m_s^{tric}
$N_f = 3$
? phys. point
?
crossover
$N_f = 1$
m_s
2nd order Z(2)
1st
0
0
m_u, m_d
∞

. $m \simeq 0$ EFFECTIVE CHIRAL ACTION

[WILCZEK et al 82]

$$\underset{\sim}{\Phi} \; : \; \Phi_{ij} \equiv \langle \bar{\psi}_i (1+\gamma_5) \psi_j \rangle \quad i,j = 1 \ldots N_F$$

$$\underset{\sim}{\Phi} \longrightarrow e^{i\alpha} U_L(N_F) \Phi \, U_R(N_F) \qquad O(2) \otimes SU(N_f) \otimes SU(N_f)$$

$$\mathcal{L}_{eff} = Tr\{\partial_\mu \Phi^+ \partial^\mu \Phi\} - m_\Phi^2 Tr\{\Phi^+\Phi\} - g_1 Tr\{(\Phi^+\Phi)^2\}$$
$$- g_2 \left(Tr\{\Phi^+\Phi\}\right)^2 + c\left[\det\Phi + \det\Phi^+\right]$$

REN. GROUP ⊕ $4-\varepsilon$: LOOK FOR I.R. STABLE FIXED POINTS (2nd ORDER PHASE TRANSITIONS)

- $N_F \geq 3$ NO FIXED POINTS $\Rightarrow$ 1st ORDER
- $N_F = 2$
  - (A) $c \simeq 0$ AT T_c ($m_{\eta'} = 0$) $O(2) \times O(4)$ NO FIXED POINT (1st ORDER, 1ST ORDER ALSO at $m \neq 0$
  - (B) $c \neq 0$ AT T_c ($m_{\eta'} \neq 0$) : $O(4)$ I.R. FIXED POINT $\exists$. 2nd ORDER POSSIBLE : $O(4)$ UNIVERSALITY CLASS. CROSS-OVER at $m \neq 0$.

FIG 2

(A). NO TRICRITICAL POINT IN (μ, T) PLANE [STEPHANOV, SHURYAK]
AN ORDER PARAMETER $\exists$ FOR CONFINEMENT

(B.) TRICRITICAL POINT IN (μ, T) PLANE.
GO CONTINUOUSLY FROM CONFINED TO DECONFINED

- EXPERIMENTAL LIMITS ON CONFINEMENT

$$\left(\frac{n_q}{n_p}\right)_{obs} < 10^{-15}\left(\frac{n_q}{n_p}\right)_{S.C.M}$$

$$\sigma(p+p \to q(\bar{q})+x) < 10^{-15}\,\sigma_{Tot}(p+p)$$

. NATURAL EXPLANATION $\left(\frac{n_q}{n_p}\right) = 0$

$$\sigma(p+p \to q(\bar{q})+x) = 0$$

CONFINEMENT $\Longleftrightarrow$ SYMMETRY OF THE VACUUM

['tHOOFT 78]

$\Downarrow$

ORDER PARAMETER

- THE ISSUE IS FUNDAMENTAL AND CAN BE DECIDED BY LATTICE SIMULATIONS.

BIG NUMERICAL EFFORT. [

PREVIOUS WORK

~ INCONCLUSIVE

- COLUMBIA
- BIELEFELD
- TSUKUBA
- MILC

II. SCALING ANALYSIS

- LATTICE $L_t \times L_s^3$ — $L_t = 4$, $L_s = 12, 16, 20, 24\ 32$ | 1 Teraflop × year APE 1000.

$$\boxed{\tau \equiv 1 - \frac{T}{T_c}}$$

$$; \quad T = \frac{1}{L_t\, a(\beta, m)}$$

$$T_c = \frac{1}{L_t\, a(\beta_c, 0)}$$

$$\boxed{\tau \underset{m \to 0}{\simeq} C\left(\beta_c - \beta + K m\right)}$$

$$C = \left.\frac{\partial \ln a}{\partial \beta}\right|_{(\beta_c, 0)} \qquad C \cdot K = \left(\frac{\partial \ln a}{\partial m}\right)_{(\beta_c, 0)}$$

TERM Km OVERLOOKED BY PREVIOUS ANALYSES.

$\xi \equiv$ CORRELATION LENGTH OF THE ORDER PARAMETER

$$\boxed{\xi \underset{\tau \to 0_+}{\simeq} \tau^{-\nu}}$$

ν, α, γ, y_h CRITICAL INDICES

$\frac{a}{\xi} \simeq 0 \longrightarrow$ SCALING

$$\Downarrow$$

$$\boxed{\begin{array}{ll} (\times\times) & c_V - c_0 \simeq L_s^{\alpha/\nu}\, \phi_c\left(\tau L_s^{1/\nu}, m L_s^{y_h}\right) \\ (\times) & \chi_{\bar\psi\psi} \simeq L_s^{\gamma/\nu}\, \phi_\chi\left(\tau L_s^{1/\nu}, m L_s^{y_h}\right) \\ & \ldots \end{array}}$$

(××) ALWAYS TRUE: PREJUDICE INDEPENDENT

(×) TRUE IF $\bar\psi\psi$ IS THE CORRECT ORDER PARAMETER

(××) CAN BE USED TO LEGITIMATE THE ORDER PARAMETER

CRITICAL INDICES IDENTIFY ORDER & UNIVERSALITY CLASS OF THE TRANSITION

INDEX / TRANSITION	ν	y_h	α	γ
O(4)	.75(1)	2.49(1)	-.25(1)	1.47(2)
O(2)	.67(1)	2.48(1)	-.013(1)	1.31(2)
WEAK 1st ORDER	1/3	3	1	1
MEAN FIELD	2/3	9/4	0	1

- RUN AT $m L_s^{y_h}$ FIXED=F & $y_h = 2.49 \begin{cases} O(4) \\ O(2) \end{cases}$

$$C_V - C_0 \simeq L_s^{\alpha/\nu} \phi_c(\tau L_s^{1/\nu}, F) \Rightarrow \boxed{(C_V - C_0)_{peak} \propto L_s^{\alpha/\nu}}$$

$$\chi \simeq L_s^{\gamma/\nu} \phi_\chi(\tau L_s^{1/\nu}, F) \Rightarrow \boxed{\chi_{peak} \propto L_s^{\gamma/\nu}}$$

O(4) EXCLUDED $\chi^2/\mathrm{dof} \simeq 15$; O(2) $\chi^2/\mathrm{dof} \simeq 10$

$4\times 12^3, 4\times 16^3, 4\times 32^3, 4\times 20^3$ — INFRARED DOMINATED [Fig 3]

- CONTINUITY ARGUMENT

$$(C_V - C_0) \underset{m\to 0}{\simeq} m^{-\alpha/y_h\nu} f_c(\tau L_s^{1/\nu}) \Rightarrow \boxed{m^{\alpha/y_h\nu} \cdot (C_V - C_0)_{peak} = \mathrm{const}}$$

$$\chi \underset{m\to 0}{\simeq} m^{-\gamma/\nu y_h} f_\chi(\tau L_s^{1/\nu}) \Rightarrow \boxed{m^{\gamma/y_h\nu} \chi_{peak} = \mathrm{const.}}$$

O(2) O(4) EXCLUDED. COMPATIBLE WITH 1st ORDER [Fig 4,5]

- PEAK POSITION β_{max}

$$\boxed{\beta_c - \beta_{max} + K m = C/L^{1/\nu}} \quad m a \leq .08$$

COMPATIBLE WITH O(4) + FIRST ORDER & PREVIOUS WORKS [P4]

III DUAL-SUPERCONDUCTING VACUUM

$\langle \mu \rangle$ DISORDER PARAMETER

μ MAGNETICALLY CHARGED

[A.dG 93; AdG et al 2000; 2002]

$\langle \mu \rangle \neq 0 \quad T < T_c$ (MONOPOLE CONDENSATION; DUAL-SUPERCONDUCTIVITY)

$\langle \mu \rangle = 0 \quad T > T_c$ (NORMAL VACUUM)

[J. Carmona et al]

$$\langle \mu \rangle \underset{T \to T_c}{\simeq} L_s^{\rho} \, \Phi_\mu \left(\tau L_s^{1/\nu}, \, m L_s^{y_h} \right)$$

$$\langle \mu \rangle \underset{m \to 0}{\simeq} m^{\rho/y_h} \, \tilde{\Phi}_\mu \left(\tau L_s^{1/\nu} \right)$$

$$\langle \rho \rangle = \frac{\partial}{\partial \beta} \ln \langle \mu \rangle$$ (SUSCEPTIBILITY)

$$\boxed{\langle \rho \rangle / L_s^{1/\nu} = \Phi_\rho \left(\tau L_s^{1/\nu} \right)}$$

fig. [6, 7]

O(4) EXCLUDED

1st ORDER O.K.

$\langle \mu \rangle$ COMPATIBLE WITH c_V.

- ALREADY TESTED IN QUENCHED [AdG et al 2000]

Similar results in [CEA, COSMAI, D'ELIA 04]

THE (DIS)ORDER PARAMETER $\langle\mu\rangle$

WORKING HYPOTHESIS: CONFINEMENT PRODUCED BY DUAL SUPERCONDUCTIVITY OF THE VACUUM ['tHooft 75, Mandelstam 75, 'tHooft 81]

$\Downarrow$

DETECT HIGGS BREAKING WITH CONDENSATION OF MAGNETIC CHARGES.

— TWO MAIN ~~PHI~~ PHYLOSOPHIES (NOT EXCLUSIVE

a) LOOK FOR THE SYMMETRY PATTERN POSTPONING ANY COMMITMENT ON THE RELEVANT EXCITATIONS [PISA, BARI, ITEP]

b) LOOK FOR AN EFFECTIVE THEORY OF DOMINANT EXCITATIONS [KANAZAWA, ITEP, BERLIN..]

— TEST GROUND: U(1) G.T.

(a) [MARINO, SHROR, SWIECA, 82]

(b) [FRÖHLICH & AL 86] $\longrightarrow$ POLIKARPOV, WIESE 90
THEOREM

$\rightarrow$ DEL DEBBIO, A.DG., PAFFUTI 94 ; A.D.G, G.PAFFUTI 97

$$\mu(\vec{x},t) = \exp\left[-\int d^3y\, \vec{E}(\vec{y},t)\, \vec{b}_\perp(\vec{x}-\vec{y})\right]$$

$$\vec{\nabla}\vec{b}_\perp = 0 \qquad \vec{\nabla}\wedge\vec{b}_\perp = \frac{\pi}{e}\frac{\vec{r}}{r^3} + \text{Dirac String}$$

(a) - (b) EQUIVALENT : ADG, G PAFFUTI 97

(b) MORE PRACTICAL — ISING MODEL J.M CARMONA et al
— XY 3d DIGECIO ET AL

↑ • μ IS A MAGNETICALLY CHARGED GAUGE INVARIANT OPERATOR À LA DIRAC [FRÖHLICH-SPENCER 82]: GLOBAL SYMMETRY BREAKING [Schwinger, Elitzur]

↓ • μ IS ILL-DEFINED IN THE PRESENCE OF ELECTRIC CHARGES [FROLICH, MARCHETTI 2001]

— SU(N) GAUGE THEORY ; QCD

WITH OR WITHOUT DYNAMICAL QUARKS

$$\mu^a(\vec{x},t) = \exp\left(\int d^3y\, \mathrm{Tr}\left\{\overbrace{U(\vec{y},t)\,\phi^a_{diag}\,U^+(\vec{y},t)}^{\phi^a(\vec{y},t)}\vec{E}(\vec{y},t)\right\}\cdot \vec{b}_\perp(\vec{x}-\vec{y})\right)$$

$a = 1 \ldots N-1$

[A.DiG 93
Del Debbio et al 95; A. D.G et al 2000]

$$\vec{\nabla}\cdot\vec{b}_\perp = 0 \quad \vec{\nabla}\wedge\vec{b}_\perp = \frac{\pi}{g}\frac{\vec{r}}{r^3} + \text{Dirac string}$$

$$\phi^a_{diag} = \mathrm{diag}\Big(\underbrace{\tfrac{N-a}{N} \ldots \tfrac{N-a}{N}}_{a}, \underbrace{-\tfrac{a}{N} \ldots -\tfrac{a}{N}}_{N-a}\Big) \qquad \text{ORBIT}$$

– μ^a CREATES A DIRAC MONOPOLE IN THE REPRESENTATION IN WHICH ϕ^a IS DIAGONAL ALONG THE a'th ROOT. [ABELIAN PROJECTION]

↑ – μ^a IS GAUGE INVARIANT IF $\phi^a \underset{V}{\rightarrow} V\phi^a V^+$

↑ – $\langle\mu^a\rangle \neq 0$ SIGNALS DUAL SUPERCONDUCTIVITY

↑ – $\langle\mu^a\rangle \neq 0$ ABELIAN PROJECTION (U) INDEPENDENT STATEMENT. [JM CARMONA ET AL 2001]

DENSITY OF MONOPOLES IS FINITE : [FIG]

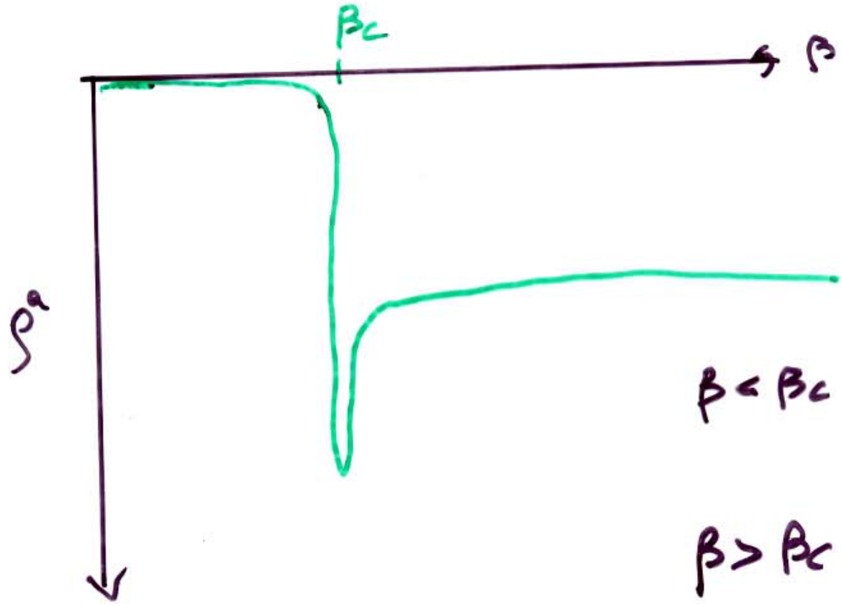
GAUGE TRANSFORMATION BETWEEN ANY TWO ABELIAN PROJECTIONS IS CONTINUOUS EXCEPT IN .. FEW ISOLATED POINTS.

↓ – ELECTRIC CHARGES PRESENT : μ^a AMBIGUOUS BY TERMS $O(a^2)$ [FROHLICH . MARCHETTI LATTICE 2001]
HOWEVER μ^a ITSELF DEFINED $O(a^2)$.
CONTINUUM LIMIT

— DETECTION TECHNIQUE $(V \to \infty)$ [Adda et al 2000 P.R] [cee comment]

$$\langle \mu^a \rangle = \frac{\langle Z^a \rangle}{\langle Z \rangle} \Rightarrow \rho^a = \frac{d}{d\beta} \ln \langle \mu^a \rangle$$

$$\langle \mu^a(\beta=0) \rangle = 1 \qquad \langle \mu^a \rangle_\beta = \exp\left[\int_0^\beta \rho^a \, d\beta\right]$$



$\beta < \beta_c$ ρ^a V INDEPENDENT $\langle \mu^a \rangle \underset{\to \infty}{\neq} 0$

$\beta > \beta_c$ $\rho^a \simeq -K L_s + K'$ [fig]
$\rho^a \xrightarrow[L_s \to \infty]{} -\infty$ $\langle \mu^a \rangle = 0$

$\beta \simeq \beta_c$ FINITE SIZE SCALING : QUENCHED

$$\langle \mu^a \rangle \simeq \tau^\delta \, \Phi_{\mu^a}(\tau L_s^{1/\nu})$$

$$\rho^a \simeq \frac{\delta \, L_s^{1/\nu}}{\tau L_s^{1/\nu}} + L^{1/\nu} \frac{\Phi'}{\Phi_{\mu^a}} \Longrightarrow \rho^a / L_s^{1/\nu} = f(\tau L_s^{1/\nu})$$

$(\rho^a_{peak} \,|\, \propto L_s^{1/\nu}$

SU(2) $\longrightarrow$ $\nu = .63(3) \to$ 3d ISING
SU(3) $\nu = 1/3$ 1ST ORDER [Adda et al 2000 P.R]
CONSISTENT WITH $\langle L \rangle$ & C_V [Bielefeld]

IR7

IV CONCLUSIONS & OUTLOOK

- $N_F = 2$ QCD RELEVANT FOR CONFINEMENT

- IF CHIRAL TRANSITION IS 1st ORDER DECONFINEMENT CAN BE ORDER - DISORDER

  NO TRICRITICAL POINT AT $\mu \neq 0$

  $m_{\eta'} \simeq 0$ AT T_c

- IF 2nd ORDER + CROSSOVER

  $m_{\eta'} \neq 0$ AT T_c ; A TRICRITICAL POINT $\exists$ AT $\mu \neq 0$.

  DECONFINEMENT NOT A TRANSITION

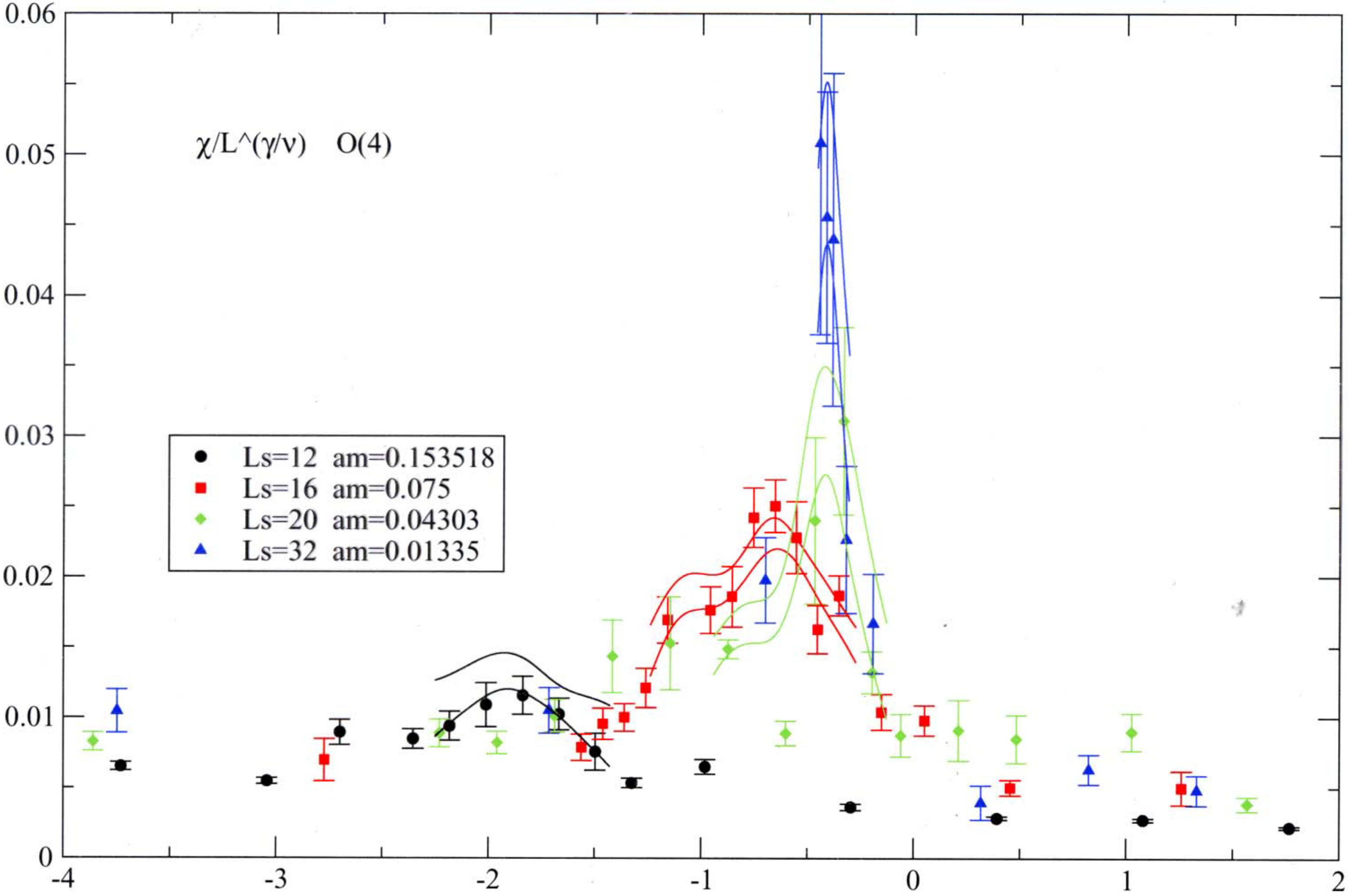
- STRONG EVIDENCE THAT THE TRANSITION IS 1st. ORDER

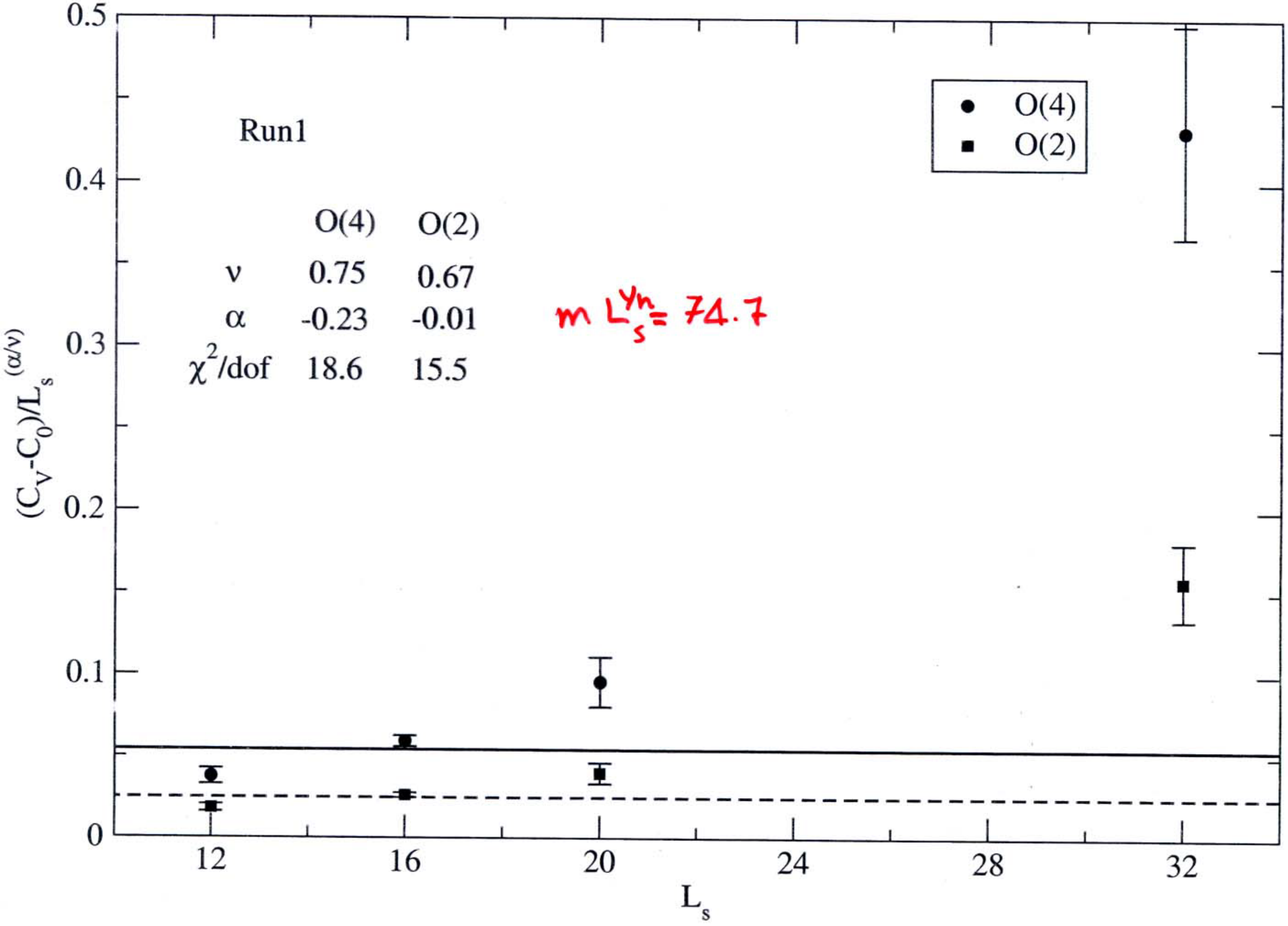
  $\langle \mu \rangle$ A GOOD ORDER PARAMETER

- REPEAT THE ANALYSIS WITH $L_t = 6$ & IMPROVED ACTION

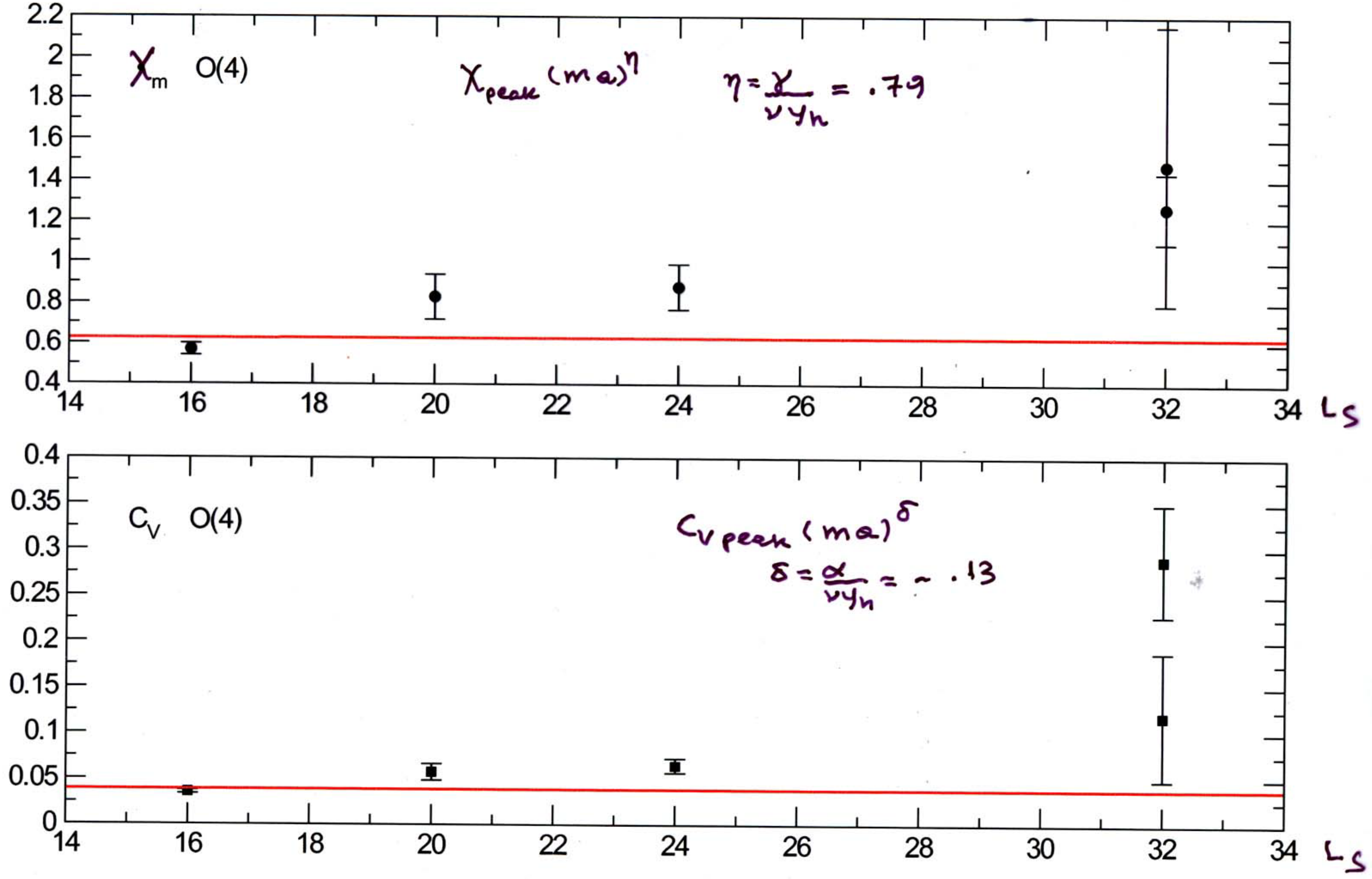
  CHECK $m_{\eta'}$ AT T_c.

RUN 1 $amL_s^{2.44} = const = 74.46$



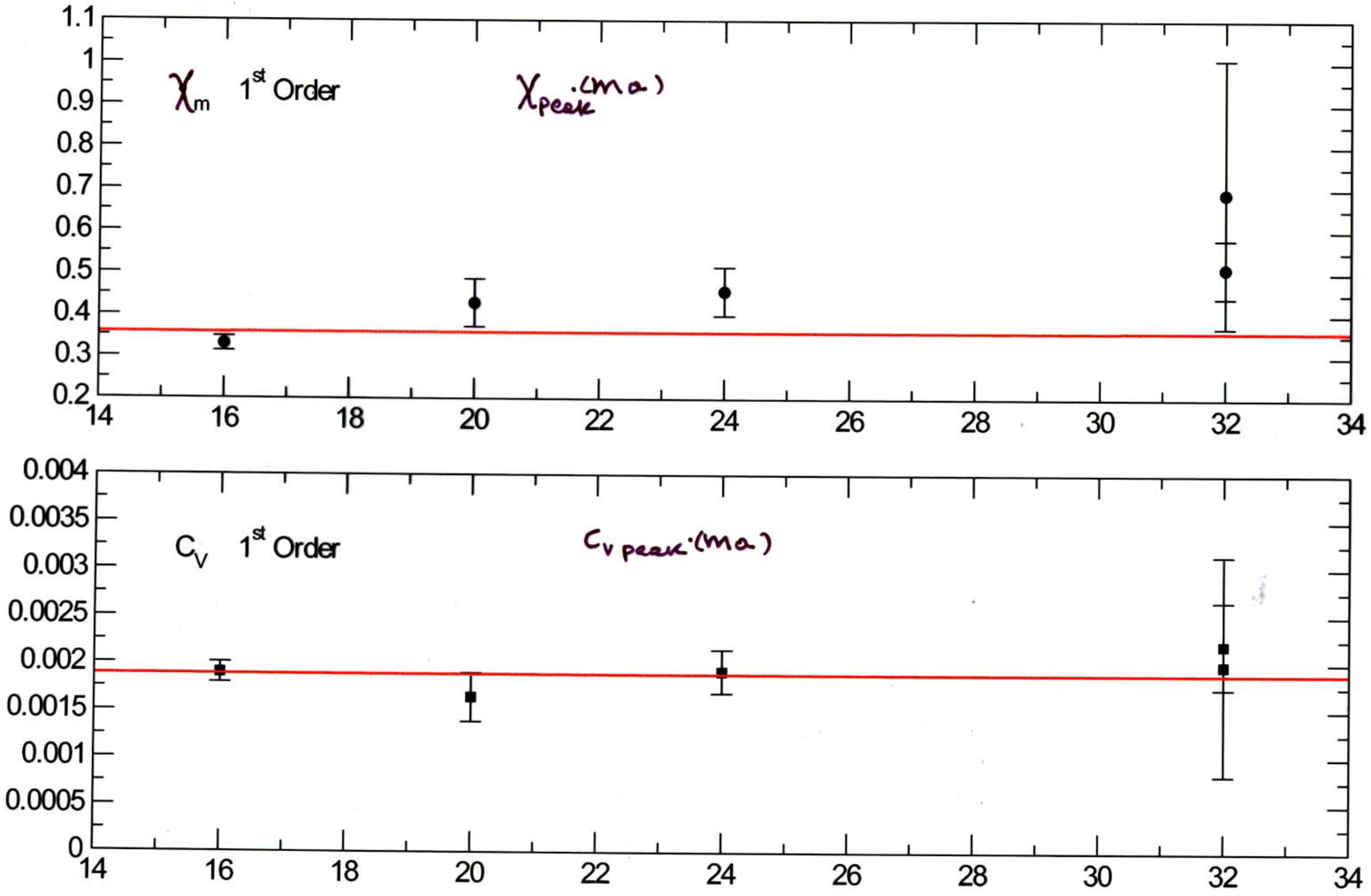


Run1
O(4) O(2)
ν 0.75 0.67
α -0.23 -0.01
χ²/dof 18.6 15.5
$m L_s^{y_h} = 74.7$
O(4)
O(2)
$(C_V - C_0)/L_s^{(\alpha/\nu)}$
L_s
0
0.1
0.2
0.3
0.4
0.5
12
16
20
24
28
32



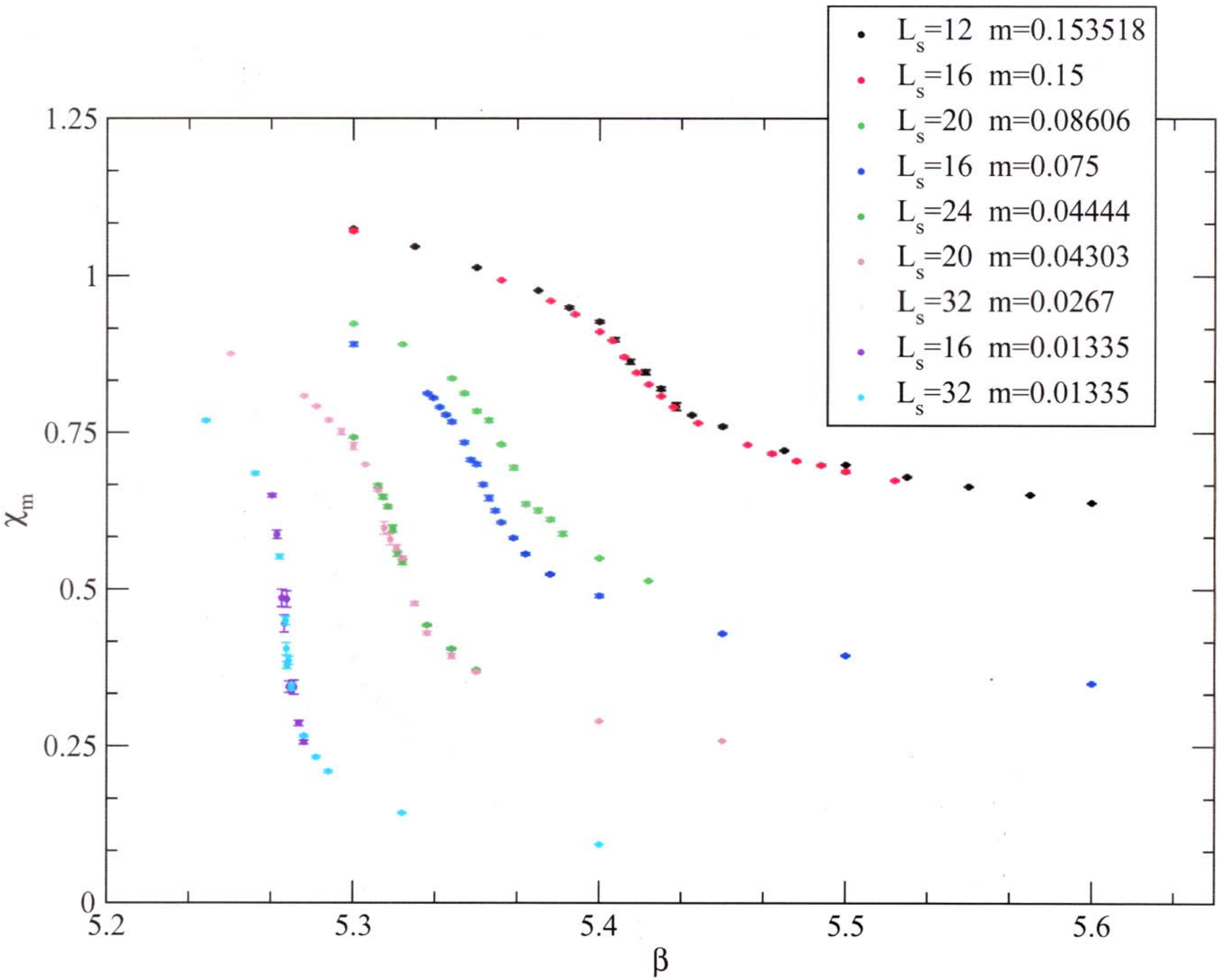
χ_m O(4)
$\chi_{peak}(ma)^{\eta}$
$\eta = \frac{\gamma}{\nu y_h} = .79$
C_V O(4)
$C_{V\,peak}(ma)^{\delta}$
$\delta = \frac{\alpha}{\nu y_h} = \sim .13$
L_s

FIG 4

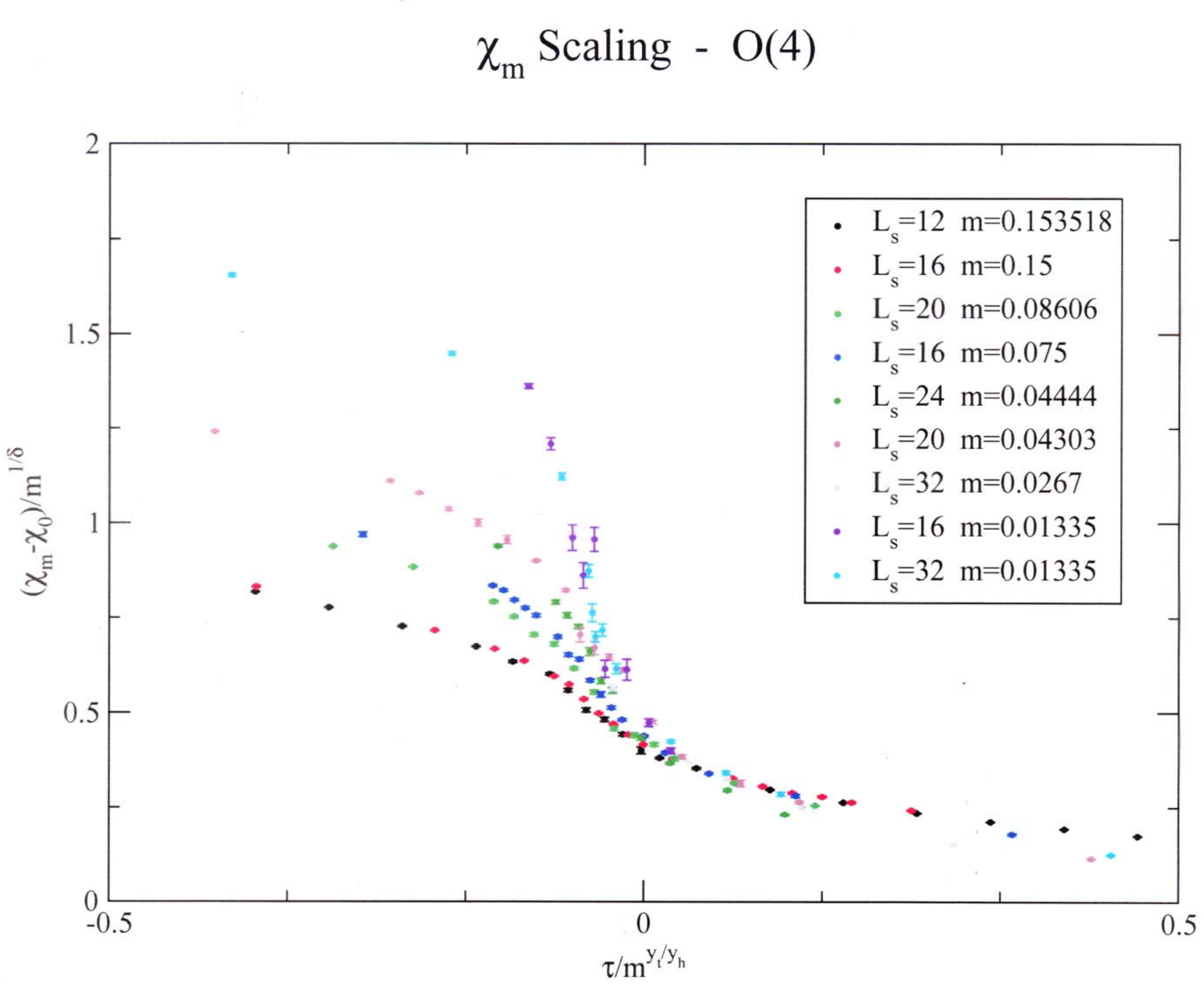


χm 1st Order
χpeak·(ma)
Cv 1st Order
Cv peak·(ma)
1.1
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.004
0.0035
0.003
0.0025
0.002
0.0015
0.001
0.0005
0
14
16
18
20
22
24
26
28
30
32
34

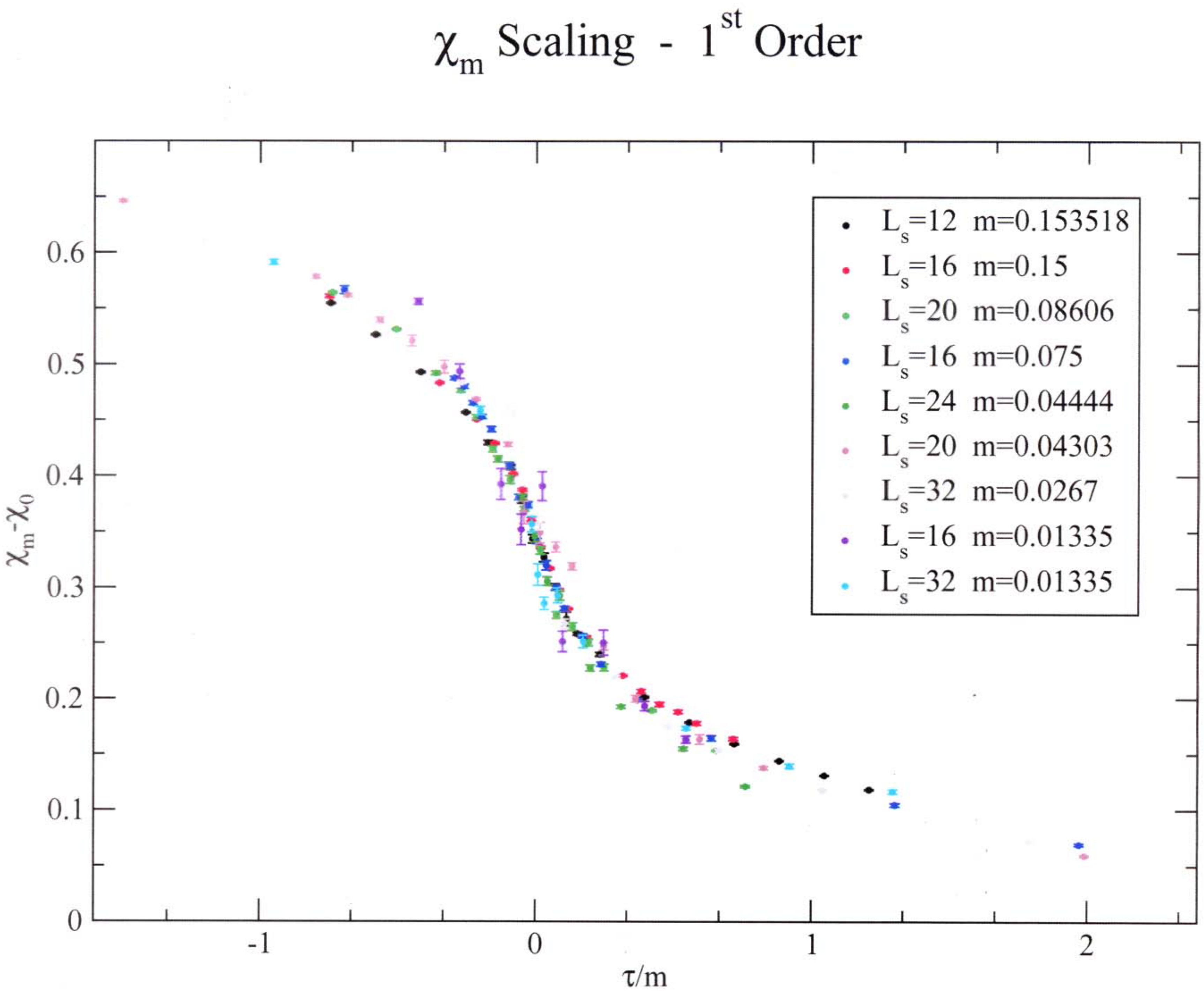
FIG 5



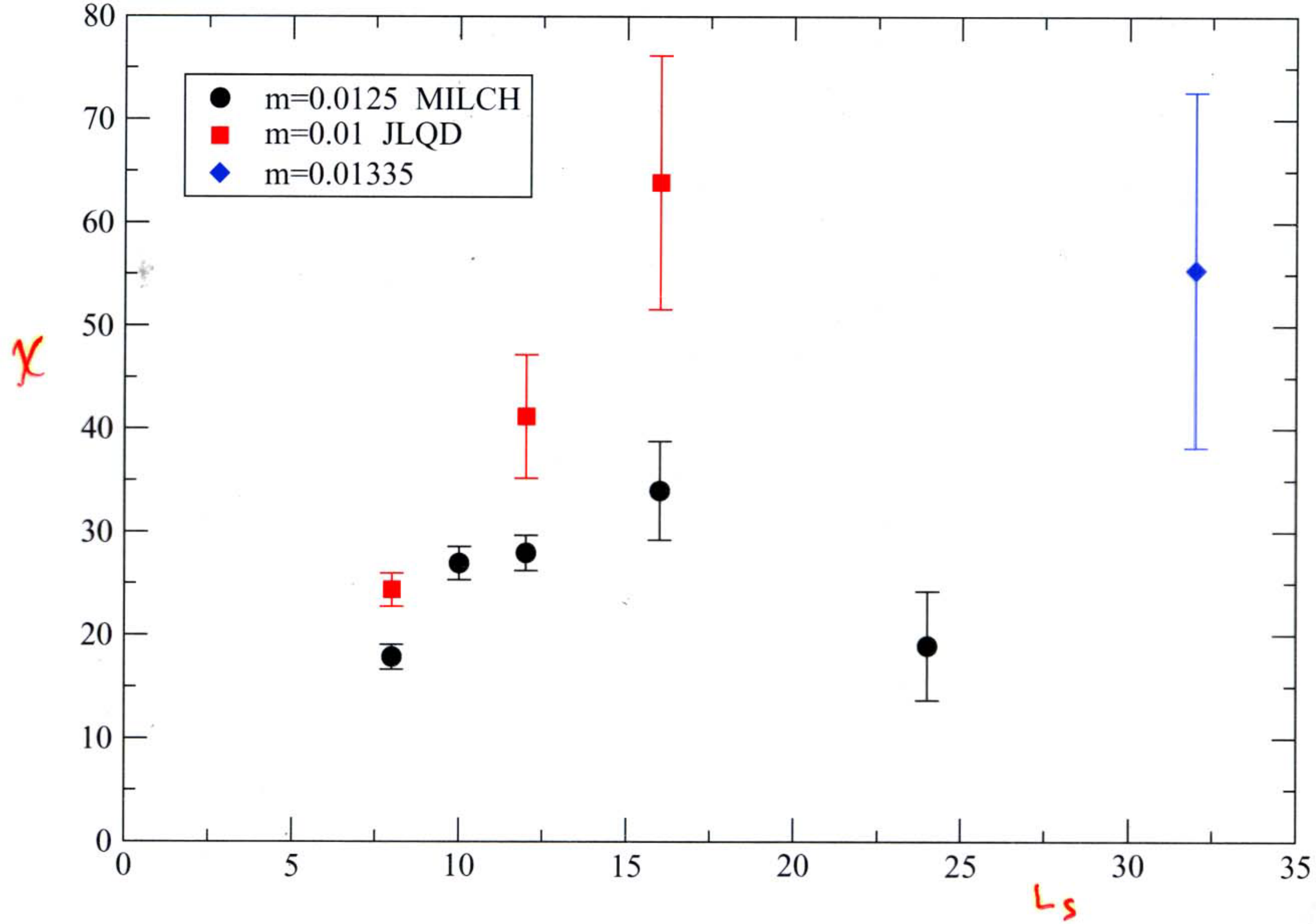
L_s=12 m=0.153518
L_s=16 m=0.15
L_s=20 m=0.08606
L_s=16 m=0.075
L_s=24 m=0.04444
L_s=20 m=0.04303
L_s=32 m=0.0267
L_s=16 m=0.01335
L_s=32 m=0.01335
1.25
1
0.75
0.5
0.25
0
5.2
5.3
5.4
5.5
5.6
χm
β



χm Scaling - O(4)
(χm-χ0)/m1/δ
τ/m^(yt/yh)
0
0.5
1
1.5
2
-0.5
0
0.5
Ls=12 m=0.153518
Ls=16 m=0.15
Ls=20 m=0.08606
Ls=16 m=0.075
Ls=24 m=0.04444
Ls=20 m=0.04303
Ls=32 m=0.0267
Ls=16 m=0.01335
Ls=32 m=0.01335

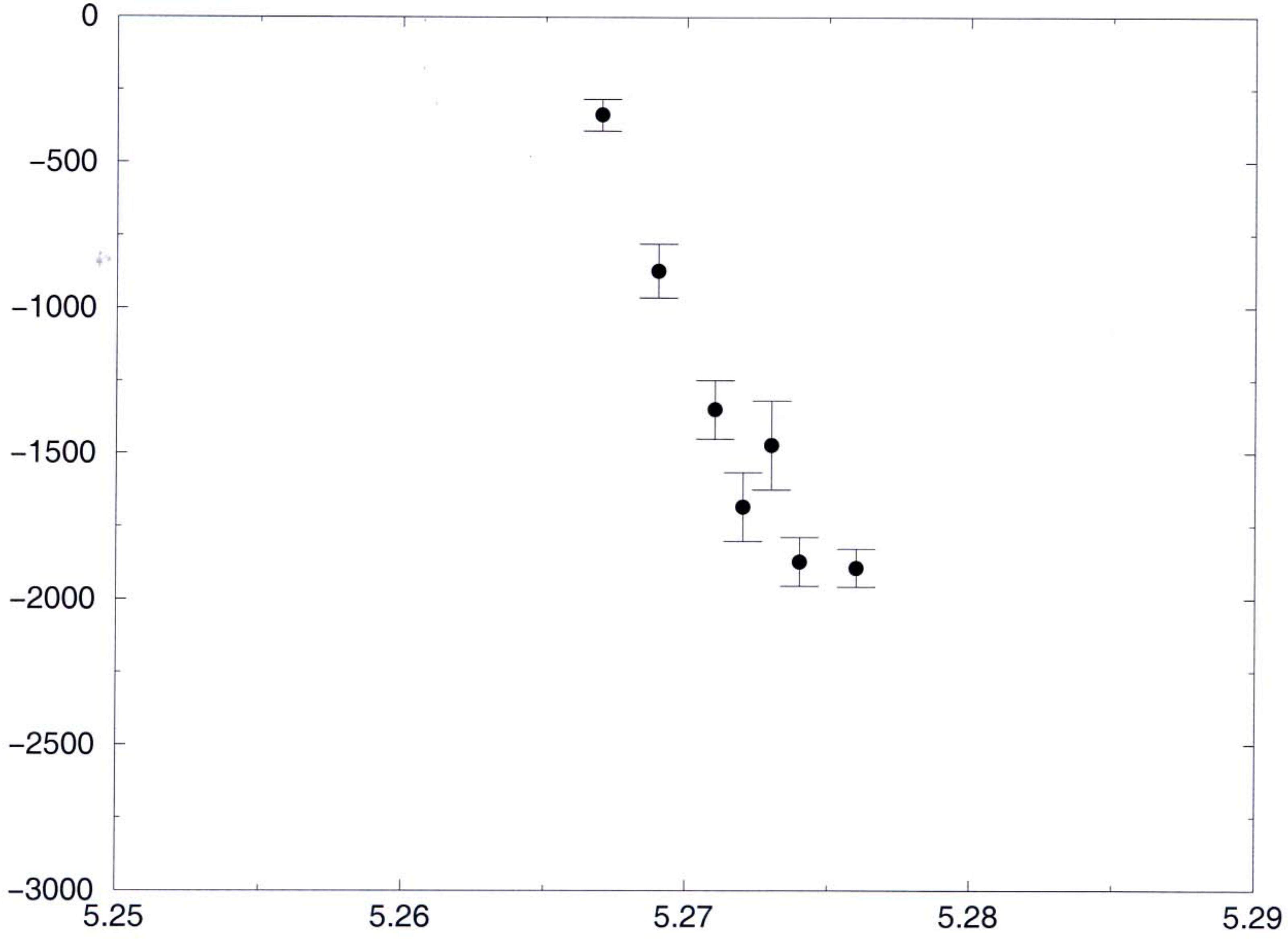


χm Scaling - 1st Order
χm-χ0
τ/m
0
0.1
0.2
0.3
0.4
0.5
0.6
-1
0
1
2
Ls=12 m=0.153518
Ls=16 m=0.15
Ls=20 m=0.08606
Ls=16 m=0.075
Ls=24 m=0.04444
Ls=20 m=0.04303
Ls=32 m=0.0267
Ls=16 m=0.01335
Ls=32 m=0.01335

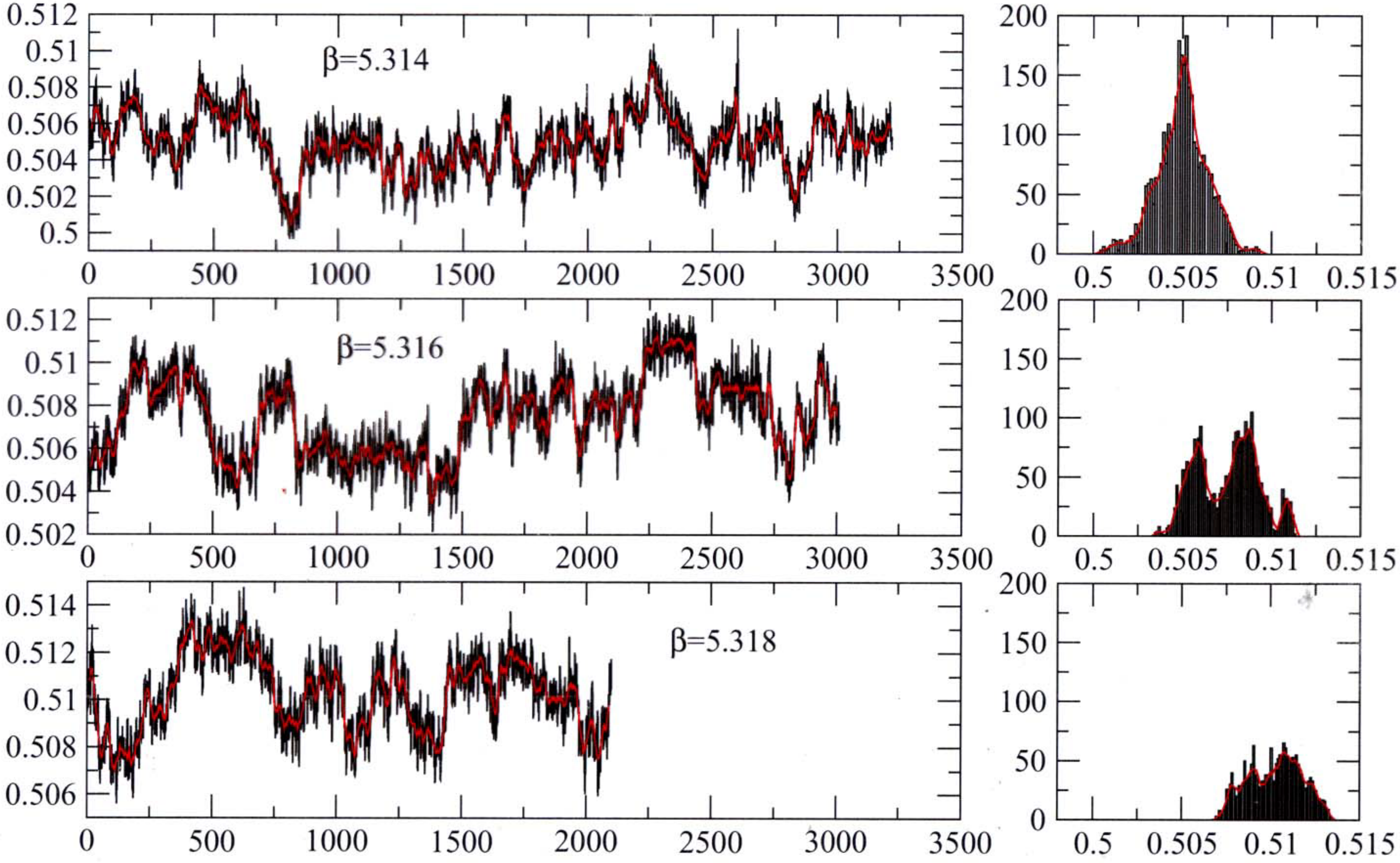


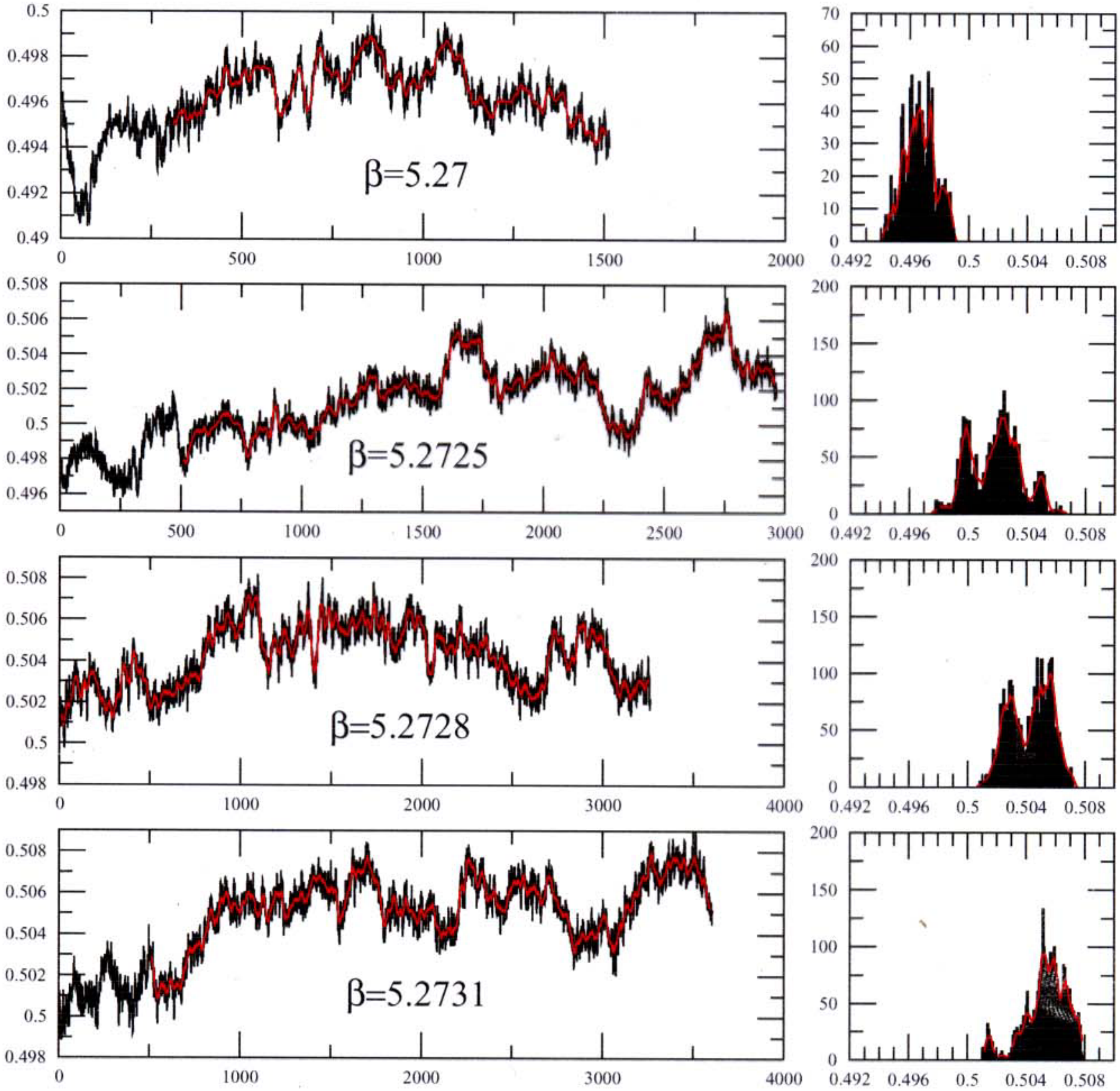
m=0.0125 MILCH
m=0.01 JLQD
m=0.01335
χ
L_s
0
10
20
30
40
50
60
70
80
5
15
25
35

rho L = 16 am = 0.01335



24^3x4 m=0.04444




β=5.27
β=5.2725
β=5.2728
β=5.2731
0
500
1000
1500
2000
2500
3000
4000
0.49
0.492
0.494
0.496
0.498
0.5
0.502
0.504
0.506
0.508
0
10
20
30
40
50
60
70
100
150
200

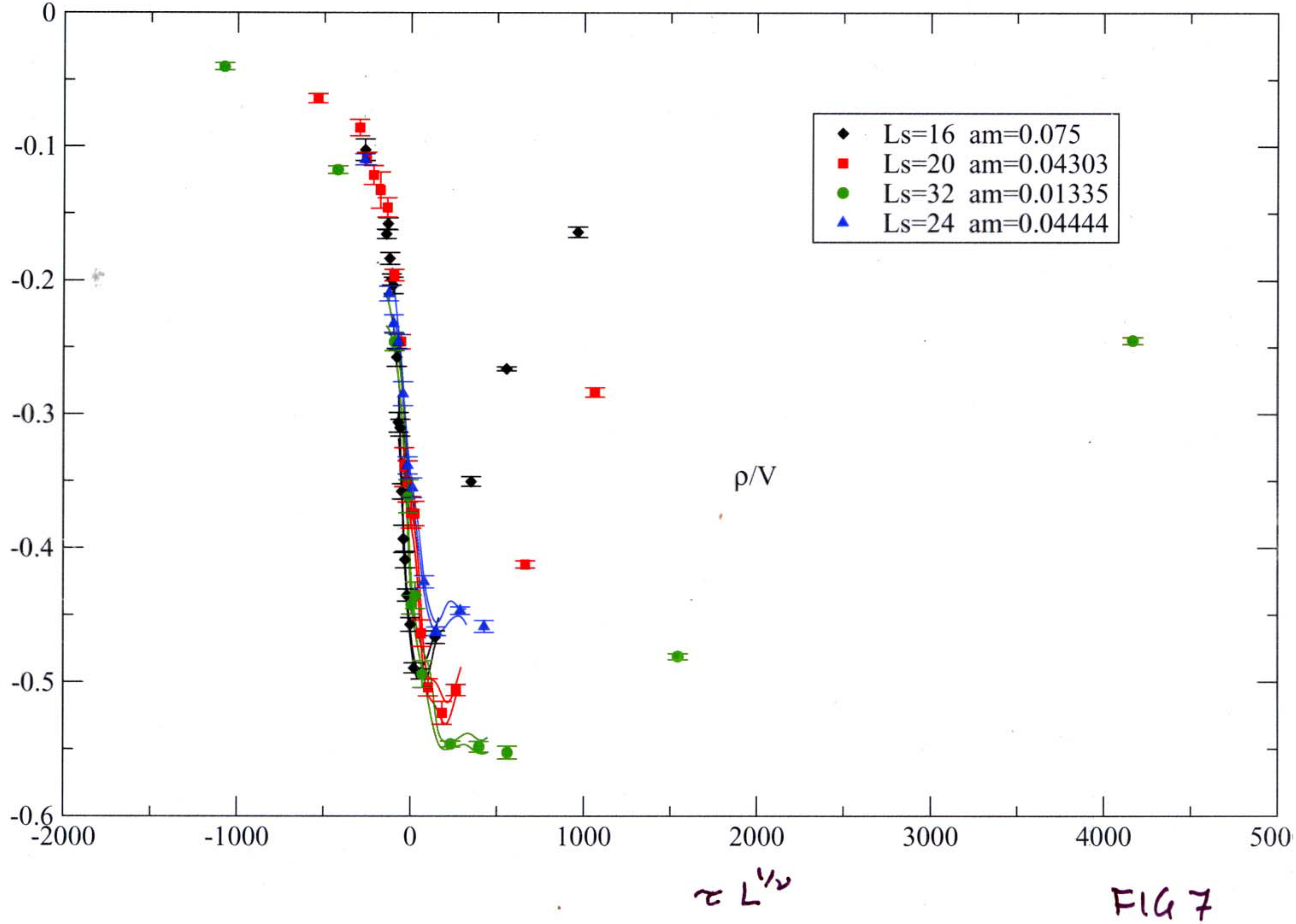
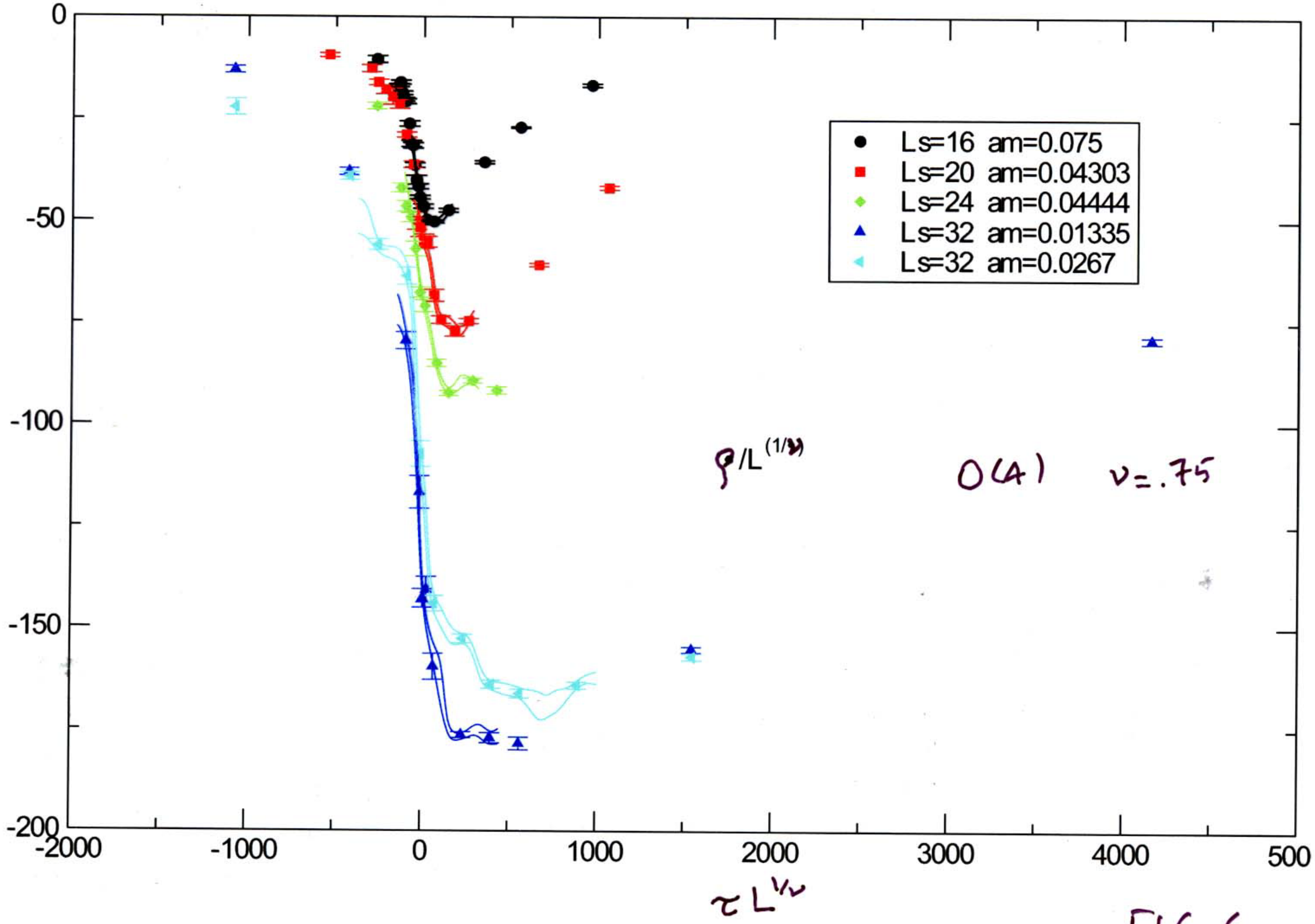

Ls=16 am=0.075
Ls=20 am=0.04303
Ls=32 am=0.01335
Ls=24 am=0.04444
ρ/V
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-2000
-1000
0
1000
2000
3000
4000
500
$\tau L^{1/\nu}$

FIG 7



0
-50
-100
-150
-200
-2000
-1000
0
1000
2000
3000
4000
500
Ls=16 am=0.075
Ls=20 am=0.04303
Ls=24 am=0.04444
Ls=32 am=0.01335
Ls=32 am=0.0267
$\rho/L^{(1/\nu)}$
O(4) $\nu=.75$
$\tau L^{1/\nu}$

FIG 6