QED_3 on a space-time lattice: compact versus noncompact formulation

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in collaboration with

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SMFT 2004

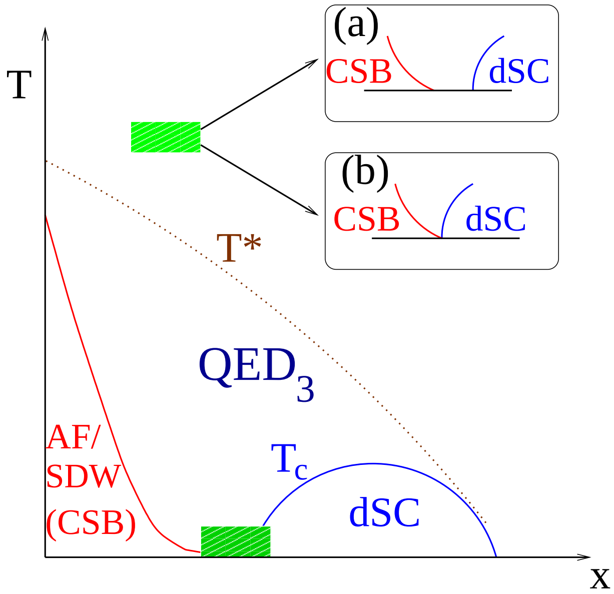
Bari (Italy), September 29 - October 1, 2004

Plan of the talk

- **Introduction**
  - **QED_3 as an effective theory for high-T_c superconductors**
  - **QED_3 as a laboratory for the theoretical investigation of mechanisms of confinement**
- **The model and its properties**
- **Lattice formulation**
- **Compact versus noncompact formulation**
  [H.R. Fiebig and R.M. Woloshyn, Phys. Rev. D **42**, 3520-3523 (1990)]
- **Numerical results**
- **Summary and conclusions**

QED_3 as an effective theory for high-T_c superconductors

- **Cuprates are superconductors which manifest a superconducting behaviour at high temperature and exhibit a pseudogap phase**



$\Rightarrow \quad \langle \bar{\psi}\psi \rangle = 0$

$\Rightarrow \quad \langle \bar{\psi}\psi \rangle \neq 0$

[M. Franz et al., Phys. Rev. B 66, 054535 (2002)]

AF/SDW $\rightarrow$ **antiferromagnet/spin density wave**
d**-SC** $\rightarrow$ d**-wave superconductor**

- **What is the nature of the $T = 0$ phase?** $\rightarrow$ **At $T = 0$, when the phase coherence is lost, the phase incoherent d-wave pseudogap phase is nothing but the insulating SDW, that is a weak antiferromagnet**

- **An effective theory of underdoped cuprates is provided by QED_3 where**

  1. **Fermion fields $\rightarrow$ spin $1/2$ excitations of the superconducting states (SPINONS) minimally coupled to a massless gauge field**
  2. **Gauge field $\rightarrow$ field derived from the fluctuating topological defects in the superconducting phase**

- **The quantum phase transition in cuprates is an example of spontaneous breaking of continuous global symmetry in QED_3, the chiral symmetry**

- **Chiral symmetry breaking (CSB) in QED_3 is strongly related to N_f $(1 \div 3)$. There are several studies and numerical attempts to state this, but the question is still open**

  [S.J. Hands et al., Nucl. Phys. B645, 321-336 (2002)]

QED_3 and confinement

- **A particular formulation of QED in 3 Euclidean dimensions is proven to possess the property of permanent confinement [A.M. Polyakov, Nucl. Phys. B120 (1977) 429-458]**

- **Any pair of test electric charge and anti-charge is confined by a linear potential**

- **This has been obtained by considering instanton solutions of the theory, which in 3 dimensions are magnetic monopoles**

- **Confinement of electrically charged particles is caused by a plasma of monopoles**

- **Monopoles are topological defects which appear when we consider a compact gauge group ($cQED_3$)**

- **Moreover, this is true for $cQED_3$ pure gauge, but it is not clear what happens when the gauge field is coupled to matter**

- **For example Herbut et al. argue that $cQED_3$ with massless fermions is always in the confined phase [I.F. Herbut and B.H. Seradjeh, Phys. Rev. Lett. 91, 171601 (2003)]**

Compact and noncompact QED_3

In order to study the chiral condensate and the confinement in QED_3, which are nonperturbative issues, we need to discretize the theory on a lattice. There are two formulations

- **Compact QED_3 $\rightarrow$ The d.o.f. are the link variables**
- **Noncompact QED_3 $\rightarrow$ The d.o.f. are the phases of the link variables**

Noncompact QED_3 is widely used as an effective theory for high-temperature superconductors

[S.J. Hands, J.B. Kogut and C.G. Strouthos, Nucl. Phys. B645 (2002) 321-336]

[I.F. Herbut, Phys. Rev. B 66 (2002) 094504]

[N.E. Mavromatos, J. Papavassiliou, cond-mat/0311421]

[T. Appelquist, L.C.R. Wijewardhana, hep-ph/0403250]

[J. Alexandre, K. Farakos, S.J. Hands, G. Koutsoumbas and S.E. Morrison, Phys. Rev. D 64 (2001) 034502]

The compact formulation of the theory is suitable for the study of the mechanism of confinement

[A.M. Polyakov, Nucl. Phys. B120 (1977) 429-458]

[I.F. Herbut and B.H. Seradjeh, Phys. Rev. Lett. 91 (2003) 171601]

[P.D. Coddington, A.J.G. Hey, A.A. Middleton, J.S. Townsend, Phys. Lett. B 175 (1986) 64]

There are attempts to study some aspects of high-temperature superconductors within the compact formulation of QED_3

The model

- **The continuum Lagrangian density describing QED$_3$ is given in Euclidean metric by**
  [T.W. Appelquist et al., Phys. Rev. D 33 (1986) 3704]

$$\mathcal{L} = \frac{1}{4} F_{\mu\nu}^2 + \overline{\psi}_i D_\mu \gamma_\mu \psi_i + m_0 \overline{\psi}_i \psi_i$$

  where $D_\mu = \partial_\mu - ieA_\mu$ and $F_{\mu\nu}$ is the field strength

- **The fermions ψ_i $(i = 1, \ldots, N_f)$ are 4-component spinors**

- **Note that QED$_3$ is a super-renormalizable theory, dim$[e] = +1/2$, so the coupling does not display any energy dependence**

- **A convenient representation for γ_μ $(\mu = 0, 1, 2)$ matrices is:**

$$\gamma^0 = \begin{pmatrix} \sigma_3 & 0 \\ 0 & -\sigma_3 \end{pmatrix}; \qquad \gamma^1 = \begin{pmatrix} i\sigma_1 & 0 \\ 0 & -i\sigma_1 \end{pmatrix}; \qquad \gamma^2 = \begin{pmatrix} i\sigma_2 & 0 \\ 0 & -i\sigma_2 \end{pmatrix};$$

  where σ_i are 2×2 Pauli matrices

- **There are two 4×4 matrices that anticommute with $\gamma^0, \gamma^1, \gamma^2$:**

$$\gamma^3 = i \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \qquad \gamma^5 = i \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix};$$

- **So the massless theory is invariant under the chiral transformations:**

$$\psi \Rightarrow e^{i\alpha\gamma^3}\psi, \qquad \psi \Rightarrow e^{i\beta\gamma^5}\psi$$

- **Writing a 4-component spinor as $\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$ the mass term becomes:**

$$m\overline{\psi}\psi = m\psi_1^\dagger \sigma_3 \psi_1 - m\psi_2^\dagger \sigma_3 \psi_2$$

  and under parity transformation $\psi_1(x_0, x_1, x_2) \Rightarrow \sigma_1 \psi_2(x_0, -x_1, x_2)$ and $\psi_2(x_0, x_1, x_2) \Rightarrow \sigma_1 \psi_1(x_0, -x_1, x_2)$ so $m\overline{\psi}\psi$ is parity conserving

Lattice formulation

- **The lattice action using staggered fermion field $\overline{\chi}, \chi$ is given by**

$$S = S_G + \sum_{i=1}^{N} \sum_{n,m} \overline{\chi}_i(n) M_{n,m} \chi_i(m)$$

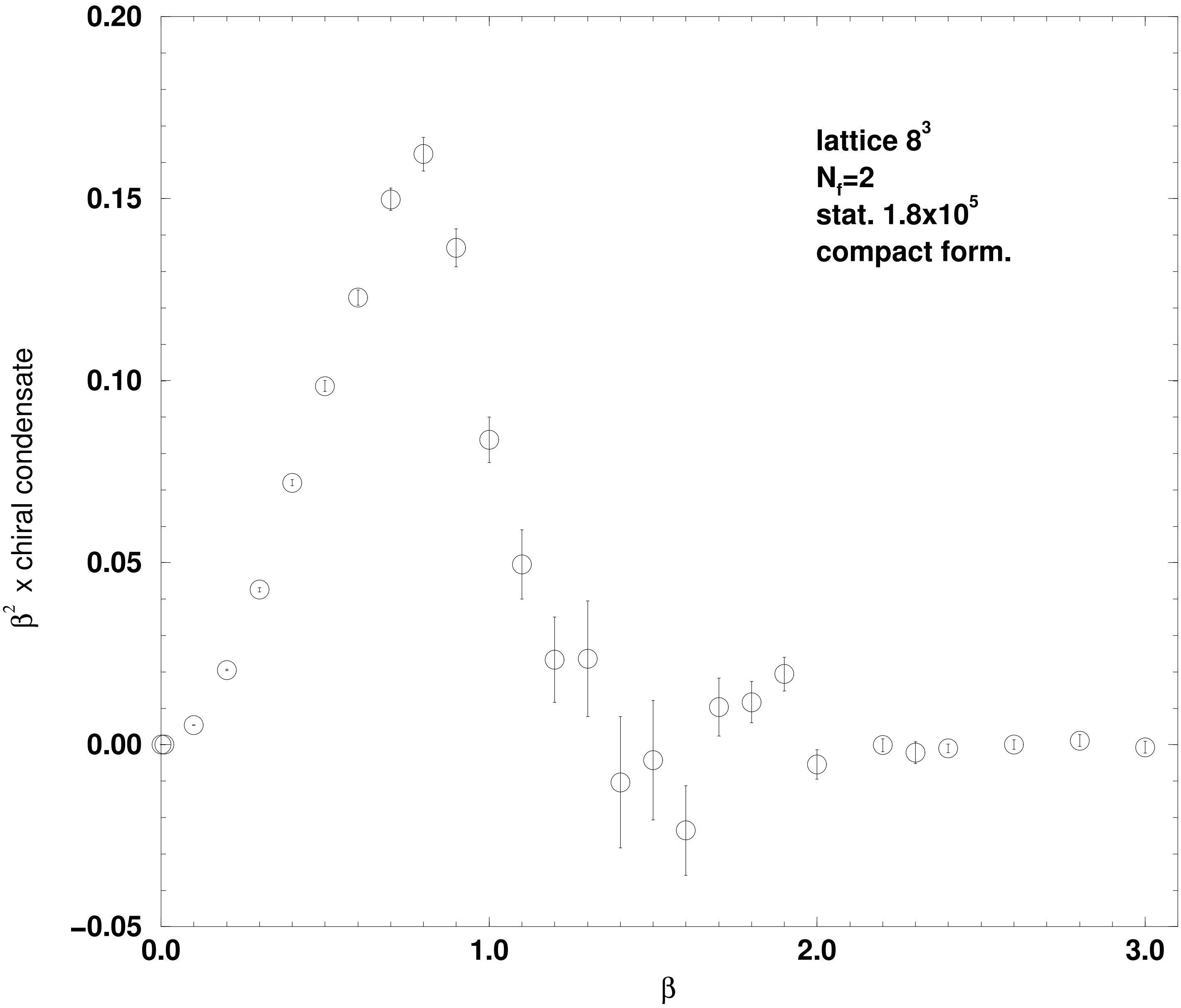
$$M_{n,m}[U] = \sum_{\nu=1,2,3} \frac{\eta_\nu(n)}{2} \left\{ [U_\nu(n)]\delta_{m,n+\hat{\nu}} - [U^\dagger_\nu(m)]\delta_{m,n-\hat{\nu}} \right\}$$

- **This action allows to simulate $N = 1, 2$ flavors of staggered fermions corresponding to $N_f = 2, 4$ flavors of the 4-component fermions ψ considered in the previous slide**
  [C. Burden and A.N. Burkitt, Europhys. Lett. 3 (1987) 545]

- **S_G depends on the type of formulation:**
  - **Compact:** $$S_G[U] = \beta \sum_{\mu<\nu,n} \left[1 - \frac{1}{2} \left(U_{\mu\nu}(n) + U^\dagger_{\mu\nu}(n) \right) \right],$$
    where $U_{\mu\nu}(n)$ is the "plaquette variable" and $\beta = \frac{1}{e^2 a}$
  - **Noncompact:** $$S_G[\alpha] = \frac{\beta}{2} \sum_{\mu<\nu,n} F_{\mu\nu}(n) F_{\mu\nu}(n)$$
    $$F_{\mu\nu}(n) = \{\alpha_\nu(n+\hat{\mu}) - \alpha_\nu(n)\} - \{\alpha_\mu(n+\hat{\nu}) - \alpha_\mu(n)\}$$
    where $\alpha_\mu(n)$ is the phase of the "link variable", $U_\mu(n) = e^{i\alpha_\mu(n)}$ and is related to gauge field by $\alpha_\mu(n) = aeA_\mu(n)$

- **We used Hybrid Monte Carlo algorithm with staggered fermions:**
  - **$N_f = 2$ (4-component fermions)**
  - **Observables:**
    - **monopole density**
      [T.A. DeGrand and D. Toussaint, Phys. Rev. D 22 (1980) 2478]
    - **chiral condensate**
  - **Lattice: 8^3 and 12^3; am=0.01, 0.02, 0.025, 0.03, 0.04, 0.05**

- **The continuum limit is reached for $\beta \to \infty$**



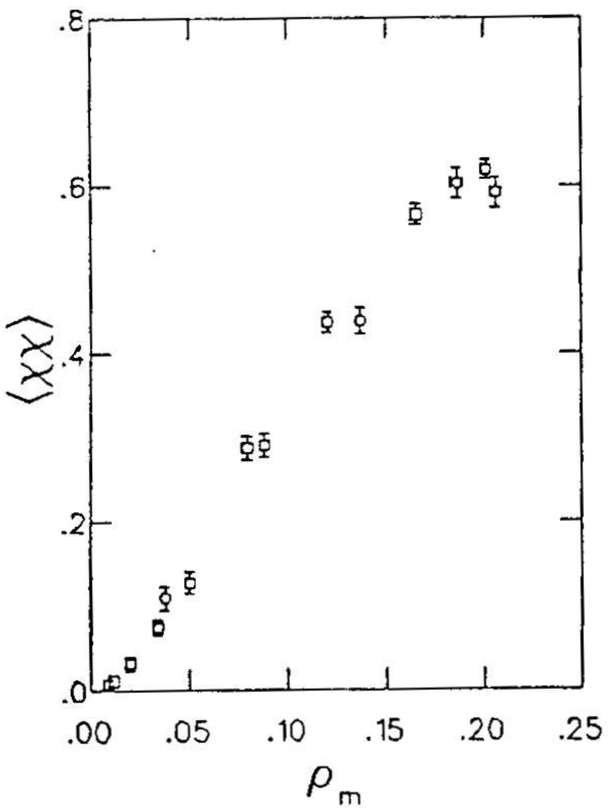
- $\beta^2 < \overline{\chi}\chi >$ **is a dimensionless variable so in the continuum limit becomes a constant**

- **We see two regimes:**
  - **For** $\beta \geq 1.4$ **the theory is in the continuum limit (plateau)**
  - **For** $\beta \leq 1.4$ **the theory describes a lattice system with finite spacing**

Compact versus noncompact QED$_3$

[H.R. Fiebig and R.M. Woloshyn, Phys.Rev. D 42, 3520-3523 (1990)]

- **The chiral condensate and the monopole density are calculated for lattice QED$_3$ ($N_f = 0$, $N_f = 2$) for both compact and noncompact formulation of the theory at finite lattice spacing**

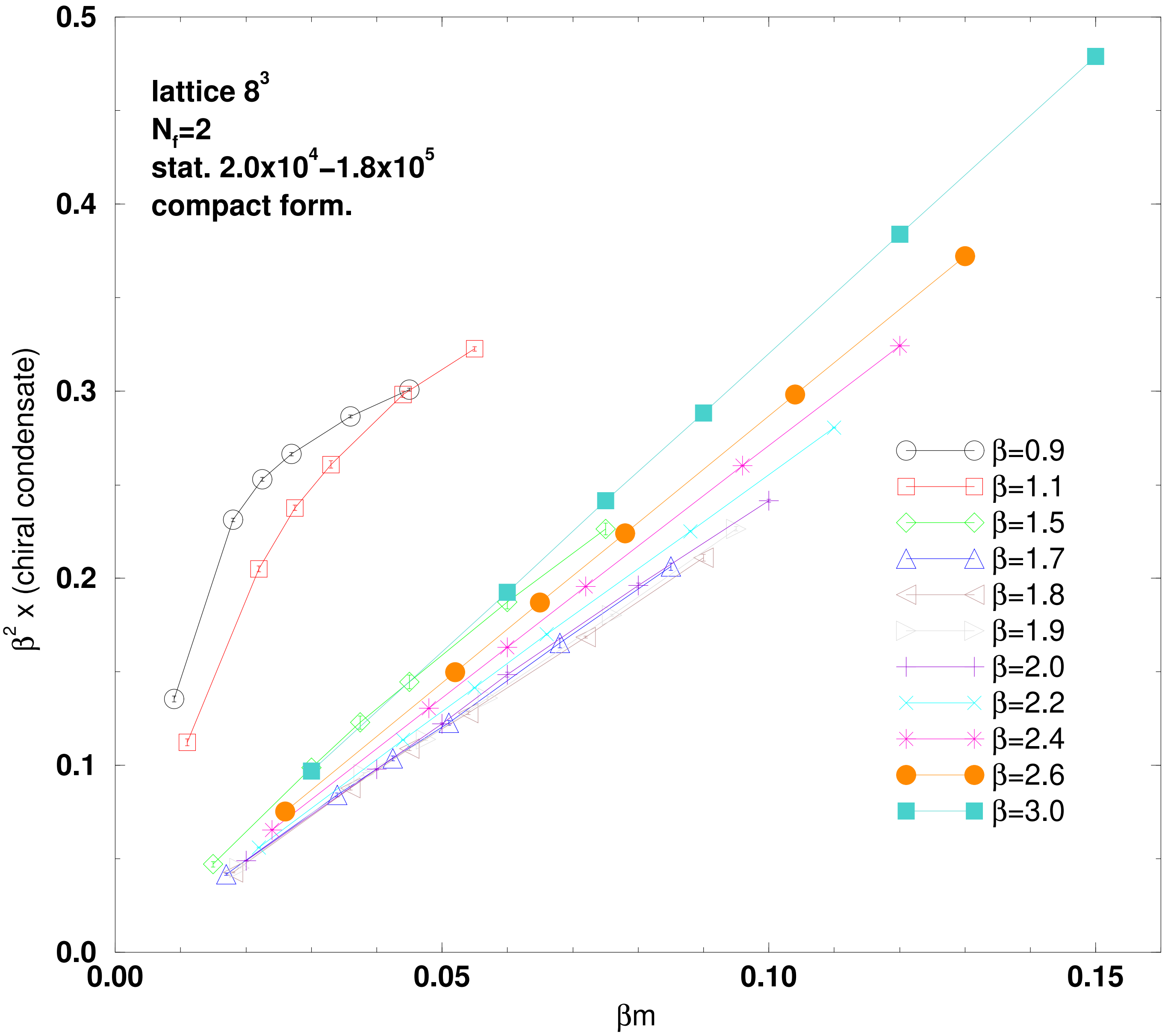


◯ compact
□ noncompact
$N_f = 2$
lattice 8^3

- **Plotting $< \overline{\chi}\chi >$ versus the monopole density ρ_m, data points for both theories fall on the same curve to a very good approximation**
- **The physics of the chiral symmetry breaking is the same in the two theories**
- **For $N_f = 2$ when $< \overline{\chi}\chi >$ vanishes ρ_m retains a small value, indicating that the quark loop effects screen the forces that produce CSB**

Our program:

- **The study of the relation between $< \overline{\chi}\chi >$ and ρ_m both in compact and noncompact QED$_3$ in the continuum limit**



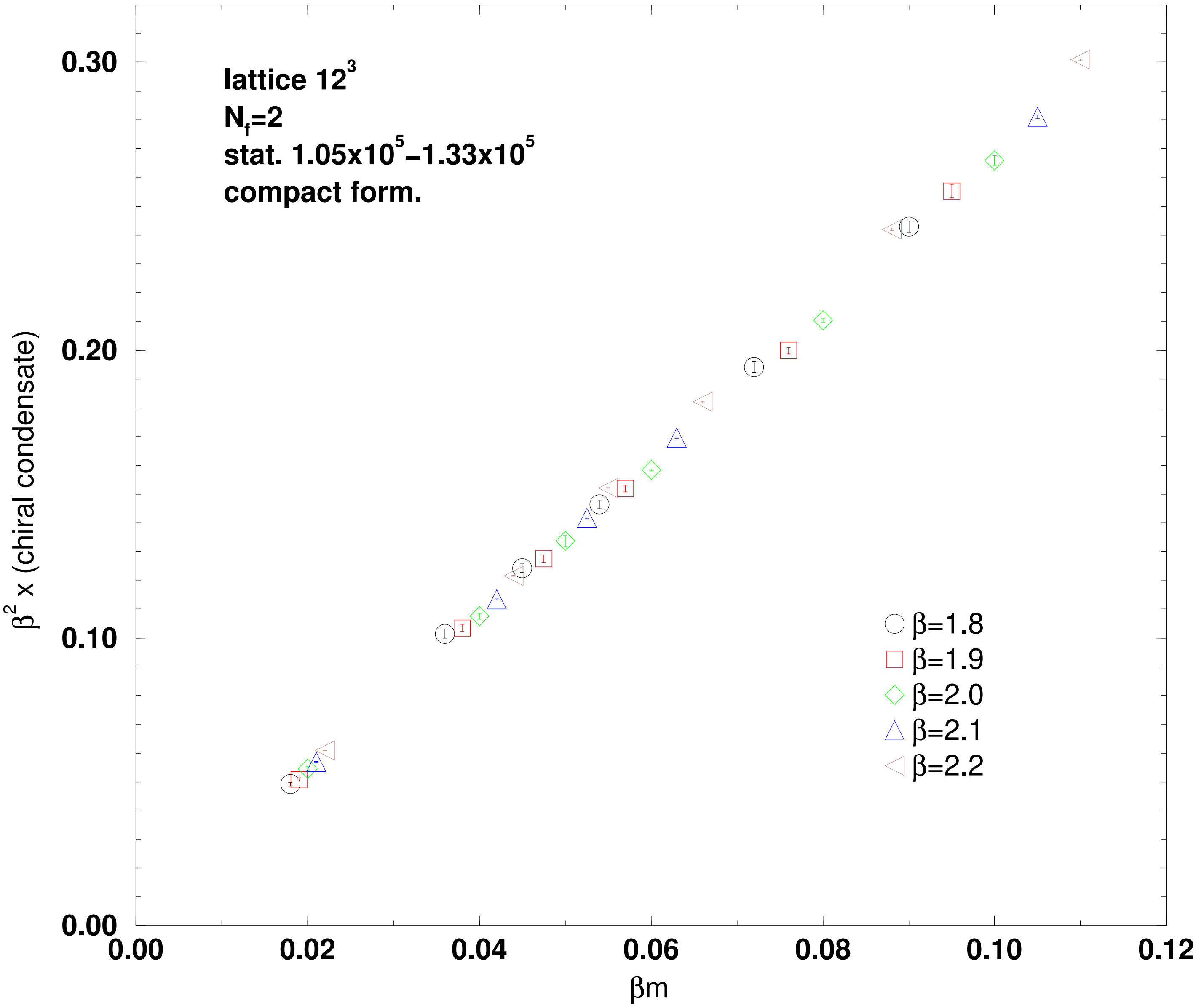
- **In the continuum limit all physical quantities are expressible in terms of the scale set by the coupling e**

- **It is natural to work in terms of dimensionless variables such as βm, L/β or $\beta^2 < \overline{\chi}\chi >$ that depend on e**

- **As the continuum limit is approached data taken at different β $(1.7 \div 1.9)$ should overlap on a single curve when plotted in dimensionless units [S.J. Hands et al., Nucl.Phys. B645, 321-336 (2002)]**

- **The effects of finite volume are significant: this is related to the presence of a massless particle, the photon**

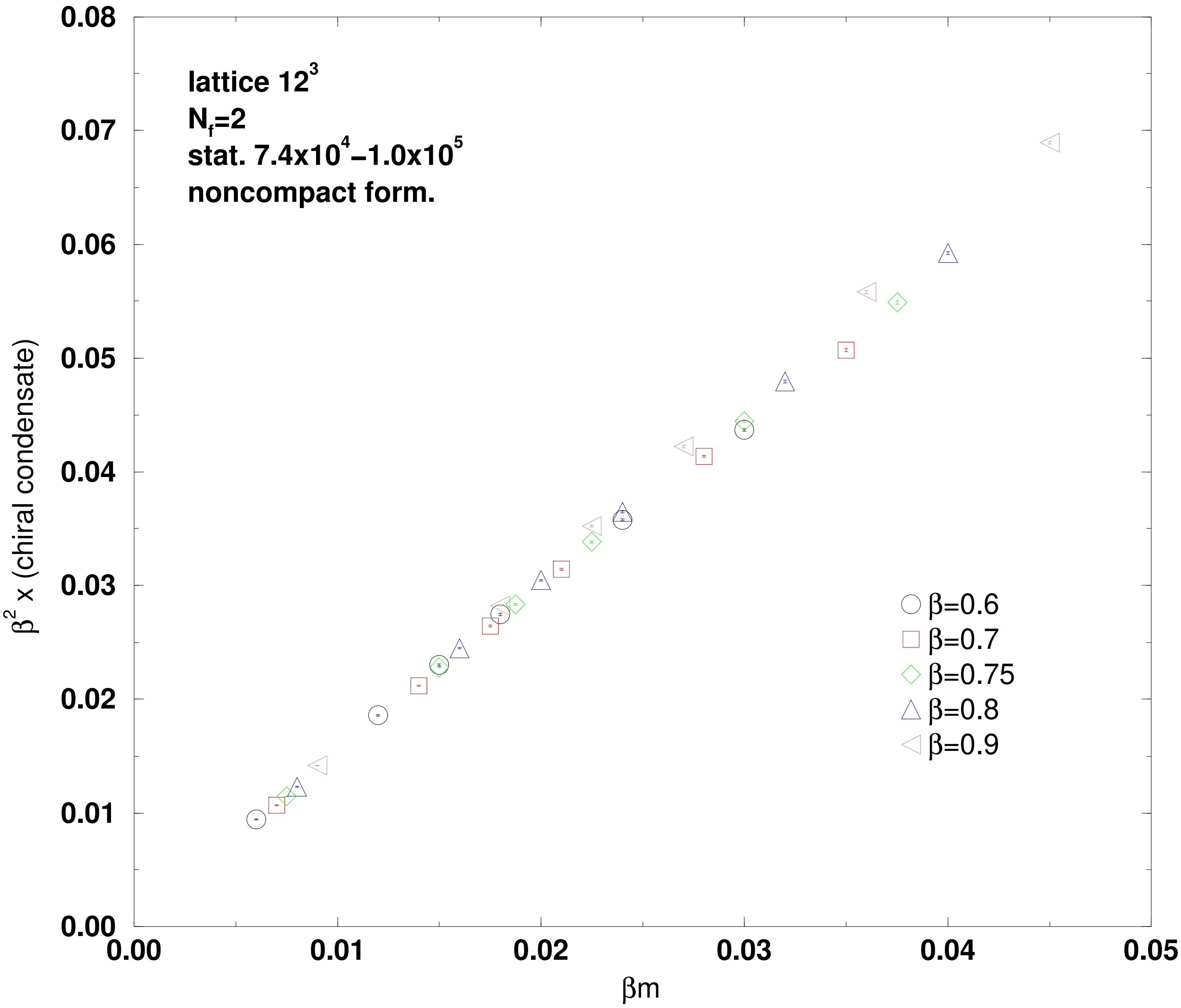
- **Thermodynamic limit $L/\beta \to \infty$** $\qquad (8/1.8 \simeq 4.4)$



- $L/\beta = 12/2.0 \simeq 6.0$

- **Linear fit with** $\beta = 1.9,\ 2.0,\ 2.1 \quad \Rightarrow \quad \beta^2 < \overline{\chi}\chi > \stackrel{\beta m \to 0}{\longrightarrow} (1.54 \pm 0.25) \times 10^{-3}$
  $\chi^2/\mathbf{d.o.f.} \simeq 8.4$

- **Linear fit with** $\beta = 2.1 \quad \Rightarrow \quad \beta^2 < \overline{\chi}\chi > \stackrel{\beta m \to 0}{\longrightarrow} (0.94 \pm 0.28) \times 10^{-3}$
  $\chi^2/\mathbf{d.o.f.} \simeq 1.3$

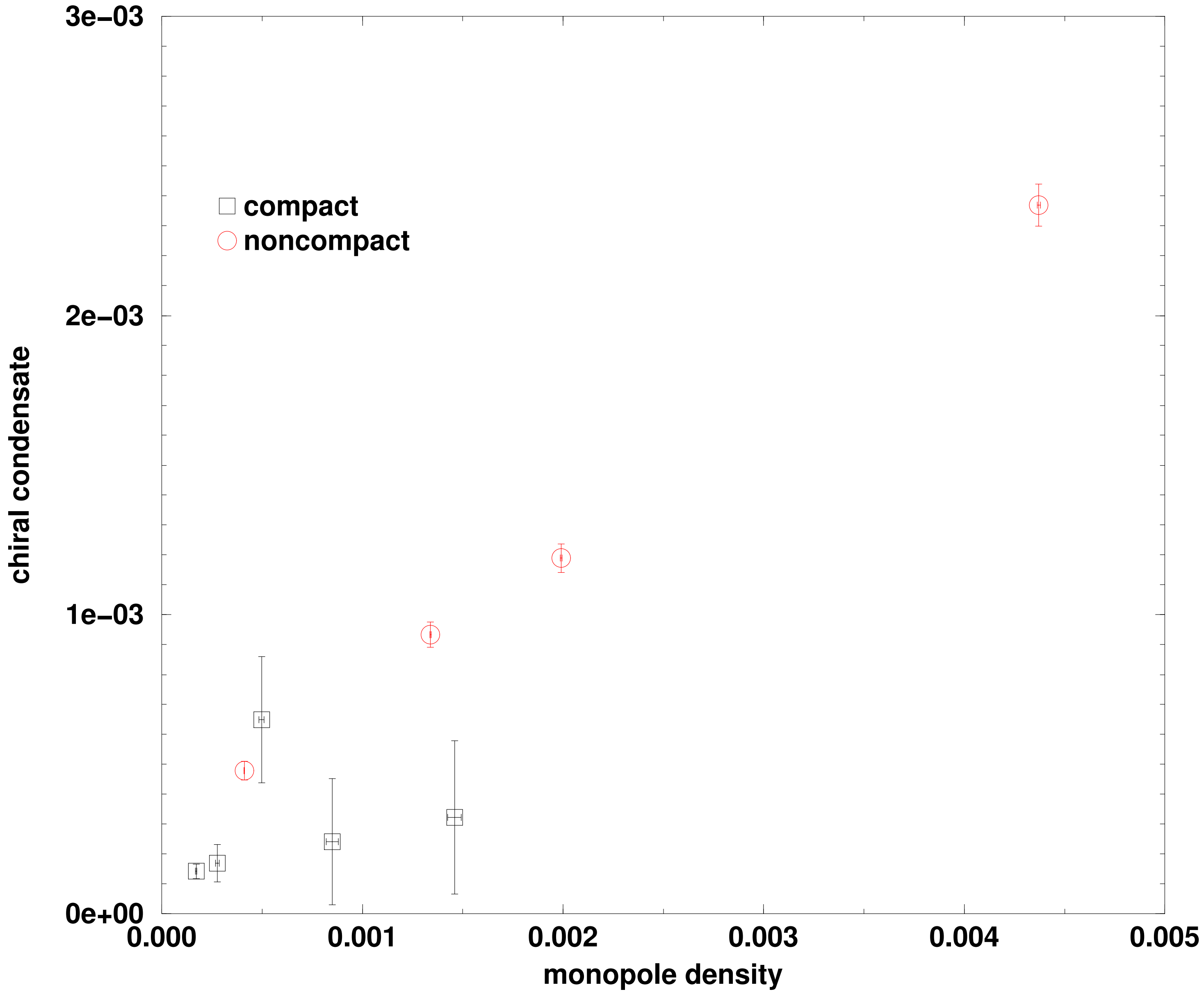


- $L/\beta = 12/0.75 \simeq 16$

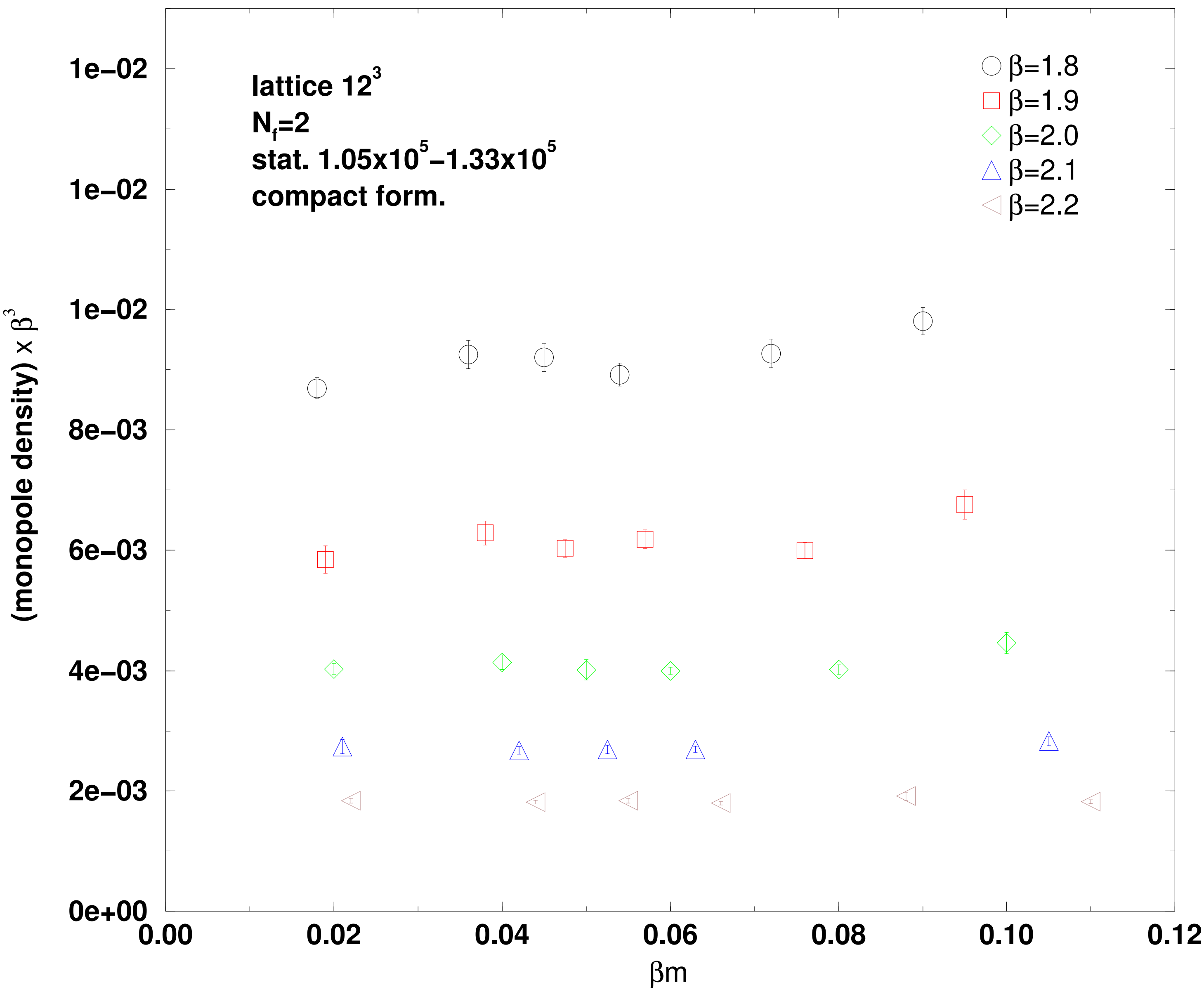
- **Linear fit with** $\beta = 0.7, 0.75, 0.8 \quad \Rightarrow \quad \beta^2 < \overline{\chi}\chi > \overset{\beta m \to 0}{\longrightarrow} (0.45 \pm 0.03) \times 10^{-3}$
  $\chi^2/$**d.o.f.** $\simeq 17$

- **Linear fit with** $\beta = 0.8 \quad \Rightarrow \quad \beta^2 < \overline{\chi}\chi > \overset{\beta m \to 0}{\longrightarrow} (0.66 \pm 0.07) \times 10^{-3}$
  $\chi^2/$**d.o.f.** $\simeq 16$

- **Linear fit with** $\beta = 0.6,\ 0.7,\ 0.8$ **and** $am = 0.02,\ 0.03,\ 0.04,\ 0.05$
  $\beta^2 < \overline{\chi}\chi > \overset{\beta m \to 0}{\longrightarrow} (1.30 \pm 0.07) \times 10^{-3}$ $\chi^2/$**d.o.f.** $\simeq 20$
  to be compared with $\beta^2 < \overline{\chi}\chi > \overset{\beta m \to 0}{\longrightarrow} (1.40 \pm 0.16) \times 10^{-3}$ **in**
  [J. Alexandre et al., Phys. Rev. D 64, 034502 (2001)]



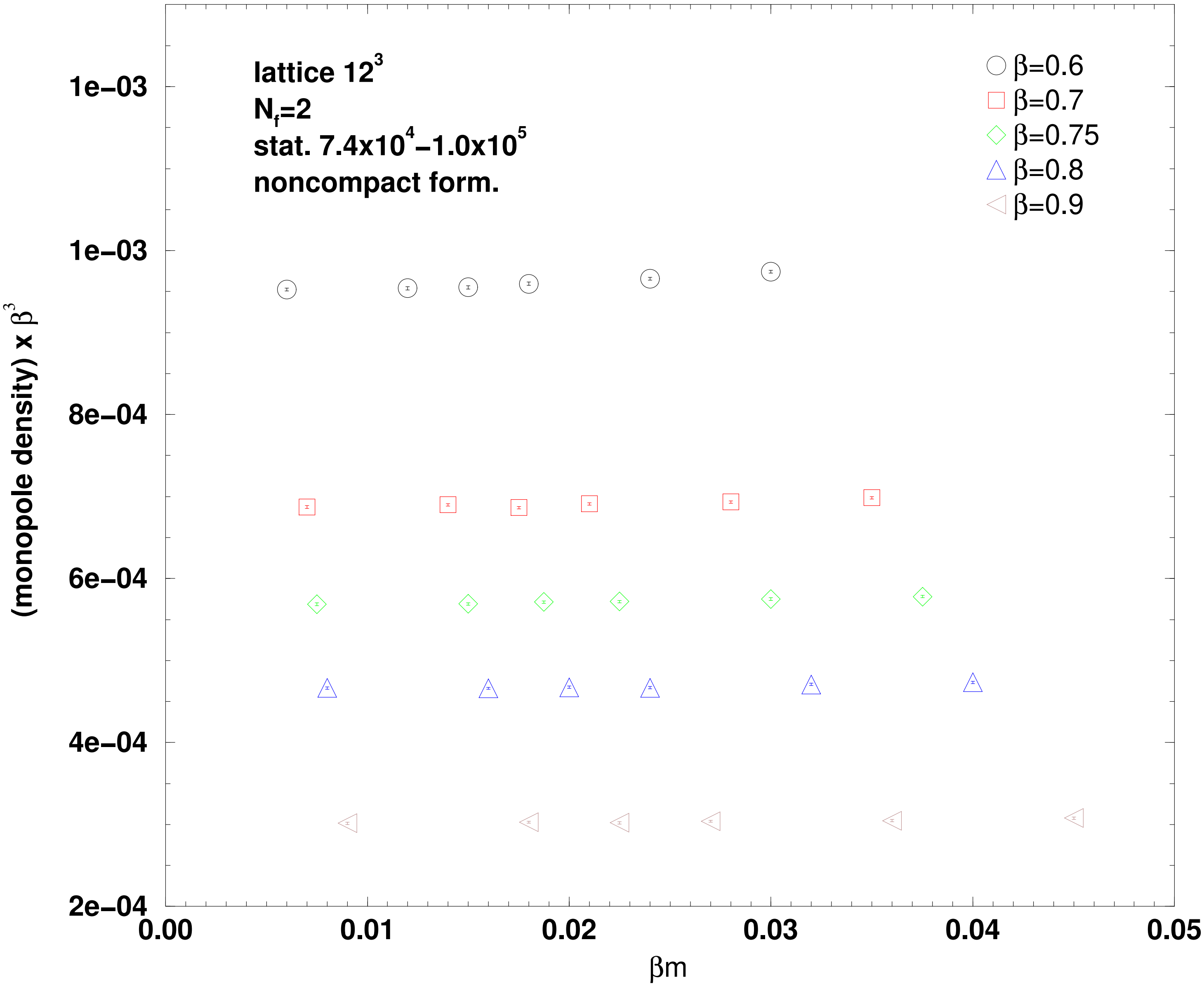
- **It is not evident that the two formulations are equivalent in the continuum limit (at least with these preliminary results)**



- **Data not fall on a universal curve ⇒ The continuum limit is not reached ?!**

- $\beta^3\rho$ **is independent from the fermion mass**

- **This supports the arguments by Herbut about the confinement in the presence of massless fermion**



lattice 12^3
$N_f=2$
stat. $7.4x10^4$–$1.0x10^5$
noncompact form.
β=0.6
β=0.7
β=0.75
β=0.8
β=0.9
(monopole density) x β^3
βm
1e−03
1e−03
8e−04
6e−04
4e−04
2e−04
0.00
0.01
0.02
0.03
0.04
0.05

Summary

- **We discussed about $\mathbf{QED_3}$ as**
  - **an effective theory for high-T_c superconductors**
  - **a laboratory for the theoretical investigation of mechanisms of confinement**
- **We showed the properties of the continuum Lagrangian and its lattice formulation**
- **We presented H.R. Fiebig and R. M. Woloshyn results (at finite lattice spacing)**
- **We discussed about our preliminary numerical results in the "continuum" limit**

Conclusions

- **Our numerical results for $\beta^2 < \overline{\chi}\chi >$ are compatible with those by other research groups, although it is still questionable if the continuum limit has been reached and if the chiral limit is stable**
- **The comparison between compact/noncompact lattice formulations is not ultimated**
- **We have verified that there is a weak dependence of the monopole density by the fermion mass, this supporting the scenario of confinement even in presence of massless fermion (Herbut's arguments)**