Spectroscopy of matter at a Quantum Critical Point

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Main idea:

Particles carrying fractionalized quantum numbers (spinons, holons, ...) are a common feature of one-dimensional correlated systems (antiferromagnets, Hubbard-like Hamiltonians, edge states of a Hall device, ...)

Is there an appropriate higher-dimensional? If yes, what is such an extension?

For simplicity, our work is confined to magnetic excitations only (spinons)

Plan of the talk:

1. Spinons as elementary excitations of one-dimensional antiferromagnets

2. Our two-dimensional model Hamiltonian H_H : phases of the model and spinon dynamics in each phase

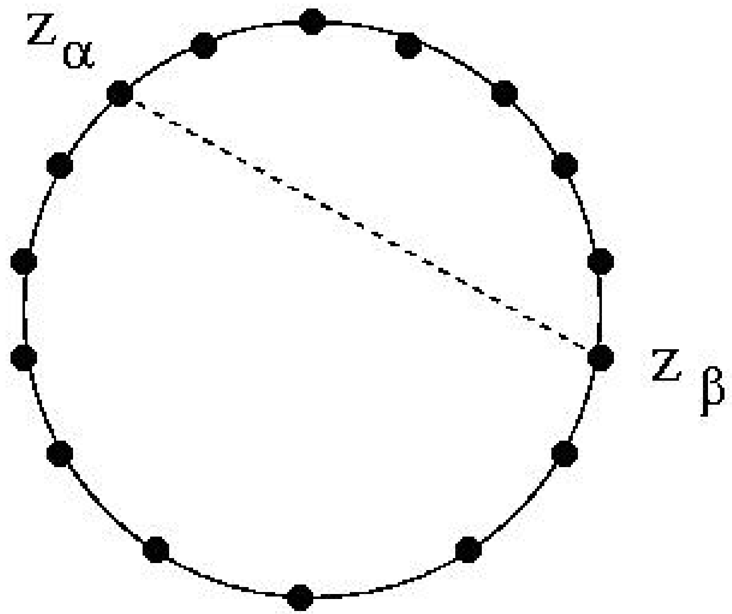
3. Quantum phase transition between the phases of H_H : critical spinon dynamics

4. Conclusions, open questions, and further developments.

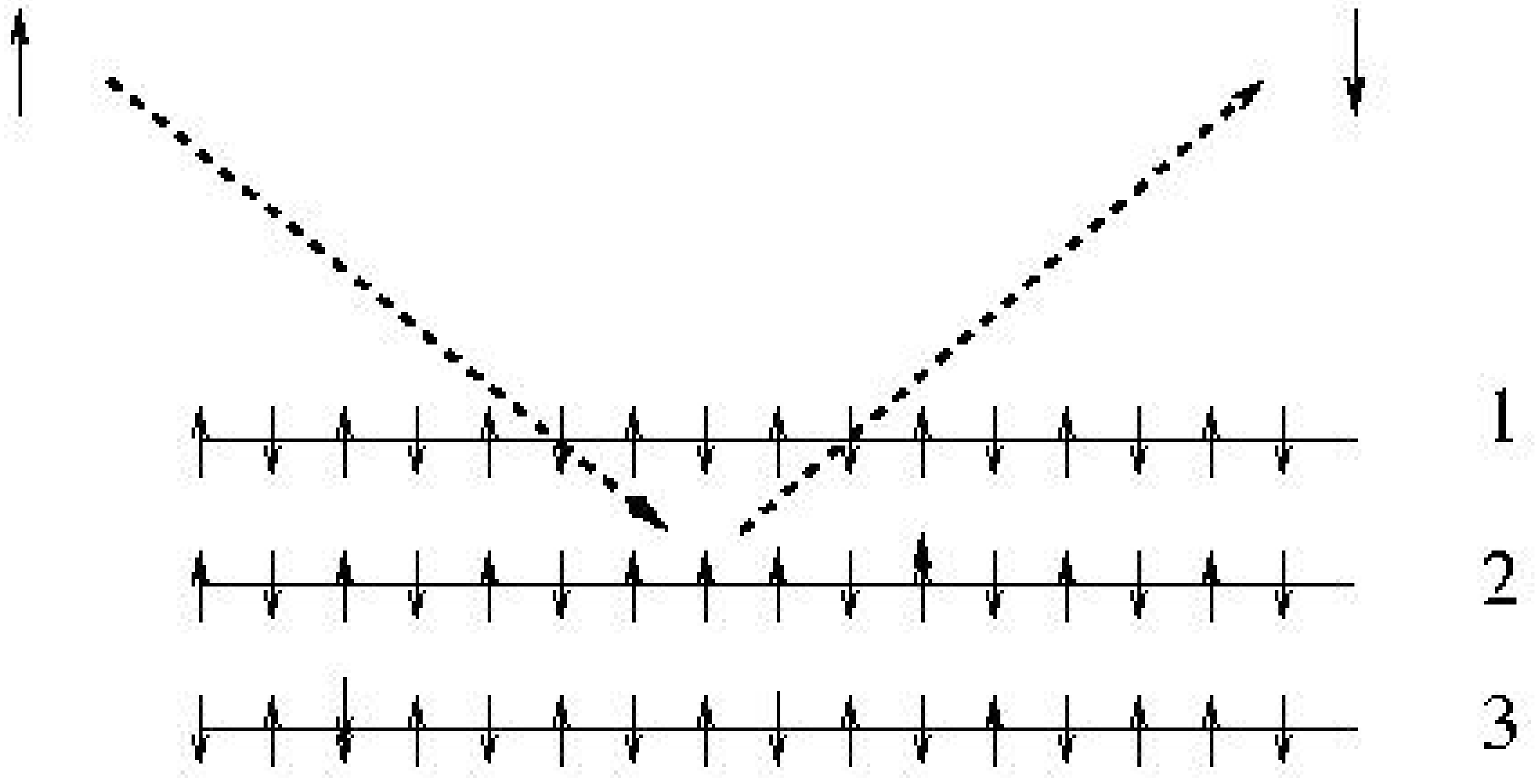
1. Spinons as elementary excitations of one-dimensional, spin-1/2, antiferromagnets

Haldane-Shastry model: prototype of one-dimensional, spin-1/2, antiferromagnet, with short-range interaction.

$$H_{HS} = \sum_{\alpha<\beta} \frac{\vec{S_\alpha} \cdot \vec{S_\beta}}{|z_\alpha - z_\beta|^2}$$



Creation of a spinon pair



Schrodinger equation for the two-spinon wavefunction $\phi_{mn}(z_\alpha, z_\beta)$

$$[H_{HS} - E_{GS}]\phi_{mn}(z_\alpha, z_\beta) =$$

$$\left\{\left[M - z_\alpha \frac{\partial}{\partial z_\alpha}\right] z_\alpha \frac{\partial}{\partial z_\alpha} + \left[M - z_\beta \frac{\partial}{\partial z_\beta}\right] z_\beta \frac{\partial}{\partial z_\beta}\right\}\phi_{mn}(z_\alpha, z_\beta)$$

$$-\frac{z_\alpha + z_\beta}{z_\alpha - z_\beta}\left[z_\alpha \frac{\partial}{\partial z_\alpha} - z_\beta \frac{\partial}{\partial z_\beta}\right]\phi_{mn}(z_\alpha, z_\beta)$$

This is exact!

Solution for two spinons

$$\phi_{mn}(z_\alpha, z_\beta) = z_\alpha^m z_\beta^n p_{mn}\left(\frac{z_\beta}{z_\alpha}\right)$$

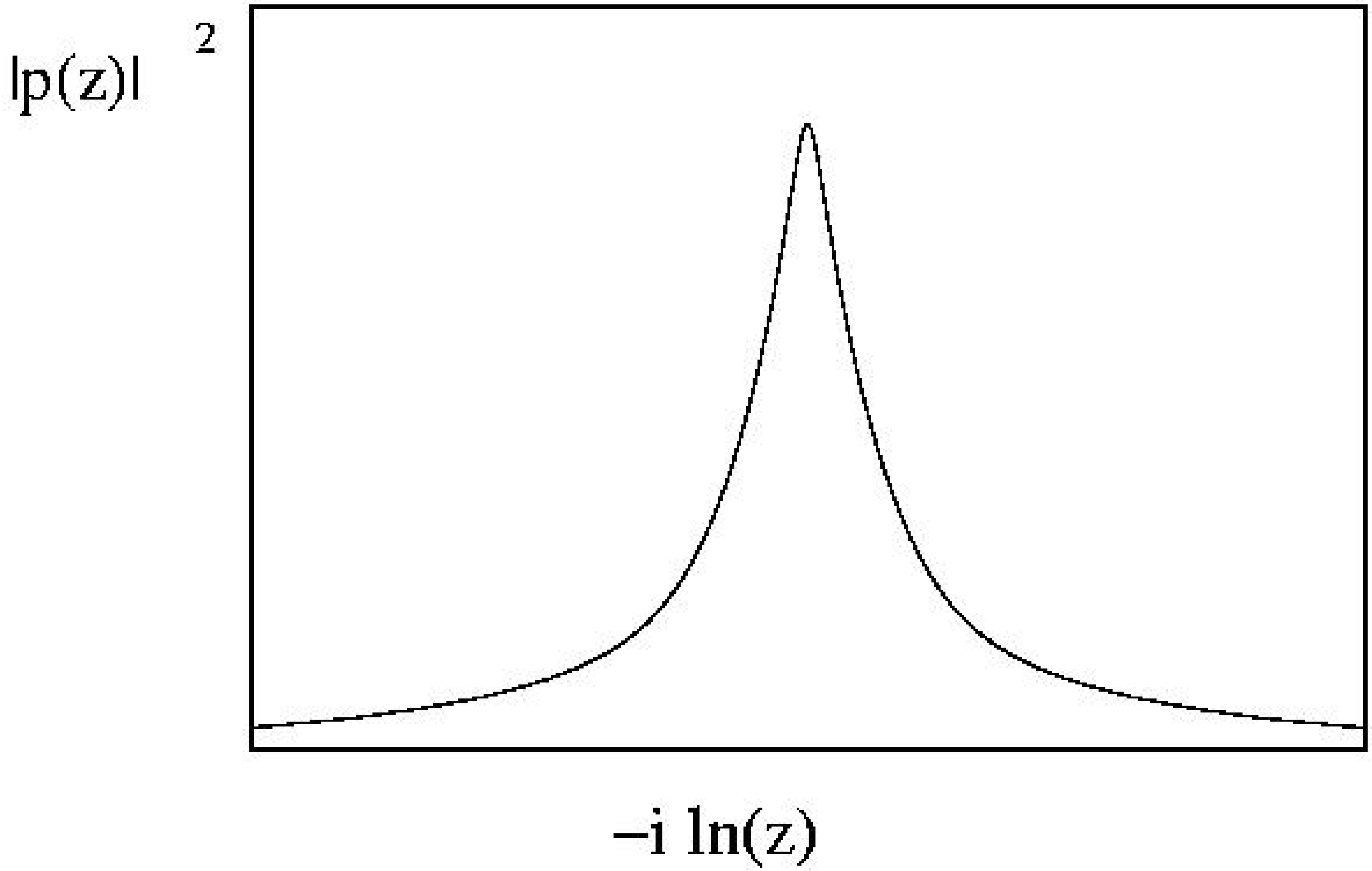
Relative wavefunction

$$p(z) = \frac{\Gamma[m-n+1]}{\Gamma[\frac{1}{2}]\Gamma[m-n+\frac{1}{2}]} \times$$

$$\sum_{k=0}^{m-n} \frac{\Gamma[k+\frac{1}{2}]\Gamma[m-n+k+\frac{1}{2}]}{\Gamma[k+1]\Gamma[m-n+k+1]} z^k$$

This is exact!

Square modulus of two-spinon wavefunction

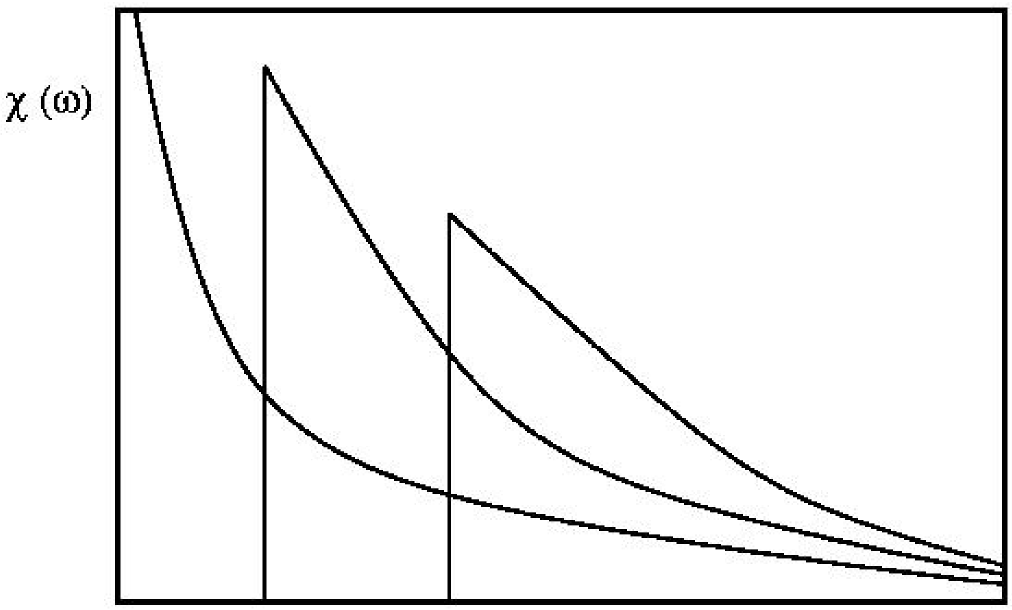


No two-spinon, spin-1 bound-states. Short-range attraction, unable to bind two spinons

Physical consequences: look at dynamical spin susceptibility

$$\chi_q(\omega) = \left\langle \overrightarrow{S_{-q}}(-\omega) \cdot \overrightarrow{S_q}(\omega) \right\rangle$$

No resonances: a sharp threshold followed by a broad feature

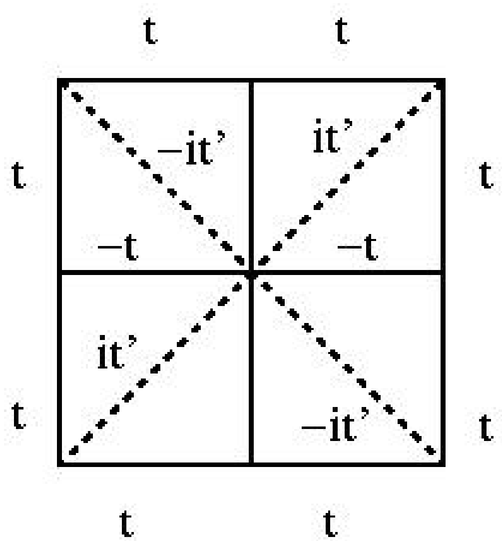


2. Our two-dimensional model Hamiltonian H_H: phases and spinon dynamics

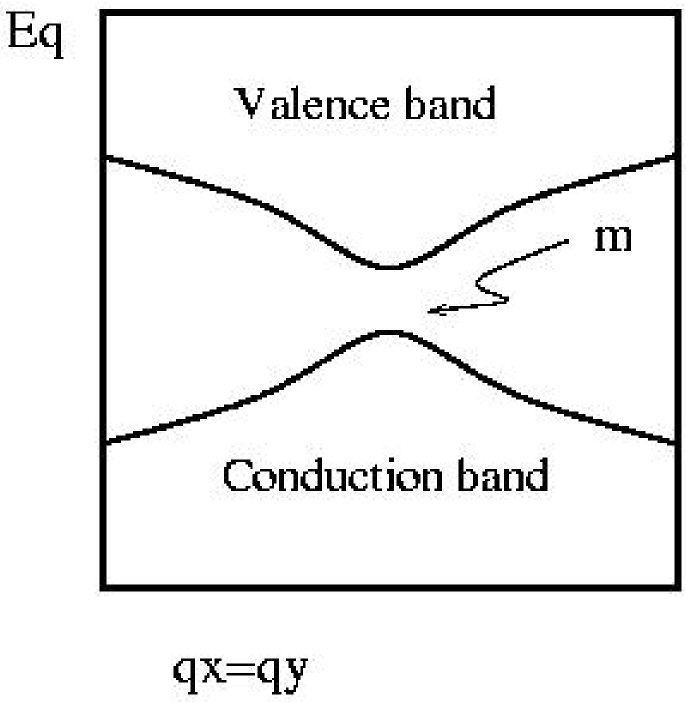
2-d Hubbard model with a T-violating second-neighbor interaction (on a square lattice) at half-filling

$$H_H = \sum_{ij,\sigma} t_{ij} c^{+}_{i,\sigma} c_{j,\sigma} + U \sum_{j} n_{j\uparrow} n_{j\downarrow}$$

Hopping amplitudes



Small U/t ⇒ Band insulating phase: (bare) gap in the single particle spectrum, m≈t'/t

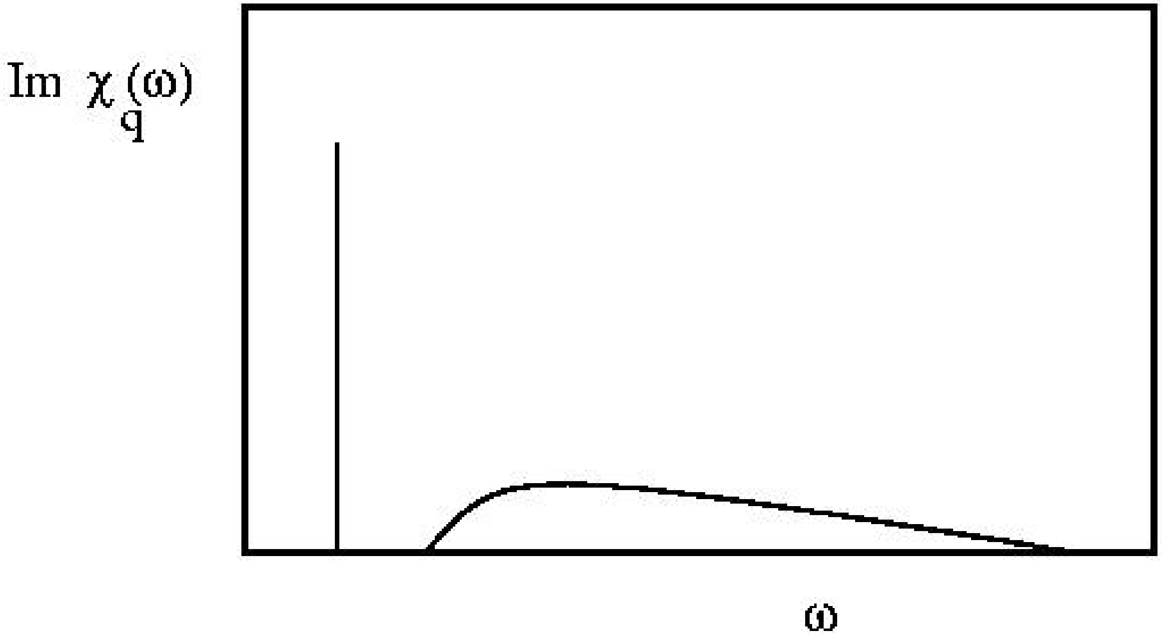


One-particle spectrum along the cut of the Brillouin zone qx=qy

Dynamical spin susceptibility

$$\chi_q(\omega) = \frac{\chi_q^{(0)}(\omega)}{1 + U\chi_q^{(0)}(\omega)}$$

$\chi_q^{(0)}(\omega)$ **is the U=0 dynamical spin susceptibility.**

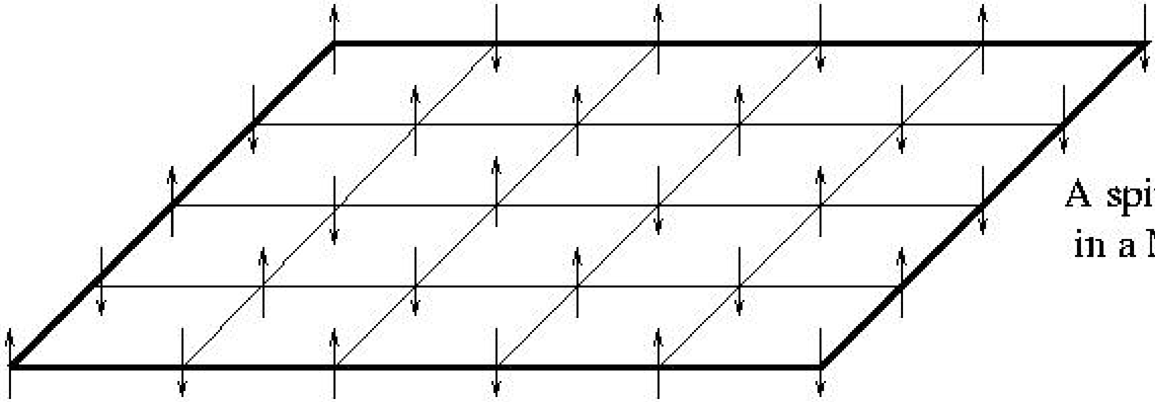


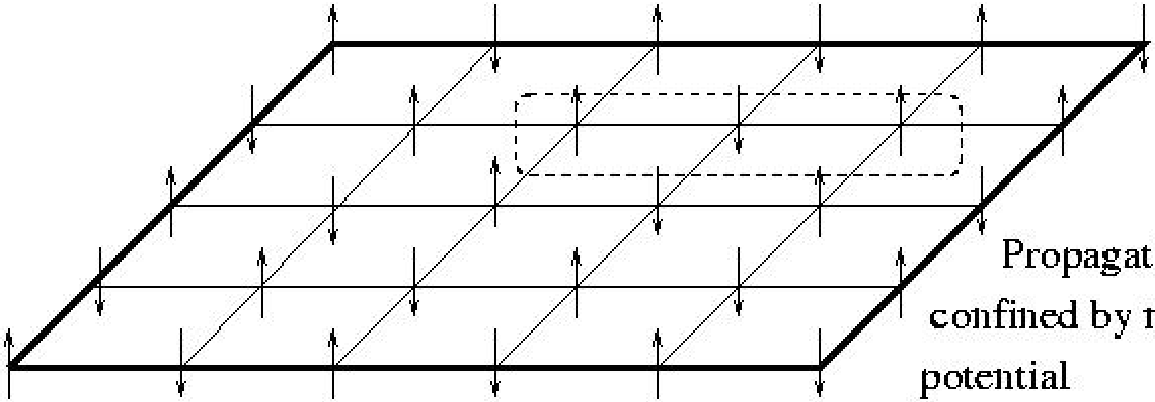
Possible spin-1/2 excitations are confined throughout all the band-insulating phase.

U/t >>1 $\Rightarrow$ mapping to the Heisemberg effective Hamiltonian via the Schrieffer-Wolff transformation

$$H_{Heis} = J_{Eff} \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j \qquad (J_{Eff} \approx t^2 / U)$$

Ground state of H_{Heis} = two-dimensional Neél state. Spin-1/2 excitations are bound to each other into transverse spin-1 spin-waves.





Possible spin-1/2 excitations are confined throughout all the antiferromagnetic phase, as well.

3. Quantum phase transition of our model

$U/t = (U/t)_{cr} \approx 1 \Rightarrow$ Phase transition between the two phases

1. At the critical value of U/t, the spin-1 excitonic mode softens at momentum $\vec{Q} = (\pi, \pi)$

2. Mean-field solution for the antiferromagnetic order parameter takes a nonzero solution only for $U/t > (U/t)_{cr}$.

3. This is a Quantum critical point, as the tuning parameter is not the temperature.

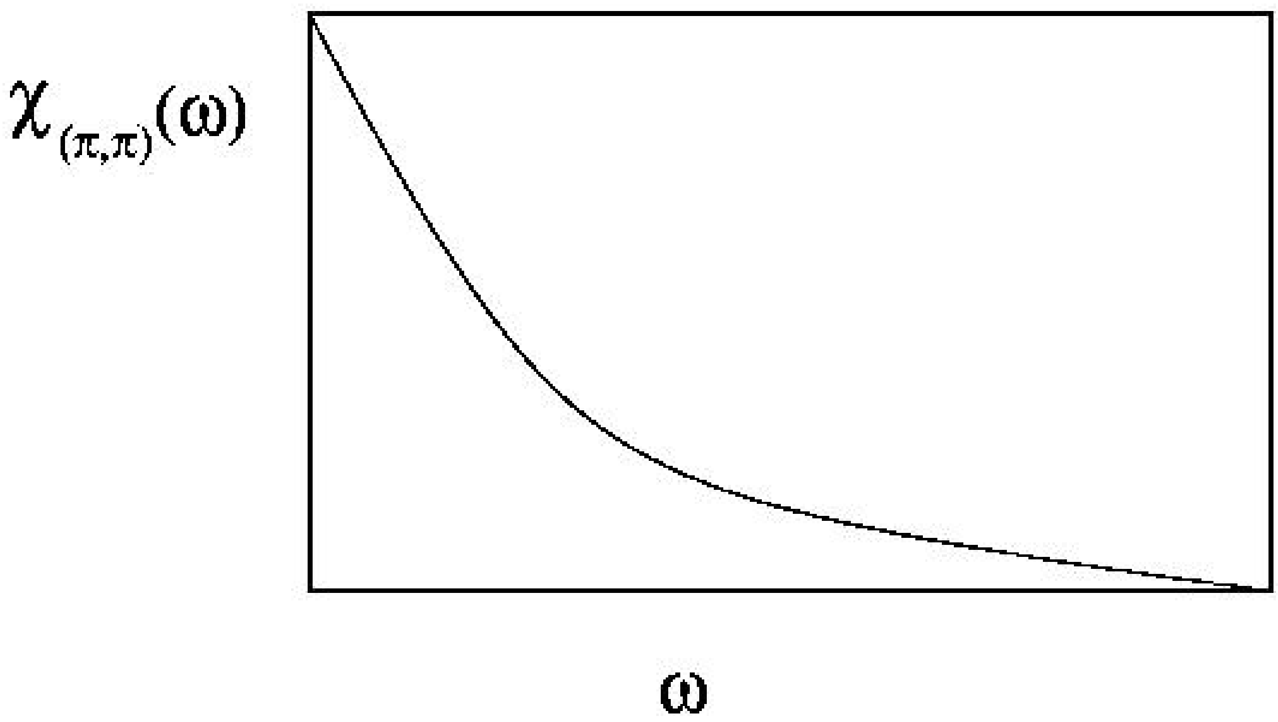
Hubbard-Stratonovitch (H-S) transformation applied to the Lattice Hamiltonian ⇒ Effective Lagrangian for the low-energy, long wavelength modes of the spin-1 H-S field, $\vec{\sigma}(\underline{x},t)$

$$L = \int d\,\underline{x}\left[\left(\partial_t \vec{\sigma}\right)^2 - \left(\vec{\partial}\vec{\sigma}\right)^2 - \mu^2\left(\vec{\sigma}\right)^2 - \lambda\left[\left(\left(\vec{\sigma}\right)^2\right)^2\right]\right]$$

The interaction term is relevant. This makes RPA calculation fully unreliable for the purpose of describing the critical theory, not even from the qualitative point of view. The Quantum critical point is Non-Gaussian.

Renormalization Group calculations + ε-expansion ⇒ the crtical dynamical spin susceptibility takes a nonzero anomalous exponent η≈0.031.

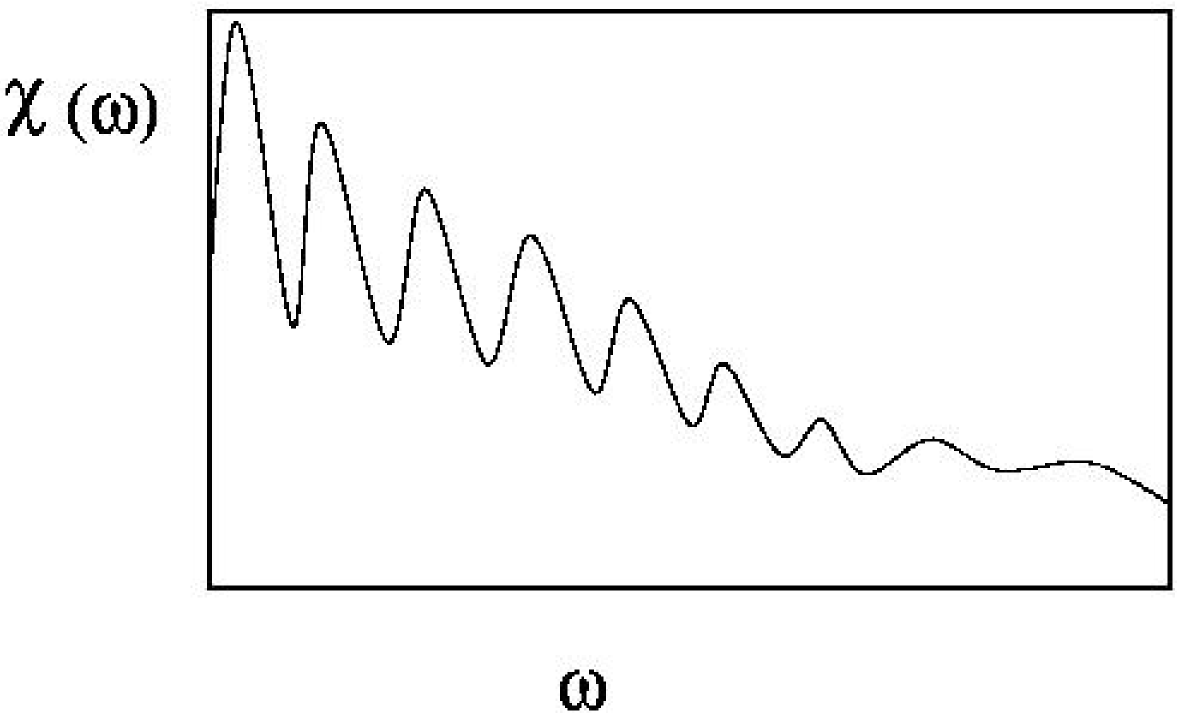
$$\chi_q(\omega) \approx \frac{1}{[\omega^2 - q^2]^{1-\eta/2}}$$



The dynamical spin susceptibility looks quite similar to its one-dimensional analog. Again, one finds sharp thresholds, followed by a broad spectrum. The nonanaliticity of the critical susceptibility proves that spin-1 modes are not the correct degrees of freedom at the Quantum critical point.

This is similar to what happens in one-dimensional systems since it describes the same physics ⇔ Spinons are deconfined at a quantum critical point.

Prediction: sequence of resonances in the dynamical spin susceptibility



The resonances should possibly be detected in a very clean neutron scattering experiment!

4. Conclusions, open questions, and further developments

1. More work to be performed on our model (fermionic self-energy, etc.)

2. Finding an effective model describing the conjectured critical behavior

3. Looking for an experiment with the desired features (two-dimensionality, transition between two insulating phases ...)

5. References, work in progress

B. A. Bernevig, D. Giuliano, and R. B. Laughlin, cond-mat/0004291, Ann. Phys. 311-1, pagg. 182-190

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