CONFINEMENT WITHOUT A CENTER: THE EXCEPTIONAL GAUGE GROUP G(2)

MICHELE PEPE
(UNIVERSITY OF BERN)

IN COLLABORATION WITH

UWE-JENS WIESE
(UNIVERSITY OF BERN)

OUTLINE

- Motivations
- The group G(2): generalities
- G(2) gauge theories → YM; → YM + Higgs
- Numerical results
- Conclusions

Motivations

- Numerical evidence: relevance of topological objects in the effective mechanism of confinement in non-Abelian gauge theories. Possible candidates: center vortices.

$$\pi_1\left(G/\mathrm{center}(G)\right) \neq \{0\}$$

$$\Downarrow$$

- Gauge theories without center vortices: study how confinement show up.

- $G(2)$: simplest group such that

$$\pi_1\left(G(2)/\{\mathbb{1}\}\right) = \{0\}$$

$$G(2) \longleftrightarrow SU(3)$$

G(2): generalities

- $G(2) \subset SO(7)$ [rank = 3; generators = 21]

$$\det \Omega = 1$$

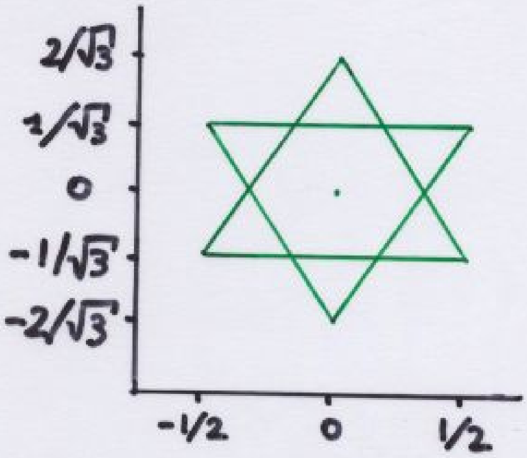
$$\delta_{ab} = \delta_{a'b'}\, \Omega_{aa'}\, \Omega_{bb'}$$

$$T_{abc} = T_{a'b'c'}\, \Omega_{aa'}\, \Omega_{bb'}\, \Omega_{cc'}\ ; \quad T = \text{antisym}$$

14 generators; real representations (fund 7×7)

G(2) - "quarks" ~ G(2) - "antiquarks"

- G(2) has rank 2



- $G(2) \supset SU(3)$ in a real representation

$$\{7\} \xrightarrow[SU(3)]{} \{3\} \oplus \{\bar{3}\} \oplus \{1\}$$

$$\Lambda_a = \begin{pmatrix} \lambda_a & 0 & 0 \\ 0 & -\lambda_a^* & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \lambda_a = \text{Gell-Mann matrices}$$

- $G(2)$: form of the matrices

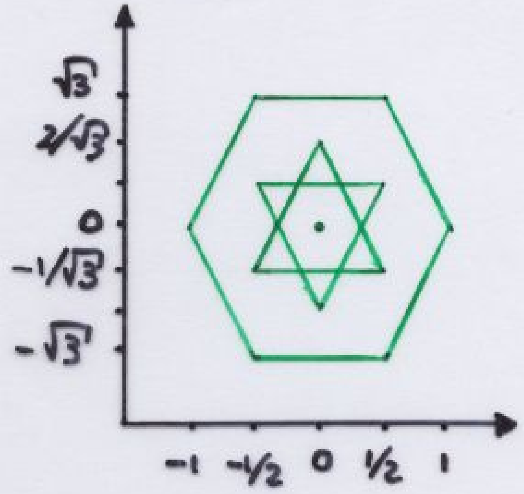
$\underset{8}{U}$: 3×3 complex matrix in $SU(3)$; $\underset{6}{K} = \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$ 3-component complex vector

$$\begin{pmatrix} C(K) & D^*(K) & K \\ D(K) & C^*(K) & K^* \\ -K^+ & -K^T & \mu(K) \end{pmatrix} \cdot \begin{pmatrix} U & 0 & 0 \\ 0 & U^* & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} CU & D^*U^* & K \\ DU & C^*U^* & K^* \\ -K^+U & -K^TU^* & \mu \end{pmatrix}$$

$C(K), D(K) = 3\times 3$ matrices ; μ = number

$$6 \quad + \quad 8 \quad = \quad 14$$

- 14 generators: adjoint rep. is $\{14\}$



$$\{14\} \xrightarrow[SU(3)]{} \{8\} + \{3\} + \{\bar{3}\}$$

14 $G(2)$- "gluons" $\xrightarrow[SU(3)]{}$ 8 gluons + "quark" + "antiquark"

- $G(2)$: its own univ. covering group $\rightarrow$ $\pi_1(G(2)/\{1\}) = \{0\}$

rank 2 $\rightarrow$ center$(G(2)) = \{1\}$ $\rightarrow$ $\pi_1(G(2)/\{1\}) = \{0\}$

$G(2) \supset SU(3)$ $\rightarrow$ center$(G(2)) = \{1\}$

"~~N-ality~~": all reps mix together in tensor product decomp.

$\Downarrow$

string breaking without dynamical $G(2)$-"quarks".

$\{7\} \otimes \{14\} \otimes \{14\} \otimes \{14\} = \{1\} \oplus \ldots$ screening

- Interesting homotopy groups

$\pi_3(G(2)) = \mathbb{Z}$ instantons] $= SU(3)$

$\pi_2(G(2)/U^2(1)) = \mathbb{Z} \otimes \mathbb{Z}$ monopoles]

$\pi_1(G(2)/\{1\}) = \{0\}$ no center vortices $\neq SU(3)$

— o — o — o — o — o — o —

$G(2) \rightarrow$ Yang-Mills

$G(2) \rightarrow$ Yang-Mills + Higgs $\{7\}$ $\rightsquigarrow$ $SU(3)$ Yang-Mills

G(2) Yang-Mills

- Pure gauge: 14 G(2)-"gluons"

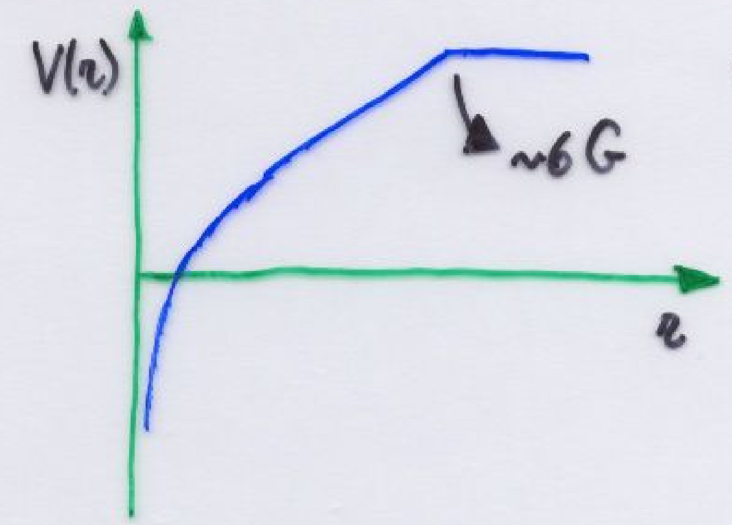
$$\{14\} \xrightarrow[SU(3)]{} \{8\} \oplus \{3\} \oplus \{\bar{3}\}$$

6 G(2)-"gluons" explicitly break $\mathbb{Z}(3) \Rightarrow$ center $(G(2)) = \{\mathbb{1}\}$
~ quarks for SU(3)

- G(2)-YM is asympotically free

low energy: confinement

string breaking



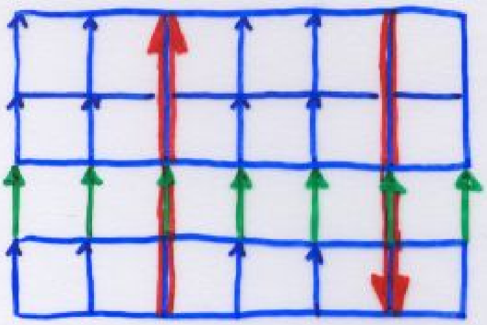
$\Rightarrow \sigma = 0$
(QCD)

- G(2)-"laboratory": confinement similar to QCD without complications related to fermions
- Wilson loop → perimeter law

Fredenhagen-Marcu → confining/Higgs phase (strong coupling expansion)

Finite temperature

- centre$(G(2)) = \{1\}$ different behaviour w.r.t. SU(N)-YM



$$P \to z P$$

$$\langle P_0 P_r^* \rangle \xrightarrow[r \to \infty]{} \langle P \rangle^2$$

$\langle P \rangle \sim e^{-F_q/T}$

$\mathbb{Z}(N)$ unbroken	$\mathbb{Z}(N)$ broken
$\langle P \rangle = 0$	$\langle P \rangle \neq 0$
$\sigma \neq 0$	$\sigma = 0$

- In SU(N)-YM there is a symmetry that breaks down in $G(2)$ NO $\Rightarrow$ no 2^{nd} order phase transition

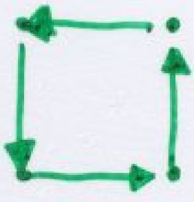
$$\Downarrow$$

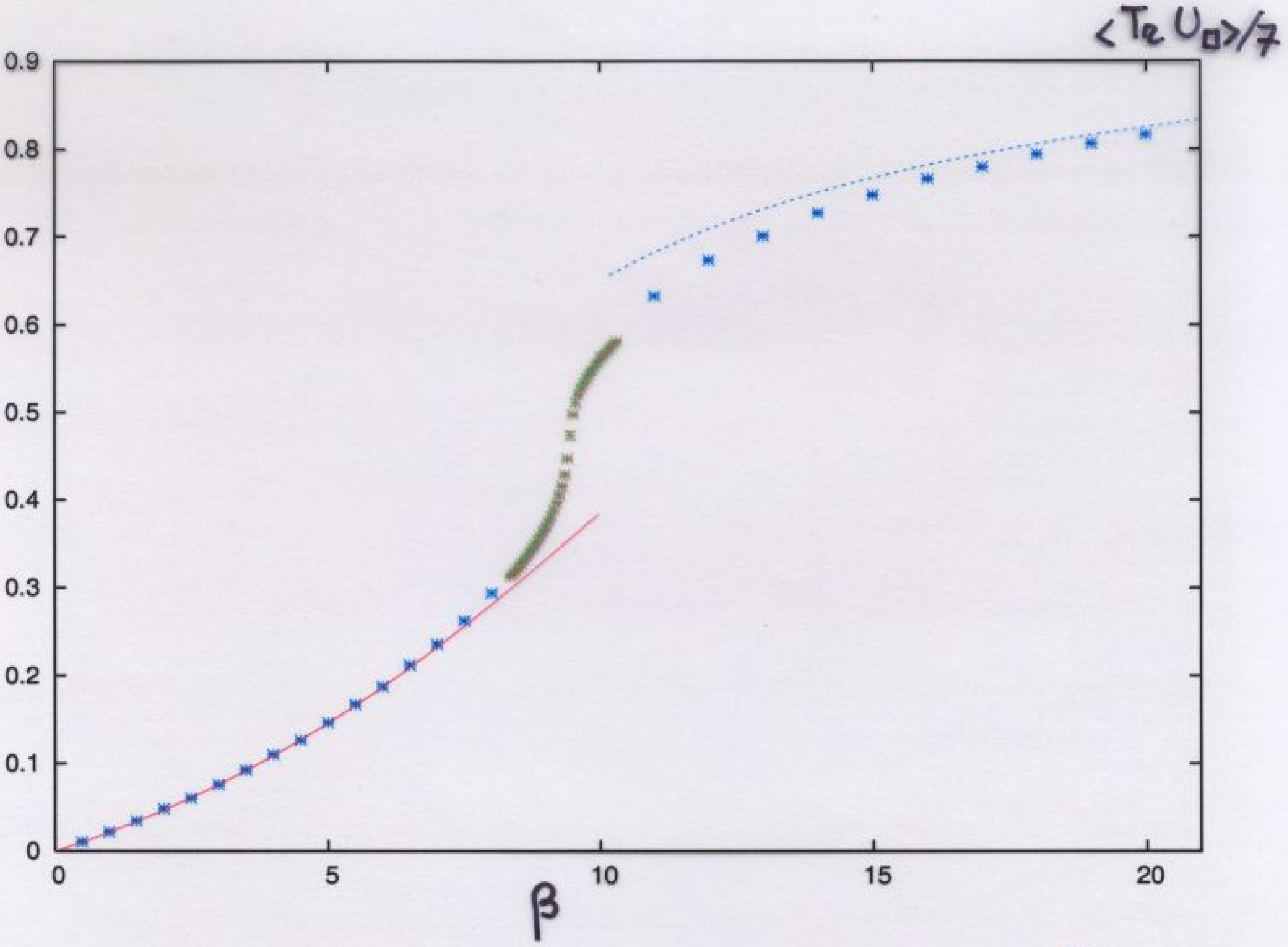
1st order or crossover ?

$$\Downarrow$$

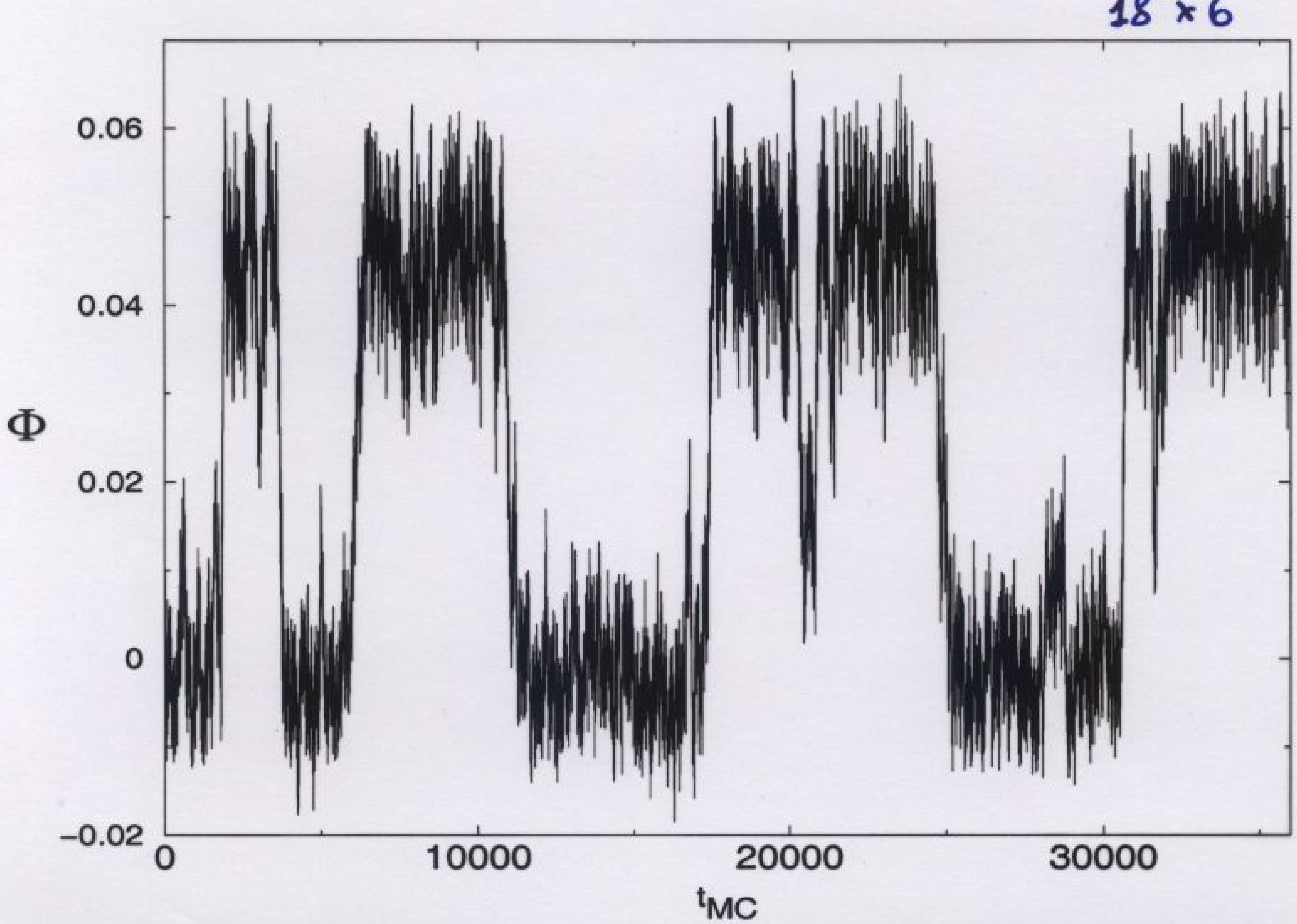
dynamical issue : numerical simulations

$$S_{YM} = -\frac{\beta}{7} \sum_{\square} \mathrm{Tr}\, U_{\square}$$

$U_{\square} =$ 



<Tr U□>/7
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
5
10
15
20
β



18³ × 6
0.06
0.04
Φ
0.02
0
−0.02
0
10000
20000
30000
t_MC

G(2) Yang-Mills + Higgs {7}

- Higgs {7}: G(2) $\longrightarrow$ SU(3) $\quad \langle \phi \rangle = v$

$$\{14\} \xrightarrow[SU(3)]{} \{8\} \oplus \{3\} \oplus \{\bar{3}\}$$

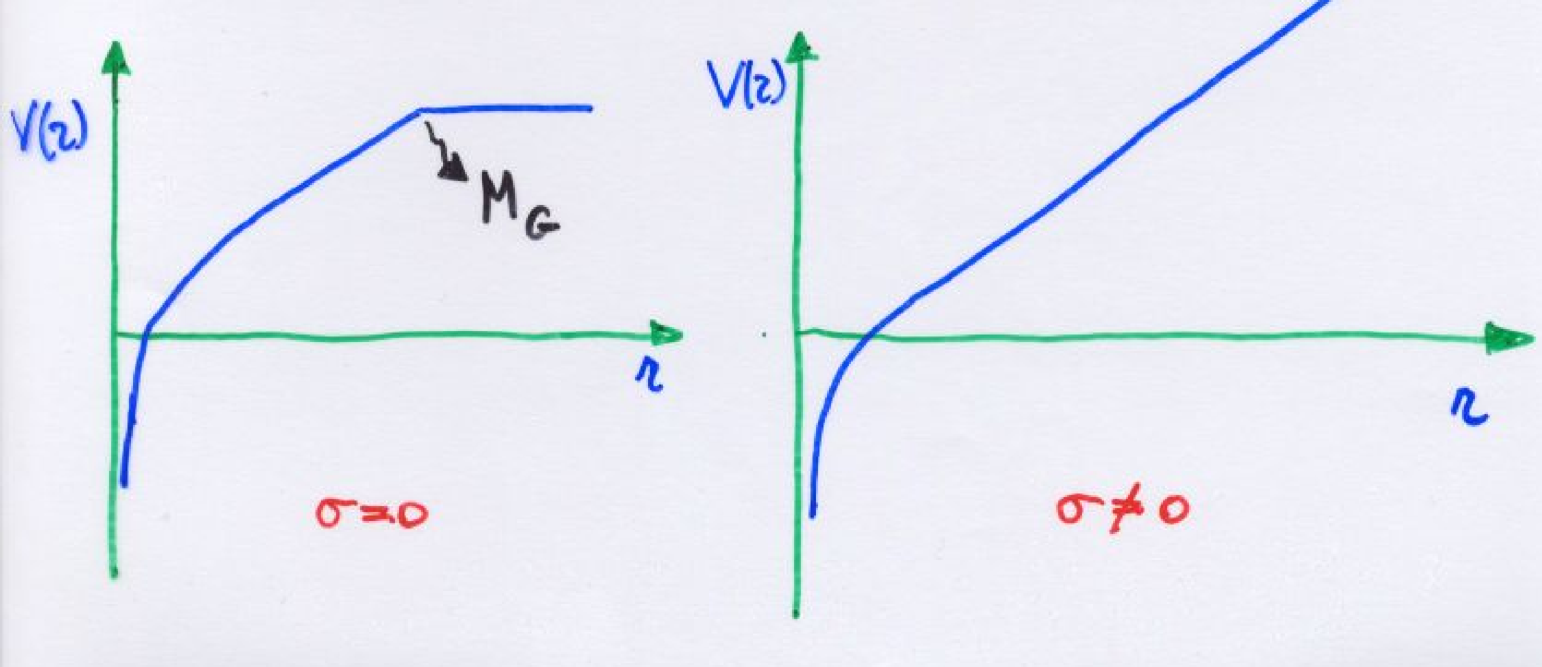
6 G(2)-"gluons" with mass $\propto v$ and 8 massless

- For $v \ll \Lambda_{QCD}$ the 6 massive G(2)-"gluons" participate to dynamics for $v \gg \Lambda_{QCD}$ they decouple $\Rightarrow$ SU(3)

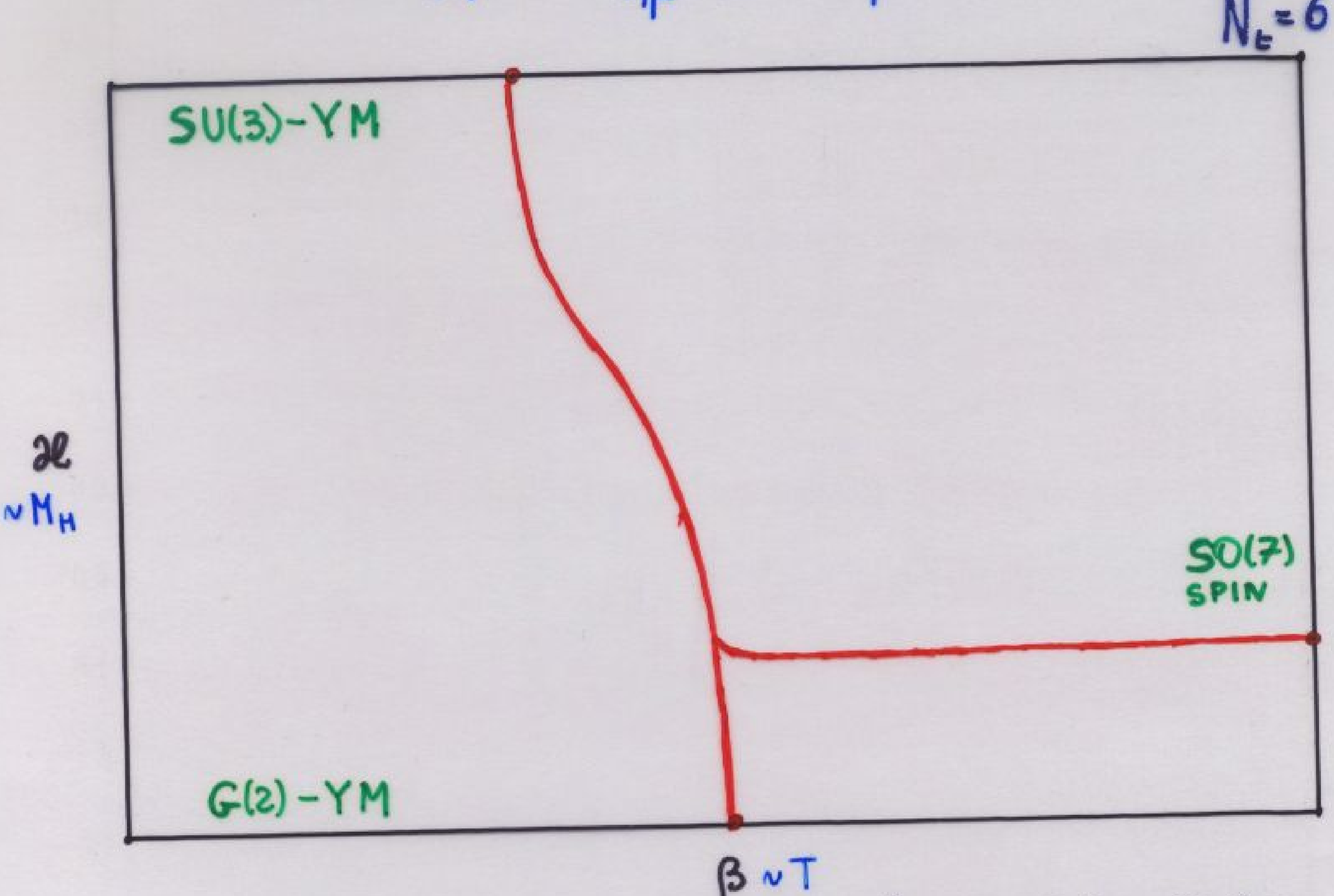
$$\Downarrow$$

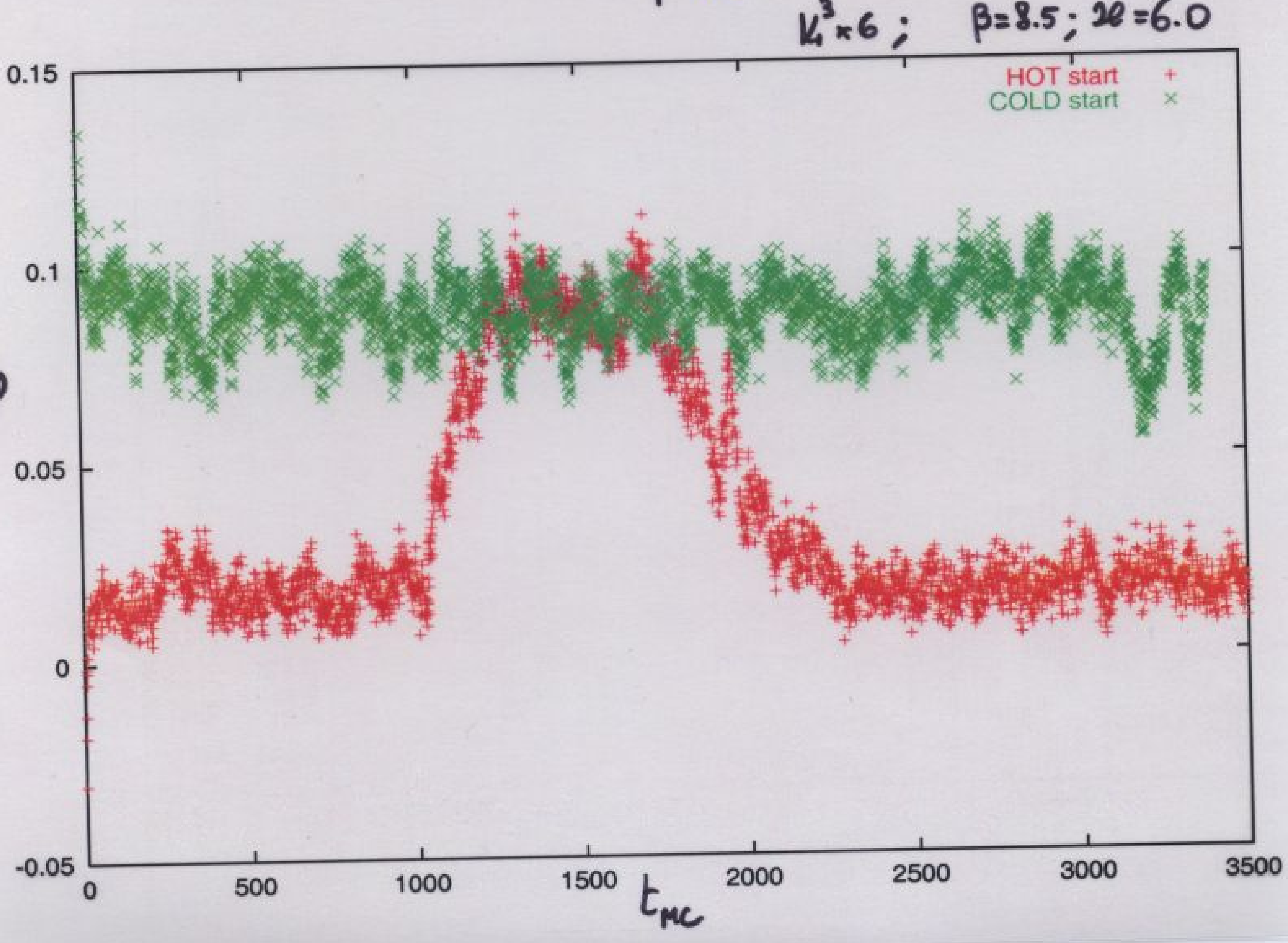
Higgs {7}: handle for G(2) $\longleftrightarrow$ SU(3)

- Confinement G(2) $\longrightarrow$ SU(3). 6 massive G(2)-"gluons" are $\{3\} \oplus \{\bar{3}\}$ ~ quarks $\Rightarrow$ string breaking



$$S_H = S_{YM} - \varkappa \sum_{x,\mu} \phi^{+}(x)\, U_{\mu}(x)\, \phi(x+\hat{\mu})$$





Conclusions

- We have studied confinement in gauge theories with symmetry group $G(2)$.
    - investigate confinement without the luxury of a center

      $$\pi_1(G(2)/\{1\}) = \{0\}$$

    - study $SU(3)$ gauge theory in an unusual context

      $$SU(3) \xleftrightarrow[\text{Higgs } \{7\}]{} G(2)$$

- $G(2)$-YM : we show numerical evidence for a 1^{st} order transition at finite temperature
- $G(2)$-YM + Higgs$\{7\}$: study of the phase diagram in the (β, κ) plane finding that the finite temperature phase transition of $SU(3)$-YM and $G(2)$-YM are connected.