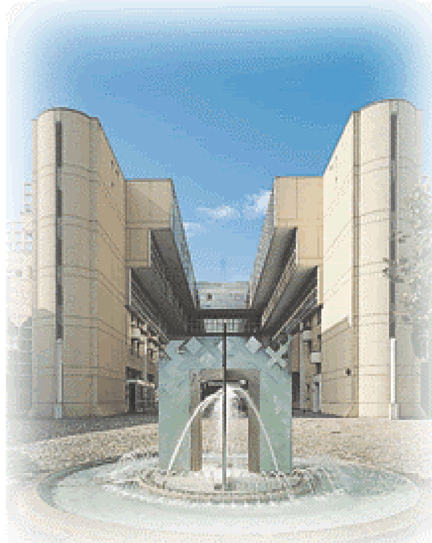
Bari 3-5 September 2008



# Fluctuations near a critical point

**S. Ciliberto, S. Joubaud, B. Persier, A. Petrossyan**

**Laboratoire de Physique, ENS de Lyon - UMR5672 CNRS**

## Fluctuations near a critical point

# Motivations

Why is interesting to study these fluctuations near a second order phase transition?

1) The probability density functions of global variables

S. T. Bramwell, P. Holdsworth, J.-F. Pinton, Nature vol. 396, 552 (1998)
*Universality of rare fluctuations in turbulence and critical phenomena*

E. Bertin, Phys. Rev. Lett, 95 170601 (2005).
*Global fluctuations in Gumbel Statistics*

2) Aging at critical point

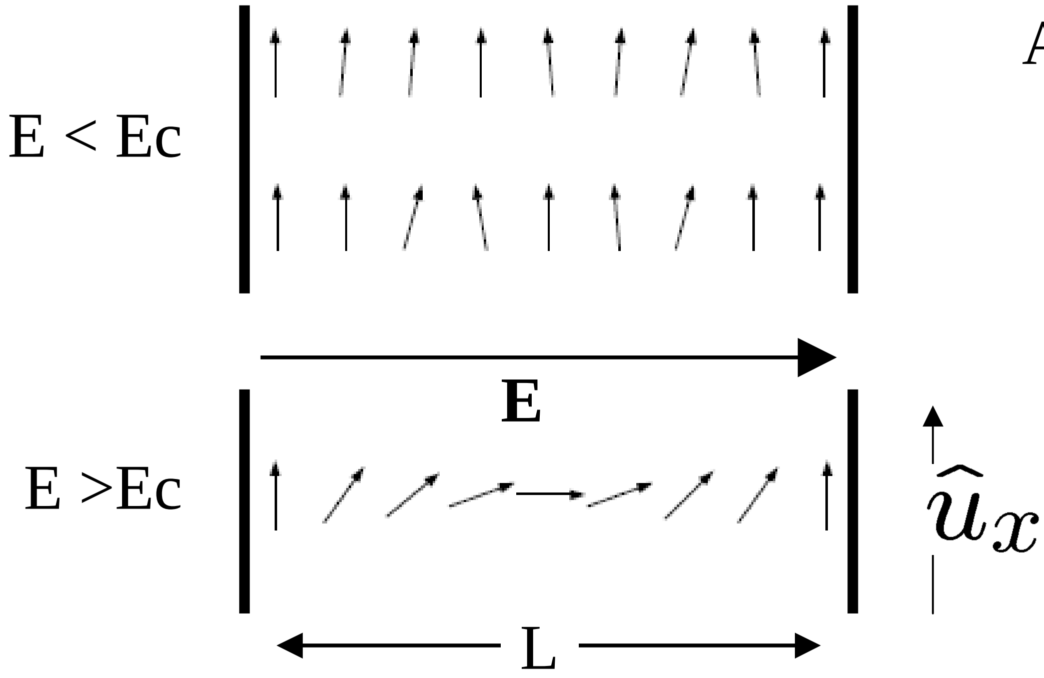
L. Berthier, P. Holdsworth, Europhys. Lett. 58, 35 (2002)
*Surfing on a critical line: Rejuvenation without chaos, memory without a hierarchical phase space*

P. Calabrese and A. Gambassi, cond-mat/0410357V2
*Aging Properties of Critical Systems*

# Outline

1) The Fréedericksz transition  in liquid crystals
2) Second order phase transition and the global variable of interest
3) Experimental system
4) Experimental results on PDF
5) The universal PDF for global variables
6) Aging at critical point
7) Experimental results on aging
8) Conclusions

# Liquid Crystals and Fréedericksz transition (I)




A liquid crystal consists of elongated molecules

$\boxed{\hat{n} \text{ is the director}}$

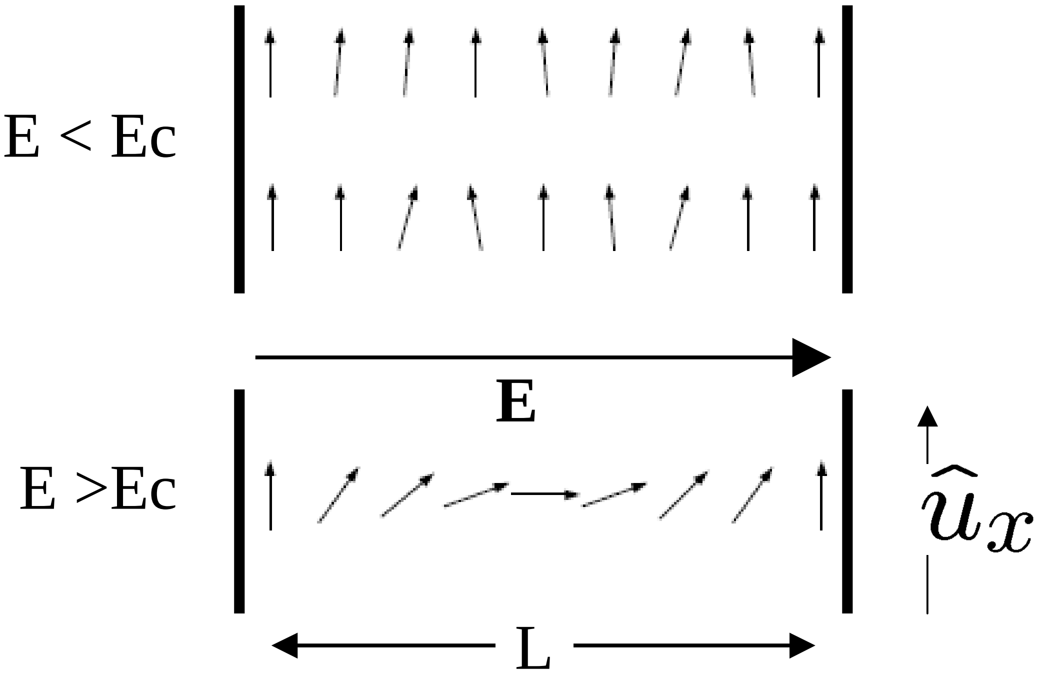
Surface treatement.
Parallel anchored
(planar allignement)

**Competition between :**

- **Elastic energy** $\widehat{n} // \widehat{u}_x$
- **Electrostatic energy** $\widehat{n} // \overrightarrow{E}$

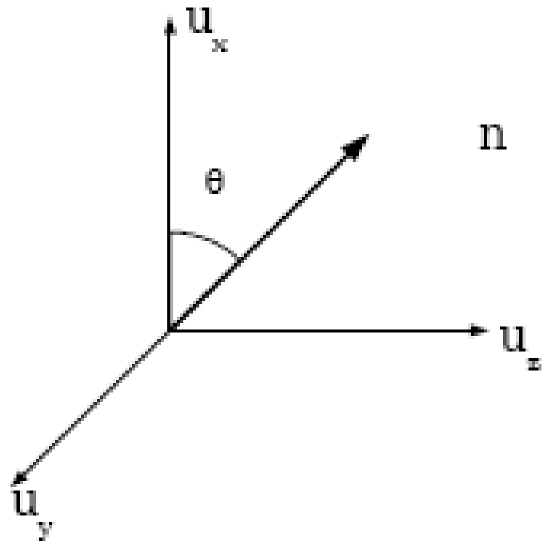
Control parameter :
voltage difference U

# Liquid Crystals and Fréedericksz transition (II)



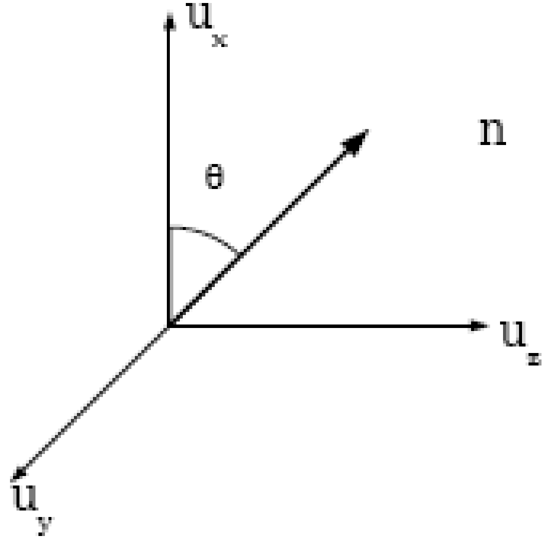

$\hat{n}$ is the director

Control parameter :
voltage difference U




Solution of the form: $\theta(z) = \theta_0(x, y)\,\sin(\frac{\pi z}{L})$

# Liquid Crystals and Fréedericksz transition (III)




$$\hat{n} = \cos(\theta)\ \hat{u}_x + \sin(\theta)\ \hat{u}_z$$

With boundary conditions

$$\theta(z=0) = \theta(z=L) = 0$$

Solution of the form: $\theta(z) = \theta_0(x,y)\sin(\frac{\pi z}{L})$

If $\theta_0 << 1$ remains small, the equation of motion of $\theta_0$ is :

$$\tau_0 \frac{d\theta_0}{dt} = \epsilon\theta_0 - \left(\kappa + \frac{\epsilon+1}{2}\right)\theta_0^3$$

$$\tau_0 = \frac{\gamma}{\epsilon_0\epsilon_a E_c^2} \quad U_c = \pi\sqrt{\frac{K_1}{\epsilon_0\epsilon_a}} \quad \kappa = \frac{K_3 - K_1}{K_1} \quad \epsilon = \frac{U^2}{U_c^2} - 1$$

Correlation length in the xy plane : $\xi_r = \frac{L}{\pi\sqrt{\epsilon}}$

(San Miguel, Phys. Rev. A, 32, 3811, 1985)

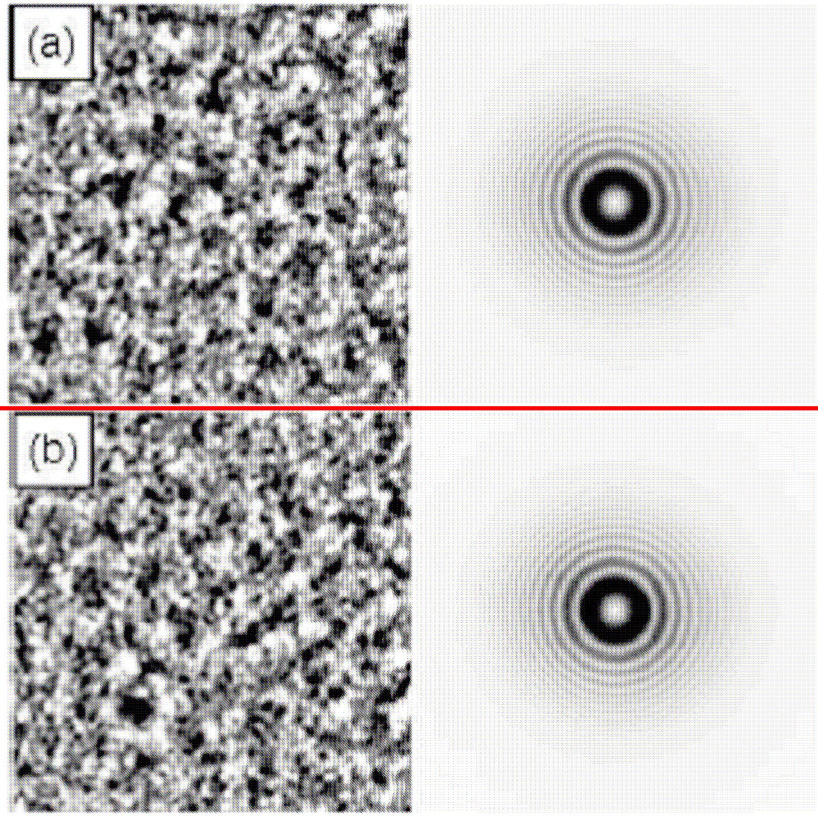
## Fréedericksz transition

- The Fréederick transition is a second order phase transition
- The order parameter is $\theta_0(x, y)$
- The control parameter is $\epsilon = U^2/U_c^2 - 1$
- The relaxation time is $\tau_{relax} = \tau_o/\epsilon$
- The correlation length $\xi_r = \frac{L}{\pi\sqrt{\epsilon}}$

# Shadowgraph image

$\nabla^2\theta(x,y)$

$\xi_r \simeq 70\mu m$




$\epsilon = -0.03$

$\epsilon = 0.02$

FIG. 1: Left column: shadowgraph snapshots of an area of $1.03 \times 1.03$ mm$^2$. Right column: corresponding structure factors, averaged over 256 images. (a): $V_0 = 3.170$ , (b): $V_0 = 3.256$ Volt.

$U_c = 3.22V$ and $L = 27\mu m$

*from* Zhou,Ahlers, arXiv: nlin/0409015v2

## Fréedericksz transition

- The Fréederick transition is a second order phase transition
- The order parameter is $\theta_0(x, y)$
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- The relaxation time is $\tau_{relax} = \tau_o/\epsilon$
- The correlation length $\xi_r = \frac{L}{\pi\sqrt{\epsilon}}$

Global variable:

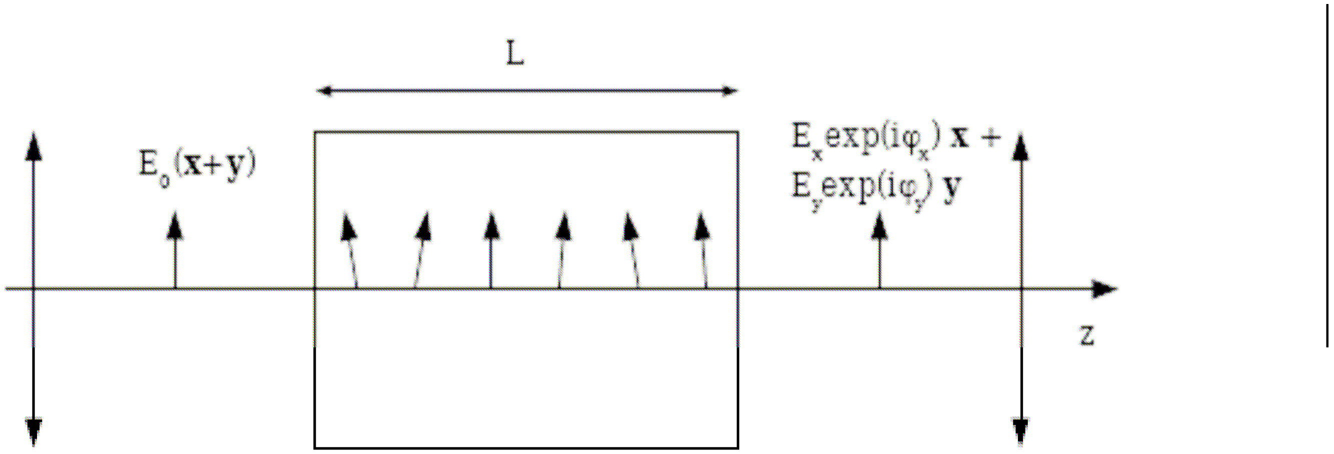
$$\zeta = \frac{2}{L}\int_0^L < (1 - n_x^2) >_{xy} dz \simeq \iint_A \theta_0^2 \, \frac{dxdy}{A}$$

where $A = \pi D^2/4$ and $< . >_{xy}$ stands for mean on $A$.

We measure de fluctuations of $\zeta$

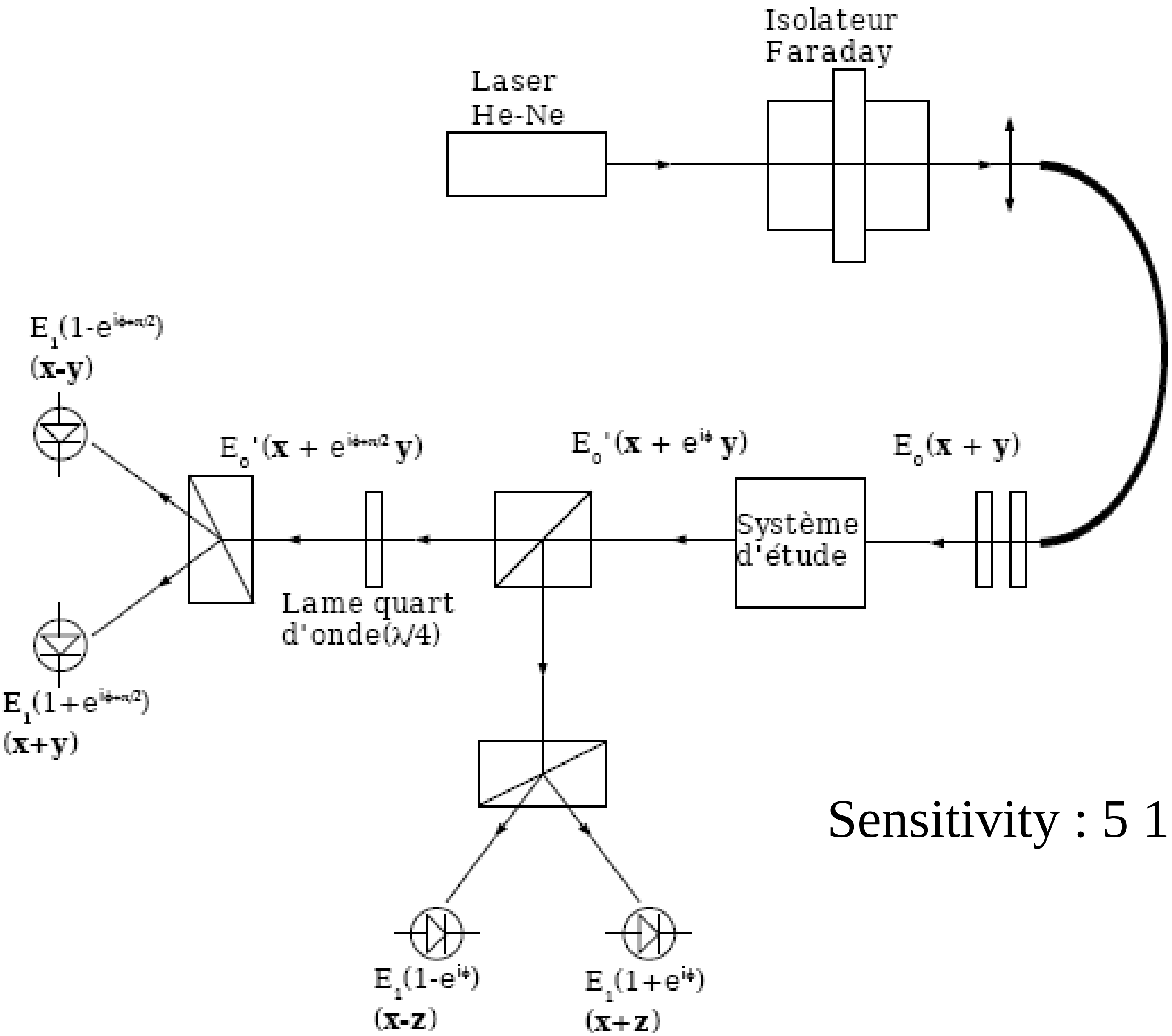
as a function of $D/\xi$

# Experimental set-up




- Two cells with different thickness: $L = 6\mu m$ and $L = 25\mu m$. Surface $S = 1cm^2$
- The liquid crystal cell is a birifrengent plate. Optical axis $// \ \hat{n}$
- Measurement of the dephasing $\Phi = \Phi_x - \Phi_y$ between the two polarisations.
- Laser diameter inside the cell $38\mu m$ (limited by diffraction)

# Experimental system: polarization interferometer




Sensitivity : 5 $10^{-14}$ m/$Hz^{1/2}$

# Measure of the dephasing

The dephasing between the Ex and Ey is :

$$\Phi = \left\langle \frac{2\pi}{\lambda} \int_0^L \left( \frac{n_o n_e}{\sqrt{n_0^2 \cos(\theta)^2 + n_e^2 \sin(\theta)^2}} - n_0 \right) \mathrm{d}z \right\rangle_{xy}$$

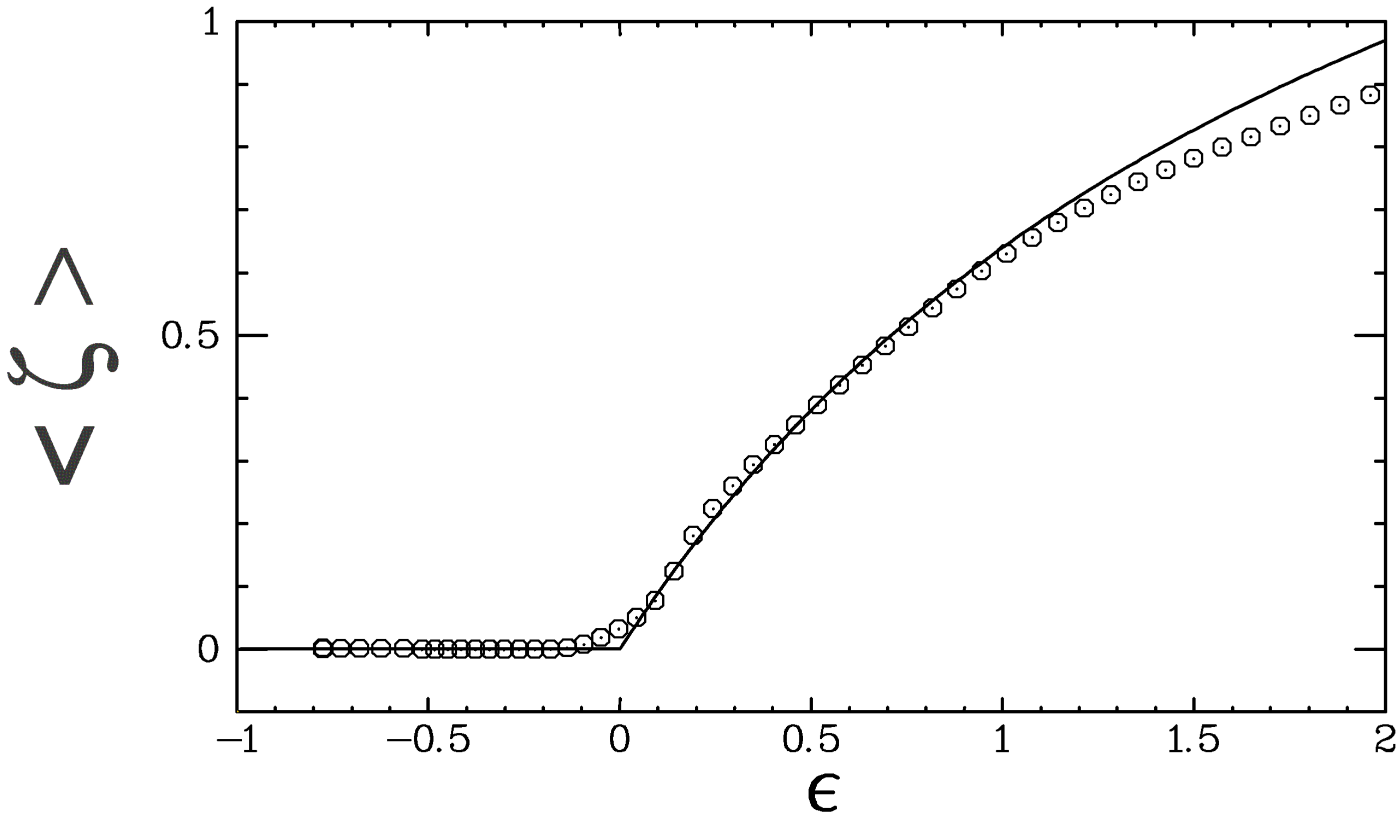
with $(n_o, n_e)$ the two anistotropic refractive indices.

If $\theta << 1$ in terms of $\zeta$ we get:

$$\Phi = \Phi_0 \left( 1 - \frac{n_e(n_e + n_o)}{4n_o^2} \zeta \right) \qquad \Phi_0 \equiv \frac{2\pi}{\lambda}(n_e - n_o)L$$

Interferometer noise : $\dfrac{(\Phi - \Phi_o)}{\Phi_o} \simeq 6\ 10^{-8} Hz^{-1/2}$

# Mean Amplitude of <ζ>



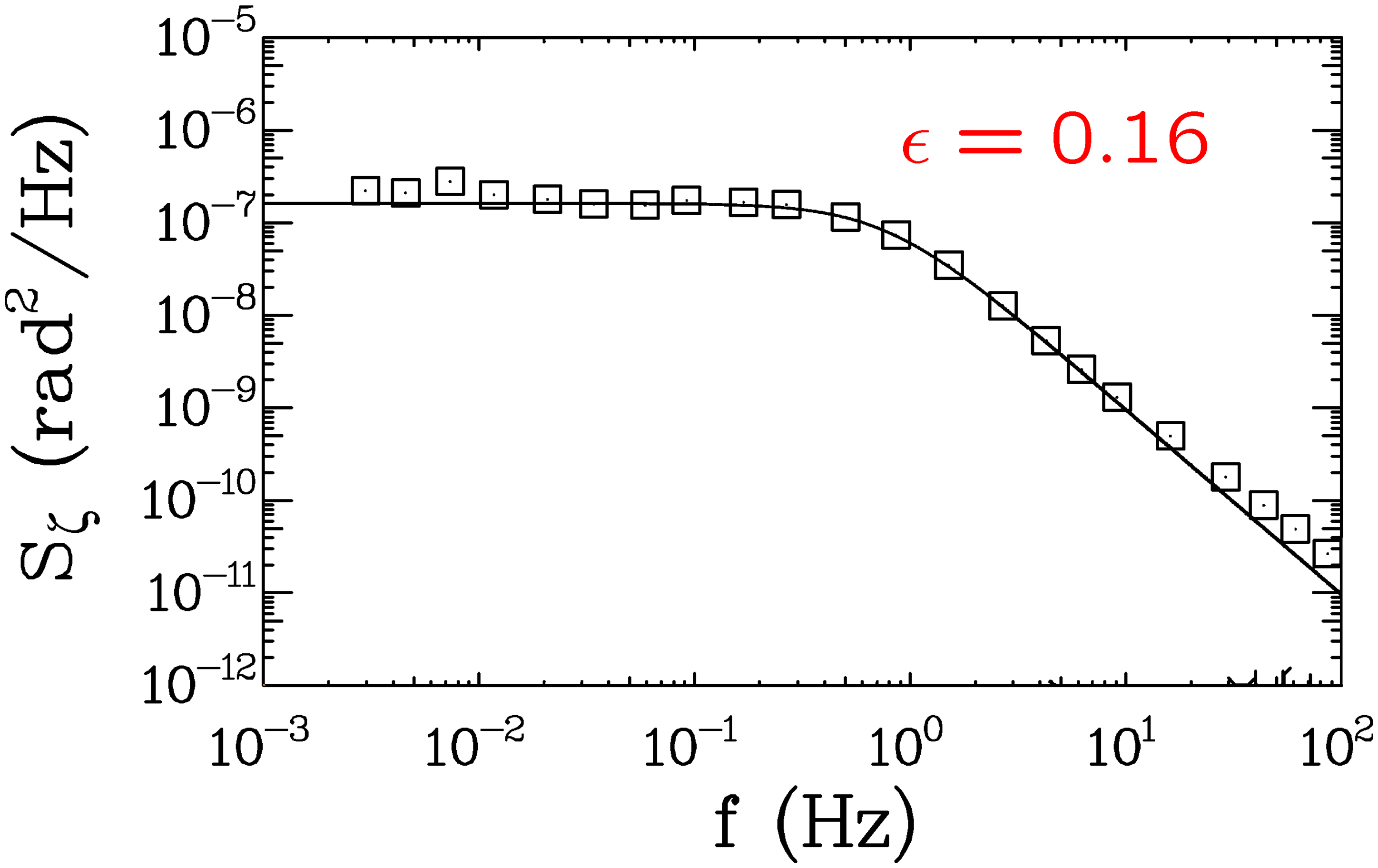

$$< \zeta >= \frac{2\epsilon}{2k + \epsilon + 1}$$

For Liquid Crystal 5CB

Computed $V_c = 0.74V$

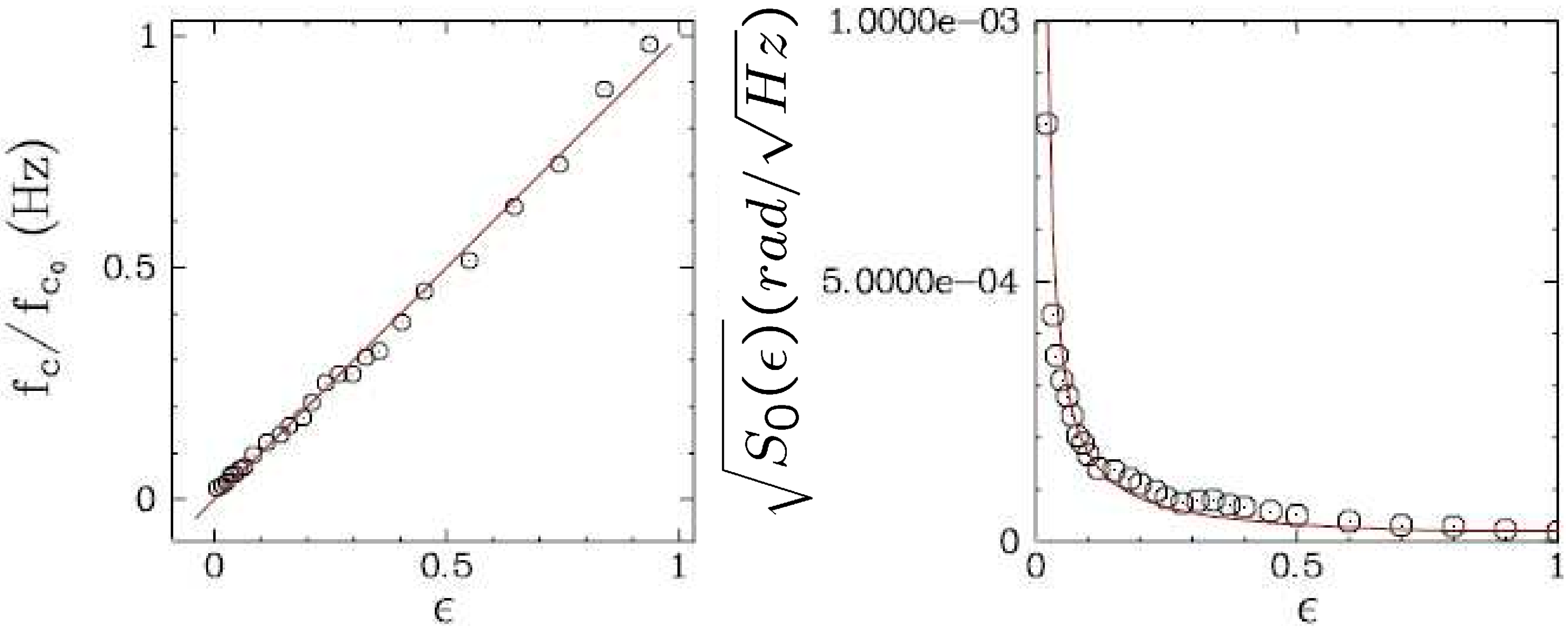
Measured $V_c = 0.73V$ by $< \zeta > \propto \epsilon = 0$

# Fluctuation spectra of ζ




$$S_\zeta(f) = \frac{S_o(\epsilon)}{1 + (\frac{f}{f_c(\epsilon)})^2}$$

# Relaxation time as a function of ε




$$f_c = \frac{1}{\tau_{relax}} = \epsilon \, \frac{1}{\tau_o}$$

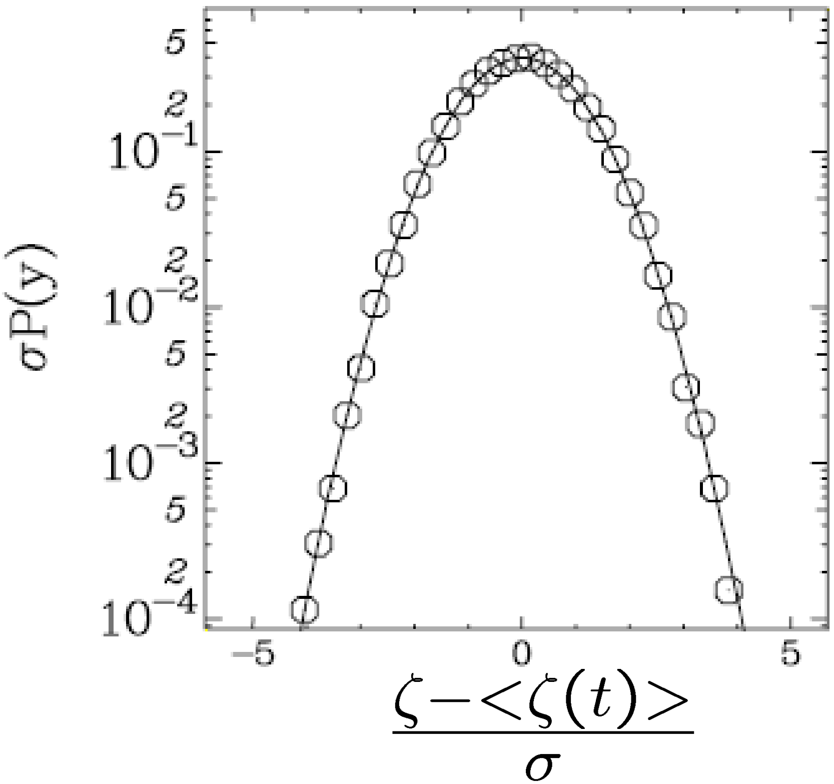
from the fit $U_c = 0.73V$ and $\tau_o = 0.1s$

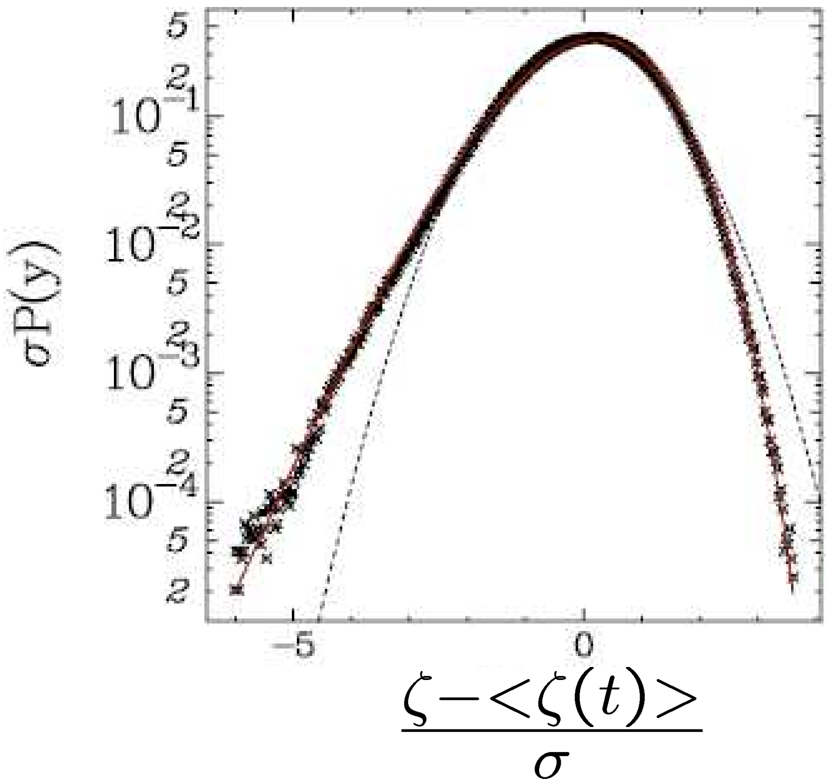
Computed $\tau_o = 0.09s$.

Variance : $\sigma^2 \propto S_0(\epsilon) f_c(\epsilon) \propto \epsilon^{-1}$

# PDF of $\zeta$

L=6.7 μm




$\epsilon = 0.02$

Gaussian fluctuations

$\xi_r = \frac{L}{\pi\sqrt{\epsilon}} = 15\mu m$

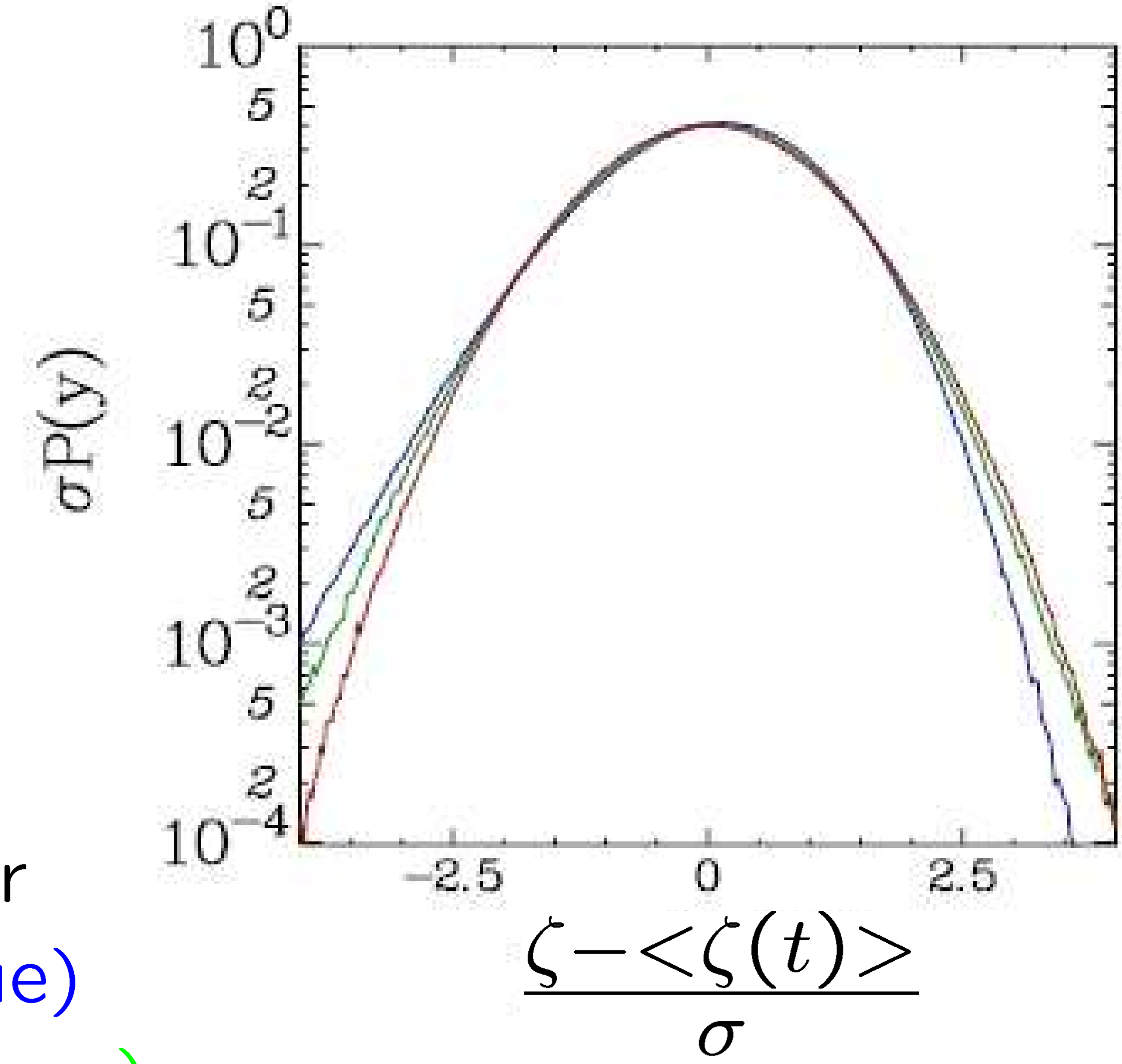
$\epsilon \simeq 0.001$

Non-Gaussian fluctuations

Gumbel model ($a \simeq 6$)

$\xi_r = \frac{L}{\pi\sqrt{\epsilon}} = 60\mu m$

# Low frequency influence




High-pass filter

- $1mHz$ (Blue)
- $50mHz$ (Green)
- $2Hz$ (Red)

# Questions

- What is the interpretation for  the PDF of $\theta_0$ ?
- Why this behavior has not been observed before?

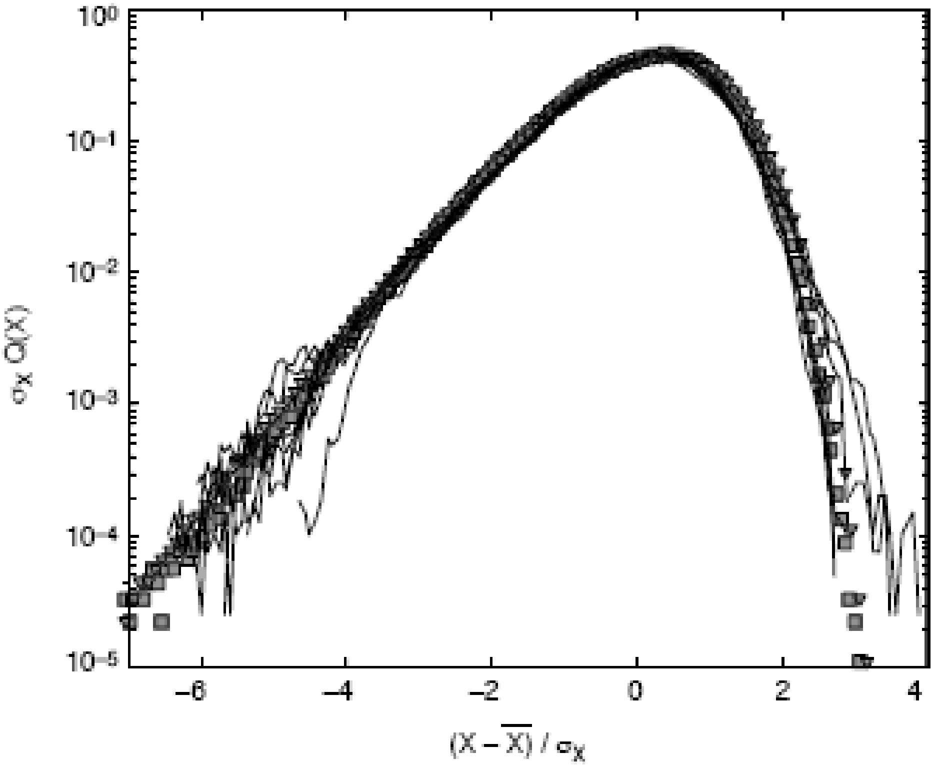
# Universality of fluctuation PDF

- In 1998 Bramwell, Holdsworth, Pinton proposed that in spatial extended systems the PDF of a global quantity $x$ may take an universal form:

$$P(x) = K \exp\{-a\ [b(x-s) - \exp(b(x-s))]\}$$

BHP distribution is equivalent to Gumbel for $a$ integer. ( Bertin 2005).

- Correlation lengths are of the order of the system size

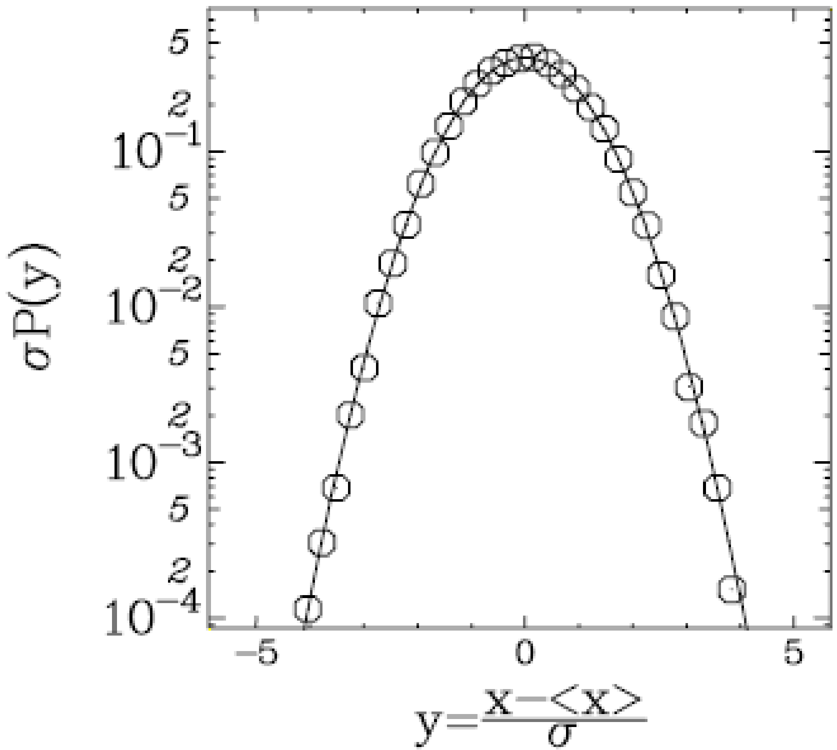


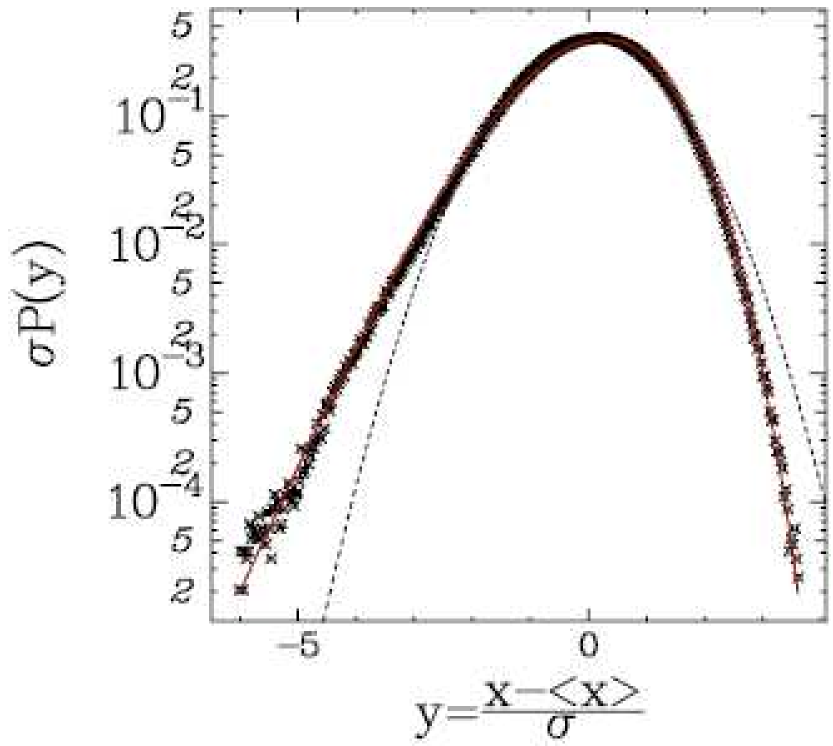

BHP has been observed in the fluctuations of :

- the injected power in an experiment of turbulence
- the magnetizaton in a xy model
- on the height of the Danube river

# PDF of $\zeta$

Cell with L=6.7 µm and $\tau_o \simeq 0.1s$




$\epsilon = 0.02$

Gaussian fluctuations

$\epsilon \simeq 0.001$

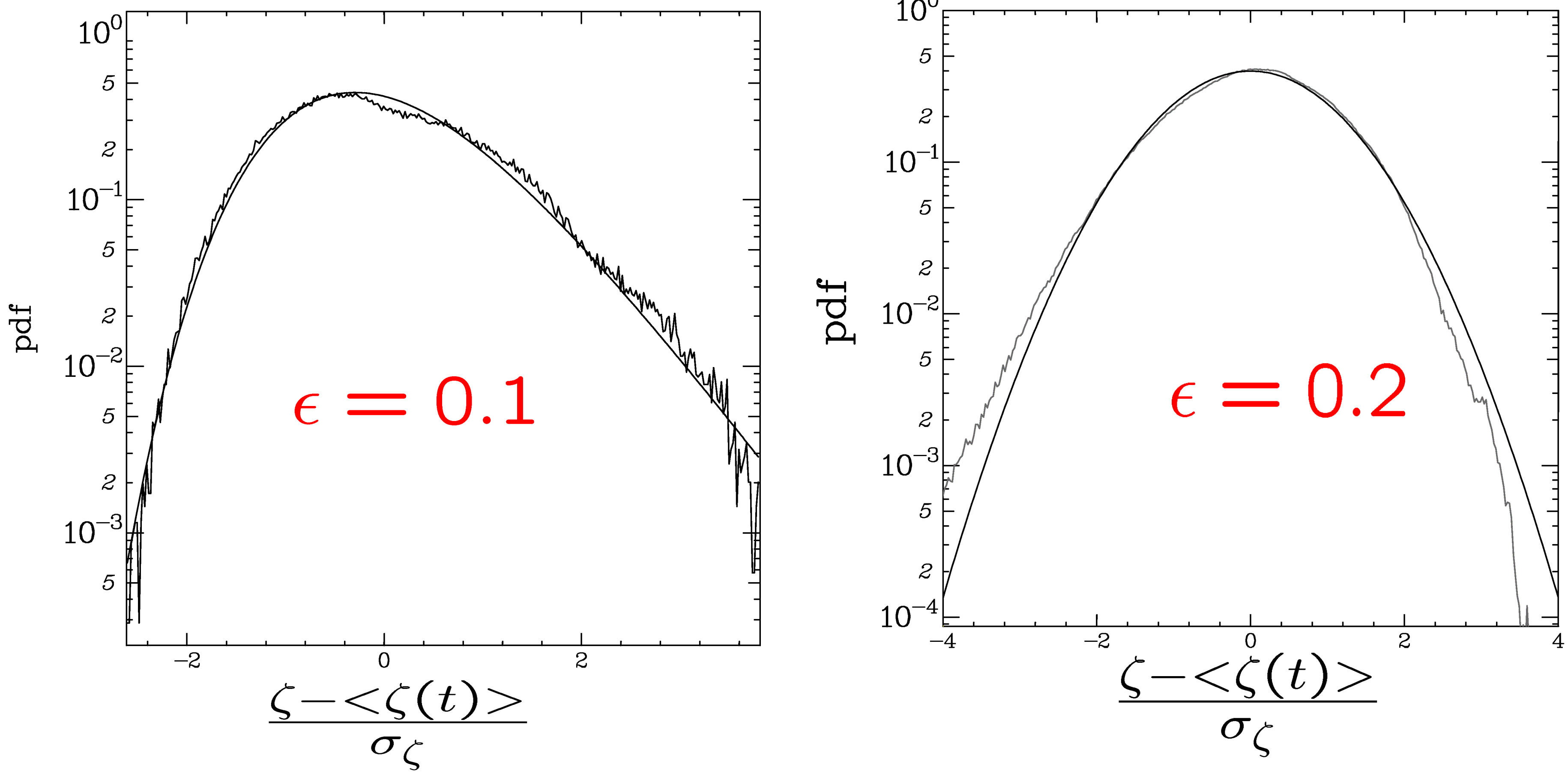
Non-Gaussian fluctuations

Gumbel model ($a \simeq 6$)

$$\xi_r = \frac{L}{\pi\sqrt{\epsilon}} = 60\mu m$$

# PDF of $\zeta$

Cell with $L = 25\mu m$ $\tau_0 \simeq 2.5s$




- The BHP distributions are observed when the diameter of the probe is smaller then the correlation length
- The BHP is determined by the very slow motion

# Aging at critical point

(Calabrese, Gambassi)

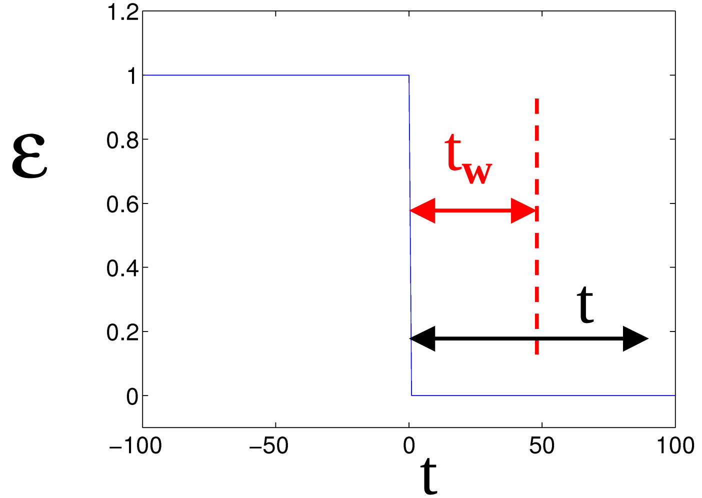
At t=0 the system is rapidly quenched from $\epsilon = \epsilon_0$ to $\epsilon_1 = 0$.

Within the Ginzburg-Landau approximation the dynamics of $\theta_0$ is ruled by:

$$\tau_o \frac{d\theta_0}{dt} = -\theta_0^3$$

whose solution is

$$< \theta_0(t)^2 >= \frac{\tau_o}{2\ (\ t + \frac{\tau_o}{2\epsilon_o})}$$




The system remains out of equilibrium forever.

Experimentally the condition $\epsilon = 0$ is never realized

$\chi(t, t_w)$ and $C(t, t_w)$

# Fluctuation dissipation theorem after a quench

We consider the fluctuations $\delta\theta(t) = \theta_0(t) - < \theta_0(t) >$

The correlation $C(t, t_w) =< \delta\theta(t)\delta\theta(t_w) >$ for $t_w < t$

$$C(t, t_w) = \frac{(t_w + \tau_m)^4 - \tau_m^4}{2(t_w + \tau_m)^{3/2}(t + \tau_m)^{3/2}}$$

$$\chi(t, t_w) = \frac{\Delta\theta_0}{\Delta h} = \frac{(t_w + \tau_m)^{3/2}}{(t + \tau_m)^{3/2}}$$ Response function

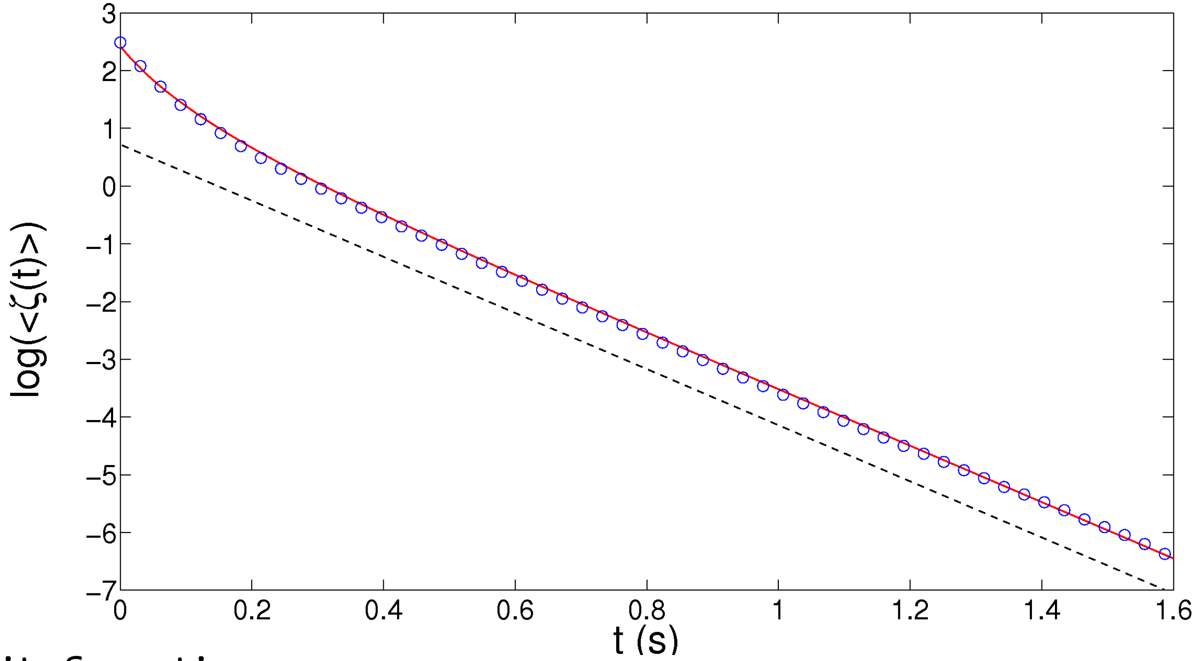
The fluctuation dissipation ratio (FDR) $X(t, t_w) = -k_B T \frac{\chi(t, t_w)}{\partial_{t_w} C(t, t_w)}$

In equilibrium $X(t, t_w) = 1$

After the quench at critical point $X(t, t_w) < 0.8 \ \forall t$ and $t_w$

# Time evolution after a quench in LC

from $\epsilon_0 = 0.3$ to $\epsilon_1 = 0.01$



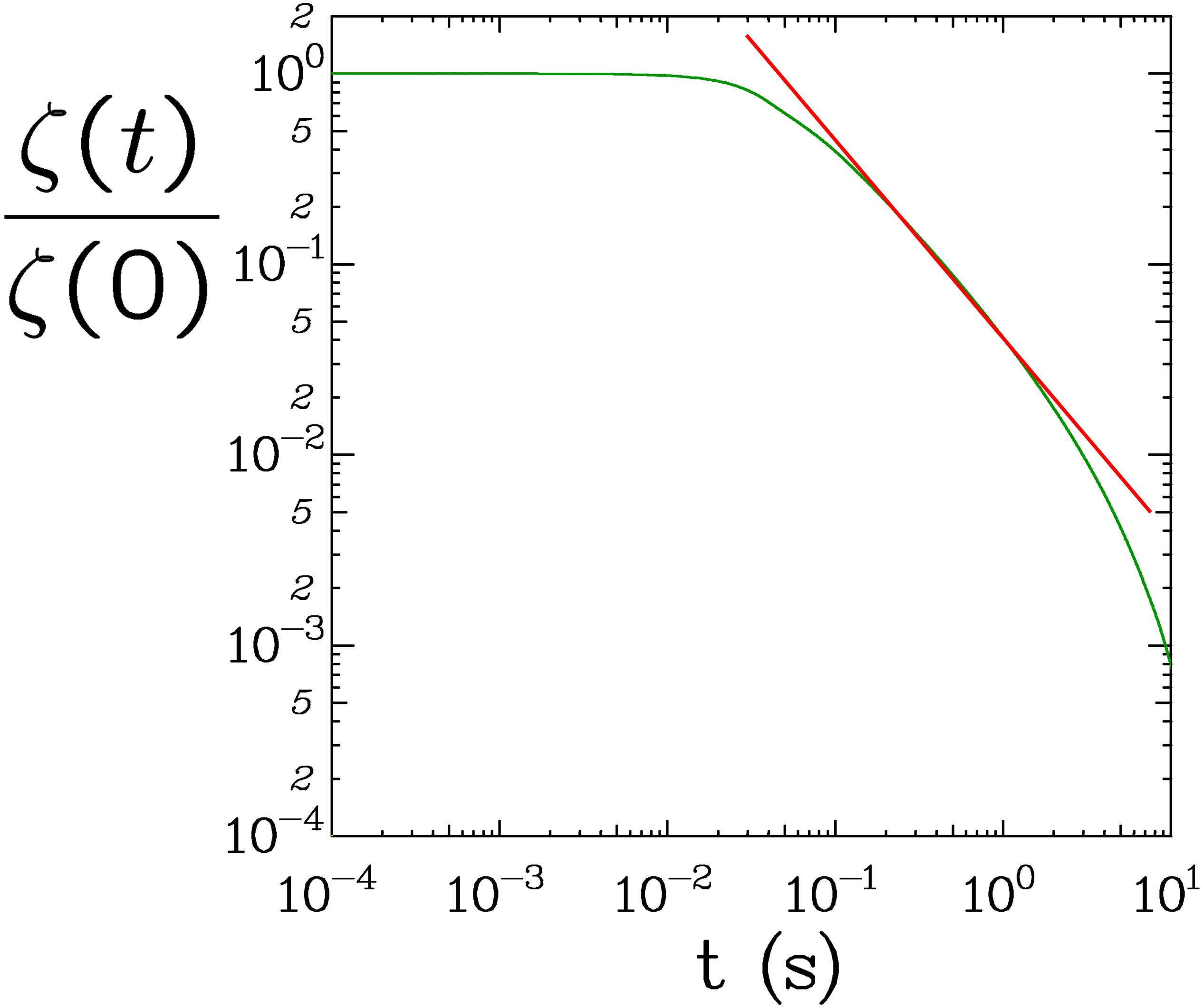

Fit function

$$\langle\ \zeta(t)\rangle = \frac{\langle\zeta(\infty)\rangle}{1 + \left(\frac{\langle\zeta(\infty)\rangle}{\langle\zeta(0)\rangle} - 1\right)\exp\left(-\frac{2\epsilon_1 t}{\tau_0}\right)},$$

with $<\zeta(\infty)>= \frac{2\epsilon_1}{(2k+\epsilon_1+1)}$ and $<\zeta(0)>= \frac{2\epsilon_0}{(2k+\epsilon_0+1)}$

# Time evolution of $\zeta$

Quench from $\epsilon_0 \simeq 1$ to $\epsilon \simeq 0.01$

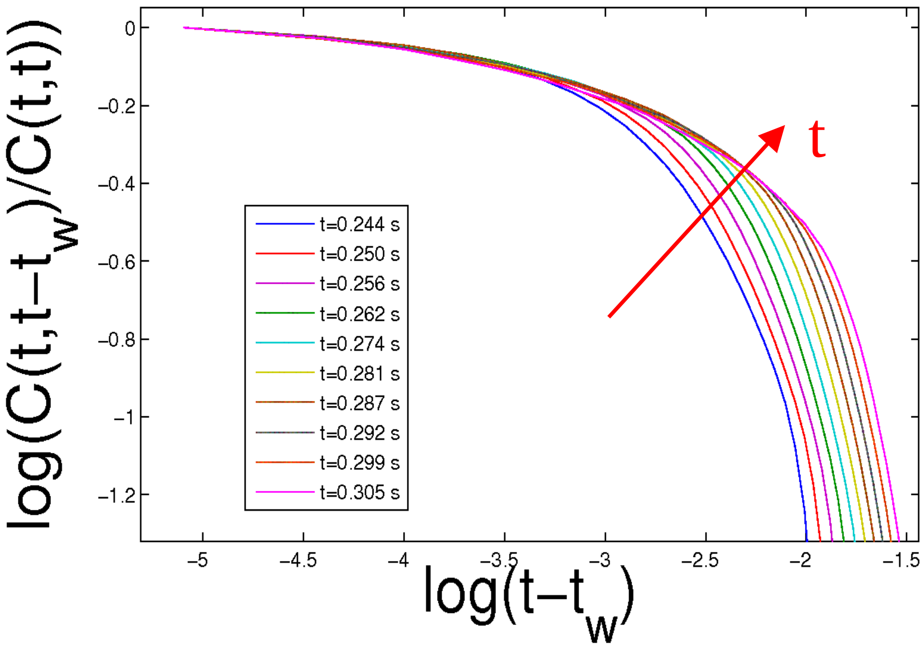


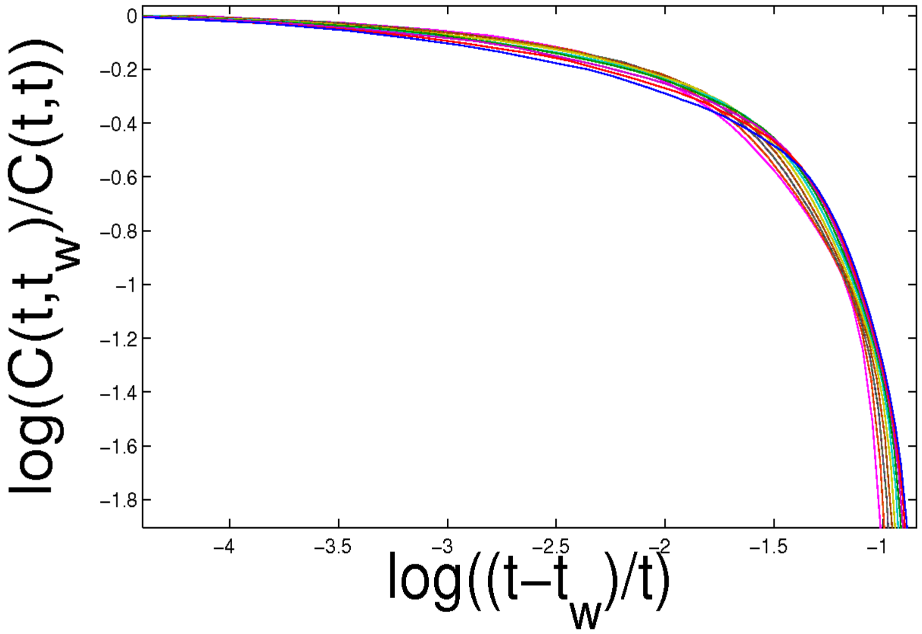

$\tau_{relax} \simeq 10s$

# Correlations

$$\zeta(t) = <\zeta(t)> + \delta\zeta(t)$$

$$C_\zeta(t, t_w) = <\delta\zeta(t)\delta\zeta(t_w)>$$




We find $C_\zeta(t,t) \propto t$ and that
it exists a master curve by scaling : $C_\zeta(t, t_w)/t$ and $(t - t_w)/t$

# FDT in the LC experiment: the measure of the response function

$$R(t,t_w) = \frac{<\Delta>}{\Gamma_{ext}} = \int_{t_w}^{t} \chi(t,t')dt' \qquad C_\theta(t,t_w) = < \delta\theta(t)\delta\theta(t_w) >$$

**FDT**

$$\boxed{R(t,t_w) = \frac{1}{k_B T_{eff}(t,t_w)}(C_\theta(t,t) - C_\theta(t,tw))}$$

In equilibrium
$T = T_{eff}$

Which is the appropriate external torque $\Gamma_{ext}$ for the LC ?

$$\frac{\gamma AL}{2}\frac{d\theta_0}{dt} = B\left[2\epsilon\theta_0 - (\kappa+\epsilon+1)\,\theta_0^3\right] + \eta \qquad \left\{\begin{array}{l} B = A\pi^2 K_1/4L \\ \epsilon = \epsilon_0 + \delta\epsilon \\ \theta_0(t) = \psi_0 + \Delta(t) \end{array}\right.$$

$$\boxed{\frac{\gamma AL}{2}\frac{d\Delta}{dt} = B\left[2\epsilon_0 - 3\,(\kappa+\epsilon_0+1)\,\psi_0^2\right]\Delta + \underbrace{2B\delta\epsilon\psi_0\left(1-\frac{\psi_0^2}{2}\right)}_{\Gamma_{ext}} + \eta}$$

# FDT in the LC experiment:

$$\zeta(t) = \theta_0(t)^2 = \psi_0^2 + 2\psi_0\Delta =< \zeta(t) > +\delta\zeta(t)$$

$$C_\zeta(t, t_w) =< \delta\zeta(t)\delta\zeta(t_w) >= 4\psi_0(t)\psi_0(t_w)\ C_\theta(t, t_w)$$

In equilibrium $\psi_0(t) = \psi_0(t_w)$

$$R(\tau) = \frac{\langle\Delta(\tau)\rangle}{\Gamma_{ext}} = \frac{R_{\zeta,\delta\epsilon}}{4B\ \psi_0^2\ \left(1 - \frac{\psi_0^2}{2}\right)}$$

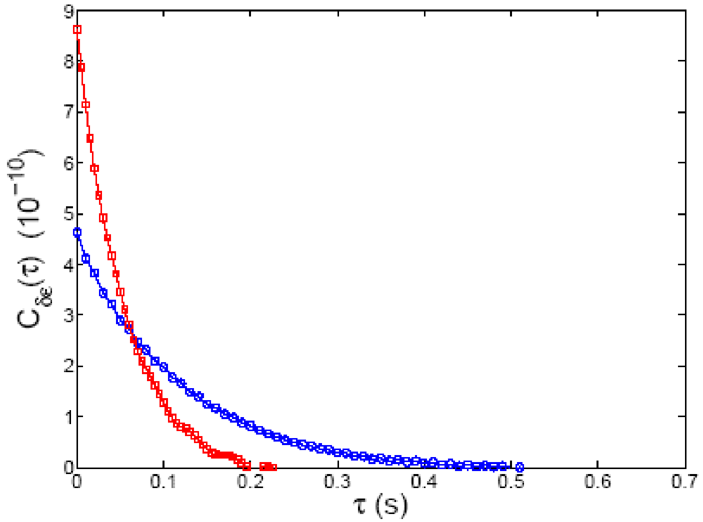
and FDT $$\frac{R_{\zeta,\delta\epsilon}}{B\left(1 - \frac{<\zeta>}{2}\right)} = \frac{1}{k_B T}(C_\zeta - C_\zeta(t, t_w))$$

with $B = \mathcal{A}\pi^2 K_1/4L$

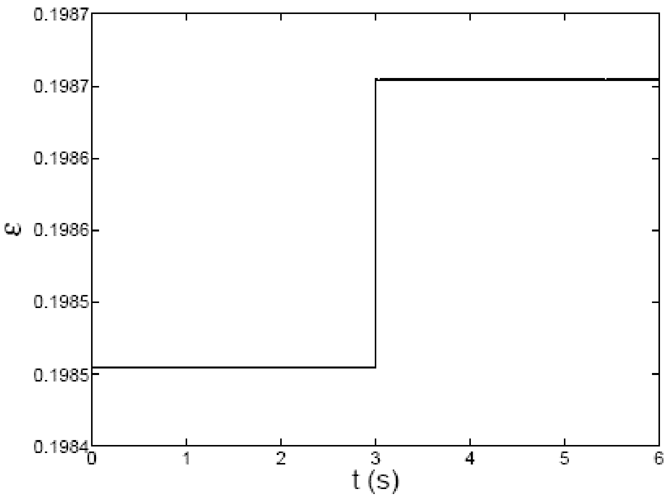
# FDT in the LC experiment in equilibrium

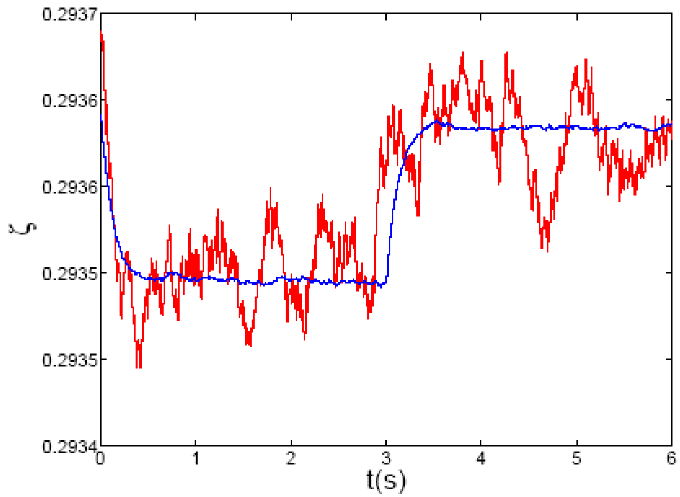
$\varepsilon_0 = 1{,}039$

$\varepsilon_0 = 0{,}1985$




• response function



# FDT in the LC experiment in equilibrium

$\varepsilon_0 = 0{,}1985$

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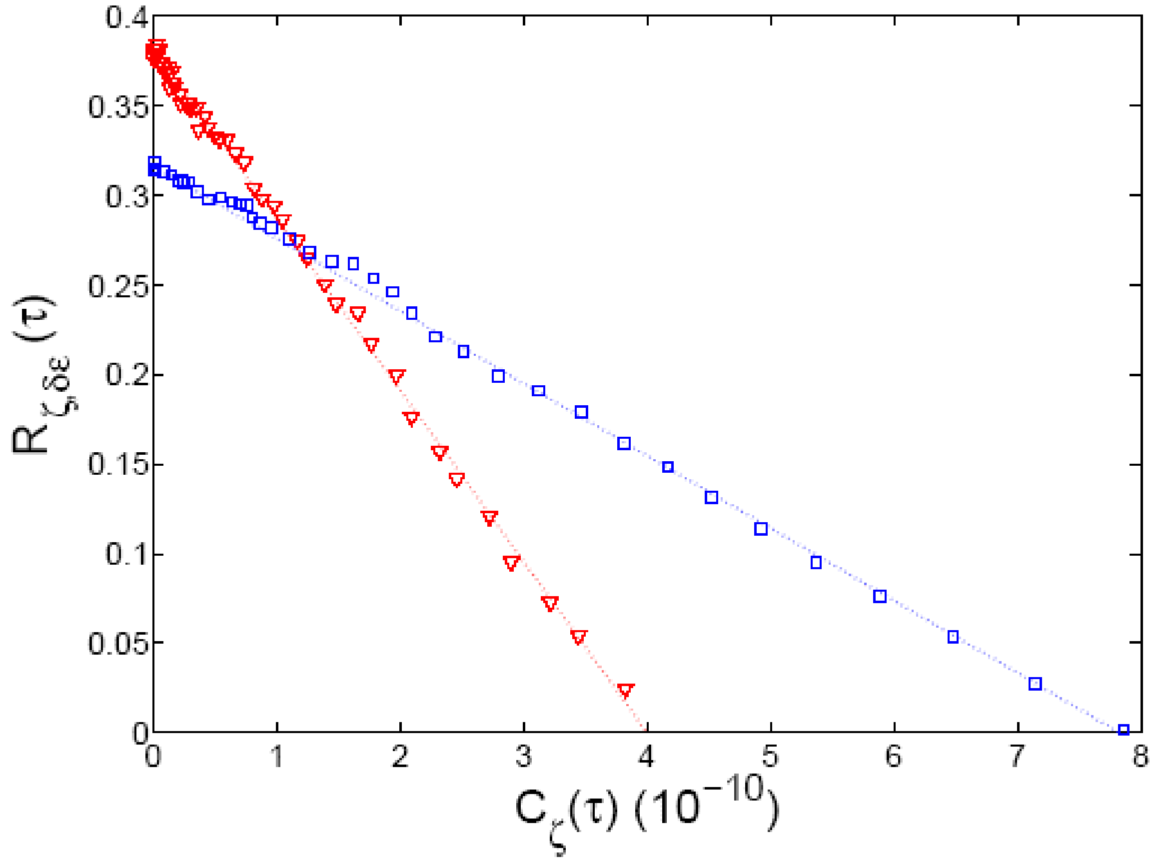
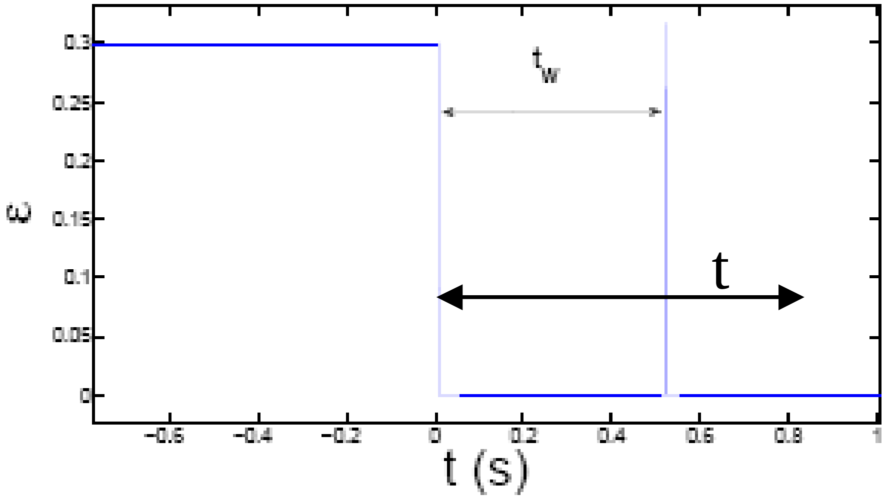
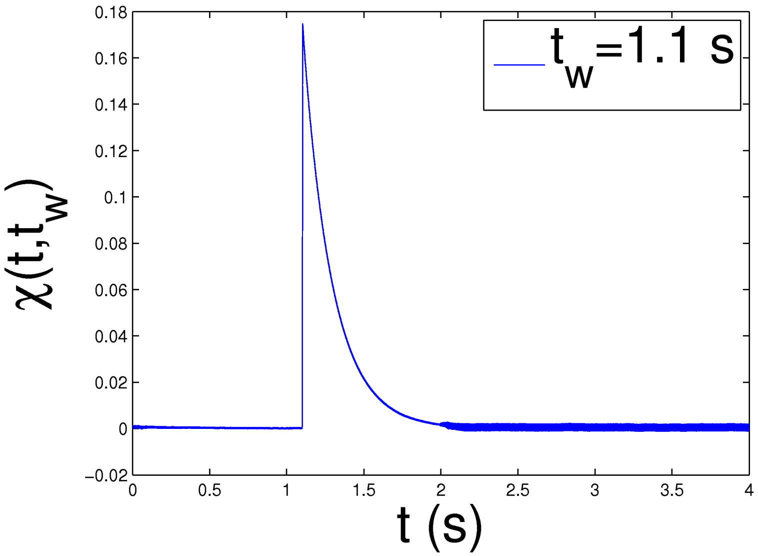



$$B' = B\left(1 - \frac{<\zeta>}{2}\right)$$

$$\begin{cases} \frac{B'}{k_B T} = 8,5.10^8 & \text{at} \quad \epsilon = 0,1985 \quad A = 2{,}4.10^{-6}\, m^2 \quad \text{Do=1.7mm} \\ \frac{B'}{k_B T} = 4.10^8 & \text{at} \quad \epsilon = 1,039 \quad A = 1{,}4.10^{-6}\, m^2 \quad \text{Do=1.3mm} \end{cases}$$

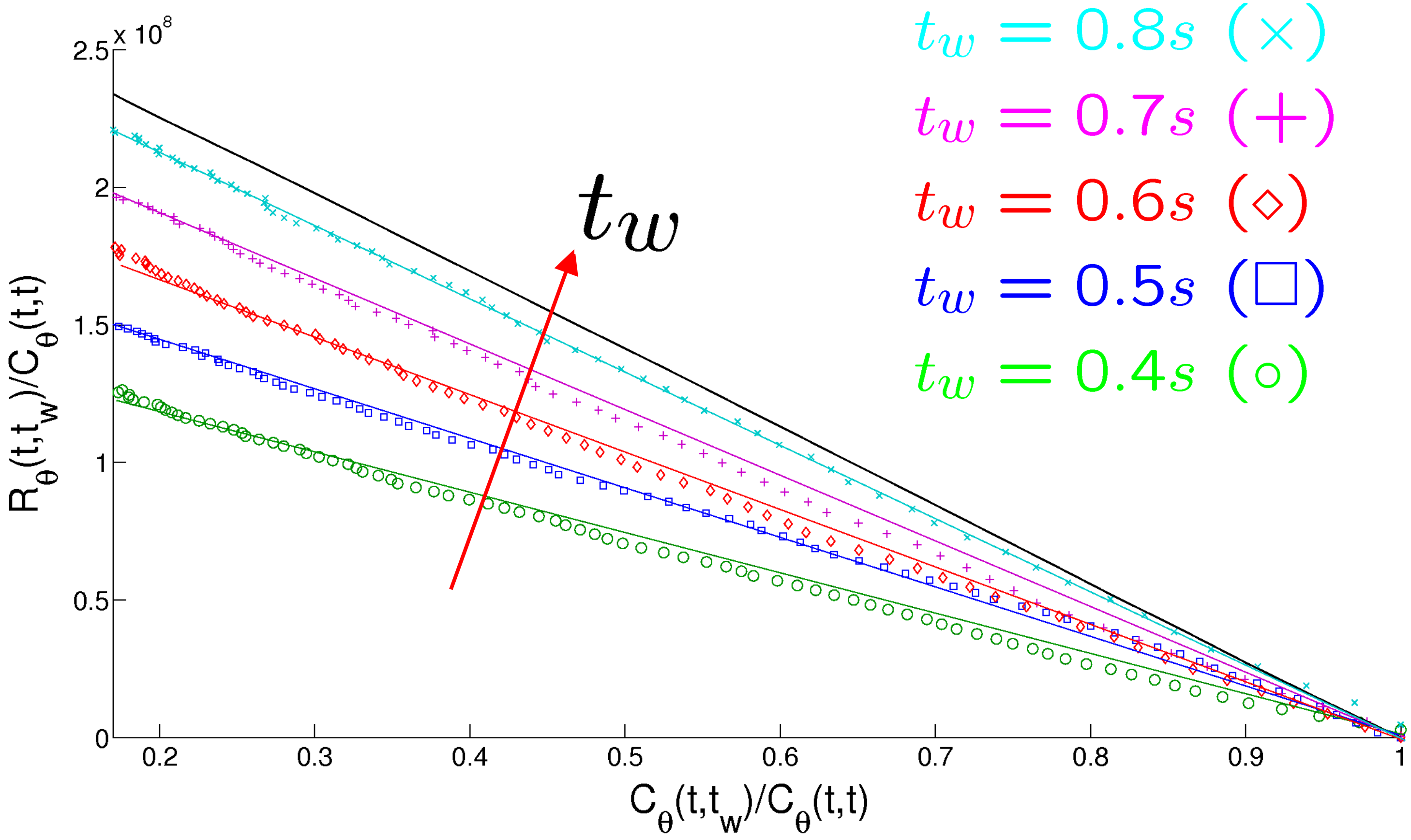
# Response function during aging



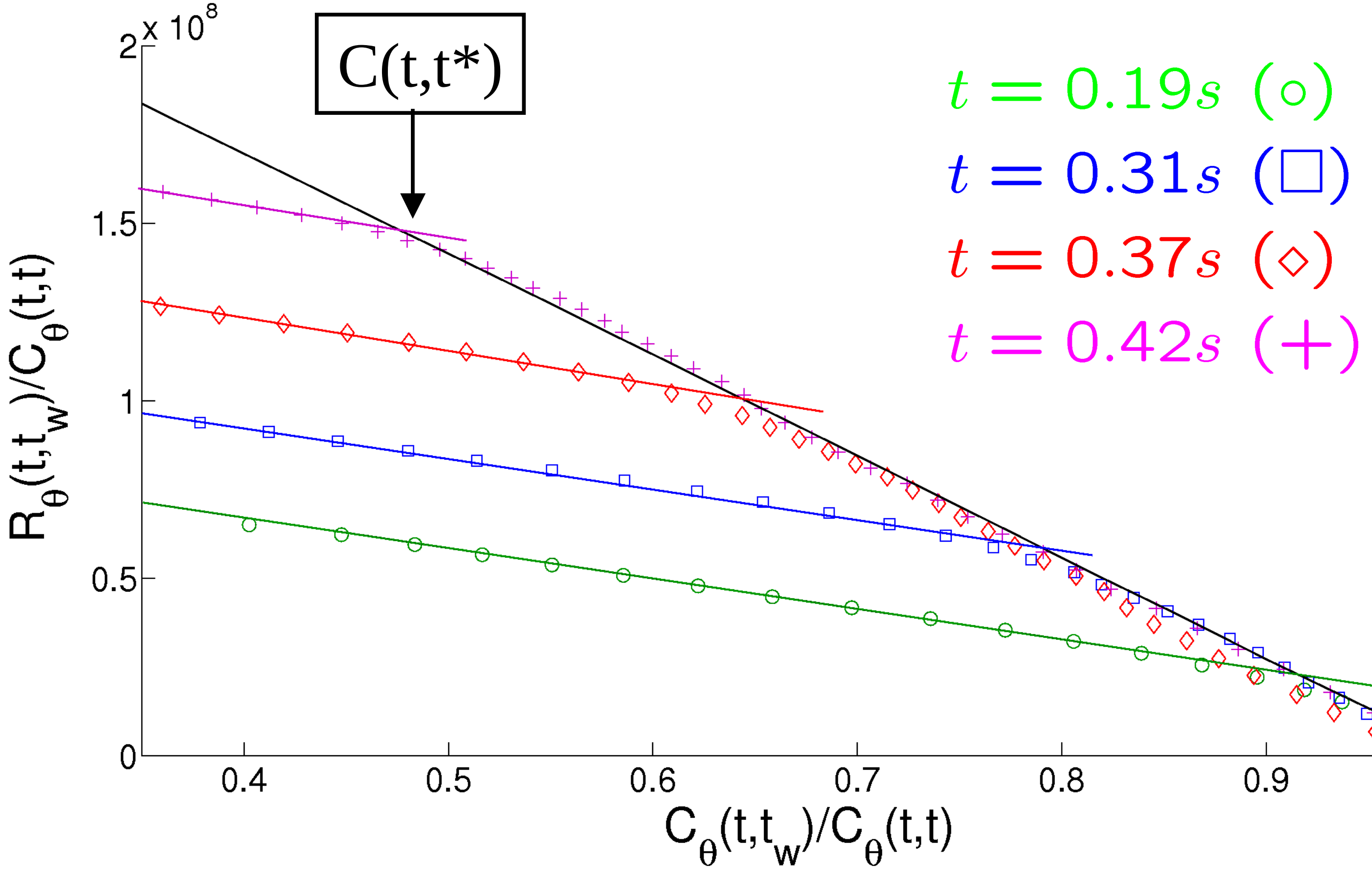



# FDT out of equilibrium: fixed tw as a function of t

$$R(t,t_w) = \frac{1}{k_B T_{eff}(t,t_w)}(C_\theta(t,t) - C_\theta(t,tw))$$

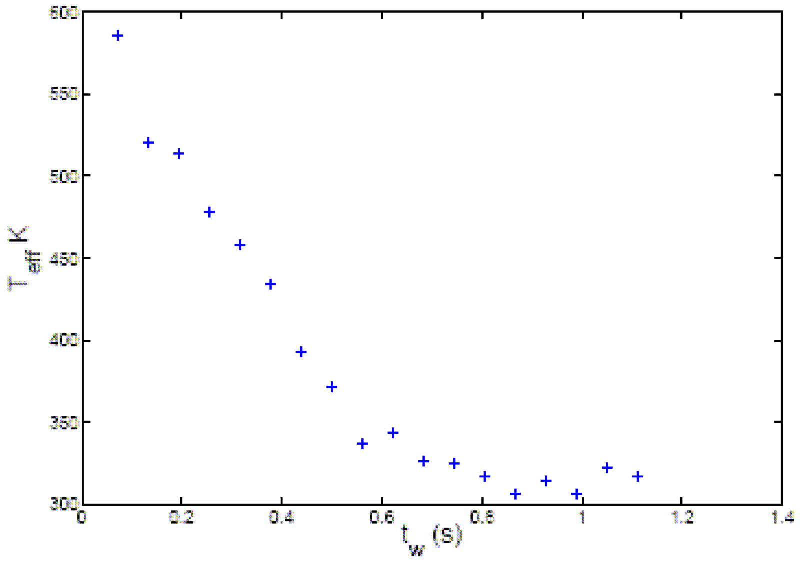


# FDT out of equilibrium: fixed t as a function of tw

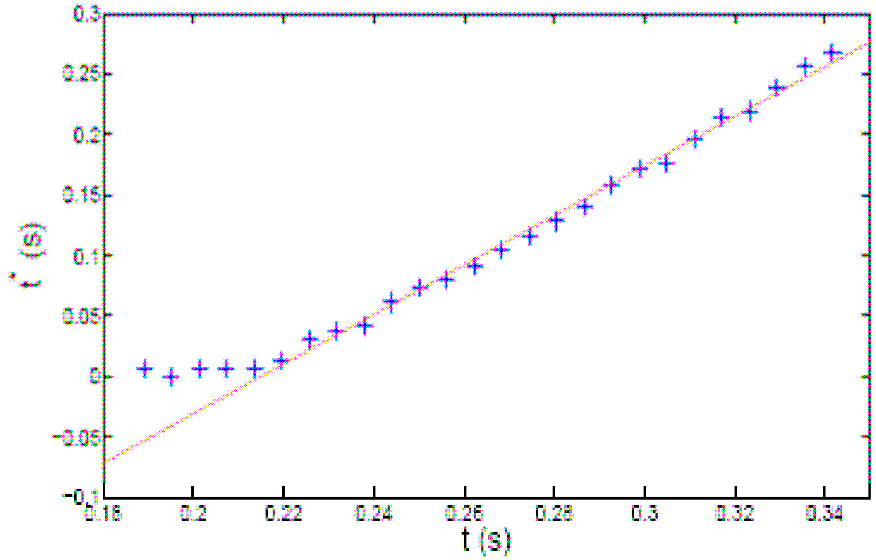


# Time evolution

fixed tw as a function of t
evolution of Teff




fixed t as a function of tw
evolution of t*



# Conclusions

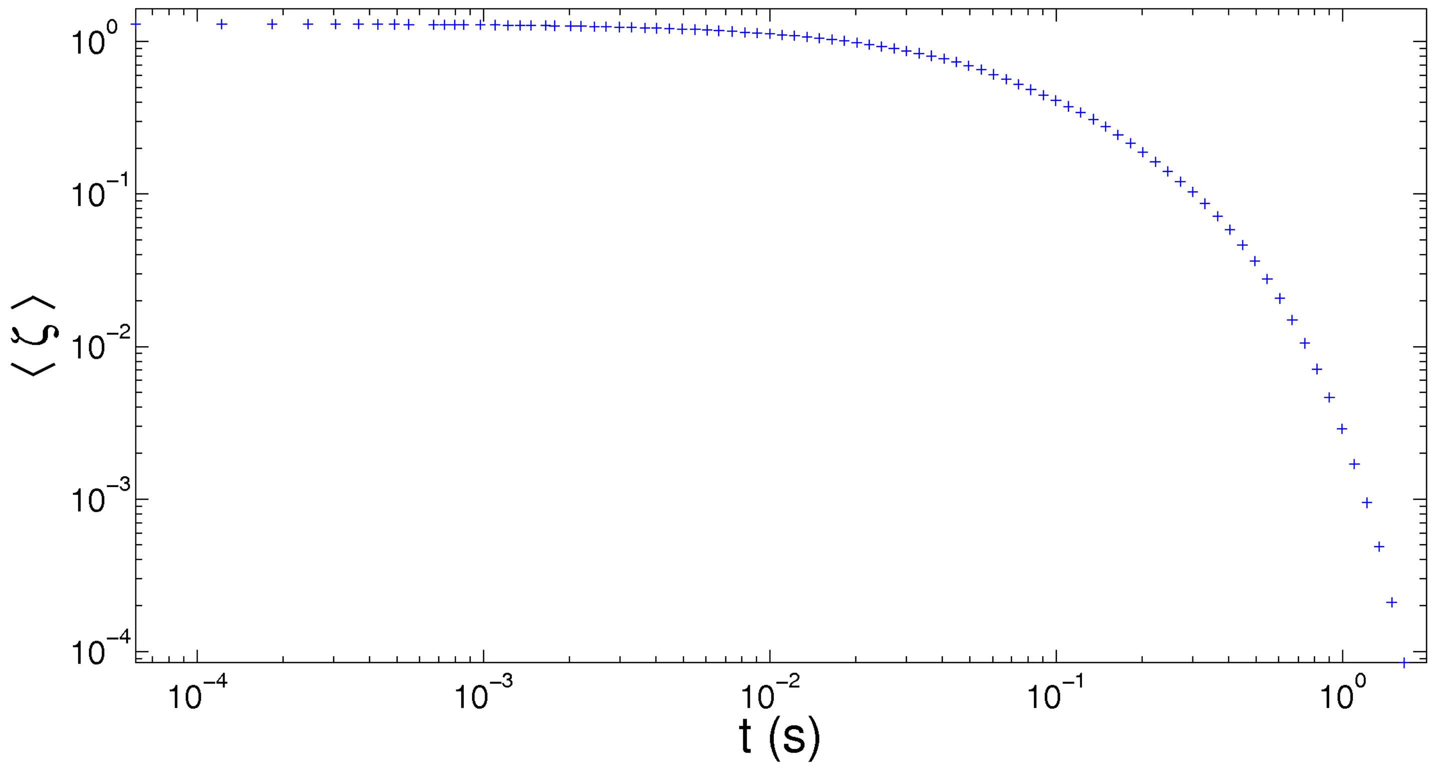
Using a liquid crystal driven by an electric field at the Fréedericksz transition we observe that :

- The probability distribution function of the order parameter fluctuations are well described by a generalized Gumbel, when the size of the measurement region is comparable to the correlation length.

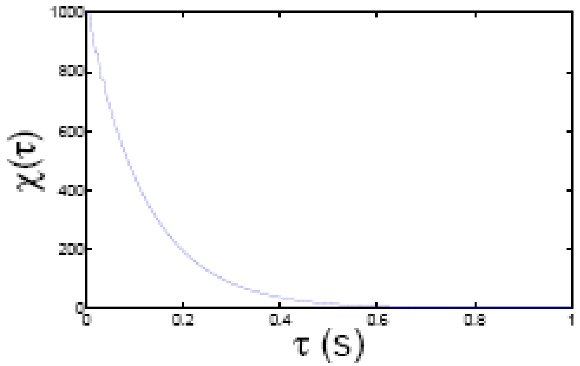
- After a quench close to the critical point the system presents power law decay. A master curve of the correlation function may be produced by a rescaling which is similar to the one used in aging materials.

- FDT is strongly violated during the decay.

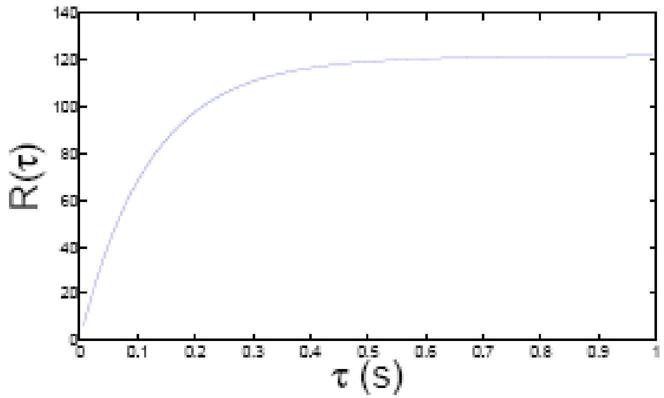
# Time evolution in loglog



# Integrated response function

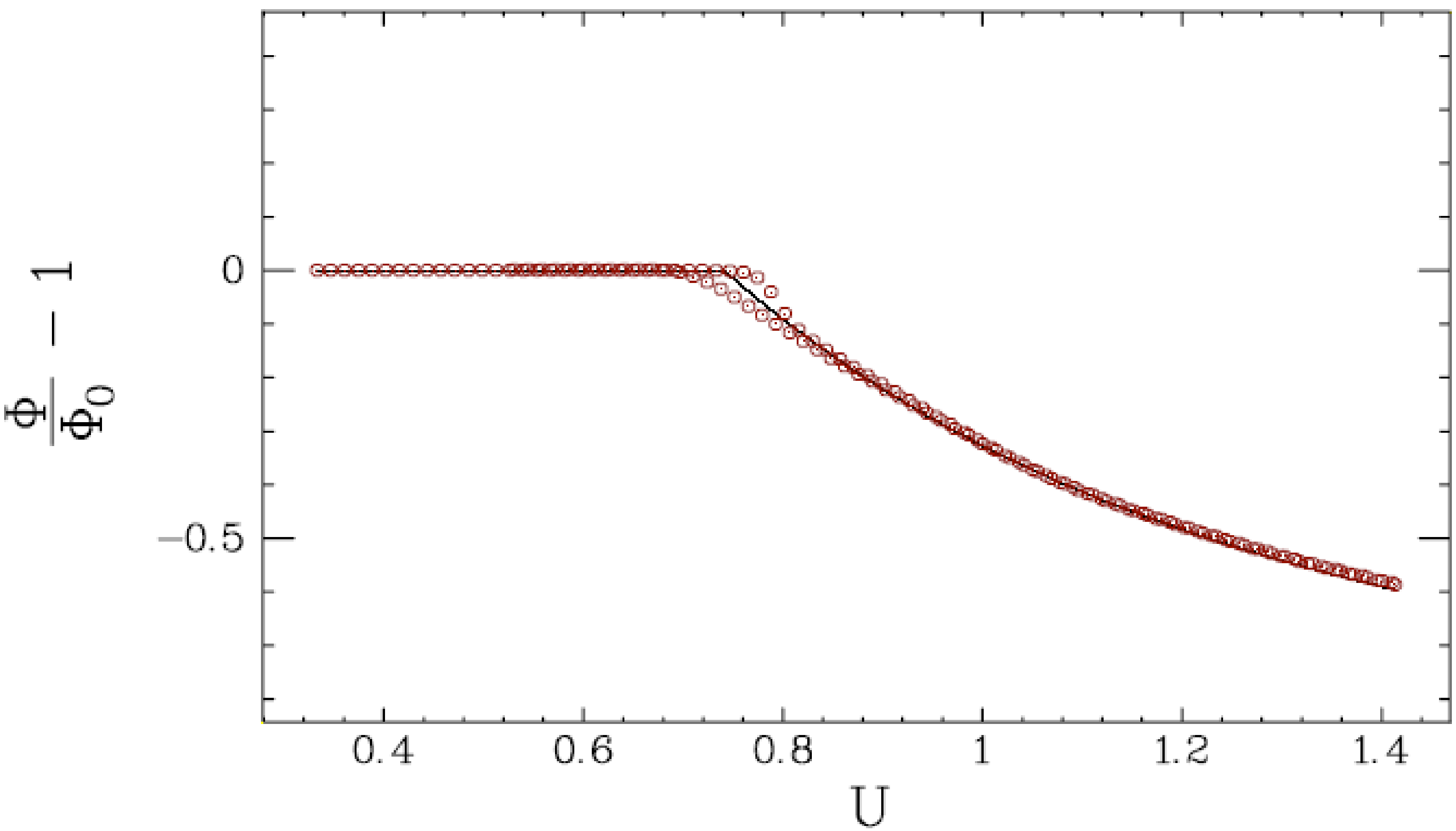



a)

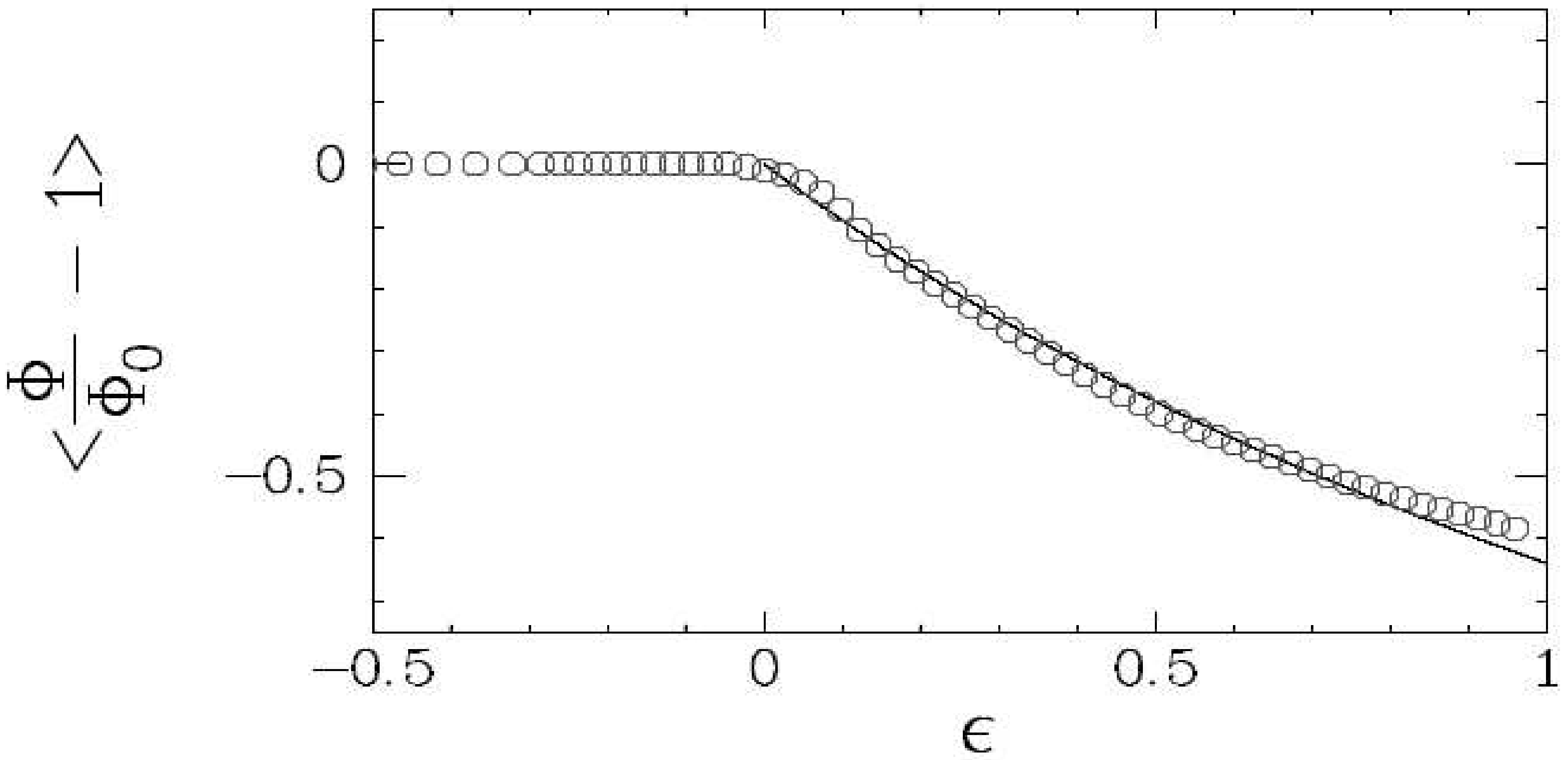



b)

# Mean Amplitude of <ζ> as a function of U



# Mean Amplitude of <ζ>



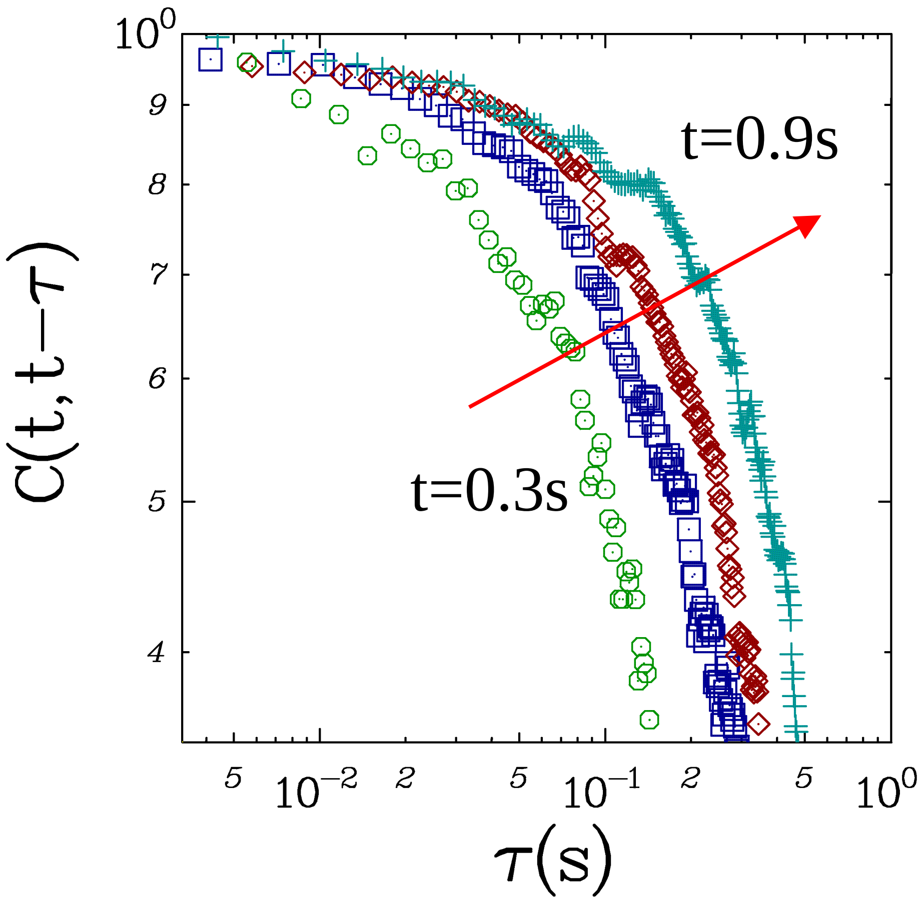

$$< \frac{\Phi}{\Phi_o} - 1 > \propto < \zeta > = \frac{2\epsilon}{2k + \epsilon + 1}$$

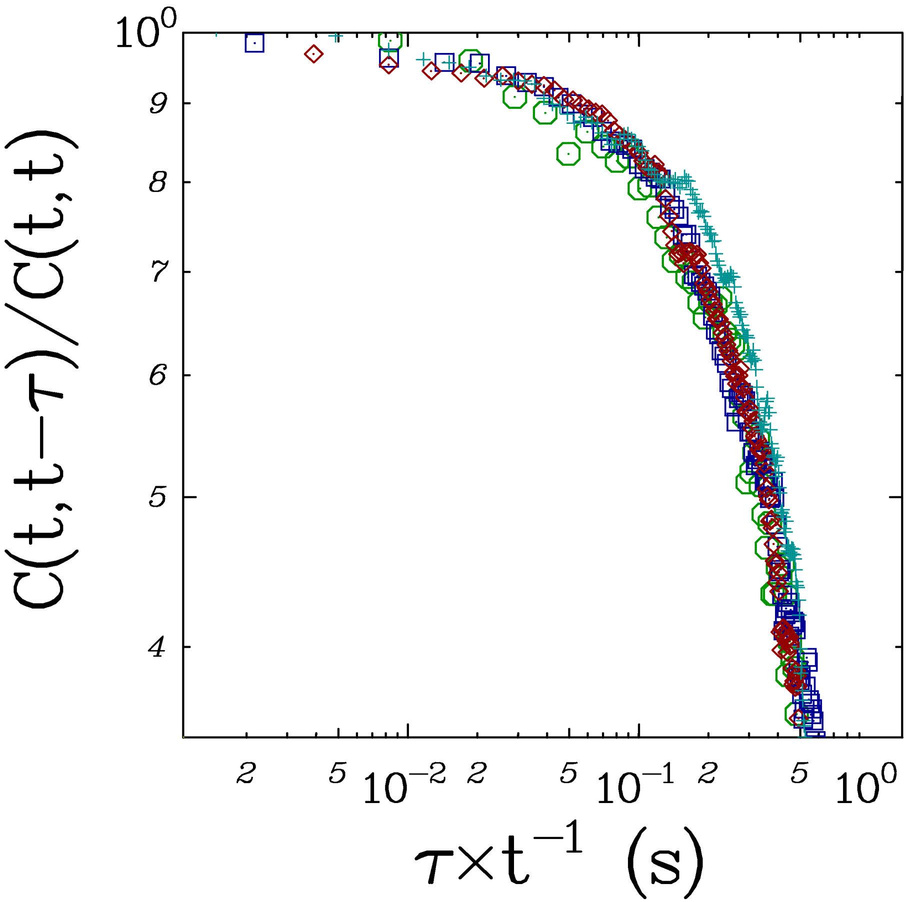
For Liquid Crystal 5CB

Computed $V_c = 0.74V$

Measured $V_c = 0.73V$ by $< \frac{\Phi}{\Phi_0} - 1 > \propto \epsilon = 0$

# Results on correlations




We find $C(t,t) \propto t$ and that

it exists a master curve by scaling : $C(t,t-\tau)/t$ and $\tau/t$