A test of first order scaling in N_f=2 QCD: progress report

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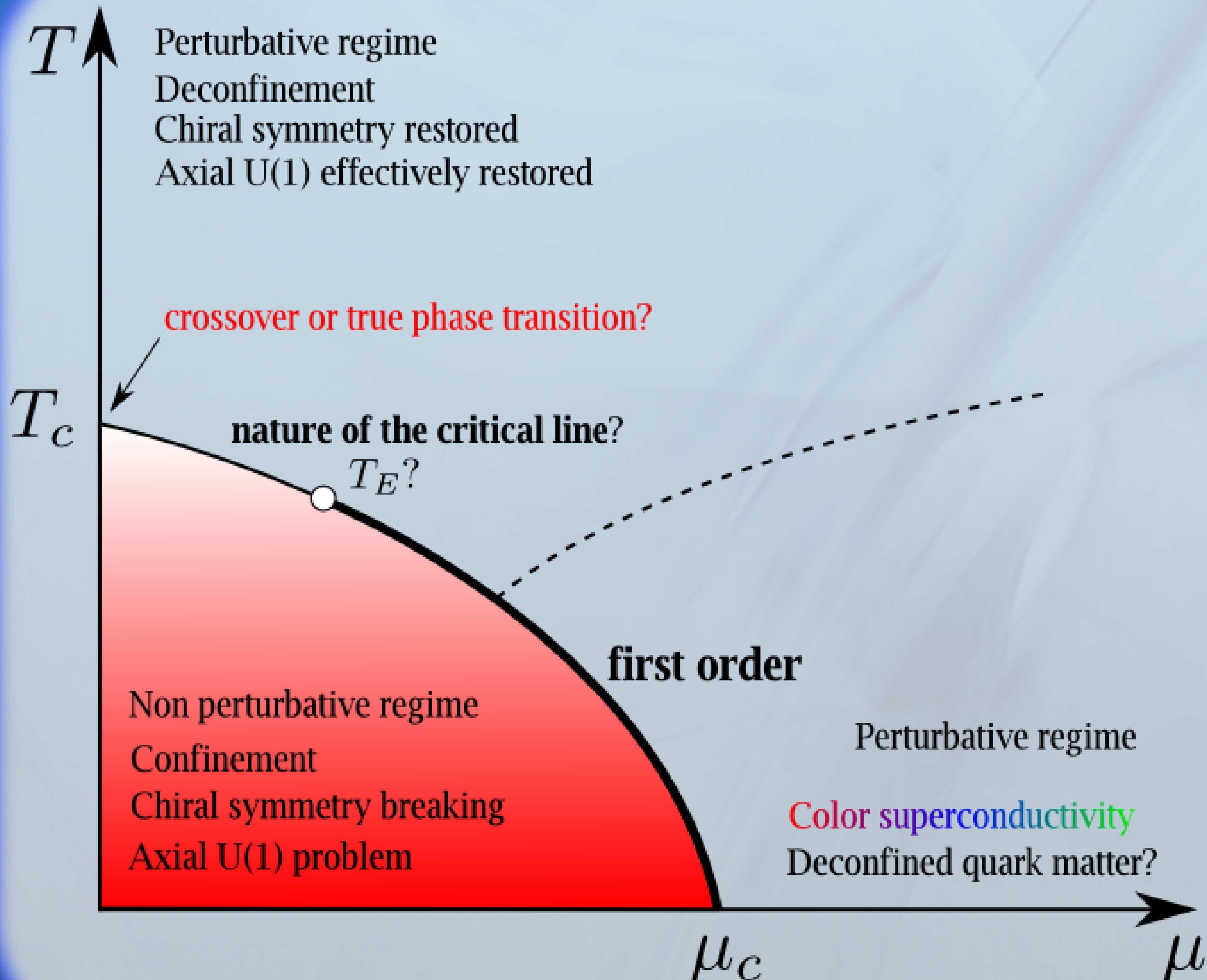
Outline

- The order of the chiral transition in $N_f{=}2$ QCD and the QCD phase diagram
- Lattice regularization of QCD
- Prediction from effective models and previous literature
- Present evidence from lattice simulations
- Preliminary new results
- Conclusion and discussion

Phase diagram



The order of the transition at $\mu_B = 0$ is of great importance: the existence of a critical endpoint at T_E stems from the hypothesis of a crossover at the massless, $\mu = 0$ theory.

There is a general tendency to accept the crossover scenario in the real QCD case ($N_f = 2+1$ with physical quark masses): it has been shown (Y. Aoki, Z. Fodor, S. D. Katz and K. K. Szabo, Phys. Lett. B 643, 46 (2006); Nature 443, 675 (2006)) that the susceptibility of a possible order parameter for the transition (the chiral condensate) does not show any signal of growing with the spatial volume, till $L_S \sim 6$ fm.

However is interesting to deeply address the problem because experimental evicences still lack.
Two flavor QCD in the light quark mass limit is an interesting case:

- there are theoretical predictions about the order (eff. Chiral models)
- no definite answer from lattice simulations yet

M. D'Elia, A. Di Giacomo, C. Pica, PRD 72, 114510 (2005)
G. C., M. D'Elia, A. Di Giacomo, C. Pica, arXiv:0706.4470 + unpublished results

Model predictions for chiral $N_f=2$

- $U_A(1)$ anomaly effective (no light η', $c \neq 0$): effective model has a fixed point, i.e. second order transition in the $O(4)$ universality class or a first order.
- $U_A(1)$ anomaly not effective (light η', $c = 0$): no $O(4)$ stable f. p.

F. Basile, A. Pelissetto, E. Vicari, 2005 $\Longrightarrow$ $U(2)_L \otimes U(2)_R / U(2)_V$ or first order

Second order in the chiral limit $\Longrightarrow$ crossover at small quark masses

First order in the chiral limit $\Longrightarrow$ first order in a small region around the chiral point

Lattice regularizaton of QCD - 1

The thermal QCD partition function is rewritten in terms of a path integral over a discretized euclidean lattice with periodic boundary condition in the time direction (antiperiodic for fermion fields):

$$Z = \int DU D\psi D\bar{\psi} e^{-(\beta S_G + \bar{\psi} M[U,m_q]\psi)} = \int DU e^{-\beta S_G} \det M[U,m_q]$$

where $\beta \cdot S_G$ is the pure gauge (e.g. plaquette) action, $\beta \equiv 2N_c/g^2$ is the inverse bare gauge coupling and M is the fermion matrix.
In the staggered (Kogut-Susskind) formulation:

$$M_{i,j} = am\delta_{i,j} + \frac{1}{2}\sum_{\nu=1}^{4} \eta_{i,\nu}(U_{i,\nu}\delta_{i,j-\hat{\nu}} - U^{\dagger}_{i-\hat{\nu},\nu}\delta_{i,j+\hat{\nu}})$$

In this formulation thermal expectation values are computed by Monte Carlo simulations, using $e^{-\beta S_G} \det M[U,m_q]$ as a probability distribution function for gauge configurations.

Lattice regularizaton of QCD - 2

The physical temperature is given by the inverse temporal extension

$$T = \frac{1}{aN_t}$$

and the approach to the continuum field theory links the lattice spacing a to the bare parameters of the theory, $a \equiv a(\beta, m_q)$. Therefore at fixed N_t the temperature is a function of β and the bare quark mass m_q.

A regular lattice formulation of fermions in general breaks chiral symmetry explicitely (see Nielsen-Ninomiya no-go theorem), therefore, in case of a second order phase transition, one expects to see the universal behaviour (e.g. O(4)) only in the continuum limit.

However in the staggered fermion formulation (in contrast to Wilson fermions) a residual symmetry of the lattice action leads to a predicition for O(2) universal behaviour also at finite lattice spacing.

Determining the order: strategies

- Try the easiest (?) thing: look for metastabilities and double peak structure of the order parameter and of the energy density around the transition, i.e. coexistence of phases, which is a clear signature for first order. This search failed in the past leading to a preference for the second order scenario.

- Perform an accurate Finite Size Scaling analysis of various thermodynamical quantities around the chiral critical point to extract critical indexes ($O(2)$ is expected rather than $O(4)$ for the staggered fermion formulation).

	y_t	y_h	ν	α	γ
$O(4)$	**1.336(25)**	**2.487(3)**	**0.748(14)**	**-0.24(6)**	**1.479(94)**
$O(2)$	**1.496(20)**	**2.485(3)**	**0.668(9)**	**-0.005(7)**	**1.317(38)**
1st Order	**3**	**3**	**1/3**	**1**	**1**

Finite Size Scaling

Approaching the transition the correlation length of the order param. ξ goes large and one can write the following scaling ansatzs:

free enery density $\Longrightarrow L/kT \simeq L_s{}^{-d}\, \phi\, (\tau L_s{}^{1/\nu}, am_q L_s{}^{y_h})$

specific heat $\Longrightarrow C_V - C_0 \simeq L_s{}^{\alpha/\nu}\, \phi_c\, (\tau L_s{}^{1/\nu}, am_q L_s{}^{y_h})$

order parameter suscept. $\Longrightarrow \chi \simeq L_s{}^{\gamma/\nu}\, \phi_\chi\, (\tau L_s{}^{1/\nu}, am_q L_s{}^{y_h})$

Technical difficulties:

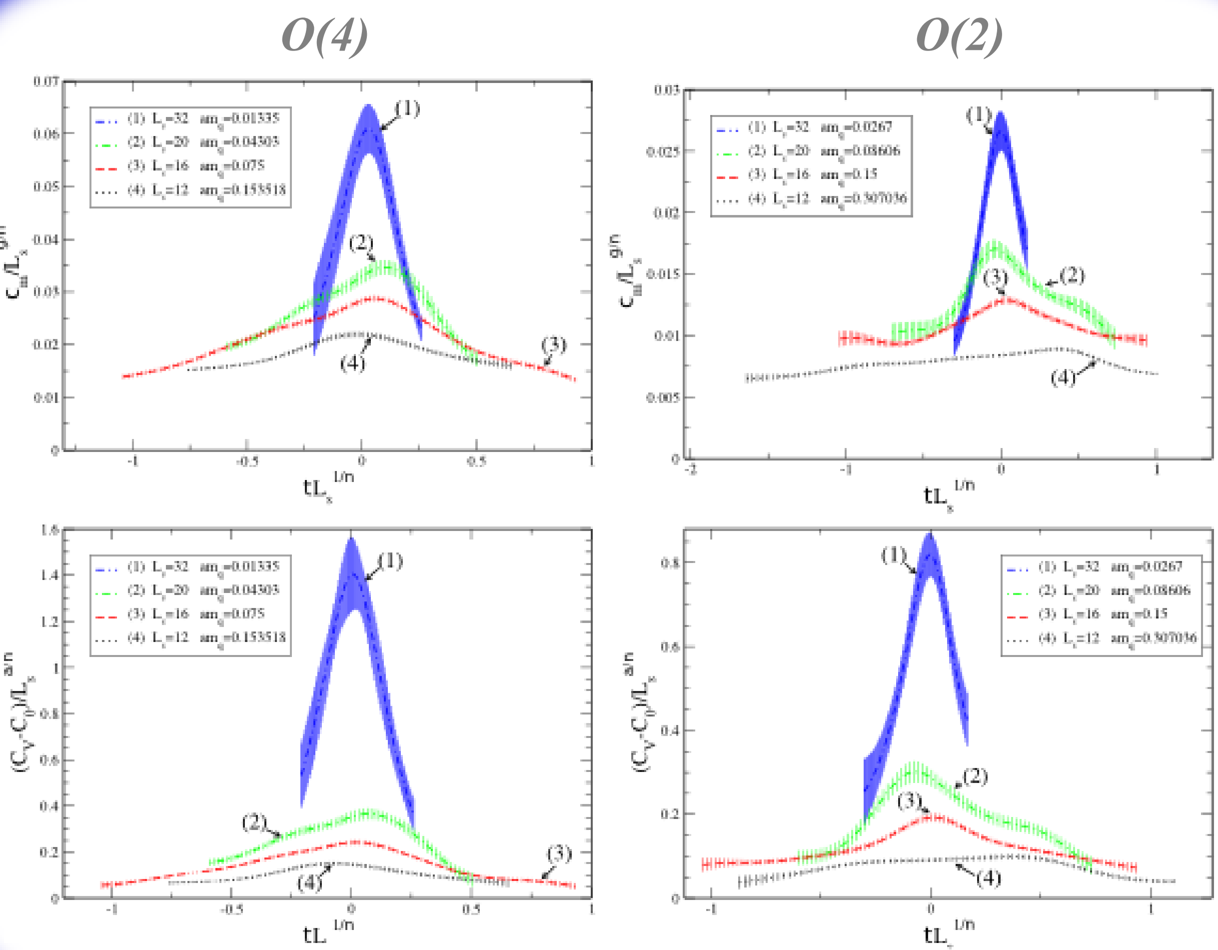
- Simulations on large volumes and with light quark masses are necessary for a reliable f.s.s. analysis $\Longrightarrow$ huge computational power required
- f.s.s. behavior is given in terms of two different scales (two scaling variables).

<u>No clear answer from previous literature</u> (see our works for references).

Previous work: second order check

CHIRAL COND.

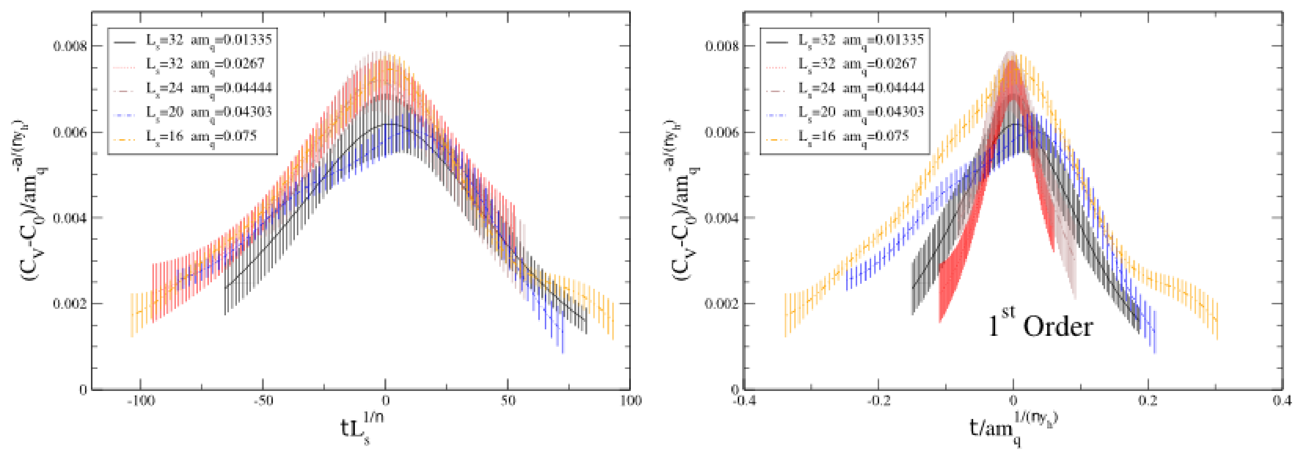
SPECIFIC HEAT



$O(4)$ and $O(2)$ are ruled out by our data. Notice that $\alpha<0$ for $O(4)$

Previous work: first order check

An approximate check for a first order scaling on the collected data was perfomed.

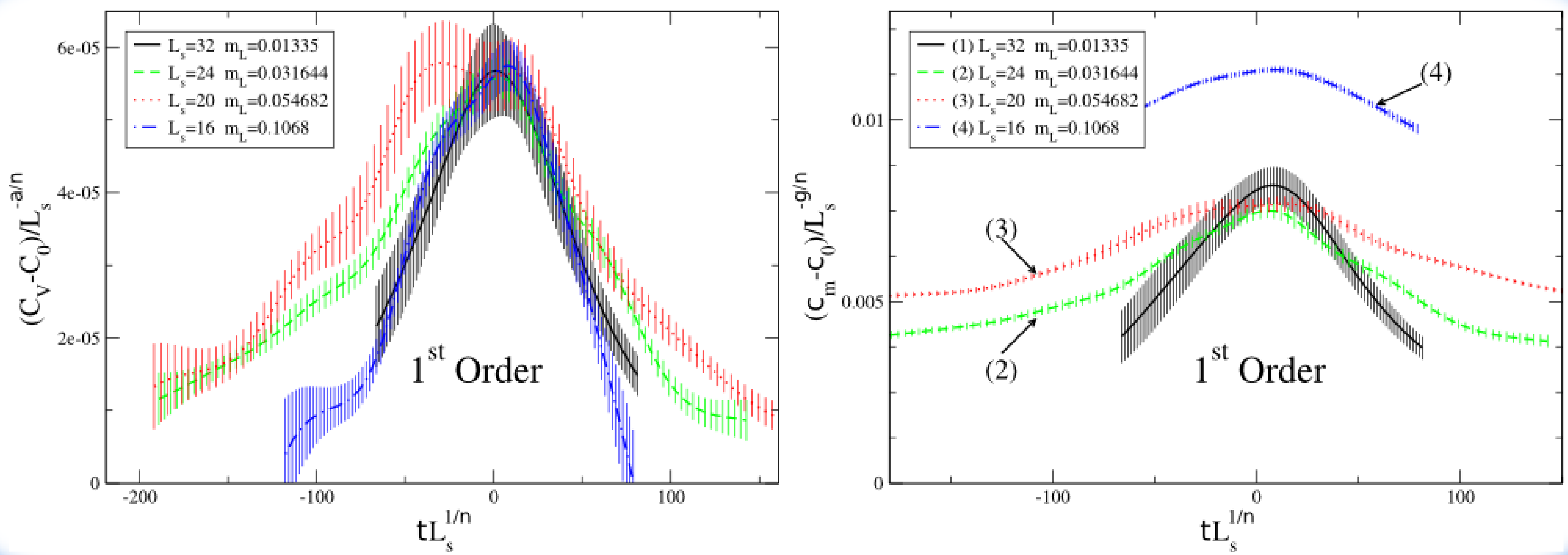


Indication that the transition could be first order. So we decided to give a chance to this hypothesis. But:

- where is the growth linear with the spatial volume expected for a first order transition at fixed mass?
- where are the double peaks?

Further tests: 1^{st} order direct check

G. C., M. D'Elia, A. Di Giacomo, C. Pica, arXiv:0706.4470



- Chiral susceptibility shows deviations, possibily due to the large mass range explored (up to 0.1) which could be outside the scaling region.
- Good scaling of the specific heat: not only the peak heights but also the widths are well described by the first order hypothesis.
- Results with the “inexact” R algorithm confirmed by simulation with RHMC

First order scaling analysis

Consider again the scaling law $C_V - C_0 \simeq L_s^{\alpha/\nu} \phi_c(\tau L_s^{1/\nu}, am_q L_s^{y_h})$.

- Continuous transition $\Longrightarrow$ L_s dependence must cancel as $L_s \to \infty$ at finite m_q . The scaling function can be expanded in terms of 1/($am_q L_s^{y_h}$) : the leading term must be 1/$(am_q L_s^{y_h})^{\alpha/\nu y_h}$ $\Longrightarrow$ no discontinuity (no latent heat) at finite m_q.
- First order chiral transition $\Longrightarrow$ a first order singularity is expected also at some $m_q \neq 0$, leading to a non-zero latent heat: we can allow for a constant term in the expansion in powers of 1/($am_q L_s^{y_h}$)

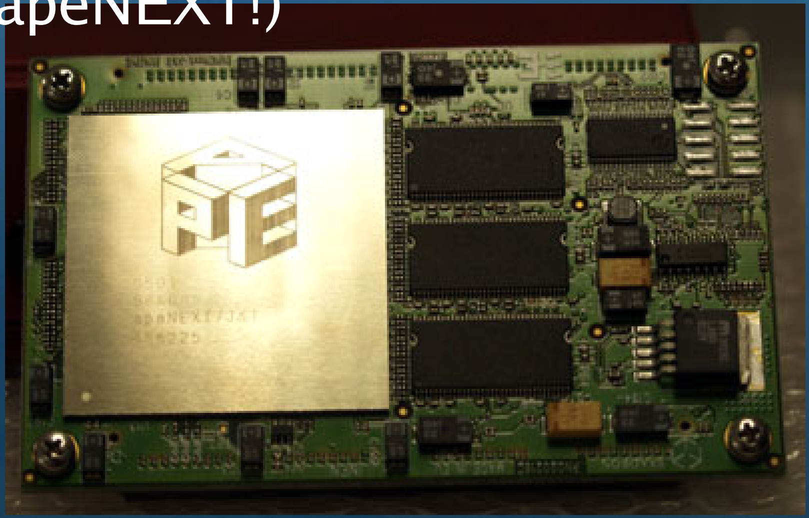
$$C_V - C_0 \sim am_q^{-1} \phi_c(\tau V) + V \phi'_c(\tau V)$$

In the second case the relative weight of the singular to the regular contribution is not known apriori, may be very small for small volumes and weak first order transitions.

There are various possibilities:

- There is really a first order transition which however is so weak that metastabilities will not show up but on very large, still unexplored volumes.
- We observe the “wrong” critical indexes because the scaling region around the chiral point is so small that the “correct” O(4) indexes will not show up but at very small, still unexplored quark masses.

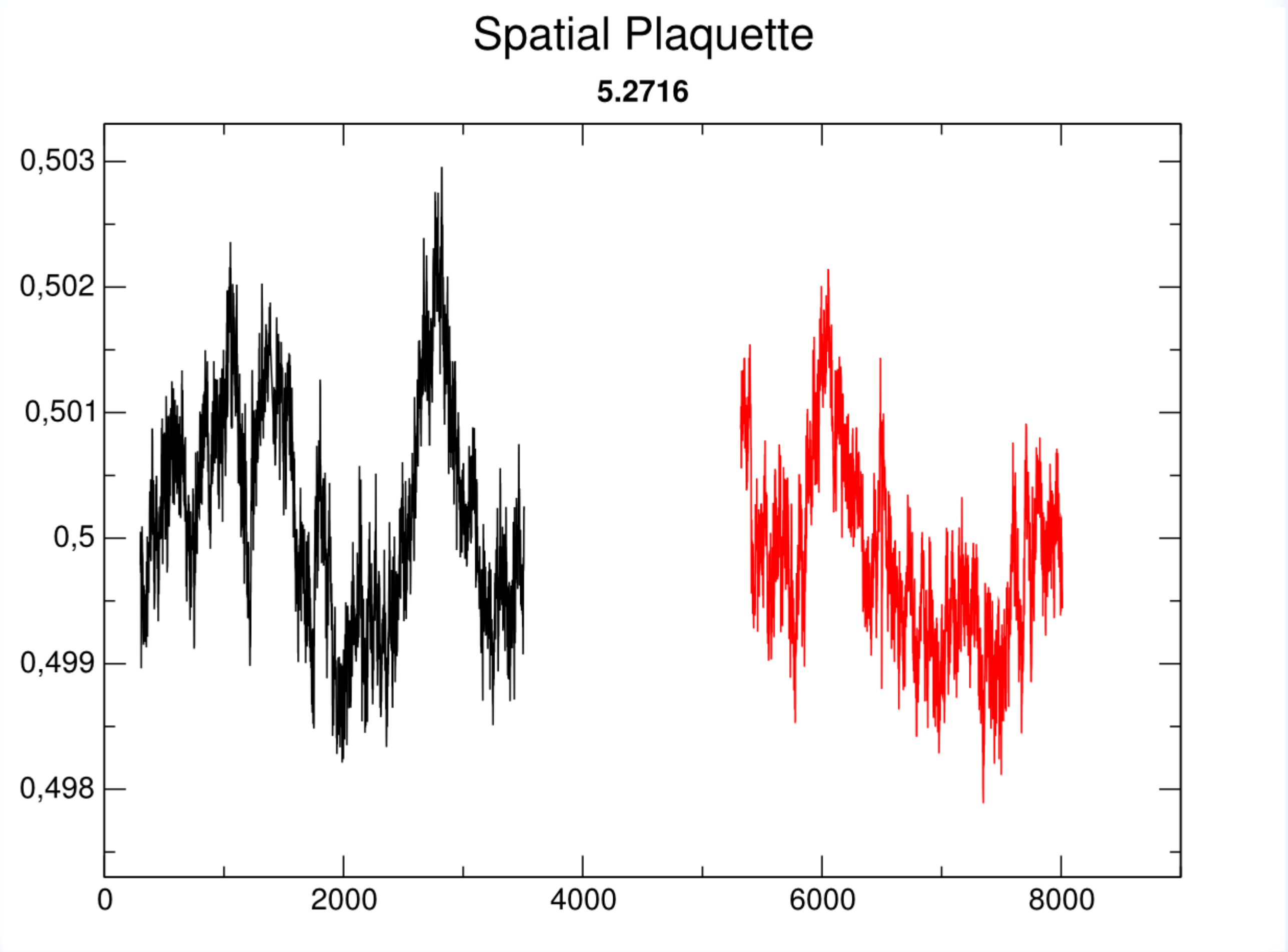
In order to clarify the issue, we have judged worth dedicating a large numerical effort to a run at am_q=0.01335 on a $48^3 \times 4$ lattice (thanks to apeNEXT!)



That corresponds to $m_\pi \sim$ twice the physical value and to a spatial size $\sim$13-14 fm.

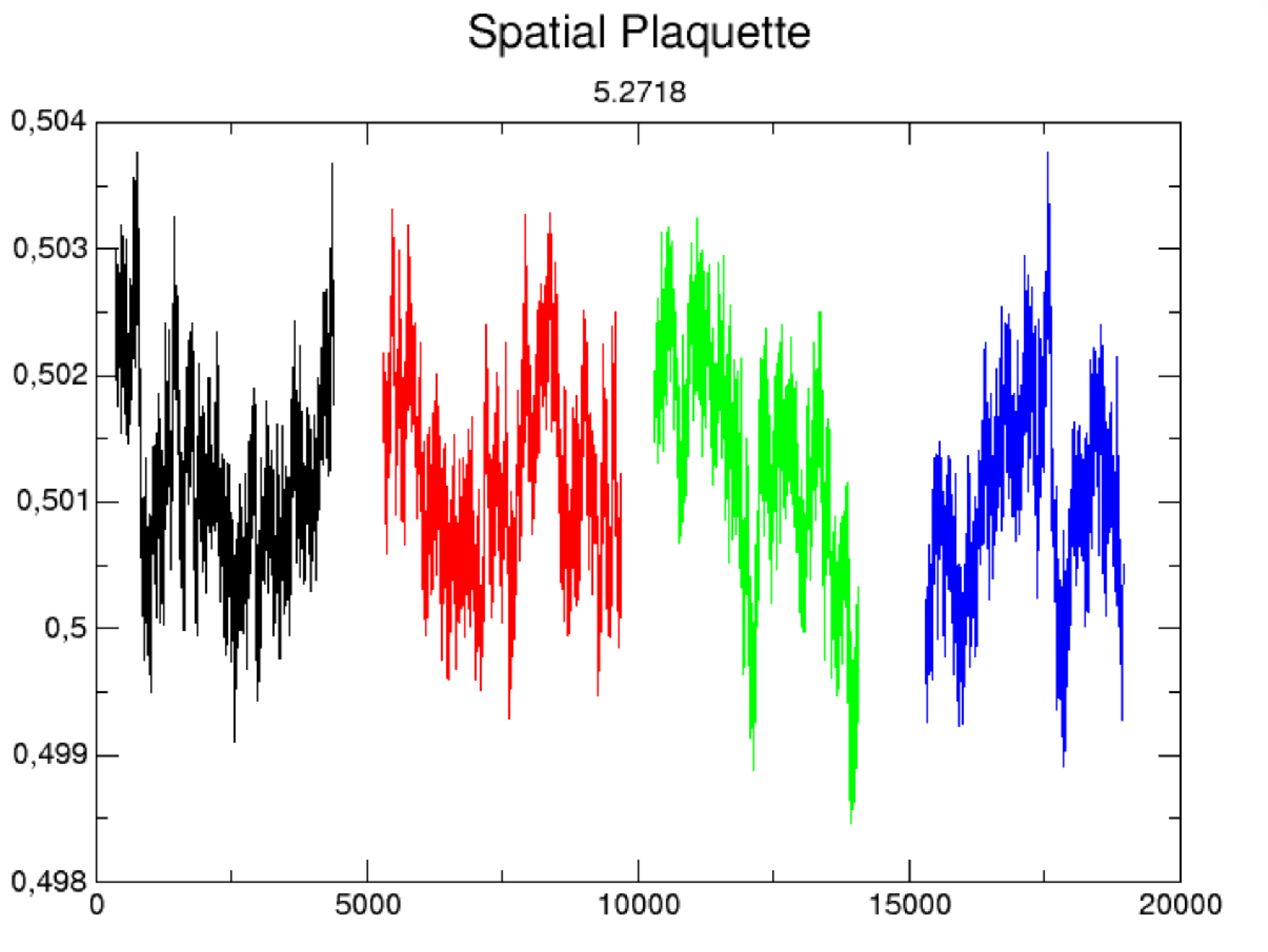
About 3.5k measurements/month/beta using 4 crates of apeNEXT.
Really computationally demanding!

Preliminary results - Histories



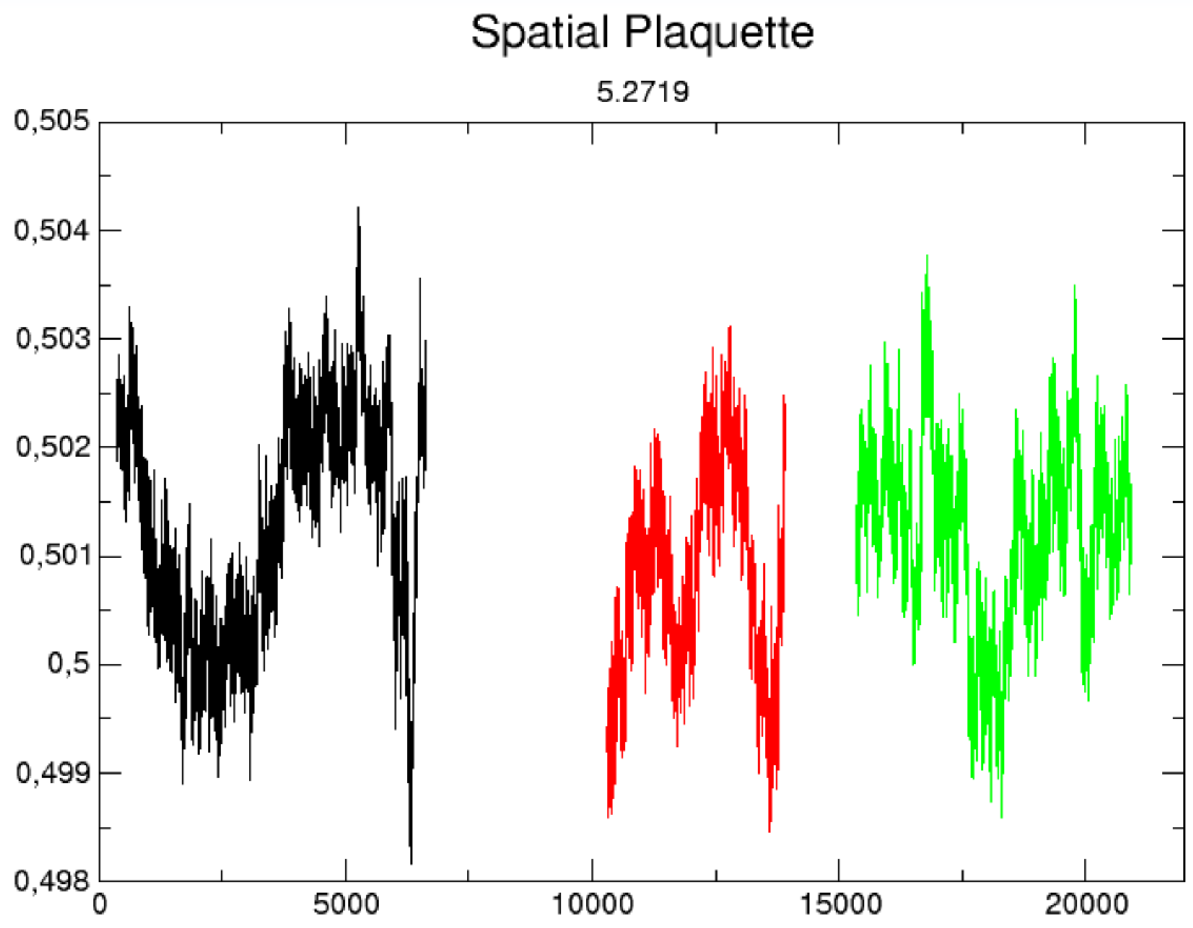
We collected $\sim 50k$ trajectories in total (4 betas)

Preliminary results - Histories

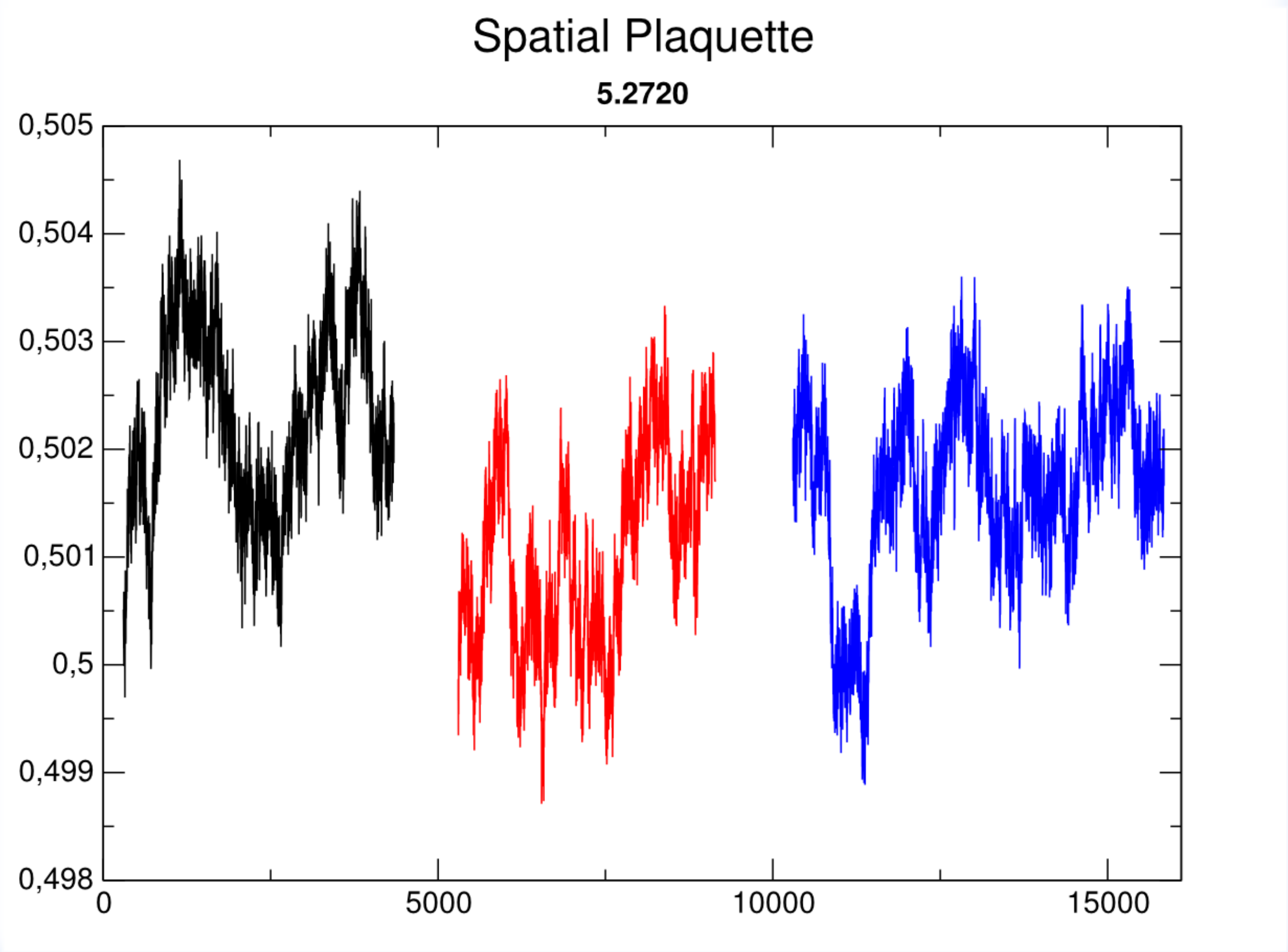


Different colors for different runs

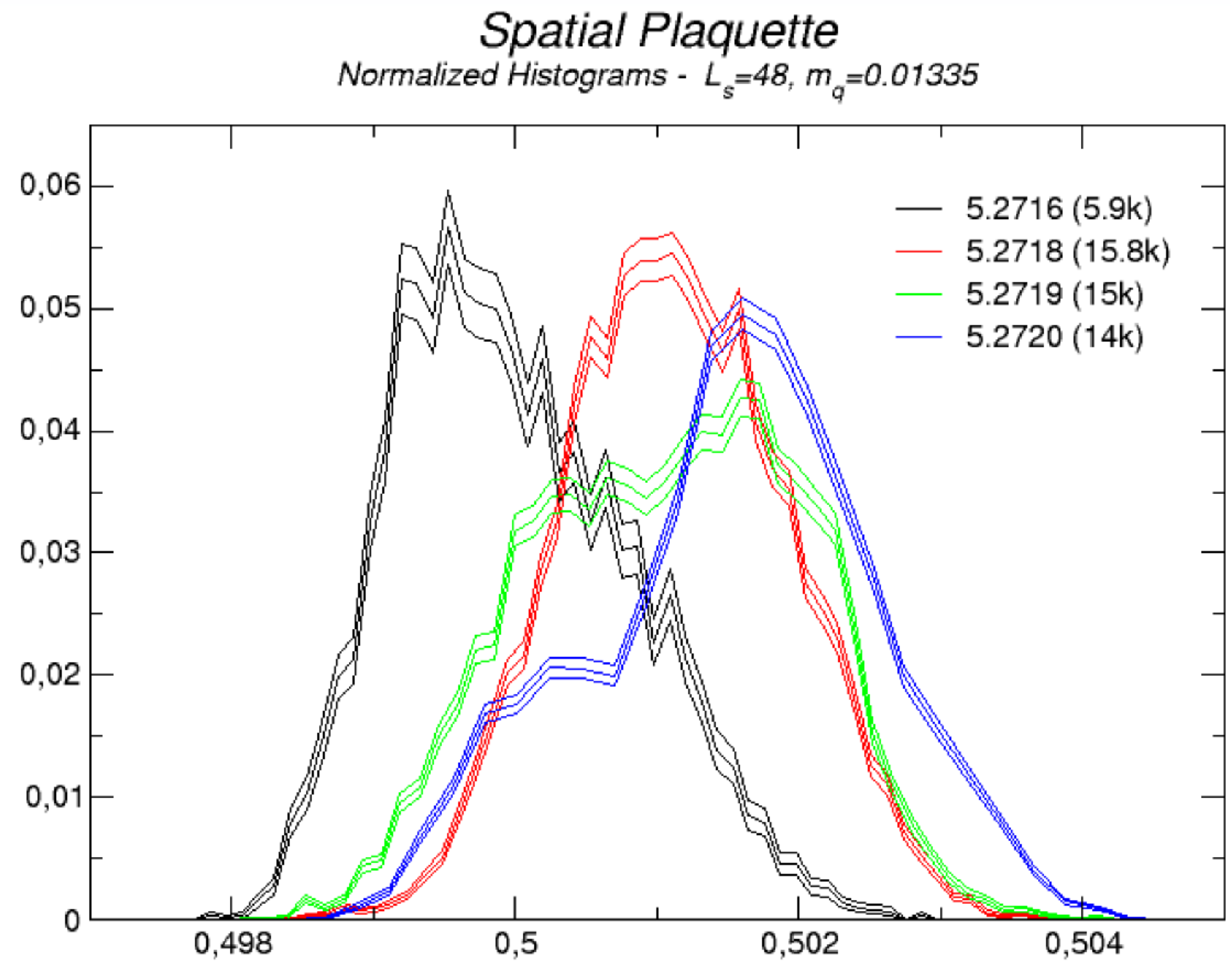
Preliminary results - Histories



Preliminary results - Histories

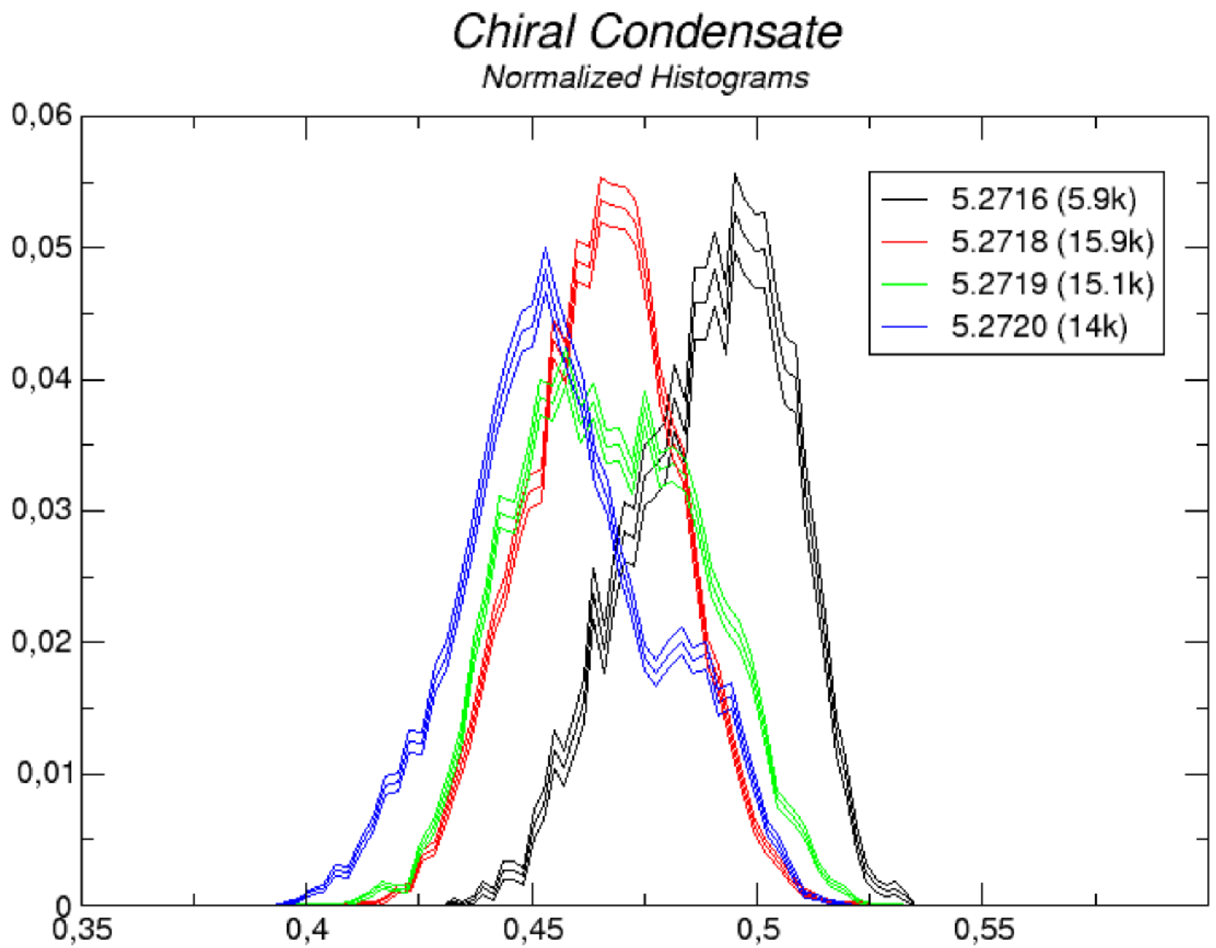


Preliminary results - Histograms

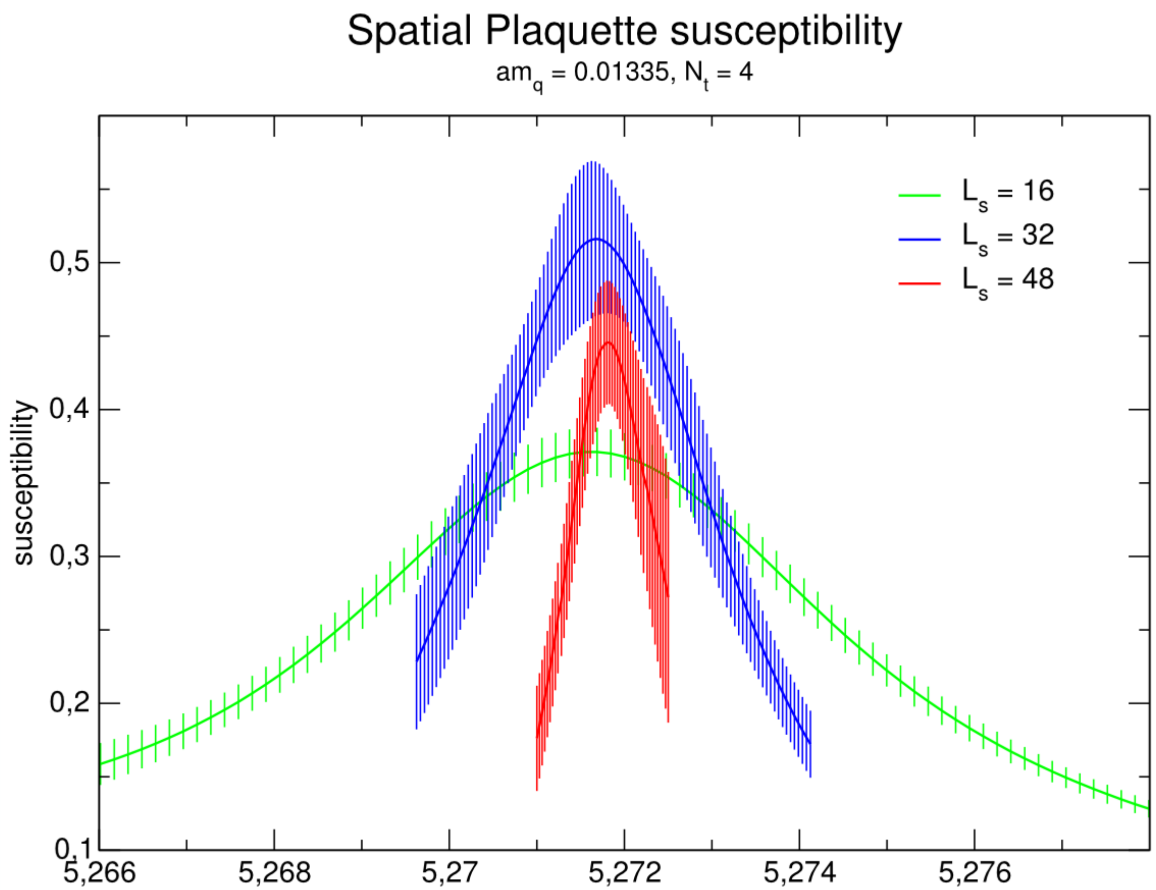


Error bars by bootstrap

Preliminary results - Histograms



Preliminary results - Susceptibility



Shrinks with the correct factor but doesn't grow. We need more statistics. We hope to completely clarify this issue in the next months.

Discussion and conclusions

Conclusion 1: With present UV cutoff effects ($N_t=4$, non-improved action) and within the present quark mass range a second order chiral transition in the O(4) (and O(2) and $U(2)_L \otimes U(2)_R/U(2)_V$) seems to be excluded.
Conclusion 2: First order critical indexes seem to be preferred.
Preliminary: we have some signals (to be confirmed!!) for a first order bistability at $am_q=0.01335$ - the bistability does not show up until $L_s=12/T \sim 13$–14 fm.
STATISTICS STILL LOW TO DRAW CONCLUSIONS!
Needs more investigation!

Our results have been obtained with a quite large lattice spacing $N_t=4$, 0.3 fm (lattice spacing) and with a non-improved action.
If our results will be confirmed on $N_t=6$ and/or using an improved lattice action, then the crossover scenario must be changed.

Thank you!

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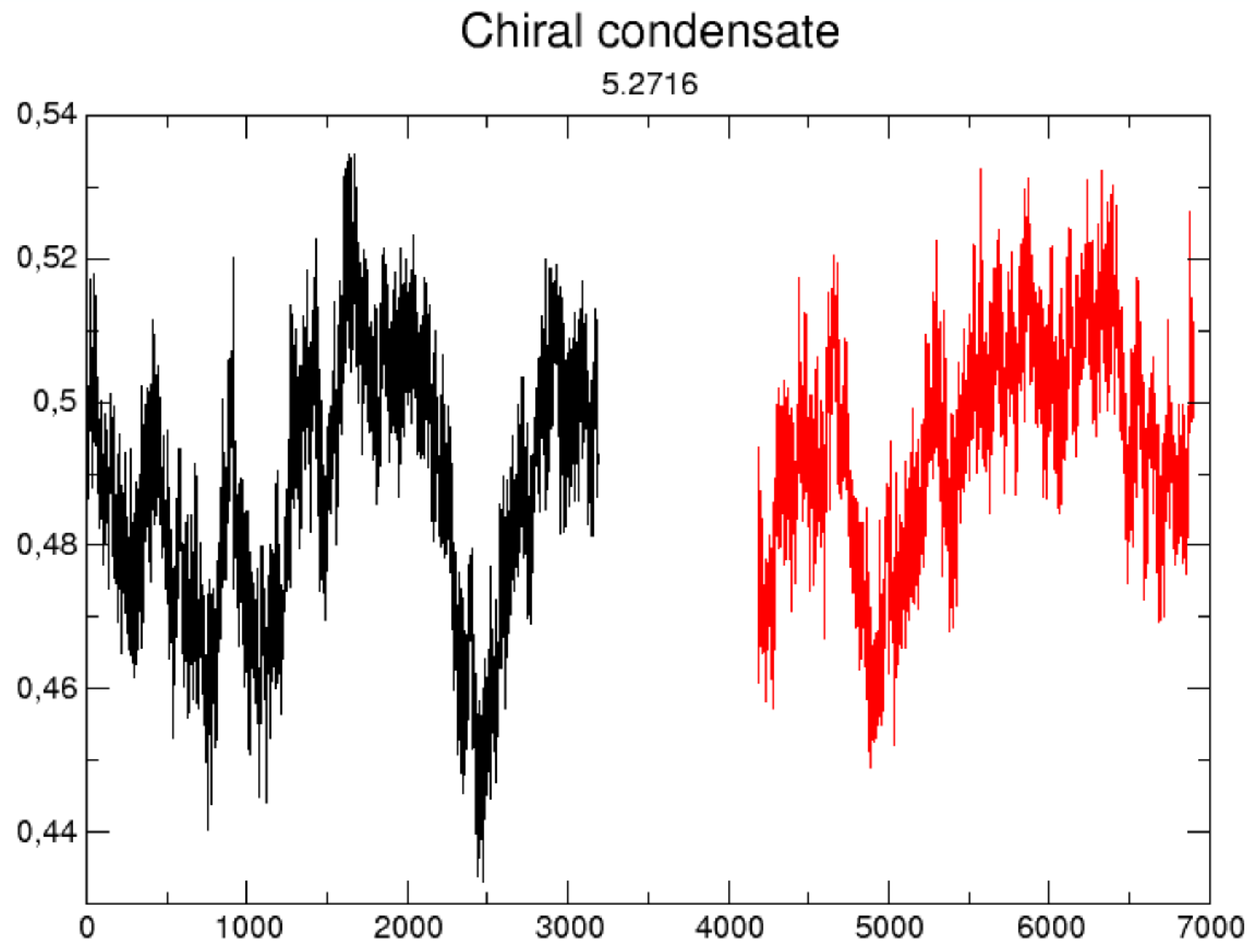
Preliminary results - Histories



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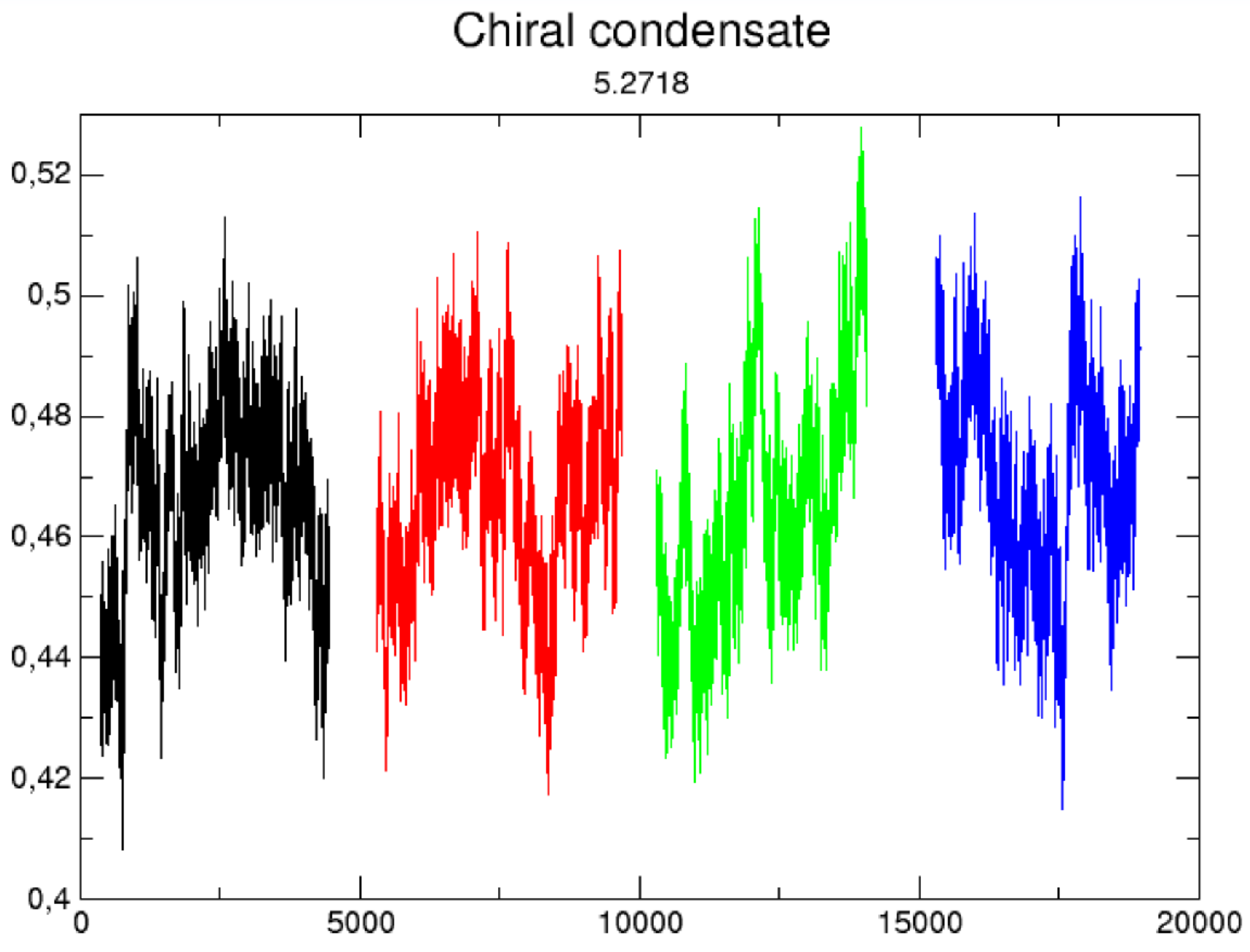

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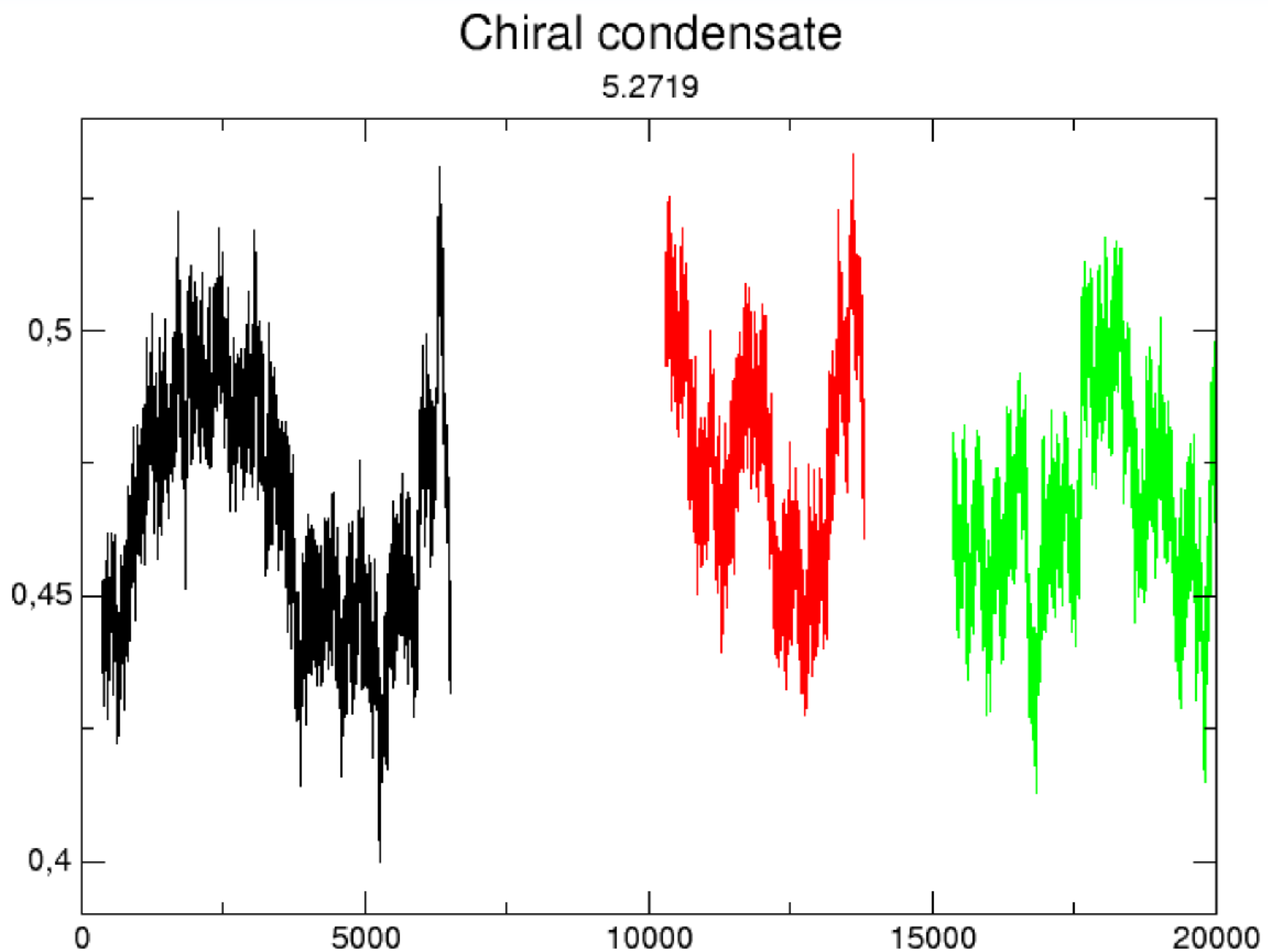
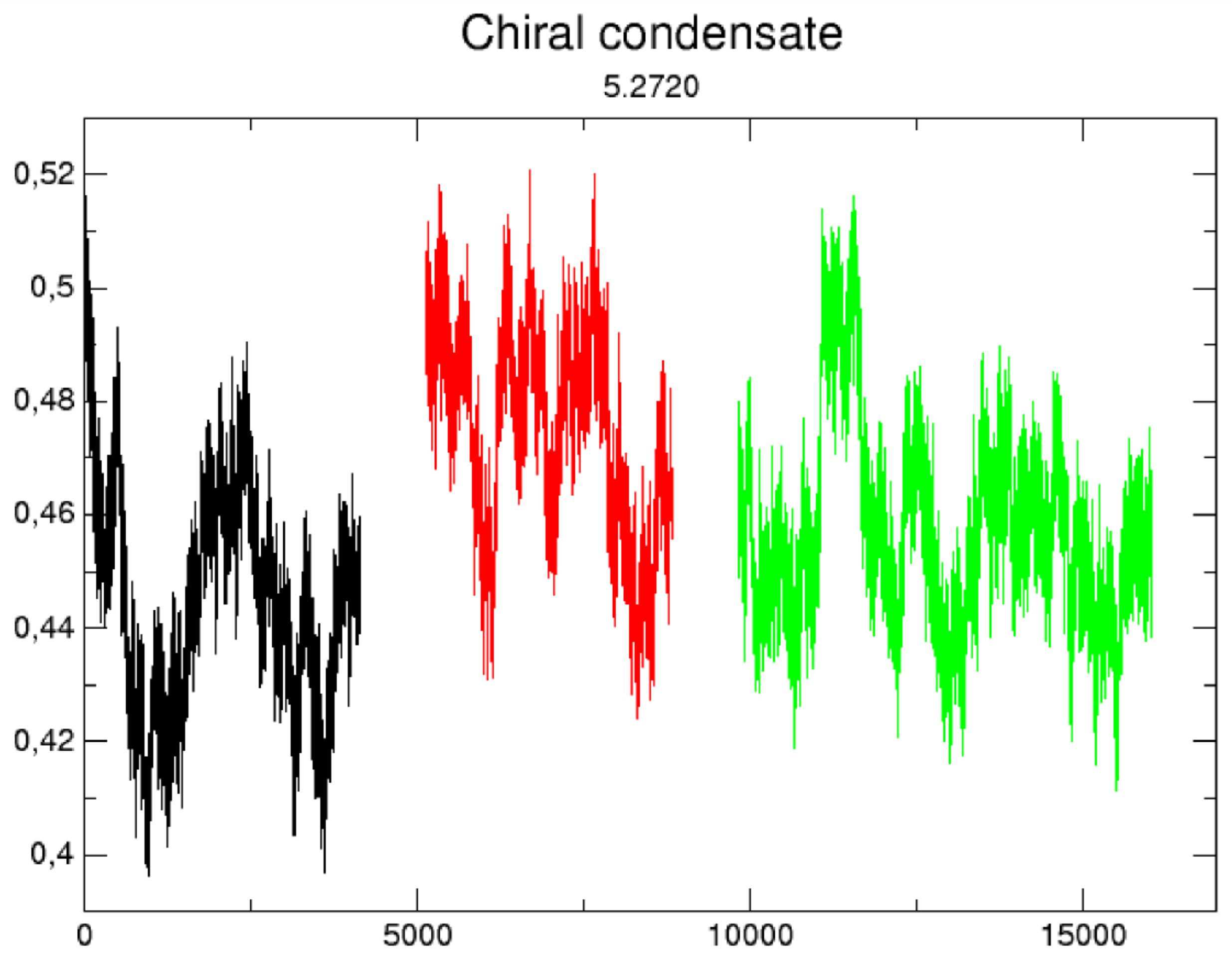


Preliminary results - Histories



Example – SU(3) weak 1st order

Plaquette

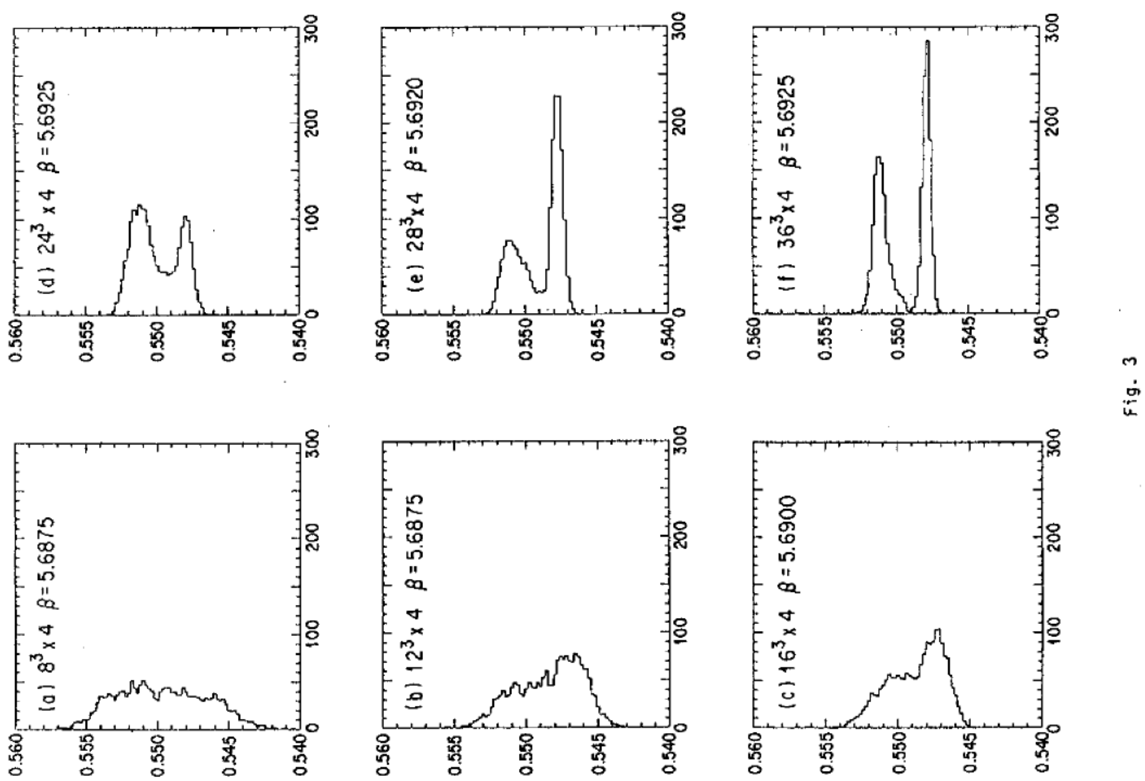


Image from Nucl. Phys. B337:181, 1990

Example – SU(3) weak 1st order

Polyakov Loop

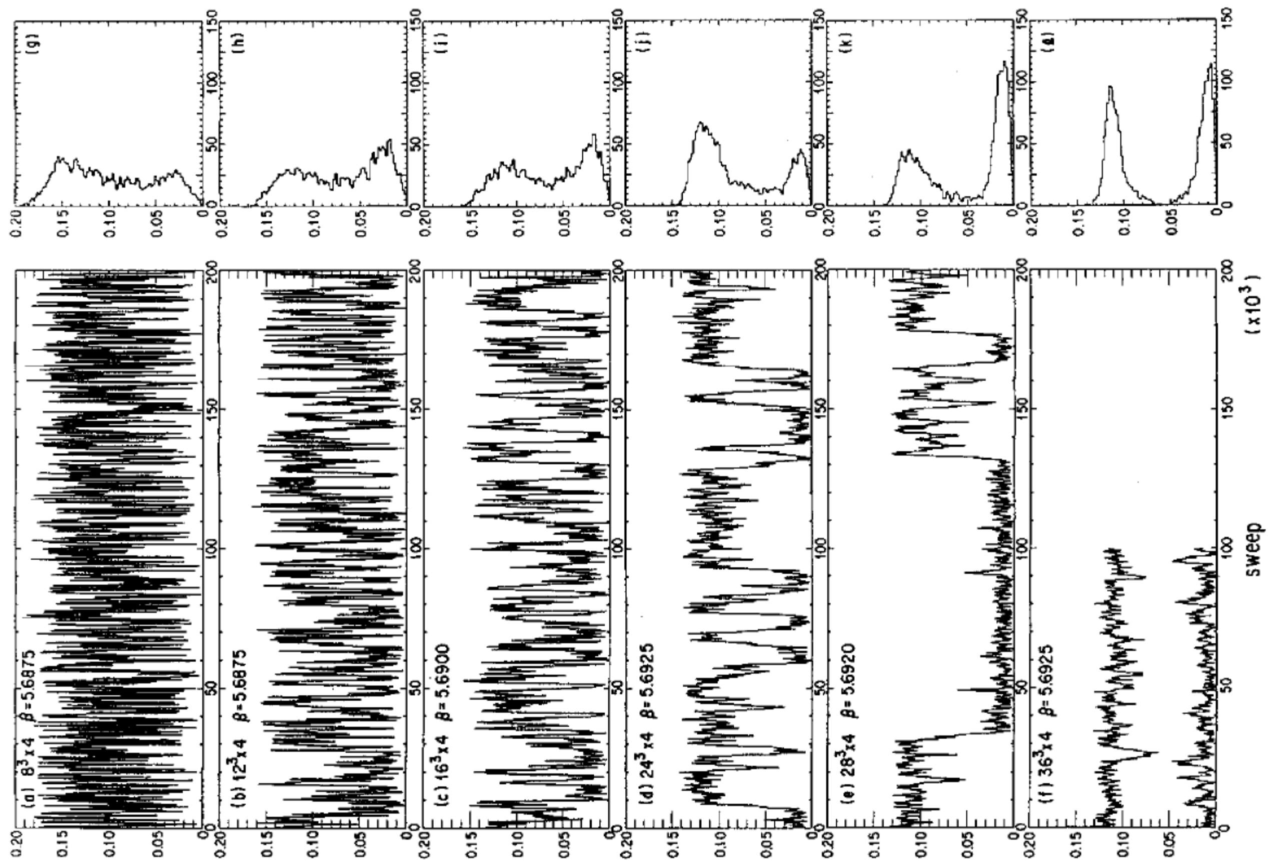


Image from Nucl. Phys. B337:181, 1990