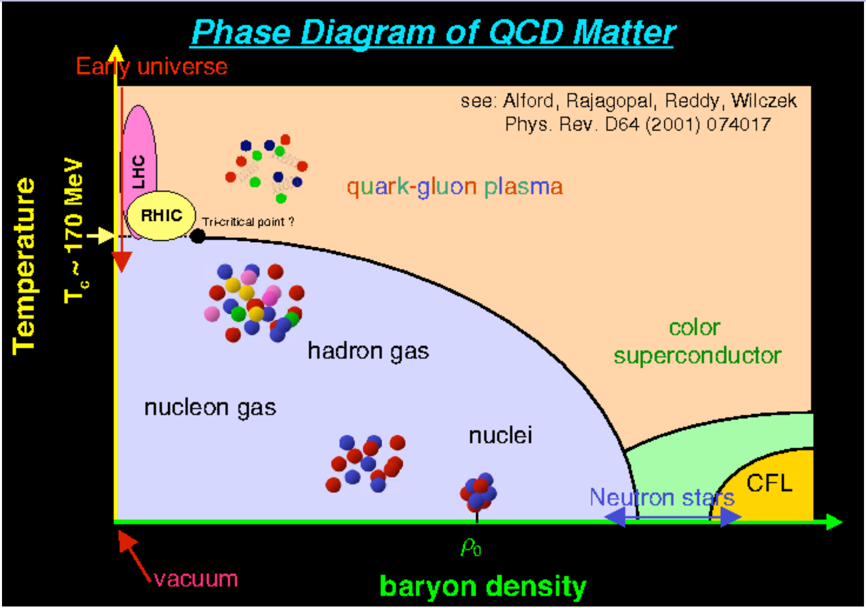
Lattice study of the QCD critical point

Philippe de Forcrand
ETH Zürich and CERN

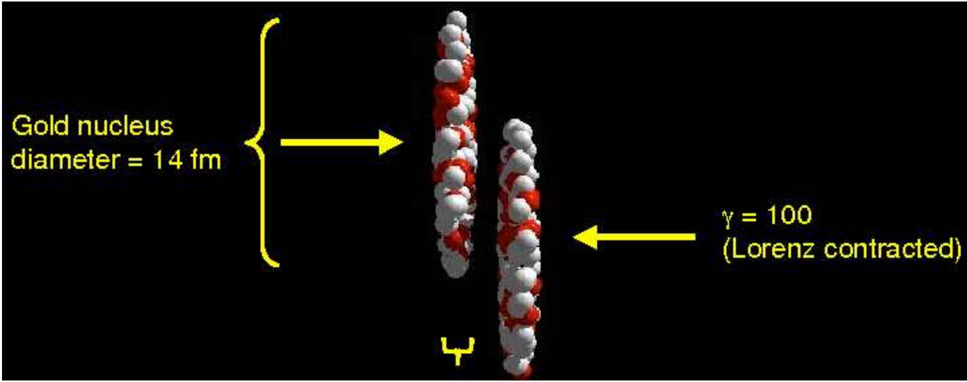
original work in collaboration with Owe Philipsen (Münster)

Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

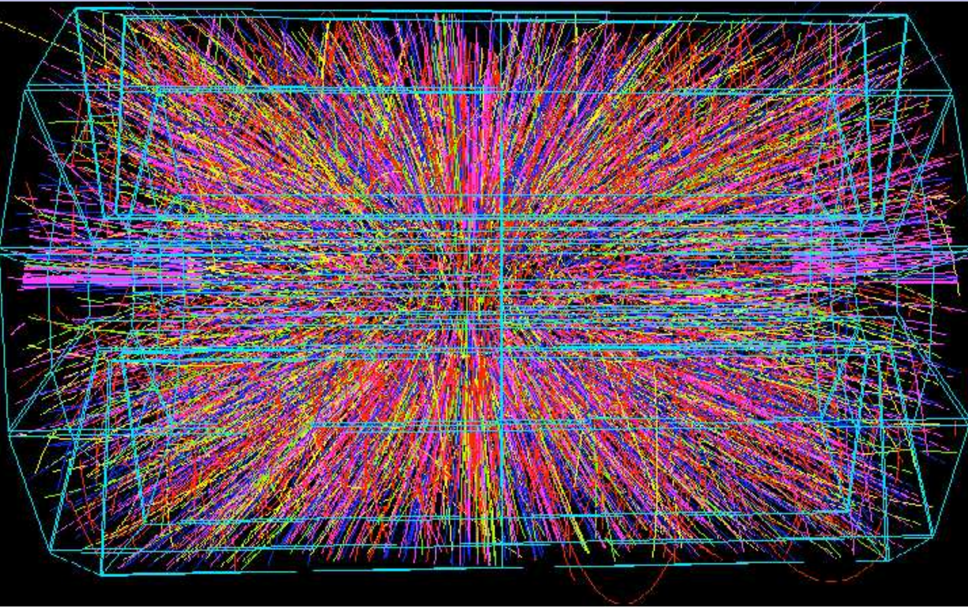
QCD Phase diagram



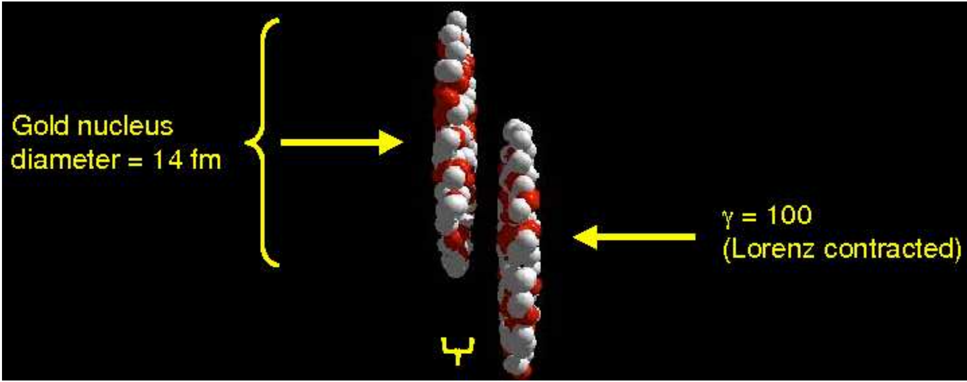
Heavy-ion collisions



Heavy-ion collisions



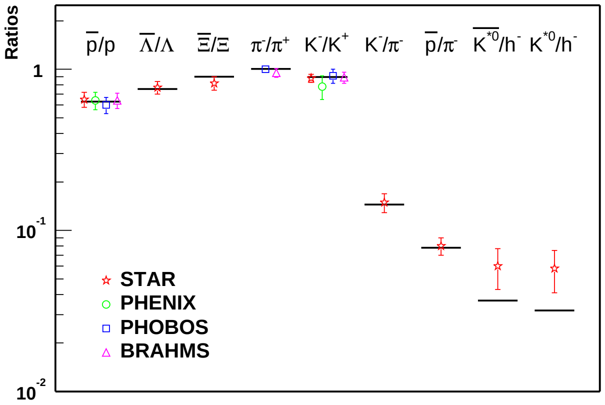
Heavy-ion collisions



- does not behave like superposition of $N - N$ collisions
- well described by relativistic hydrodynamic fluid

Strongly Interacting Quark-Gluon Plasma found

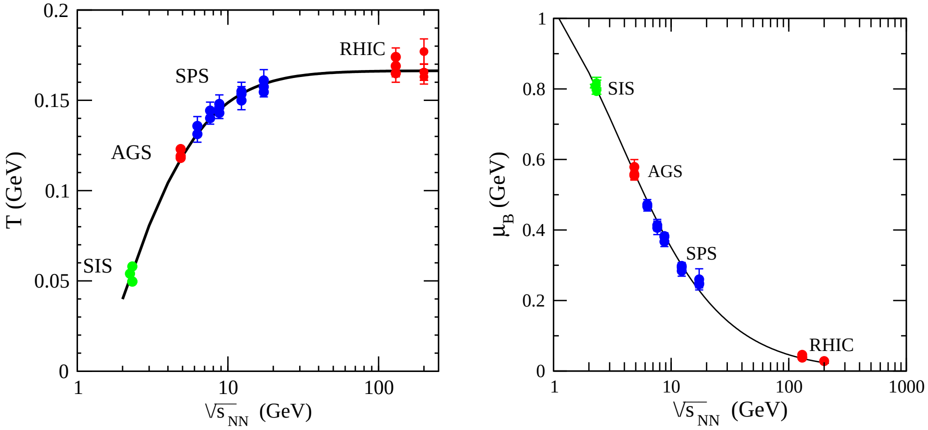
Phase boundary versus freeze-out temperature?



At fixed $\sqrt{s}$, relative abundances of hadrons fitted well with (T, μ_B)

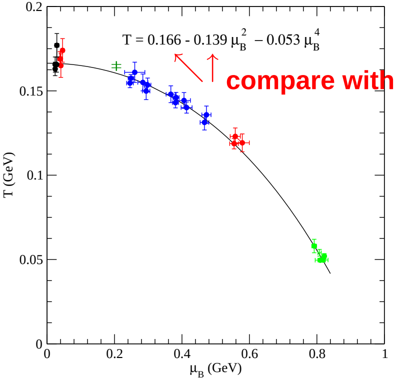
Phase boundary versus freeze-out temperature?

Repeat for successive $\sqrt{s}$:



Phase boundary versus freeze-out temperature?

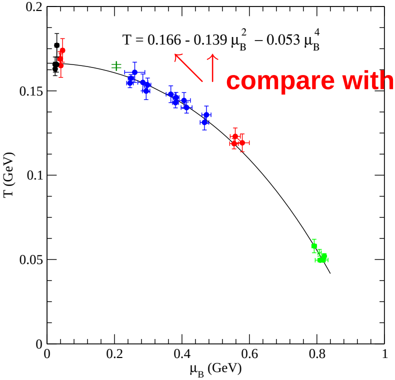
J. Cleymans et al., hep-ph/0607164



compare with lattice QCD

Phase boundary versus freeze-out temperature?

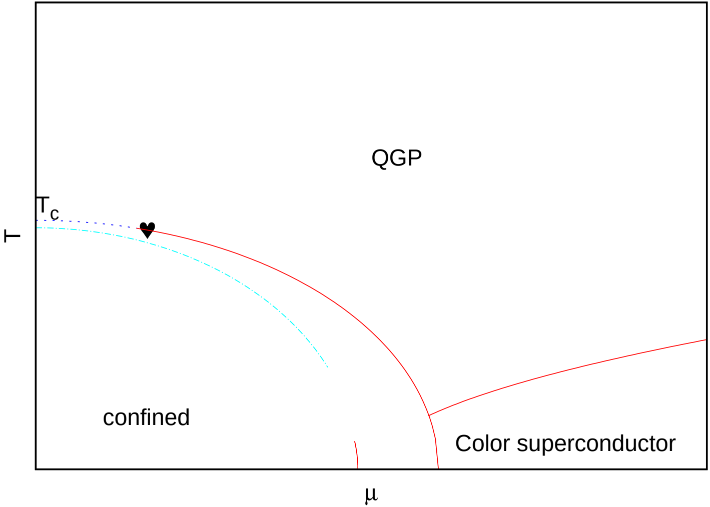
J. Cleymans et al., hep-ph/0607164



- $T = 0$ when $\mu_B \approx m_N$: boundary of *inelastic* collisions
  T(freeze-out), not related to T_c(QGP)
- T(freeze-out) $\leq$ T_c(QGP) but very close?

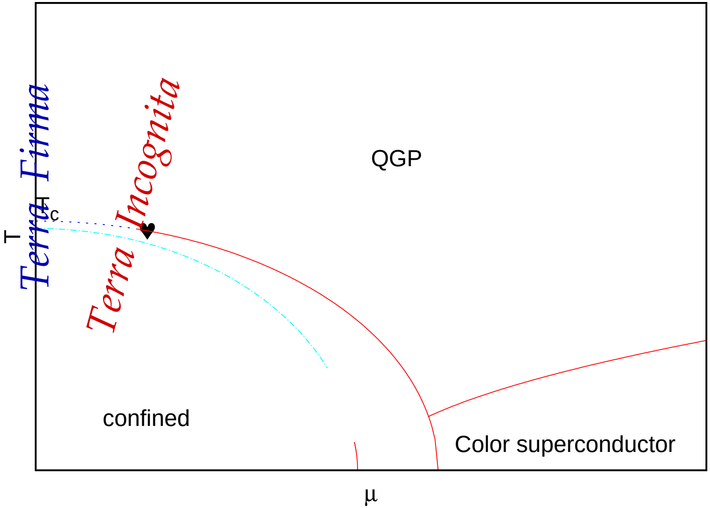
Braun-Munzinger, Stachel & Wetterich, nucl-th/0311005

Schematic phase diagram – perhaps



Can one locate the critical point (μ_E, T_E) by lattice simulations?

Schematic phase diagram – perhaps



Can one locate the critical point (μ_E, T_E) by lattice simulations?

The sign and overlap problems

- Integrate over fermions: $\det(\not{D} + m + \mu\gamma_0)$ complex *unless* $\mu = 0$ *or* $\mu = i\mu_i$

$\longrightarrow$ standard importance sampling impossible

The sign and overlap problems

- Integrate over fermions: $\det(\not{D} + m + \mu\gamma_0)$ complex *unless* $\mu = 0$ *or* $\mu = i\mu_i$

  $\longrightarrow$ standard importance sampling impossible

- **Reweighting**: - simulate theory with no sign pb., eg. $\mu = 0$
  - reweight each measurement with $\rho(U) = \frac{\det(U,\mu\neq 0)}{\det(U,\mu=0)}$ complex
  - $\langle\rho(U)\rangle = \frac{Z(\mu\neq 0)}{Z(\mu=0)} \sim \exp(-V\frac{\Delta f(\mu)}{T})$ $\longrightarrow$ large V ?, large μ ?

1. maintain statistical accuracy on $\langle\rho\rangle$: **sign** pb.
2. ensure that $Z(\mu \neq 0)$ is properly sampled: **overlap** pb.

1 and **2** require statistics $\propto$ $\exp(+V)$

The sign and overlap problems

- Integrate over fermions: $\det(\not{D} + m + \mu\gamma_0)$ complex *unless* $\mu = 0$ *or* $\mu = i\mu_i$

  $\longrightarrow$ standard importance sampling impossible

- **Reweighting**: - simulate theory with no sign pb., eg. $\mu = 0$
  - reweight each measurement with $\rho(U) = \frac{\det(U,\mu\neq 0)}{\det(U,\mu=0)}$ complex
  - $\langle\rho(U)\rangle = \frac{Z(\mu\neq 0)}{Z(\mu=0)} \sim \exp(-V\frac{\Delta f(\mu)}{T})$ $\longrightarrow$ large V ?, large μ ?

  1. maintain statistical accuracy on $\langle\rho\rangle$: **sign** pb.
  2. ensure that $Z(\mu\neq 0)$ is properly sampled: **overlap** pb.

  1 and **2** require statistics $\propto$ $\exp(+V)$

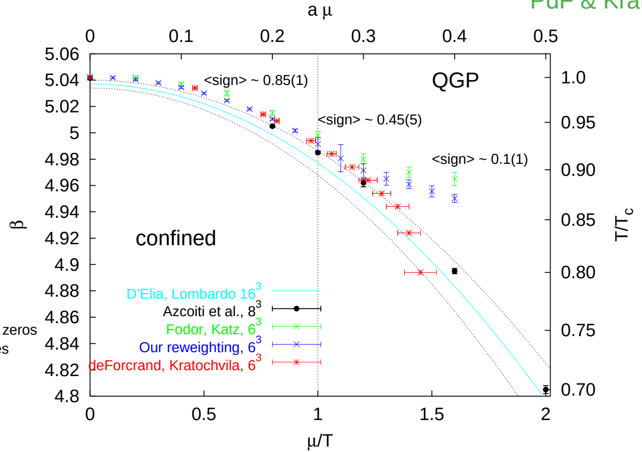
- Measure **derivatives** w.r.t. μ at $\mu = 0$: $\boxed{\langle W(\mu)\rangle = \langle W(\mu=0)\rangle + \sum_k \mathbf{c_k}\left(\frac{\mu}{\pi T}\right)^k}$
  - directly at $\mu = 0$ MILC, TARO, Bielefeld-Swansea, Gavai-Gupta,..
  - by fitting polynomial to $\mu = i\mu_i$ results D'Elia-Lombardo, PdF-Philipsen,..

Controlled thermodynamics and continuum limits $\Rightarrow$ **derivatives** only

The good news: curvature of the pseudo-critical line

All with $N_f = 4$ staggered fermions, $am_q = 0.05, N_t = 4 \quad (a \sim 0.3$ fm)

PdF & Kratochvila



imaginary μ
2 param. imag. μ
dble reweighting, LY zeros
Same, susceptibilities
canonical

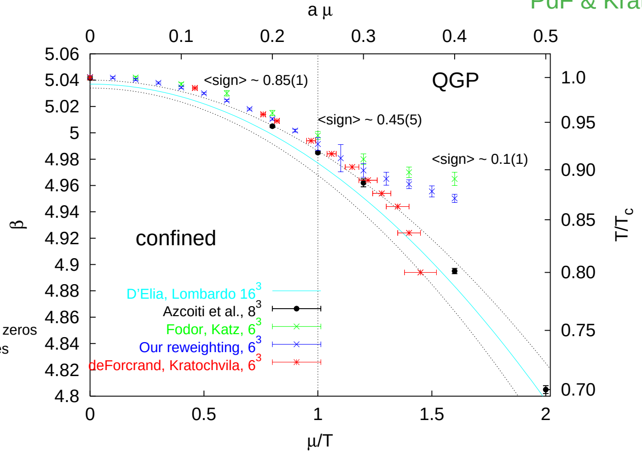
Agreement for $\mu/T \lesssim 1$

The good news: curvature of the pseudo-critical line

All with $N_f = 4$ staggered fermions, $am_q = 0.05, N_t = 4 \quad (a \sim 0.3$ fm)

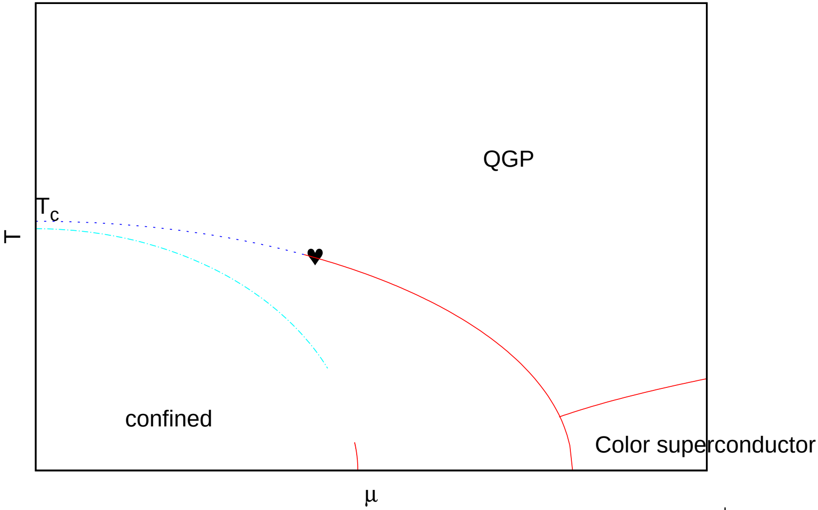
PdF & Kratochvila

imaginary μ
2 param. imag. μ
dble reweighting, LY zeros
Same, susceptibilities
canonical



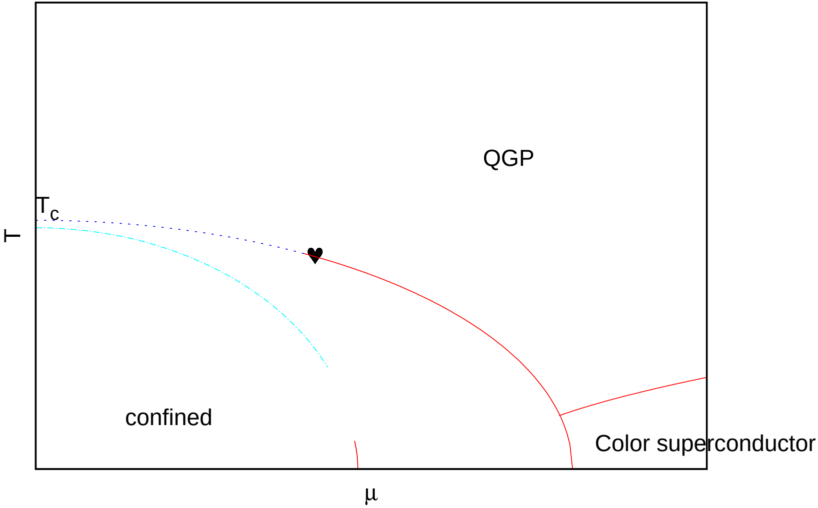
$T_c(\mu)$ considerably flatter than **freeze-out** curve (factor ~ 3 in $\left.\frac{d^2 T_c}{d\mu^2}\right|_{\mu=0}$)

The good news: curvature of the pseudo-critical line



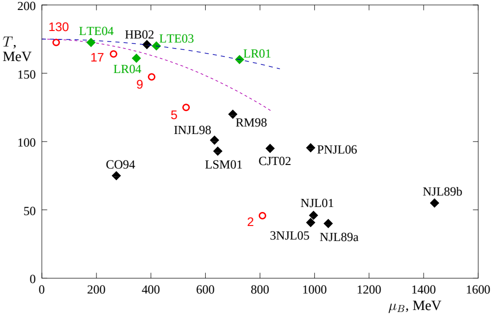
$T_c(\mu)$ considerably flatter than **freeze-out** curve (factor ~ 3 in $\left.\frac{d^2T_c}{d\mu^2}\right|_{\mu=0}$)

The good news: curvature of the pseudo-critical line



- Signal from critical pt. washed out by evolution until freeze-out
- Only control parameters: $\sqrt{s}$ and A

The bad news: locating the critical point



M. Stephanov, hep-lat/0701002

- Challenging task:

  detect divergent correlation length (2nd order)

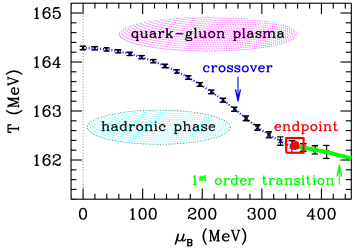
  vs finite but large (crossover, 1rst order)

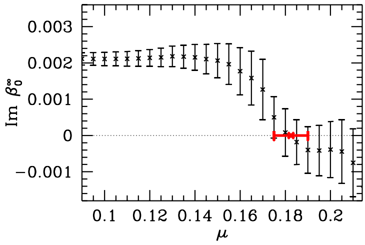
  on small lattice

Mission impossible? finite-size scaling crucial – more control parameters

Critical point already determined, but...

Fodor & Katz: hep-lat/0402006 ($\sim$ physical quark masses)





$$(\mu_E^q, T_E) = (120(13), 162(2))\ \text{MeV}$$

Strategy: reweight from $(\mu = 0, T_c)$ along pseudo-critical line

Legitimate **concerns**:

- Discretization error? $N_t = 4 \implies a \sim 0.3$ fm
- Abrupt qualitative change near μ_E:

  abrupt change of physics or breakdown of algorithm (Splittorff)?

$\longrightarrow$ repeat with conservative approach (**derivative**)

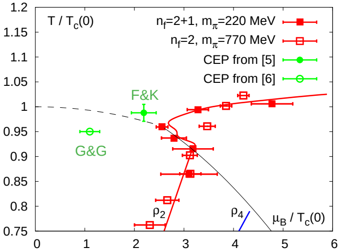
Critical point from radius of convergence?

$$\frac{p}{T^4} = \sum_{n=0}^{\infty} c_{2n}(T)\left(\frac{\mu}{T}\right)^{2n}$$

Singularity $(\mu_E, T_E) \Rightarrow$ $$\boxed{\frac{\mu_E}{T_E} = \lim_{n\to\infty}\sqrt{\left|\frac{c_{2n}}{c_{2n+2}}\right|}}$$ Karsch et al.

- Need $n \to \infty$, not $n = 1$ or 2

$\sqrt{\left|\frac{c_2}{c_4}\right|}$ is not a lower or upper bound



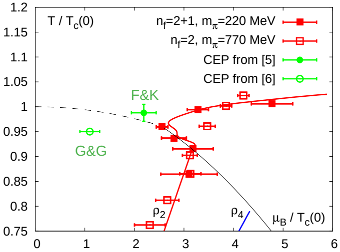
Critical point from radius of convergence?

$$\frac{p}{T^4} = \sum_{n=0}^{\infty} c_{2n}(T) \left(\frac{\mu}{T}\right)^{2n}$$

Singularity $(\mu_E, T_E) \Rightarrow$ $\boxed{\frac{\mu_E}{T_E} = \lim_{n\to\infty} \sqrt{\left|\frac{c_{2n}}{c_{2n+2}}\right|}}$ Karsch et al.

- Need $n \to \infty$, not $n = 1$ or 2

$\sqrt{\left|\frac{c_2}{c_4}\right|}$ is not a lower or upper bound



- Other definitions just as good, eg. $\lim_{n\to\infty} \left|\frac{c_0}{c_{2n}}\right|^{\frac{1}{2n}}$

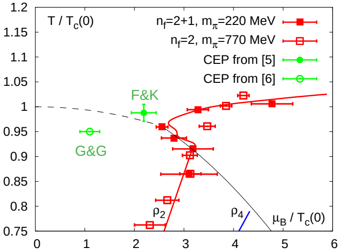
Critical point from radius of convergence?

$$\frac{p}{T^4} = \sum_{n=0}^{\infty} c_{2n}(T) \left(\frac{\mu}{T}\right)^{2n}$$

Singularity $(\mu_E, T_E) \Rightarrow$ $\boxed{\frac{\mu_E}{T_E} = \lim_{n\to\infty} \sqrt{\left|\frac{c_{2n}}{c_{2n+2}}\right|}}$ Karsch et al.

- Need $n \to \infty$, not $n = 1$ or 2

$\sqrt{\left|\frac{c_2}{c_4}\right|}$ is not a lower or upper bound



- Also $\frac{n_q}{T^3} = \sum_{n=1}^{\infty} 2n\, c_{2n} \left(\frac{\mu}{T}\right)^{2n-1} \longrightarrow \frac{\mu_E}{T_E} = \lim_{n\to\infty} \sqrt{\left|\frac{2n\, c_{2n}}{(2n+2)c_{2n+2}}\right|}$

  $n = 1 \longrightarrow$ factor $1/\sqrt{2}$

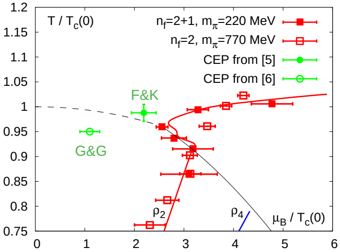
Critical point from radius of convergence?

$$\frac{p}{T^4} = \sum_{n=0}^{\infty} c_{2n}(T) \left(\frac{\mu}{T}\right)^{2n}$$

Singularity $(\mu_E, T_E) \Rightarrow$ $\boxed{\frac{\mu_E}{T_E} = \lim_{n\to\infty} \sqrt{\left|\frac{c_{2n}}{c_{2n+2}}\right|}}$ Karsch et al.

- Need $n \to \infty$, not $n = 1$ or 2

$\sqrt{\left|\frac{c_2}{c_4}\right|}$ is not a lower or upper bound



- Or $\frac{\chi_q}{T^2} = \sum_{n=1}^{\infty} 2n(2n-1)\, c_{2n} \left(\frac{\mu}{T}\right)^{2n-2} \longrightarrow \frac{\mu_E}{T_E} = \lim_{n\to\infty} \sqrt{\left|\frac{2n(2n-1)c_{2n}}{(2n+2)(2n+1)c_{2n+2}}\right|}$

$n = 1 \longrightarrow$ factor $1/\sqrt{6}$ Gavai & Gupta

Critical point from radius of convergence?

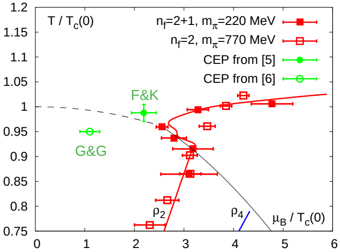
$$\frac{p}{T^4} = \sum_{n=0}^{\infty} c_{2n}(T) \left(\frac{\mu}{T}\right)^{2n}$$

Singularity $(\mu_E, T_E) \Rightarrow$ $$\boxed{\frac{\mu_E}{T_E} = \lim_{n\to\infty} \sqrt{\left|\frac{c_{2n}}{c_{2n+2}}\right|}}$$

Karsch et al.

- Need $n \to \infty$, not $n = 1$ or 2

$\sqrt{\left|\frac{c_2}{c_4}\right|}$ is not a lower or upper bound

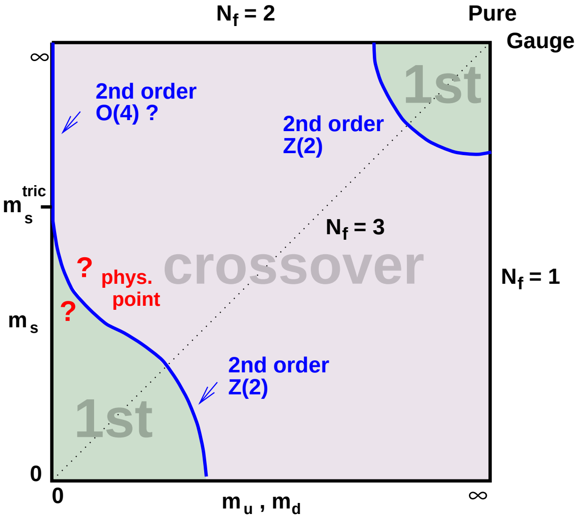


Systematic error uncontrolled

Better strategy?

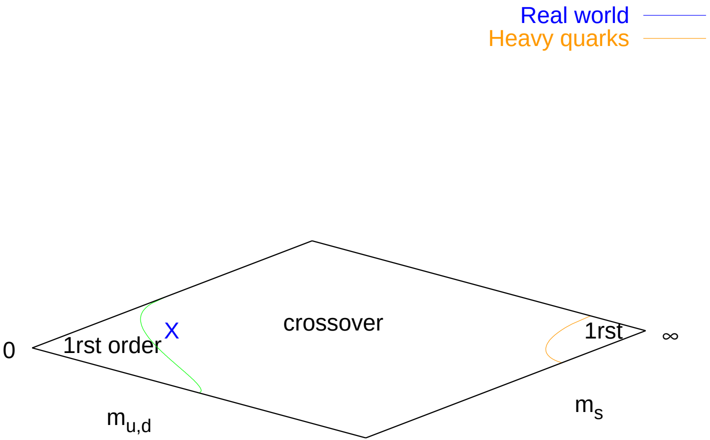
Generalize QCD to arbitrary $(m_{u,d}, m_s)$, T: phase diagram

$\mu = 0$



Generalize QCD to arbitrary $(m_{u,d}, m_s)$, T: phase diagram

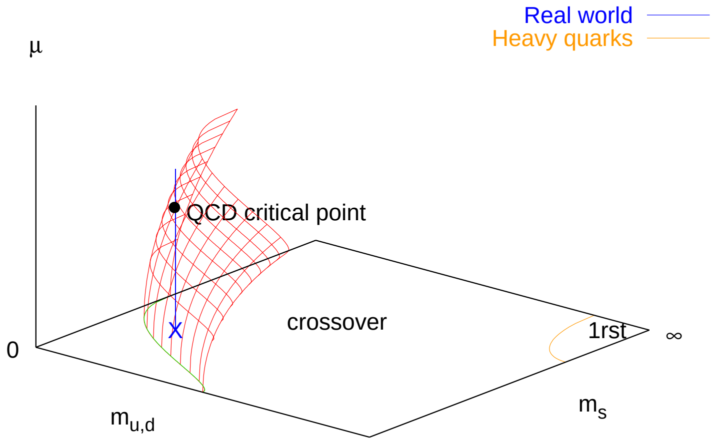
$\mu = 0$



Now turn on μ

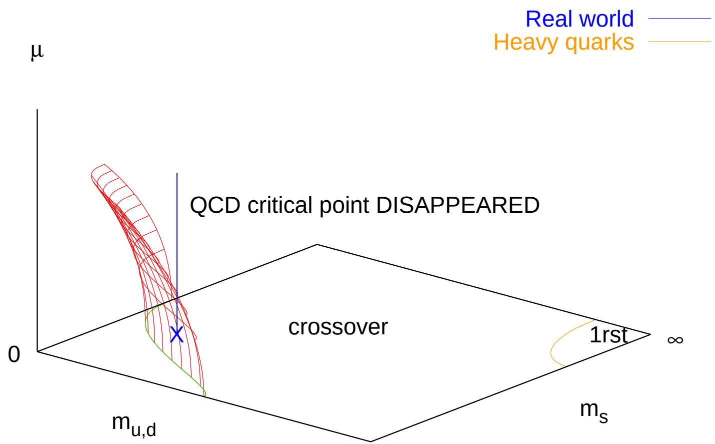
Generalize QCD to arbitrary $(m_{u,d}, m_s)$, T: phase diagram

$\mu \neq 0$



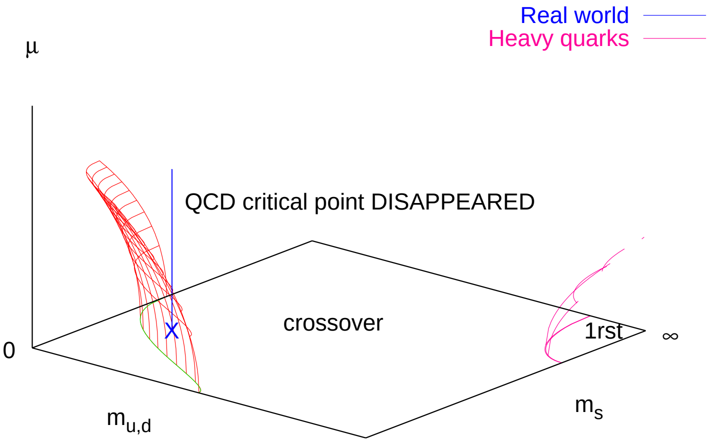
Conventional wisdom: first-order region expands with real $|\mu|$

Generalize QCD to arbitrary $(m_{u,d}, m_s)$, T: phase diagram



Exotic scenario: first-order region shrinks with real $|\mu|$ $\frac{d\ m_c}{d\mu^2}|_{\mu=0} < 0$

Generalize QCD to arbitrary $(m_{u,d}, m_s)$, T: phase diagram



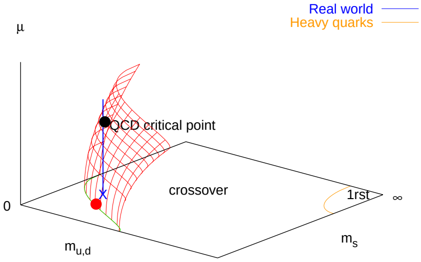
For heavy quarks, first-order region shrinks (PdF, Kim, Takaishi, hep-lat/0510069)

Strategy, with Owe Philipsen

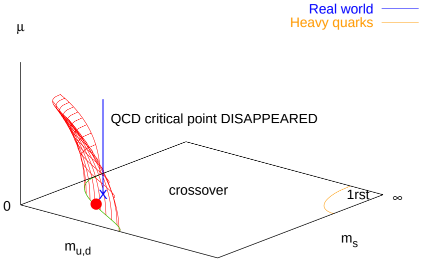
1. Tune quark mass(es) to $m_c(0)$: 2nd order transition at $\mu = 0, T = T_c$
   known universality class: 3d Ising

2. Measure derivatives $\frac{d^k m_c}{d\mu^{2k}}|_{\mu=0}$:

$$\frac{m_c(\mu)}{m_c(0)} = 1 + \sum_{k=1} c_k \left(\frac{\mu}{\pi T}\right)^{2k}$$



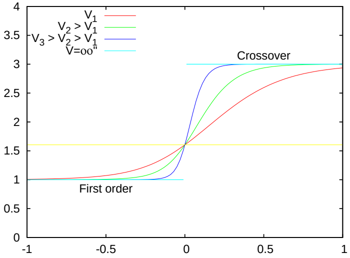
$c_1 > 0$

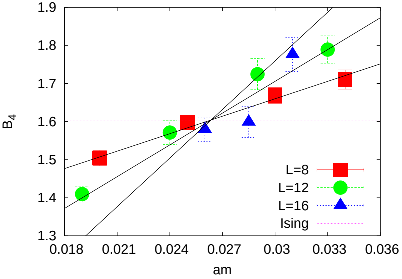


$c_1 < 0$

Observable: Binder cumulant

- Probability distribution of order parameter
  - distinguishes crossover (Gaussian) vs 1rst order (2 peaks)
  - 2nd order: scale-invariant distribution with known Ising exponents
  - encoded in Binder cumulant
- Measure $B_4(\bar\psi\psi) \equiv \frac{\langle(\delta\bar\psi\psi)^4\rangle}{\langle(\delta\bar\psi\psi)^2\rangle^2}\Big|_{\langle(\delta\bar\psi\psi)^3\rangle=0} = \begin{cases} 3 & \text{crossover} \\ 1 & \text{first-order} \\ 1.604 & \text{3d Ising} \end{cases}$ for $V \to \infty$

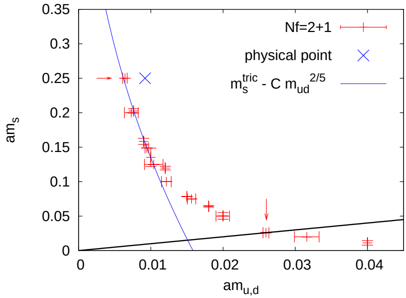


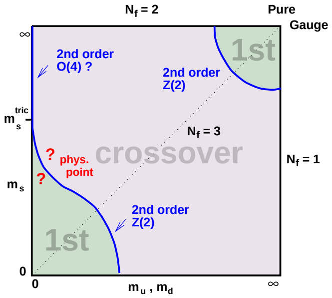


- Finite volume, $\mu = 0$: $B_4(am) = 1.604 + c(L)(am - am_0^c) + \ldots$, $c(L) \propto L^{1/\nu}$

Results: hep-lat/0607017, 0808.1096

1. Line of second-order phase transitions in the quark mass plane $(m_{u,d}, m_s)$ via Binder cumulant $B_4 = \langle(\delta\bar{\psi}\psi)^4\rangle / \langle(\delta\bar{\psi}\psi)^2\rangle^2$



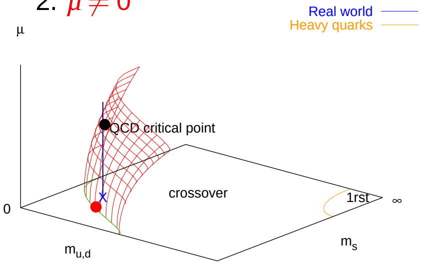


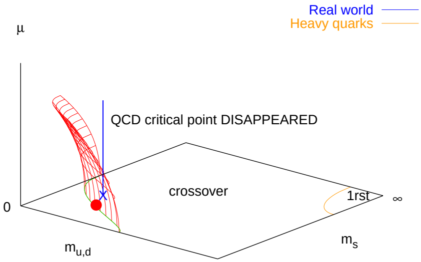
$\mu = 0$:

- data consistent with tricritical point at $m_{u,d} = 0$, $m_s \sim 2.8 T_c$
- physical point in crossover region cf. Fodor & Katz

Results: hep-lat/0607017, 0808.1096

2. $\mu \neq 0$





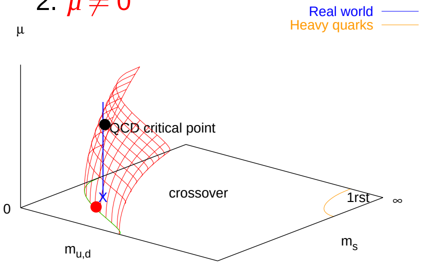
Strategy: tune m_q for 2nd-order P.T. at $\mu = 0$, then turn on [imaginary] μ
Does the transition become 1rst-order (left) or crossover (right)?

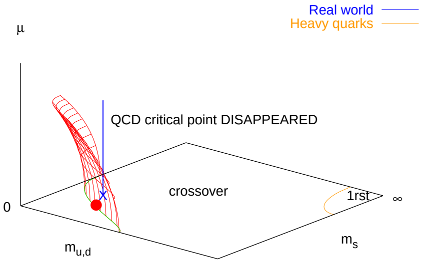
$$B_4(am, a\mu) = 1.604 + \sum_{k,l=1} b_{kl} \, (am - am_0^c)^k \, (a\mu)^{2l}$$

$$\frac{d\,am^c}{d(a\mu)^2} = -\frac{\partial B_4}{\partial (a\mu)^2} \Big/ \frac{\partial B_4}{\partial am} = -b_{01}/b_{10}, \quad \text{hard / easy}$$

Results: hep-lat/0607017, 0808.1096

2. $\mu \neq 0$





Strategy: tune m_q for 2nd-order P.T. at $\mu = 0$, then turn on [imaginary] μ

Does the transition become 1rst-order (left) or crossover (right)?

$$B_4(am, a\mu) = 1.604 + \sum_{k,l=1} b_{kl}\,(am - am_0^c)^k (a\mu)^{2l}$$

$$\frac{d\,am^c}{d(a\mu)^2} = -\frac{\partial B_4}{\partial (a\mu)^2} \Big/ \frac{\partial B_4}{\partial am} = -b_{01}/b_{10}, \quad \text{hard / easy}$$

Answer: very little change ($\to$ surface almost vertical)

Two methods to measure change in B_4: $\frac{\partial B_4}{\partial (a\mu)^2}$

1. Finite-μ: MC at several $\mu = i\mu_i$, fit $B_4(\mu_i)$ with truncated Taylor series in μ^2

Danger: truncation error?

Two methods to measure change in B_4: $\frac{\partial B_4}{\partial (a\mu)^2}$

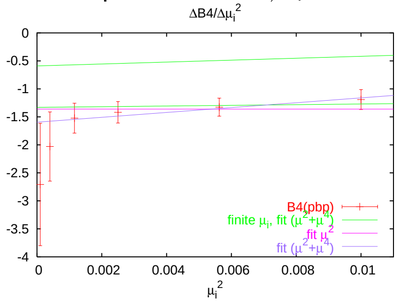
1. **Finite-μ:** MC at several $\mu = i\mu_i$, fit $B_4(\mu_i)$ with truncated Taylor series in μ^2
   Danger: truncation error?
2. **Derivative:** MC at $\mu = 0$, reweight to small $\mu = i\mu_i$, measure $\frac{\Delta B_4}{\Delta \mu^2}$
   Advantage: fluctuations cancel in ΔB_4

Two methods to measure change in B_4: $\frac{\partial B_4}{\partial (a\mu)^2}$

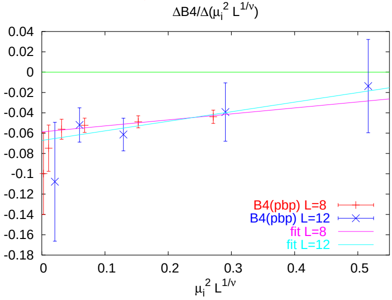
1. **Finite-μ:** MC at several $\mu = i\mu_i$, fit $B_4(\mu_i)$ with truncated Taylor series in μ^2
   Danger: truncation error?

2. **Derivative:** MC at $\mu = 0$, reweight to small $\mu = i\mu_i$, measure $\frac{\Delta B_4}{\Delta \mu^2}$
   Advantage: fluctuations cancel in ΔB_4

Comparison $8^3 \times 4, N_f = 3$:



Scaling 8^3 and 12^3 $\times 4$:



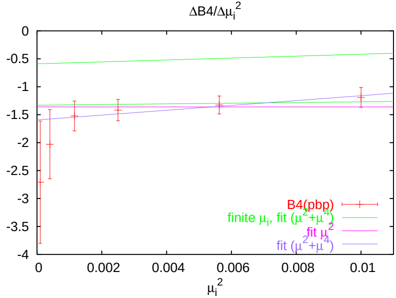
No doubt about sign $\longrightarrow$ non-standard scenario!?

Two methods to measure change in B_4: $\frac{\partial B_4}{\partial (a\mu)^2}$

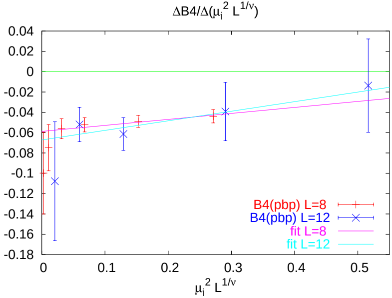
1. Finite-μ: MC at several $\mu = i\mu_i$, fit $B_4(\mu_i)$ with truncated Taylor series in μ^2
Danger: truncation error?

2. Derivative: MC at $\mu = 0$, reweight to small $\mu = i\mu_i$, measure $\frac{\Delta B_4}{\Delta \mu^2}$
Advantage: fluctuations cancel in ΔB_4

Comparison $8^3 \times 4, N_f = 3$:

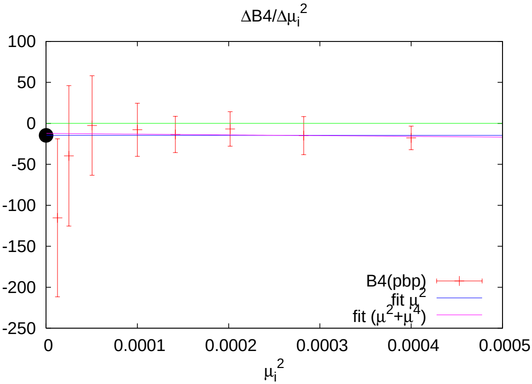


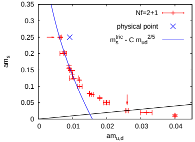
Scaling 8^3 and $12^3 \times 4$:



$$N_t = 4,\ N_f = 3: \quad \frac{m_c(\mu)}{m_c(0)} = 1 - 3.3(3)\left(\frac{\mu}{\pi T}\right)^2 - 47(20)\left(\frac{\mu}{\pi T}\right)^4 + \ldots$$

$N_t = 4, N_f = 2+1$: moving along the critical line





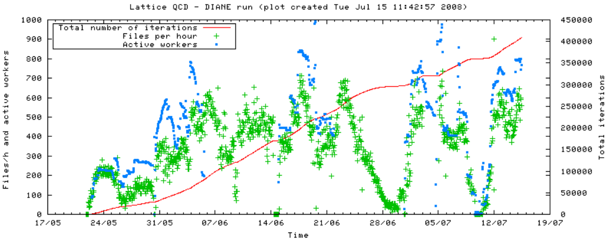
- $16^3 \times 4, am_s = 0.25, am_{u,d} = 0.005$, *lighter than in nature* $(m_\pi L = 3.4)$
  700k trajectories, 2 months of Grid computing
- $b_{01} = -14(4)$ (μ^2 fit) $\rightarrow \partial am^c / \partial(a\mu^2) = -0.13(4)$

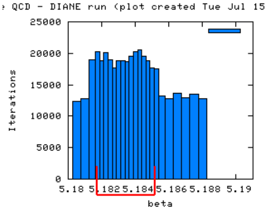
[or $b_{01} = -13(11)$ ($\mu^2 + \mu^4$ fit)]

- $c_1 = -16(5)$, ie. $\frac{m_c(\mu)}{m_c(0)} = 1 - \mathbf{16(5)} \left(\frac{\mu}{\pi T}\right)^2$ almost conclusive

LQCD on the Computing Grid

- 725k trajectories (2 quark masses) in 2 months $\longrightarrow$ 115 CPU years
- on average 700 CPUs active at all times
- 330k files = 3 TB of data transferred
- computing support provided by CERN IT/GS: *thanks a lot!*





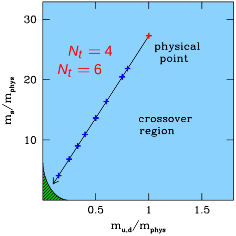
prioritized scheduling

- calculations on EGEE Grid
- resources provided by CERN, CYFRONET (Poland), CSCS (Switzerland), NIKHEF (Holland) + 10 more across Europe

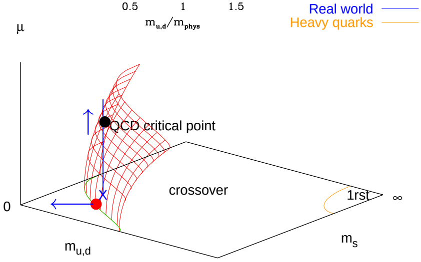
$N_t = 6, N_f = 3$: towards the continuum limit

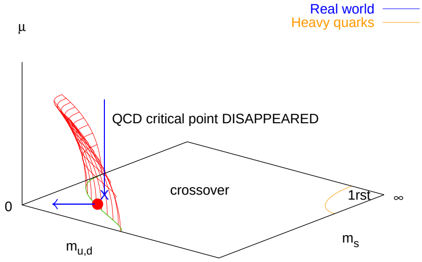
1. $\mu = 0$: re-tune the quark mass for 2nd-order transition at $T = T_c$

$\longrightarrow$ At $T = 0$, $\frac{m_\pi}{T_c} = 0.954(12)$ instead of $1.680(4)$ ($N_t = 4$)



cf. Endrodi, Fodor et al., arXiv:0710.0998

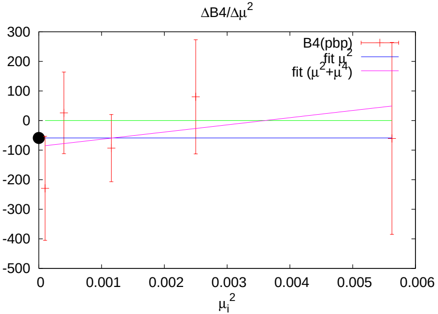




Critical surface moves further away from physical point

$N_t = 6, N_f = 3$: towards the continuum limit

2. Measure $\frac{\partial B_4}{\partial(am)}$ (easy) and $b_1 \equiv \frac{\partial B_4}{\partial(a\mu)^2}$ (hard)



- $18^3 \times 6, am = 0.003$, $m_\pi = 0.95 T_c \sim$ 170 MeV $(m_\pi L = 2.9)$
  120k trajectories, 6 months of SX-8
- $b_{01} = -58(49)$ (μ^2 fit) $\longrightarrow c_1 = -28(23)$, ie. $\frac{m_c(\mu)}{m_c(0)} = 1 - \mathbf{28(23)} \left(\frac{\mu}{\pi T}\right)^2$

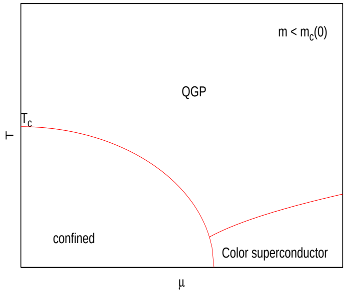
[or $b_{01} = -88(75)$ ($\mu^2 + \mu^4$ fit)]

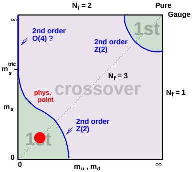
- Assume $c_1 = +18$, ie. 2 sigmas away; then $\frac{\mu_E}{T_E} = 1 \Rightarrow \frac{m_c(\mu_E)}{m_c(0)} \sim 3$,

insufficient to reach physical point

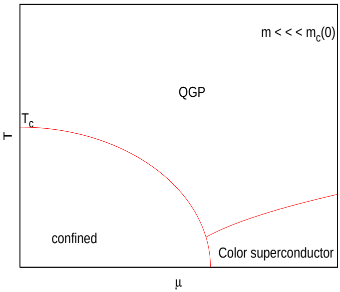
Resulting phase diagram (simplest possibility)

Standard scenario



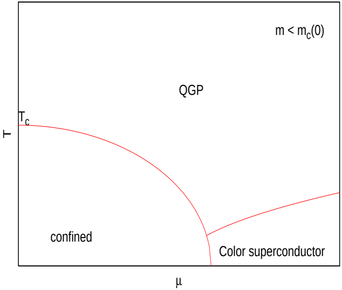


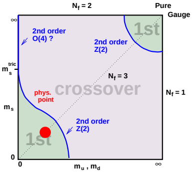
Exotic scenario



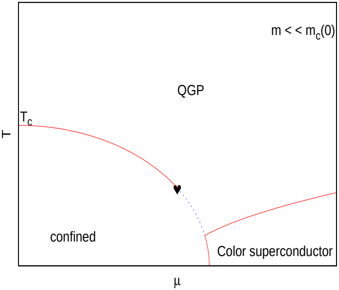
Resulting phase diagram (simplest possibility)

Standard scenario



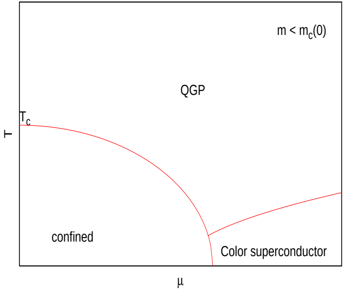


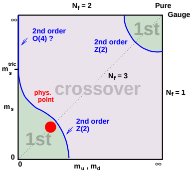
Exotic scenario



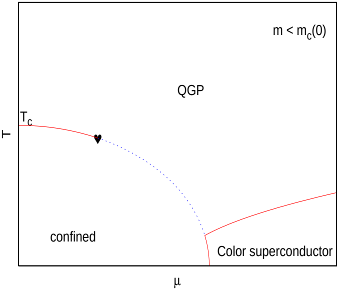
Resulting phase diagram (simplest possibility)

Standard scenario





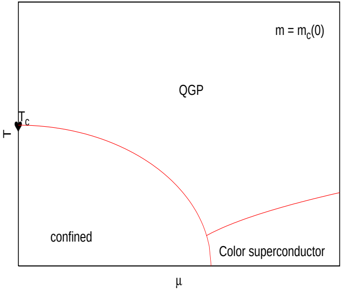
Exotic scenario

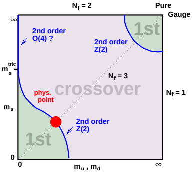


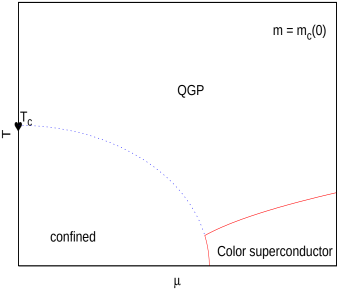
Resulting phase diagram (simplest possibility)

Standard scenario

Exotic scenario

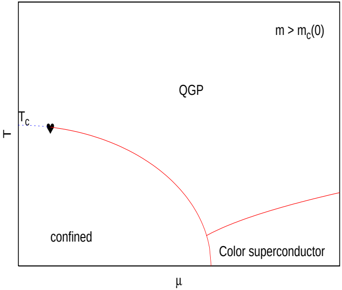


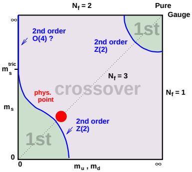




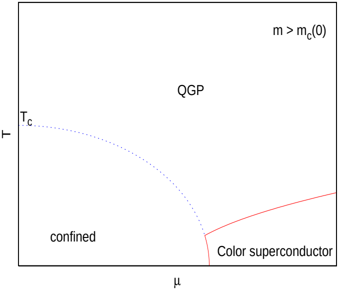
Resulting phase diagram (simplest possibility)

Standard scenario



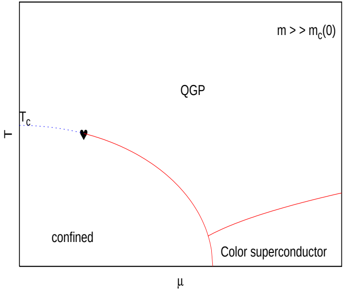


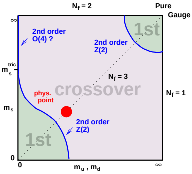
Exotic scenario



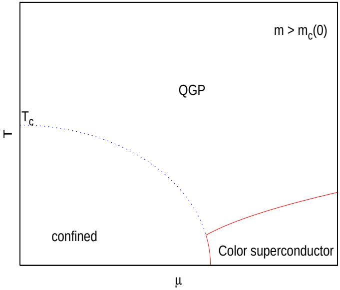
Resulting phase diagram (simplest possibility)

Standard scenario



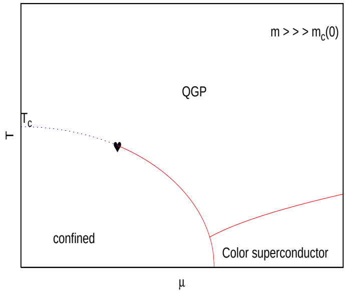


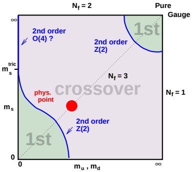
Exotic scenario



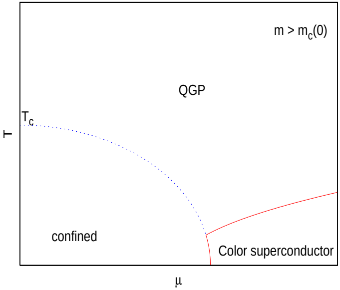
Resulting phase diagram (simplest possibility)

Standard scenario





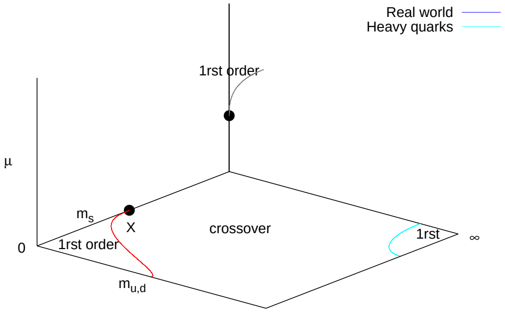
Exotic scenario



Arguments for standard wisdom?

- $O(4)$ transition for 2 massless flavors — Pisarski & Wilczek

$\Rightarrow$ tricritical points $(m_{u,d} = 0, m_s = \infty, \mu = \mu^*)$ and $(m_{u,d} = 0, m_s = m_s^*, \mu = 0)$

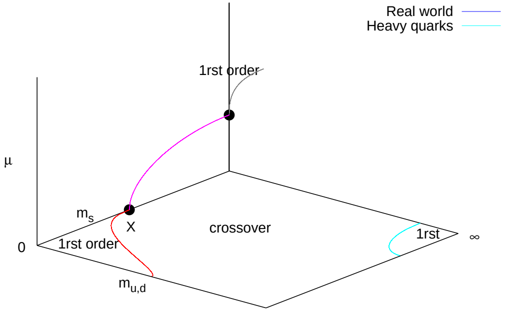


Arguments for standard wisdom?

- $O(4)$ transition for 2 massless flavors — Pisarski & Wilczek

$\Rightarrow$ tricritical points $(m_{u,d} = 0, m_s = \infty, \mu = \mu^*)$ and $(m_{u,d} = 0, m_s = m_s^*, \mu = 0)$

- $N_f = 2$ and $N_f = 2+1$ analytically connected

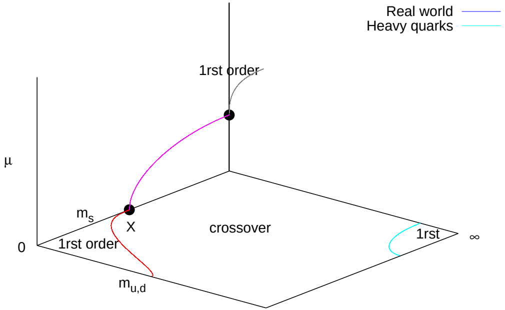


Arguments for standard wisdom?

- $O(4)$ transition for 2 massless flavors — Pisarski & Wilczek

$\Rightarrow$ tricritical points $(m_{u,d} = 0, m_s = \infty, \mu = \mu^*)$ and $(m_{u,d} = 0, m_s = m_s^*, \mu = 0)$

- $N_f = 2$ and $N_f = 2+1$ analytically connected



Critique:

- $O(4)$ if strong enough $U_A(1)$ anomaly, otherwise first-order

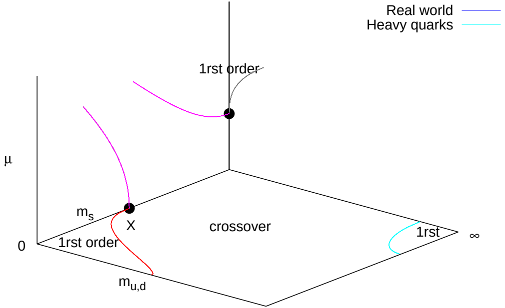
Chandrasekharan & Mehta

Arguments for standard wisdom?

- $O(4)$ transition for 2 massless flavors — Pisarski & Wilczek

$\Rightarrow$ tricritical points $(m_{u,d} = 0, m_s = \infty, \mu = \mu^*)$ and $(m_{u,d} = 0, m_s = m_s^*, \mu = 0)$

- $N_f = 2$ and $N_f = 2+1$ analytically connected



Critique:

- $O(4)$ if strong enough $U_A(1)$ anomaly, otherwise first-order

Chandrasekharan & Mehta

- $N_f = 2$ and $N_f = 2+1$ need not be connected

Conclusions

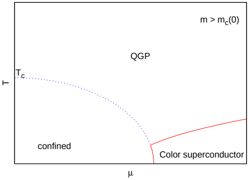
- Race between theory and experiment: no finish line?

- $\frac{m_c(\mu)}{m_c(0)} = 1 + \mathbf{c_1} \left(\frac{\mu}{\pi T}\right)^2 + \ldots$: *can control systematics*

$N_t = 4$,	$N_f = 3$	LO+NLO	0808.1096
	$N_f = 2+1$	LO	soon
$N_t = 6$,	$N_f = 3$	LO	underway
$\cdots$	**Non-standard scenario $c_1 < 0$ favored**		

- $a \to 0$: critical surface *far* from physical point
  $\Longrightarrow$ need $c_1 > 0$ *and large* for $\frac{\mu_E}{T_E} \lesssim 1$, disfavored by data

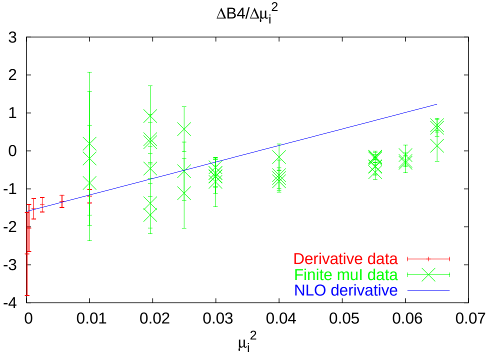
- **QCD critical point?**



$\mu_E^B \lesssim 500$ MeV unlikely, or non-chiral

$N_t = 4, N_f = 3$: combining the two methods

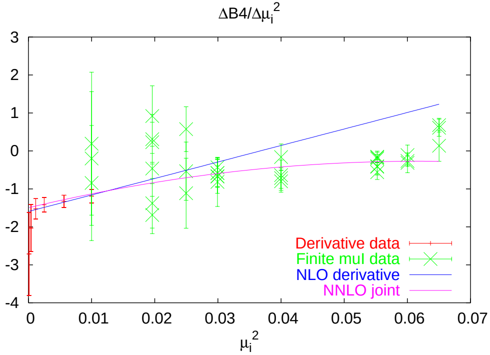
Methods 1 and 2 cover different ranges of μ_i $\longrightarrow$ combine them



$$\frac{B_4(\mu_i)-B_4(0)}{\mu_i^2} = \underbrace{b_{01}}_{<0} + \underbrace{b_{02}}_{>0} \mu_i^2$$

$N_t = 4, N_f = 3$: combining the two methods

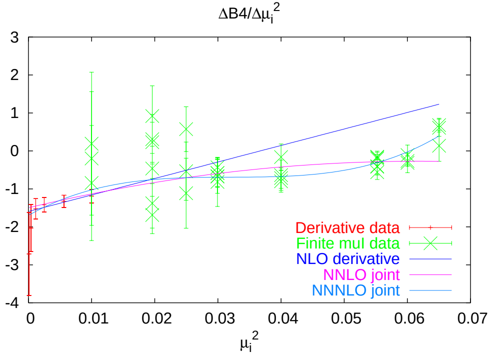
Methods 1 and 2 cover different ranges of μ_i $\longrightarrow$ combine them



$$\frac{B_4(\mu_i)-B_4(0)}{\mu_i^2} = \underbrace{b_{01}}_{<0} + \underbrace{b_{02}}_{>0}\mu_i^2 + \underbrace{b_{03}}_{<0}\mu_i^4$$

$N_t = 4, N_f = 3$: combining the two methods

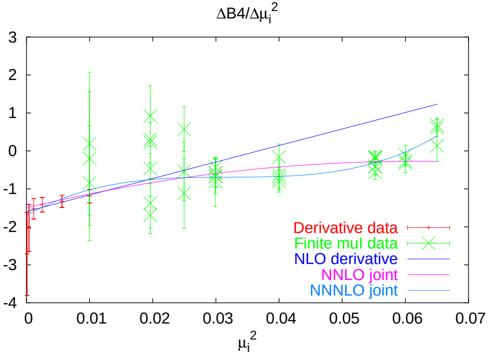
Methods 1 and 2 cover different ranges of μ_i $\longrightarrow$ combine them



$$\frac{B_4(\mu_i)-B_4(0)}{\mu_i^2} = \underbrace{b_{01}}_{<0} + \underbrace{b_{02}}_{>0}\mu_i^2 + \underbrace{b_{03}}_{<0}\mu_i^4 + \underbrace{b_{04}}_{>0}\mu_i^6$$

$N_t = 4, N_f = 3$: combining the two methods

Methods 1 and 2 cover different ranges of μ_i $\longrightarrow$ combine them



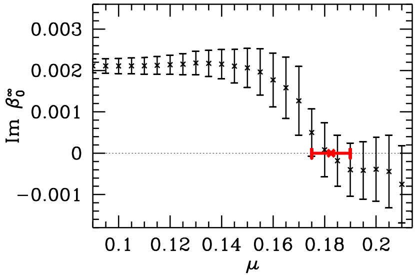
$$\frac{B_4(\mu_i)-B_4(0)}{\mu_i^2} = \underbrace{b_{01}}_{<0} + \underbrace{b_{02}}_{>0}\mu_i^2 + \underbrace{b_{03}}_{<0}\mu_i^4 + \underbrace{b_{04}}_{>0}\mu_i^6$$

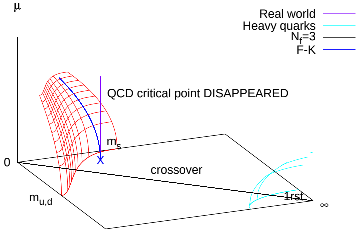
Real μ: $B_4(\mu) = B_4(0) + \underbrace{(-b_{01})}_{>0}\mu^2 + \underbrace{(+b_{02})}_{>0}\mu^4 + \underbrace{(-b_{03})}_{>0}\mu^6 + \underbrace{(+b_{04})}_{>0}\mu^8$

B_4 increases with $\mu \rightarrow$ crossover: all terms reinforce exotic scenario!

Contradiction with other lattice studies?

- Fodor & Katz: $(T_E, \mu_E) = (162(2), 120(13))$ MeV ?

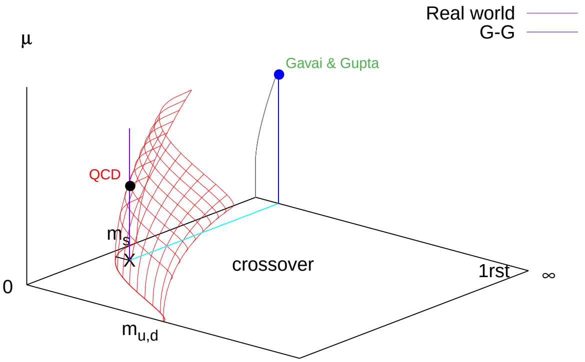




- Very little μ-dependence until $\mu \sim \mu_E \longrightarrow$ need high-degree Taylor expansion
- $m_q a$, ie. $\frac{m_q}{T_c}$ fixed, while $T_c(\mu)$ decreases for $\mu \neq 0 \Rightarrow$ non-const. physics
  Lighter quarks at larger μ favor first-order transition

Contradiction with other lattice studies?

- Gavai & Gupta: $\mu_E / T_E \lesssim 1$?
  different theory $N_f = 2$



- Agreement with isospin μ Kogut-Sinclair, PdF-Stephanov-Wenger