Critical behavior of the compact $3d$ U(1) theory in the limit of zero spatial coupling[1]

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[1] J. Stat. Mech. (2008) P08009

Out of line

1. Introduction
   - Why $3d$ U(1) LGT?
   - BKT transition

Out of line

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$3d$ U(1) LGT

$T = 0$

The theory is confining at all values of the bare coupling constant.

Finite T

Deconfining phase transition.

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Finite T

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$3d$ U(1) LGT is one of the simplest model
with continous gauge symmetry which possess
the same fundamental properties as QCD.

$3d$ U(1) LGT

Critical behavior

DUAL LATTICE

- $3d$ U(1) LGT in the Villain formulation coincides with the $2d$ XY model in the leading order of the high-temperature expansion.

SVETITSKY-YAFFE

- If correlation lenght diverges, the finite-temperature phase transition in the $3d$ U(1) LGT should belong to the $2d$ XY universality class.

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$3d$ U(1) LGT and $2d$ XY
in the same universality class?

XY: Berezinskii-Kosterlitz-Thouless transition

MERMIN-WAGNER Theorem

The global U(1) symmetry cannot be spontaneously broken in $2d$.
A local order parameter does not exist.

Transition is understood in terms of the unbinding of topological object:

- SPIN ($2d$ XY) $\leftrightarrow$ POLYAKOV LOOP ($3d$ U(1) LGT)
- VORTICES ($2d$ XY) $\leftrightarrow$ MONOPOLES ($3d$ U(1) LGT)

Berezinskii-Kosterlitz-Thouless transition

”Bound” phase

$$\Gamma(R) \sim \frac{1}{R^{\eta(T)}}$$

Logarithmic non-confining potential between charges

”Unbound” phase

$$\Gamma(R) \sim \exp[-R/\xi(t)]$$

Linear confining potential between charges

ESSENTIAL SCALING

$$\xi \sim e^{bt^{-\nu}} \qquad t = T/T_c - 1$$

Berezinskii-Kosterlitz-Thouless transition

XY UNIVERSALITY CLASS

$$\nu = 1/2 \qquad \eta(T_c) = 1/4$$

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An approximated RG calculation[a] indicates $\eta(T_c) = 1/4$
A numerical check[b] confirms the BKT nature of the phase transition, but concludes $\eta(T_c) \sim 0.78$!!!

[a] Svetitsky and Yaffe (1982)
[b] Coddington et al. (1986)

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WE PROPOSE A NUMERICAL STUDY OF THE CRITICAL PROPERTIES OF $3d$ U(1) LTG THEORY

$3d$ U(1) LGT: definitions

Partition function:

$$Z(\beta_t, \beta_s) = \int_0^{2\pi} \prod_{x\in\Lambda} \prod_{n=0}^{2} \frac{d\omega_n(x)}{2\pi} \exp S[\omega]$$

Action

$$S[\omega] = \beta_s \sum_{p_s} \cos\omega(p_s) + \beta_t \sum_{p_t} \cos\omega(p_t)$$

$\Lambda = L^2 \times N_t$

Parameters:

$$\beta_t = \frac{1}{g^2 a_t}, \quad \beta_s = \frac{\xi}{g^2 a_s} = \beta_t \xi^2, \quad \xi = \frac{a_t}{a_s}$$

g is the continuum coupling constant.

Finite-temperature limit

$$\xi \to 0, \quad N_t, L \to \infty, \quad a_t N_t = \frac{1}{T}$$

Correlator:

$$\Gamma(R) \; = \langle P_x^\dagger P_{x+R} \rangle = \left\langle \exp \left[i \sum_{x_0=0}^{N_t-1} (\omega_0(x_0, x_1, x_2) - \omega_0(x_0, x_1, x_2 + R)) \right] \right\rangle$$

In this talk $\beta_s = 0$

$$Z(\beta_t, \beta_s = 0) = \int_0^{2\pi} \prod_{x \in \Lambda_2} \frac{d\omega_x}{2\pi} \prod_{x,n} \left[\sum_{r=-\infty}^{\infty} I_r^{N_t}(\beta_t) \, \exp\left[ir(\omega(x) - \omega(x + e_n)) \right] \right]$$

$\Lambda_2 = L^2$ and $\omega(x) \equiv \omega(x_1, x_2)$.

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$N_t = 1$: Using $\sum_r I_r(x) e^{ir\omega} = e^{x \cos\omega}$

$$Z(\beta_t, \beta_s = 0)|_{N_t=1} = \int_0^{2\pi} \prod_x \frac{d\omega(x)}{2\pi} \exp[\beta_t \sum_{x,n} \cos(\omega(x) - \omega(x+e_n))]$$

XY Partition function

In this talk $\beta_s = 0$

$N_t \geq 2$:

The leading order of strong coupling expansion is XY model

$$Z(\beta_t \ll 1, \beta_s = 0) = \int_0^{2\pi} \prod_x \frac{d\omega(x)}{2\pi} \exp[h(\beta_t) \sum_{x,n} \cos(\omega(x) - \omega(x+e_n))]$$

$$h(\beta_t) = 2[\frac{I_1(\beta_t)}{I_0(\beta_t)}]^{N_t}, \quad \Gamma(R) = [\frac{1}{2} h(\beta_t)]^R$$

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$$Z(\beta_t \gg 1, \beta_s = 0) = \sum_{r(x)=-\infty}^{\infty} \exp[-\frac{1}{2}\tilde{\beta} \sum_x \sum_{n=1}^{2} (r(x) - r(x+e_n))^2]$$

Villain partition function with effective coupling $\tilde{\beta} = N_t/\beta_t$

Set-up

$$Z(\beta_t, \beta_s = 0) = \int_0^{2\pi} \prod_{x \in \Lambda_2} \frac{d\omega_x}{2\pi} \prod_{x,n} \left[\sum_{r=-\infty}^{\infty} I_r^{N_t}(\beta_t) \; \exp\left[ir(\omega(x) - \omega(x+e_n))\right] \right]$$

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$$Z(\beta_t, \beta_s = 0) \equiv \int_0^{2\pi} \prod_x \frac{d\omega(x)}{2\pi} \exp S'$$

Action

$$S' = \sum_{x,n} \log\left\{1 + 2\sum_{r=1}^{\infty} [b_r(\beta_t)]^{N_t} \cos r(\omega(x) - \omega(x+e_n))\right\}$$

$b_r(\beta) = I_r(\beta)/I_0(\beta)$

$N_t = 1$

The main indication of BKT critical behavior is a peculiar scaling of the pseudo-critical coupling with the spatial lattice size L:

$$\beta_{pc}(L) - \beta_c \sim \frac{1}{(\log L)^{1/\nu}},$$

this is consequence of the essential scaling.

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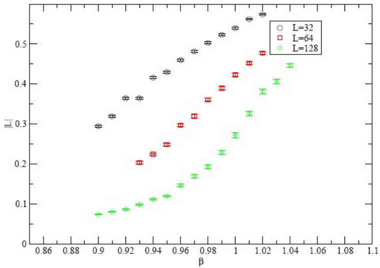
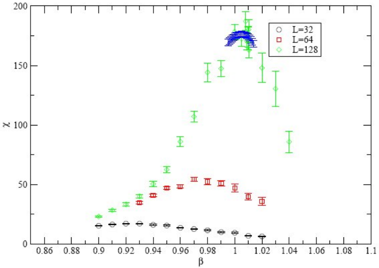
this is consequence of the essential scaling.

$\beta_{pc}(L)$ is determined with the maximum of susceptibility

$$\chi = L^2 \langle |P|^2 \rangle \ , \qquad P = \frac{1}{L^2} \sum_x P_x$$

$N_t = 1$





$N_t = 1$

$$\beta_{pc}(L) = \beta_c + \frac{A}{(\log L)^2}, \qquad \nu = 1/2$$

$L = 128, 150, 200, 256$
$\beta_c(N_t = 1) = 1.107(9)$ and $A(N_t = 1) = -2.4(2)$ (χ^2/d.o.f.=0.78)

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Best determiation: $\beta_c = 1.1199(1)$[2]

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Logarithmic corrections

$$\chi(\beta_c) \sim L^{2-\eta_c}(\log L)^{-r}, \ r = 1/8$$

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No logarithmic corrections

$$\chi(\beta_c) \sim L^{2-\eta_c}$$

$\chi(\beta = 1.12)$ for L=64, 128, 150, 200, 256

$\eta_c = 0.256(29)$ (χ^2/d.o.f.=0.2)

$N_t = 1$

Correlator

$$C(R) = \sum_{x,n} \Re\left(P_x^\dagger P_{x+R\cdot e_n}\right)$$

Logarithmic corrections

$$C(R) \sim \frac{(\log R)^{-r}}{R^{\eta_c}}$$

$N_t = 1$

Correlator

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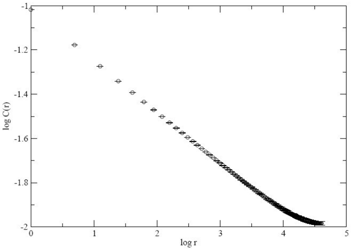
No logarithmic corrections

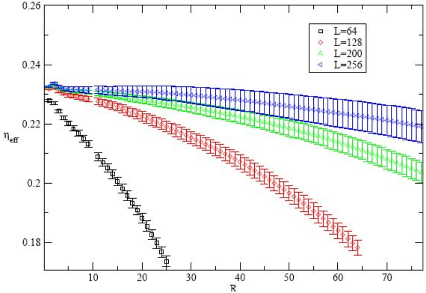
$$C(R) \sim \frac{1}{R^{\eta_c}}$$



$N_t = 1$

Effective index η_c

$$\eta_{\text{eff}}(R) \equiv \frac{\log[C(R)/C(R_0)]}{\log[R_0/R]}$$



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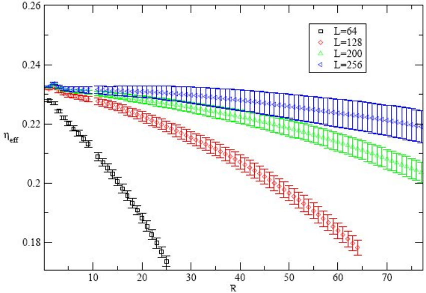


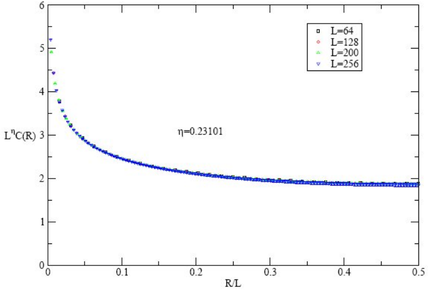
$\eta(\beta = 1.12) = \eta_{\text{eff}}(R = 6) = 0.23101(49)$

$\eta(\beta = 1.107) = 0.24085(44)$

$N_t = 1$

Rescaled correlation

$$L^{-\eta} C(R) \equiv f\left(\frac{R}{L}\right)$$



f is an universal function.

$N_t \geq 2$

Essential scaling

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$N_t = 4$: $\beta_c(N_t = 4) = 3.42(1)$ and $A(N_t = 4) = -5.1(3)$
(χ^2/d.o.f.=0.43, $L = 64, 150, 200$)

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$N_t = 8$: $\beta_c(N_t = 8) = 6.38(5)$ and $A(N_t = 8) = -15(1)$
(χ^2/d.o.f.=0.006, $L = 64, 128, 150$)

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BKT , ν OK!

Susceptibility

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$N_t = 8$: $\eta_c(N_t = 8) = 0.212(46)$ (χ^2/d.o.f.=0.43)
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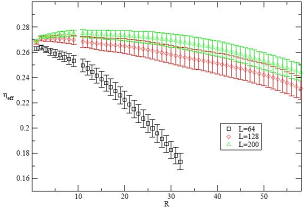
$\eta(\beta_c)$ OK!

$N_t \geq 2$

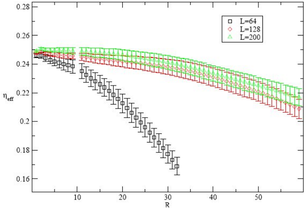
$N_t = 4$

$$\eta(\beta = 3.42) = \eta_{\text{eff}}(N_t = 4, R = 2) = 0.2724(11)$$



$N_t = 8$

$$\eta(\beta = 6.38) = \eta_{\text{eff}}(N_t = 8, R = 3) = 0.2499(11)$$



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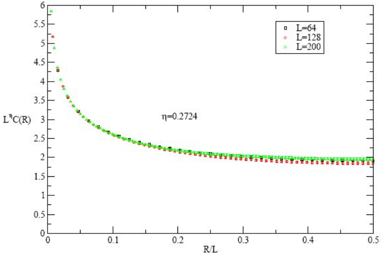

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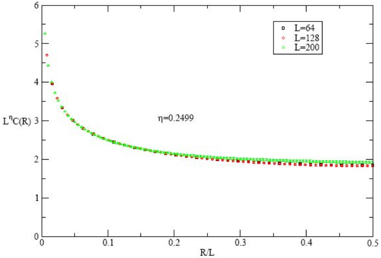

Conclusions

We give strong indications that $3d$ U(1) pure gauge theory belongs to the universality class of $2d$ XY model in the case $\beta_s = 0$, by verifing scaling laws at criticality and comparing critical inices.

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Outlooks

$\beta_s \neq 0$

We are extending this analysis to the $\beta_s \neq 0$ case and draw the phase diagram of the full theory in the (β_t, β_s) plane.

THANK YOU!