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# Antiferromagnetic two-leg spin-1/2 ladders with Mobius boundary conditions:

Antiferromagnetic two-leg spin-1/2 ladders with Mobius boundary

## a twisted CFT approach

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# 2-leg XXZ spin-1/2 ladders with Mobius boundary conditions

Let us introduce the physical system, i. e. two antiferromagnetic spin-1/2 *XXZ* chains arranged in a closed geometry for different boundary conditions imposed at the ends. Then we switch to the interacting case and describe the following perturbations: railroad, zigzag, 4-spin, 4-dimer.

1. Non interacting system: two spin-1/2 *XXZ* chains

Total number of sites

$$H_0 = J_0 \sum_{i=1}^{M} \left( S_i^x S_{i+2}^x + S_i^y S_{i+2}^y + \Delta S_i^z S_{i+2}^z \right)$$

Antiferromagnetic coupling

Anisotropy parameter

**We assume that the odd sites belong to the down leg of the ladder while the even sites belong to the up leg. The legs now are non interacting. Let us now close the chains imposing the following boundary conditions:**

$$M+1 \equiv 1, \quad M+2 \equiv 2$$

**Depending on *M*, we get two topologically inequivalent boundary conditions:**

1. ***M* even: we get two independent *XXZ* spin-1/2 chains, each one closed by gluing opposite ends, that is periodic boundary conditions (PBC).**

2. ***M* odd: the two legs are not independent, they appear to be connected at a point upon gluing the opposite ends and the system can be viewed as a single eight shaped chain, that is Mobius boundary conditions (MBC).**

For ***M* odd** the system presents a local topological defect in the gluing point: it coincides with two closed *XXZ* spin-1/2 chains, each one with $(M+1)/2$ sites, which intersect each other in the topological defect at site $M+1\equiv 1$. The general Hamiltonian in this case is written as:

$$H_0^{MBC} = H_0 + H_{MBC}$$

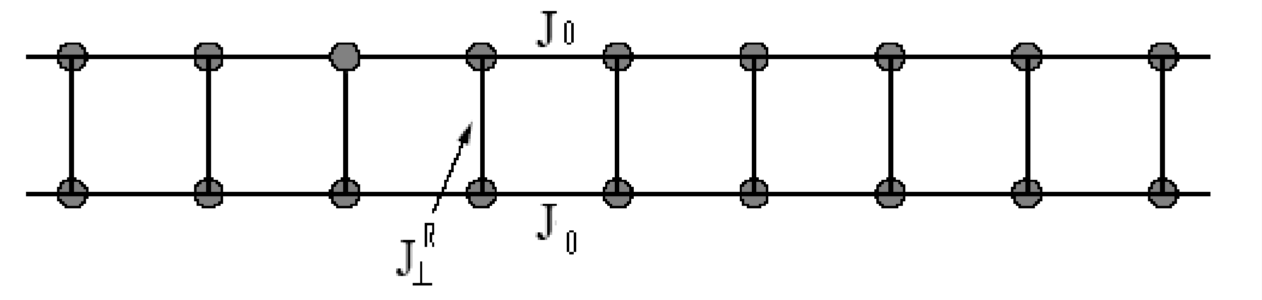
$$H_{MBC} = J_0 \sum_{i=M,M+1} \left( S_i^x S_{i+1}^x + S_i^y S_{i+1}^y + \Delta S_i^z S_{i+1}^z \right) - J_0 \left( S_M^x S_{M+2}^x + S_M^y S_{M+2}^y + \Delta S_M^z S_{M+2}^z \right)$$

2. Weak interacting system: we introduce the following interactions between the two spin-1/2 *XXZ* chains: railroad, zigzag, 4-spin, 4-dimer.

$$H_0 + H_{Railroad}$$

Railroad

$$H_{Railroad} = J_\perp^R \sum_{i=1}^{[(M+1)/2]} \left( S_{2i-1}^x S_{2i}^x + S_{2i-1}^y S_{2i}^y + \Delta S_{2i-1}^z S_{2i}^z \right)$$

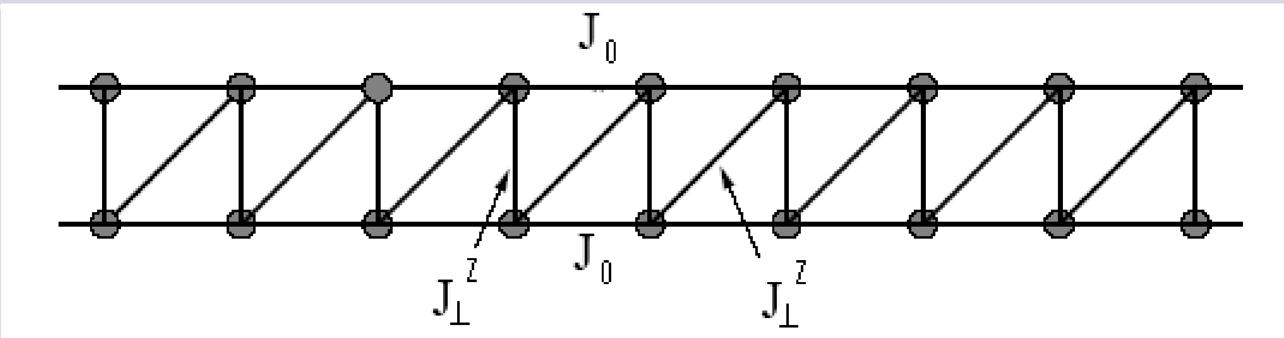



In the weak coupling limit $J_0 >> J^R$, for the isotropic *XXX* case ($\Delta$=1), we have a gapful spectrum with the formation of massive spin S=1 and S=0 particles (D. G. Shelton, A. A. Nersesian, A. M. Tsvelick, PRB 53 (1996) 8521).

Zigzag

$$H_0 + H_{Zigzag}$$

$$H_{Zigzag} = J_\perp^Z \sum_{i=1}^{M} \left( S_i^x S_{i+1}^x + S_i^y S_{i+1}^y + \Delta S_i^z S_{i+1}^z \right)$$




In the weak coupling limit $J_0 >> J^z$, for the isotropic *XXX* case ($\Delta=1$), an exponentially small gap develops and the weak interchain correlations break translational symmetry. There is a spontaneous dimerization along with a finite range incommensurate magnetic order (A. A. Nersesyan, A. O. Gogolin, F. H. L. Essler, PRL 81 (1998) 910; D. Allen, D. Senechal, PRB 55 (1997) 299).

4-Spin

$$H_{4-Spin}=J_{\varepsilon}\sum_{i=1}^{[(M+1)/2]}\varepsilon_i^{Dw}\varepsilon_i^{Up}$$

$$\varepsilon_i^{Dw}=(-1)^i\left(S_{2i-1}^x S_{2i+1}^x+S_{2i-1}^y S_{2i+1}^y+\Delta S_{2i-1}^z S_{2i+1}^z\right)$$

$$\varepsilon_i^{Up}=(-1)^i\left(S_{2i}^x S_{2i+2}^x+S_{2i}^y S_{2i+2}^y+\Delta S_{2i}^z S_{2i+2}^z\right)$$

Such a coupling can be generated either by phonons or by the conventional Coulomb repulsion between the holes.

4-Dimer

$$H_{4-Dimer}=J_{\kappa}\sum_{i=1}^{[(M+1)/2]}\kappa_i\kappa_{i+1}$$

$$\kappa_i=\varepsilon_i^{Dw}\varepsilon_i^{Up}$$

Such a term plays a crucial role in the presence of the marginal interaction $H_{ZigZag}$ only, because it gives rise to the dynamical generation of the triplet or singlet mass respectively along two different *RG* flows.

# The *XXZ* spin-1/2 chain in the continuum limit (abelian bosonization)

$$H_{XXZ} = J\sum_{j=1}^{N}\left(S_j^x S_{j+1}^x + S_j^y S_{j+1}^y + \Delta S_j^z S_{j+1}^z\right)$$

Jordan-Wigner transformation:

$$S_j^z = n_j - \frac{1}{2}, \quad S_j^- = (-1)^j c_j e^{-i\pi\sum_{l=1}^{j-1} n_l}, \quad S_j^- = (-1)^j c_j e^{-i\pi\sum_{l=1}^{j-1} n_l}$$

$$H_{XXZ} = J\sum_{j=1}^{N}\left[-\frac{1}{2}\left(c_{j+1}^+ c_j + c_j^+ c_{j+1}\right) + \Delta\left(n_j - \frac{1}{2}\right)\left(n_{j+1} - \frac{1}{2}\right)\right]$$

The Fermi surface is made of two points, $k_F=\pm\pi/2$, and linearizing the energy spectrum around them we get in the continuum:

$$c_j \to \psi(x) = e^{ik_F x}\psi_R(x) + e^{-ik_F x}\psi_L(x)$$

where $x=aj$, for lattice spacing $a$, and $\psi_R(x)$ and $\psi_L(x)$ correspond to right and left moving fermions, which can be bosonized according to:

$$\psi_{R,L}(x) \approx e^{-i\phi_{R,L}(x)}$$

*By taking the continuum limit and keeping only the marginal operators, we get the exactly solvable Luttinger Hamiltonian:*

$$H_{LL} = \frac{v}{8\pi}\int dx\left[(\partial_x\theta)^2 + (\partial_x\phi)^2\right]$$

where we defined:

$$\phi = \frac{\phi_L + \phi_R}{\sqrt{K}}, \quad \theta = \sqrt{K}(\phi_L - \phi_R)$$

$$g = \frac{(2J\Delta\sin^2 k_F)}{\pi v_F}, \quad v = \frac{v_F}{2}\sqrt{1+2g}, \quad K = \frac{1}{\sqrt{1+2g}}$$

renormalized velocity

Luttinger parameter

A comparison with exact Bethe Ansatz calculations leads to the expressions:

$$v(\Delta)=\frac{J\pi}{2}\frac{\sqrt{1-\Delta^2}}{\arccos\Delta},\quad K(\Delta)=\frac{\pi}{2(\pi-\arccos\Delta)}$$

The assumption of periodic boundary conditions implies the compactification of the boson fields:

$$\phi(x+L,t)=\phi(x,t)+2\pi m_\phi R_\phi,\quad m_\phi\in Z, R_\phi=\frac{1}{\sqrt{K}}$$
$$\theta(x+L,t)=\theta(x,t)+2\pi m_\theta R_\theta,\quad m_\theta\in Z, R_\theta=2\sqrt{K}$$

The compactified boson field has the following mode expansion:

$$\phi(z,\bar z)=q-ia_0\ln z-i\bar a_0\ln\bar z+\sum_{n\neq 0}\frac{ia_n}{n}z^{-n}+\sum_{n\neq 0}\frac{i\bar a_n}{n}\bar z^{-n}$$
$$z=e^{2\pi(\tau-ix)/L},\quad \bar z=e^{2\pi(\tau+ix)/L},\quad \tau=ivt$$
$$a_0=p+w,\quad \bar a_0=p-w$$
$$[q,p]=i,\quad [a_n,a_{n'}]=n\delta_{n,n'},\quad [a_n,\bar a_{n'}]=0,\quad [\bar a_n,\bar a_{n'}]=n\delta_{n,n'}$$

# 2-reduction technique on the plane: generalities

(G.Cristofano, G. Maiella, V.Marotta, MPLA **15**, 1679 (2000); G. Cristofano, G. Maiella, V. Marotta, G. Niccoli, NPB **641**, 547 (2002); G. Cristofano, V. Marotta, A. Naddeo, PLB **571**, 250 (2003); G. Cristofano, V. Marotta, A. Naddeo, NPB **679**, 621 (2004))

**Mother theory**: a CFT with $c$=1, described in terms of a boson field $\phi(z,\bar{z})$ compactified on a circle with general radius $R_\phi$. We can define two scalar fields, symmetric and antisymmetric under $Z_2$:

$$\tilde{X}(\omega,\overline{\omega}) = \frac{\phi(\omega,\overline{\omega}) + \phi\left(e^{i\pi}\omega, e^{-i\pi}\overline{\omega}\right)}{\sqrt{2}}, \quad \tilde{\Phi}(\omega,\overline{\omega}) = \frac{\phi(\omega,\overline{\omega}) - \phi\left(e^{i\pi}\omega, e^{-i\pi}\overline{\omega}\right)}{\sqrt{2}}$$

**Daughter theory:** we implement the map $z \to \omega^2$, getting an orbifold CFT with $c$=2. The new theory is described in terms of a $Z_2$-invariant scalar field $X$ and a twisted field $\phi$ which satisfies **twisted boundary conditions**:

$$X(z,\bar{z}) = \tilde{X}\left(z^{1/2}, \bar{z}^{1/2}\right), \quad \Phi(z,\bar{z}) = \tilde{\Phi}\left(z^{1/2}, \bar{z}^{1/2}\right)$$

The mode expansion of these two fields is:

$$X(z,\bar{z}) = q_0 - i\alpha_0 \ln z - i\bar{\alpha}_0 \ln \bar{z} + \sum_{n\in Z-\{0\}} \frac{i\alpha_n}{n} z^{-n} + \sum_{n\in Z-\{0\}} \frac{i\bar{\alpha}_n}{n} \bar{z}^{-n}$$

$$\Phi(z,\bar{z}) = -\sqrt{2}\pi w + \sum_{n\in Z} \frac{i\alpha_{n+1/2}}{n+1/2} z^{-(n+1/2)} + \sum_{n\in Z} \frac{i\bar{\alpha}_{n+1/2}}{n+1/2} \bar{z}^{-(n+1/2)}$$

$$q_0 = \sqrt{2}(q+\pi w), \quad \alpha_{n+l/2} = \frac{a_{2n+l}}{\sqrt{2}}, \quad \bar{\alpha}_{n+l/2} = \frac{\bar{a}_{2n+l}}{\sqrt{2}}, \quad n \in N; l = 0,1$$

where:

$$[\alpha_{n+l/2}, \alpha_{n'+l'/2}] = \left(n + \frac{l}{2}\right)\delta_{n,n'}\delta_{l,l'}, \quad [\bar{\alpha}_{n+l/2}, \bar{\alpha}_{n'+l'/2}] = \left(n + \frac{l}{2}\right)\delta_{n,n'}\delta_{l,l'}$$

$$[\alpha_{n+l/2}, \bar{\alpha}_{n'+l'/2}] = 0, \quad [q_0, \alpha_{n+l/2}] = i\delta_{0,n}\delta_{0,l}, \quad [q_0, \bar{\alpha}_{n+l/2}] = i\delta_{0,n}\delta_{0,l}$$

The field X is a compactified boson with compactification radius $R_X = R_\phi/\sqrt{2}$.

# 2-reduction technique on the plane: the 2-leg XXZ spin-1/2 ladder

We show that the Twisted Model (TM), generated by *2*-reduction technique, describes the continuum limit of the 2-leg *XXZ* spin-1/2 ladder with PBC and MBC. It is enough to show that the *XXZ* ladder with *M* odd and MBC is naturally mapped in the twisted sector of the TM. Such a system coincides with a system of 2 *XXZ* chains, each one with (M+1)/2 sites and size L=a(M+1)/2, which are closed with periodicity condition in one gluing site common to the 2 chains.

Upon bosonizing each chain, we obtain two boson fields $\phi_{\mathrm{Up}}$ and $\phi_{\mathrm{Dw}}$ compactified on the two circles (up and down) of the same length $L$ and with the same compactification radius $R_\phi=\sqrt{2}(\pi-\arccos\Delta)/\pi=1/\sqrt{K}$.

The topological defect implies the following b.c. for the fields at the gluing point:

$$\phi_{Up}(0,t)=\phi_{Dw}(0,t)$$

$$\phi(x,t)=\begin{cases}\phi_{Dw}(x,t), & 0\le x\le L\\ \phi_{Up}(x-L,t), & L\le x\le 2L\end{cases}$$

Folding field

The compactification space of the field $\phi$ is an eight of length $2L$ and the radius is $R_\phi$.

$$\phi(x+2L,t)=\begin{cases}\phi_{Dw}(x+L,t), & 0\le x\le L\\ \phi_{Up}(x,t), & L\le x\le 2L\end{cases}$$

$$=\begin{cases}\phi_{Dw}(x,t)+2\pi m_\phi R_\phi, & 0\le x\le L\\ \phi_{Up}(x-L,t)+2\pi m_\phi R_\phi, & L\le x\le 2L\end{cases}$$

$$\Rightarrow \quad \phi(x+2L,t)=\phi(x,t)+2\pi m_\phi R_\phi$$

2-reduction

Basic steps

$$\omega=e^{2\pi(\tau-ix)/2L},\quad \overline{\omega}=e^{2\pi(\tau+ix)/2L},\quad \tau=ivt$$

$$\tilde{X}(\omega,\overline{\omega}),\quad \tilde{\Phi}(\omega,\overline{\omega})$$

$$z=\omega^2=e^{2\pi(\tau-ix)/L},\quad \bar{z}=\overline{\omega}^2=e^{2\pi(\tau+ix)/L}$$

$$X(z,\bar{z}),\quad \Phi(z,\bar{z})$$

By means of 2-reduction technique we have transformed the boson field $\phi$ compactified on the eight of length $2L$ with radius $R_\phi$ into the two independent boson fields $X$ and $\Phi$ compactified on a circle of length $L$, where $R_X = R_\phi/\sqrt{2}$. Thus the 2-leg $XXZ$ spin-1/2 ladder with general anisotropy $\Delta$ and MBC is described by the twisted sector of the TM.

# The special XXX case

Let us put $\Delta=1$, which corresponds to the isotropic XXX ladder.

**Mother theory**: a CFT with $c=1$, described in terms of a boson field compactified on a circle with radius $R_\phi=\sqrt{2}$ (K=1/2).

**Daughter theory:** as a result of the 2-reduction procedure, we get an orbifold CFT with $c=2$. The new theory is described in terms of two boson fields, $X$ and $\phi$, which describe the spin chains of the two legs. It decomposes into a tensor product of two CFTs, a twisted invariant one with $c=3/2$, realized by the boson $X$ and a Ramond Majorana fermion, while the second has $c=1/2$ and is realized in terms of a Neveu-Schwartz Majorana fermion: $su(2)_2 \otimes I$.

# Renormalization group analysis for the XXX case

Let us consider the weakly interacting 2-leg ladder in the isotropic case, i. e. XXX, and study the different RG trajectories flowing from the UV fixed point described by our TM with central charge c=2.

The daughter fields $\mathrm{X}$ and $\Phi$ and their duals admit the following representation in terms of left and right chiral components:

$$\mathrm{X}(z,\bar{z}) = -iw_{\mathrm{X}} \ln\frac{z}{\bar{z}} + X(z) + \bar{X}(\bar{z}), \quad \mathrm{Y}(z,\bar{z}) = X(z) - \bar{X}(\bar{z})$$

$$\Phi(z,\bar{z}) = -2\pi w_{\mathrm{X}} + \varphi(z) + \bar{\varphi}(\bar{z}), \quad \Theta(z,\bar{z}) = \varphi(z) - \bar{\varphi}(\bar{z})$$

A representation in terms of four Majorana fermion fields gives (for holomorphic and anti-holomorphic components):

$$\psi_1(z) = \sin X(z), \quad \psi_2(z) = \cos X(z), \quad \psi_3(z) = \frac{\sin\varphi(z)}{\sqrt{z}}, \quad \psi_0(z) = \frac{\cos\varphi(z)}{\sqrt{z}}$$

$$\bar{\psi}_1(\bar{z}) = -\sin\bar{X}(\bar{z}), \quad \bar{\psi}_2(\bar{z}) = \cos\bar{X}(\bar{z}), \quad \bar{\psi}_3(\bar{z}) = -\frac{\sin\bar{\varphi}(\bar{z})}{\sqrt{\bar{z}}}, \quad \bar{\psi}_0(\bar{z}) = \frac{\cos\bar{\varphi}(\bar{z})}{\sqrt{\bar{z}}}$$

The Lagrangian describing the UV fixed point for the *XXX* ladder is:

$$L_0 = \frac{1}{8\pi}\left(\partial_\mu \mathrm{X}\partial^\mu \mathrm{X} + \partial_\mu \Phi \partial^\mu \Phi\right)$$

while the perturbing terms V depend on the particular system under study:

$$L = L_0 - V$$

## 1. Railroad and 4-Spin perturbations: massive flow

$$V_{Railroad} = -m_R\left(\cos \mathrm{X}(z,\bar z) - \cos\Phi(z,\bar z) + 2\cos\Theta(z,\bar z)\right), \quad m_R \propto J_\perp^R$$

$$V_{Railroad} = -im_R\sum_{i=1}^{3}\psi_i(z)\overline{\psi}_i(\bar z) + 3im_R\psi_0(z)\overline{\psi}_0(\bar z)$$

Majorana fermion representation

This is a relevant perturbation to the UV fixed point; the $\psi_i$ (i=1,2,3) fields form an Ising triplet with the same mass $m_R$ ($su(2)_2$ sector) and $\psi_0$ is an Ising singlet (*I* sector) with a larger mass $-3m_R$ (D. G. Shelton, A. A. Nersesyan, A. M. Tsvelick, PRB 53 (1996) 8521).

$$V_{4-Spin} = m_\varepsilon\left(\cos \mathrm{X}(z,\bar{z}) + \cos \Phi(z,\bar{z})\right), \quad m_\varepsilon \propto J_\varepsilon$$

$$V_{4-Spin} = i m_\varepsilon \sum_{i=0}^{3} \psi_i(z)\overline{\psi}_i(\bar{z})$$

A. A. Nersesyan, A. M. Tsvelick, PRL 78 (1997) 3939

Such a relevant mass term gives rise to the same mass contribution $m_\varepsilon$ for all the Ising fields $\psi_i$ (i=0,1,2,3). That allows the triplet or singlet mass to vanish also for finite values of the coupling constants, i.e. far from the UV conformal fixed point $c$=2

We get two possible RG flows: a flow to an IR fixed point with $c$=3/2 as a result of the Ising $I$ decoupling ($m_t$=0, $m_s \neq 0$); a flow to a different IR fixed point with $c$=1/2 as a result of the $su(2)_2$ decoupling ($m_t \neq 0$, $m_s$=0).

## 2. Zigzag and 4-Dimer perturbations: massive flow

Let us write the non interacting Lagrangian in terms of Majorana fermions only and observe that the simple continuum limit, without adding the zigzag perturbation, produces a marginal interaction $V_U$ (D. Allen, D. Senechal, PRB 55 (1997) 299):

$$L_0 = \frac{1}{2\pi}\sum_{i=0}^{3} v_i\left(\psi_i \bar{\partial} \psi_i + \overline{\psi}_i \partial \overline{\psi}_i\right), \quad v_0 = \ldots = v_3 = v \approx J_0 a$$

$$V_U = -\lambda_U\left(O_1 + O_2\right), \quad \lambda_U \approx U/|t| \geq 0$$

$$O_1 = \psi_1(z)\overline{\psi}_1(\bar{z})\psi_2(z)\overline{\psi}_2(\bar{z}) + \psi_1(z)\overline{\psi}_1(\bar{z})\psi_3(z)\overline{\psi}_3(\bar{z})$$
$$+\psi_2(z)\overline{\psi}_2(\bar{z})\psi_3(z)\overline{\psi}_3(\bar{z})$$

$$O_2 = \psi_0(z)\overline{\psi}_0(\bar{z})\left(\psi_1(z)\overline{\psi}_1(\bar{z}) + \psi_2(z)\overline{\psi}_2(\bar{z}) + \psi_3(z)\overline{\psi}_3(\bar{z})\right)$$

The zigzag interaction is, in the fermionic language:

$$V_{ZigZag} = \lambda_Z\left[\left(O_1 - O_2\right) + \sum_{i=0}^{3}\left(T^{(i)}(z) + \overline{T}^{(i)}(\bar{z})\right)\right], \quad \lambda_Z \approx J_\perp^Z/|t| \geq 0$$

Such an interaction is marginally relevant and contains a non scalar term, whose effect is simply that of renormalizing the fields and the velocities. After renormalization, the whole effect of the two terms $V_U$ and $V_{ZigZag}$ can be expressed as a marginal interaction:

$$VV_{ZigZag} = \lambda_+^0\left(O_1 + O_2\right) + \lambda_-^0\left(O_1 - O_2\right), \quad \lambda_\pm^0 = \frac{1}{2\left(1+\pi\lambda_Z\right)}\left\{\mp\frac{\lambda_Z + \lambda_U}{1 - 3\pi\lambda_Z} + \frac{\lambda_Z - \lambda_U}{1+\pi\lambda_Z}\right\}$$

The following RG equations hold:

$$\frac{d\lambda_\pm^0}{d\ln L} = 8\pi\left(\lambda_\pm^0\right)^2$$

Under the flow $\lambda^0_+$ renormalizes to zero while $\lambda^0_-$ increases and that results in a dynamical length scale $\xi \approx \exp(1/\lambda^0_-)$. The $Z_2$ symmetry of $L_0$ and $VV_{ZigZag}$ spontaneously breaks, a mass scale appears dynamically and provides a non vanishing mass for the four fermions:

$$m_i \approx v_i \xi^{-1}, i = 1,2,3; \qquad m_0 \approx v_0 \xi^{-1}$$

$$m_1 = m_2 = m_3 = m > 0; \qquad m_0 < -m$$

It is not possible to extract trajectories in the RG flow characterized by a vanishing mass in the $c$=1/2 or $c$=3/2 sector respectively. So, we need to introduce the 4-dimer perturbation:

$$V_{4-Dimer} = \lambda_\kappa \left(O_1 + O_2\right) = \lambda_\kappa \sum_{i \neq j = 0}^{3} \psi_j(z) \overline{\psi}_j(\bar{z}) \psi_i(z) \overline{\psi}_i(\bar{z}), \quad \lambda_\kappa \approx J_\kappa$$

The whole perturbation is:

$$V_{Tot} \equiv VV_{ZigZag} + V_{4-Dimer} = \lambda_+ \left(O_1 + O_2\right) + \lambda_- \left(O_1 - O_2\right)$$

$$\lambda_+ = \lambda_+^0 + \lambda_\kappa, \quad \lambda_- = \lambda_-^0$$

**It is now possible to define a path in the RG flow characterized by a vanishing singlet mass, i.e. $m_t \neq 0$, $m_s = 0$, as shown by rewriting $V_{Tot}$ as:**

$$V_{Tot} = \Lambda_1 O_1 + \Lambda_2 O_2$$

$$\Lambda_1 = \lambda_+ + \lambda_-, \quad \Lambda_2 = \lambda_+ - \lambda_-$$

**A vanishing singlet mass $m_s$ can be obtained by requiring the marginality of the operator $O_2$ along the RG flow. This selects the condition $\lambda_+ = \lambda_-$ which makes $\Lambda_2$ to vanish. The triplet mass $m_t$ is dynamically generated and reads as ($F$ is the momentum cutoff and $v_t$ is the spin triplet velocity):**

$$m_t \approx \pm v_t F \exp\left(-1/8\pi\lambda_+\right)$$

The 4-Dimer interaction allows us to describe a RG trajectory flowing from the $c$=2 UV fixed point (TM) toward the Ising $I$, the $c$=1/2 IR fixed point, as a result of the dynamical generation of the mass $m_t$ and the consequent decoupling of $su(2)_2$.