Reconciling confinement of light quarks with chiral-symmetry breaking

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Linking SCSB with confinement: A general strategy.

The two most fundamental nonperturbative phenomena in QCD are confinement and SCSB. Are they interrelated?

Confinement is characterized by the gluon condensate $\langle (gF^a_{\mu\nu})^2 \rangle$ and the vacuum correlation length λ (Pisa lattice group):

$$\sigma \propto \lambda^2 \cdot \langle (gF^a_{\mu\nu})^2 \rangle.$$

SCSB is characterized by the chiral condensate $\langle \bar{\psi}\psi \rangle$ and the constituent quark mass m.

In the heavy-quark limit (cf. SVZ sum rules): $\langle \bar{\psi}\psi \rangle_{\rm heavy} \propto -\frac{\langle (gF^a_{\mu\nu})^2 \rangle}{m}$.

In the chiral limit, NJL-type models with confinement yield

$$\langle \bar{\psi}\psi \rangle \propto -\lambda \cdot \langle (gF^a_{\mu\nu})^2 \rangle.$$

Linking SCSB with confinement: A general strategy.

Heavy quarks ⇒ an "area-squared" law for small Wilson loops.

Light quarks ⇒ an area law with a scale-dependent string tension.

The starting idea: To get $\langle\bar{\psi}\psi\rangle = -\frac{\partial}{\partial m}\langle\Gamma[A_\mu^a]\rangle$ away from the heavy-quark limit, by imposing some form of the Wilson loop interpolating between these two laws.

Linking SCSB with confinement: A general strategy.

$$\langle\Gamma[A_\mu^a]\rangle = -2N_{\rm f}\int_0^\infty \frac{ds}{s}\,{\rm e}^{-m^2 s}\int_P \mathcal{D}z_\mu \int_A \mathcal{D}\psi_\mu\,{\rm e}^{-\int_0^s d\tau\left(\frac14 \dot z_\mu^2+\frac12\psi_\mu\dot\psi_\mu\right)}\times$$

$$\times\left\{\left\langle {\rm tr}\,\mathcal{P}\,\exp\left[ig\int_0^s d\tau\, T^a\left(A_\mu^a \dot z_\mu - \psi_\mu\psi_\nu F_{\mu\nu}^a\right)\right]\right\rangle - N_c\right\}.$$

Only when $\int_P \mathcal{D}z_\mu \int_A \mathcal{D}\psi_\mu[\cdots] \to \frac{\rm const}{\sqrt{s}}$ at $s\to\infty$, is the quark condensate finite in the small-mass limit:

$$\langle\bar\psi\psi\rangle \propto \frac{\partial}{\partial m}\int_0^\infty \frac{ds}{s}{\rm e}^{-m^2 s}\cdot\frac{\rm const}{\sqrt{s}} \to -2\sqrt{\pi}\cdot{\rm const}$$

(T. Banks and A. Casher, '80).

What degree of zigzagness of quark trajectories is needed for the quark condensation?

Linking SCSB with confinement: A general strategy.

The second idea: To parametrize via $z_\mu(\tau)$ the minimal area S, which enters the area law:

$$\langle W[z_\mu]\rangle = \left\langle \mathrm{tr}\,\mathcal{P}\,\exp\left(ig\int_0^s d\tau\, T^a A_\mu^a \dot{z}_\mu\right)\right\rangle \to N_c\cdot \mathrm{e}^{-\sigma(s)\cdot S}.$$

Find an ansatz for S enabling an analytic calculation of $\langle\Gamma[A_\mu^a]\rangle$, and impose the $\int_P \mathcal{D}z_\mu \int_A \mathcal{D}\psi_\mu[\cdots] \to 1/\sqrt{s}$ asymptotic behavior $\Rightarrow \sigma(s)$.

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The area of a cone-shaped surface in 3d is generalized to 4d as

$$S_{\mathrm{3d}} = \frac{1}{2}\int_0^s d\tau |\mathbf{z}\times\dot{\mathbf{z}}| \to$$

$$\to S_{\mathrm{4d}} = \frac{1}{2\sqrt{2}}\int_0^s d\tau |\varepsilon_{\mu\nu\lambda\rho} z_\lambda \dot{z}_\rho| \geq \frac{1}{4\sqrt{3}}|\Sigma_{\mu\nu}| := S[z_\mu],$$

where $\Sigma_{\mu\nu}(s) = \varepsilon_{\mu\nu\lambda\rho}\int_0^s d\tau z_\lambda \dot{z}_\rho$.

$\langle\bar{\psi}\psi\rangle$ from the area- and the area-squared laws

The mean size of a heavy-quark trajectory is $\lesssim \lambda \Rightarrow F^a_{\mu\nu} \simeq \mathrm{const} \Rightarrow$ the nonperturbative part of a small Wilson loop is

$$\langle W[z_\mu]\rangle = N_c \cdot \mathrm{e}^{-\frac{\langle (gF^a_{\mu\nu})^2\rangle}{48N_c} S^2}.$$

Mimicing this area-squared law by a pre-exponential factor at the area law:

$$\langle W[z_\mu]\rangle = N_c \cdot \mathrm{e}^{-\sigma S} \to \frac{N_c}{2^{\alpha-1}\Gamma(\alpha)} \cdot (\sigma S)^\alpha \cdot K_\alpha(\sigma S).$$

Seeking α to provide the best approximation for

$$\mathrm{e}^{-\frac{\langle (gF^a_{\mu\nu})^2\rangle}{48N_c} S^2} \simeq \frac{1}{2^{\alpha-1}\Gamma(\alpha)} \cdot (\sigma S)^\alpha \cdot K_\alpha(\sigma S) \quad \mathrm{at} \quad \sigma S \lesssim 1$$

$$\Rightarrow \qquad \alpha = 1 + \frac{12N_c\sigma^2}{\langle (gF^a_{\mu\nu})^2\rangle} \simeq 1.90 \quad \mathrm{for} \quad \sigma = (440\,\mathrm{MeV})^2.$$

$\langle\bar{\psi}\psi\rangle$ from the area- and the area-squared laws

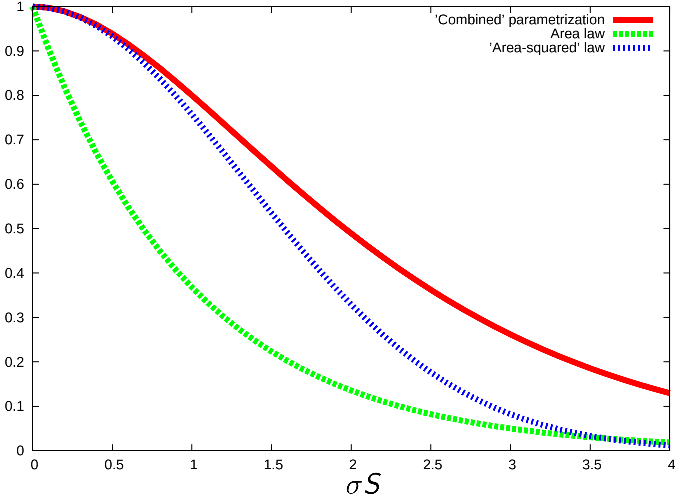

Figure: $\frac{1}{2^{\alpha-1}\Gamma(\alpha)}\cdot(\sigma S)^\alpha\cdot K_\alpha(\sigma S)$ at $\alpha=1.9$, $\mathrm{e}^{-\sigma S}$, $\mathrm{e}^{-\frac{\langle(gF^a_{\mu\nu})^2\rangle}{48N_c}S^2}$. The value $\sigma S=4$ corresponds to the radius of 0.51 fm.

$\langle\bar{\psi}\psi\rangle$ from the area- and the area-squared laws

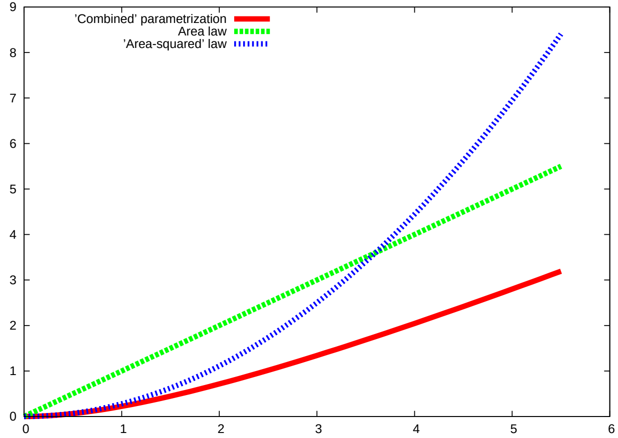

Figure: $-\ln\left[\frac{1}{2^{\alpha-1}\Gamma(\alpha)}\cdot(\sigma S)^{\alpha}\cdot K_{\alpha}(\sigma S)\right]$ at $\alpha = 1.9$, σS, $\frac{\langle (gF^a_{\mu\nu})^2\rangle}{48N_c}S^2$. The value $\sigma S = 5.5$ corresponds to the radius of 0.6 fm. The gap between $-\ln[\cdots]$ and σS, being $\sim \ln(\sigma S)$ at large distances, is irrelevant for the static potential.

$\langle\bar{\psi}\psi\rangle$ from the area- and the area-squared laws

Using this "combined" parametrization of $\langle W[z_\mu]\rangle$, with $\sigma \to \tilde{\sigma}(s)$, to calculate the quark condensate:

$$\langle\bar{\psi}\psi\rangle = -\frac{3N_{\rm f}}{4\pi^2}\cdot m\int_0^\infty ds\, {\rm e}^{-m^2 s}\cdot\frac{f[A(s),\alpha]}{2s^2A(s)},$$

where $A(s) \equiv 1/(2\tilde{\sigma}^2 s^2)$, and the function

$$f[A,\alpha] = 4\times$$

$$\times\frac{6A(1+A)^\alpha(2+A)+6[(1+A)^\alpha-1]-(2+\alpha)A[6+(1+\alpha)A(3+\alpha A)]}{3(\alpha-1)(1+A)^{\alpha+2}}$$

is continuous also at $\alpha = 1$.

In the small-mass limit, the quark condensate is only nontrivial if

$$\frac{f[A(s),\alpha]}{2s^2A(s)} \simeq \frac{\sigma_0^{3/2}}{\sqrt{s}} \quad \text{up to} \quad s_{\rm max} \gtrsim 1/m^2,$$

where $\sigma_0 = {\rm const}$.

$\langle\bar{\psi}\psi\rangle$ from the area- and the area-squared laws

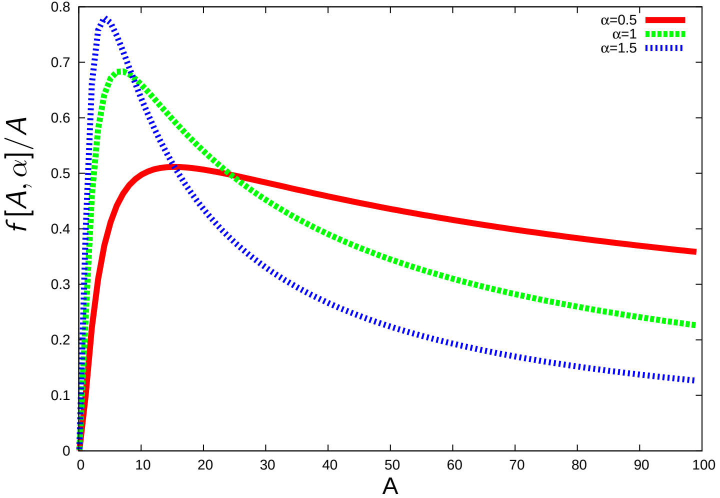

Figure: Solving $\frac{f[A,\alpha]}{A} = x$ for $A \in (0, 100)$ and $\alpha = 0.5$, $\alpha = 1$, $\alpha = 1.5$, where $x \equiv 2(\sigma_0 s)^{3/2}$.

$\langle\bar{\psi}\psi\rangle$ from the area- and the area-squared laws

The peak of $f[A,\alpha]/A$ sharpens with the increase of α, reaching the value of 1.18 at $\alpha \gtrsim 1$ $\Rightarrow$ the lower bound for the constituent quark mass

$$m_{\rm min} = 2\sqrt{\pi}\left(\frac{|\langle\bar{\psi}\psi\rangle|}{3N_{\rm f}\cdot 1.18}\right)^{1/3}\Bigg|_{N_{\rm f}=2} = 460\,{\rm MeV}.$$

The root $A(x,\alpha)$ of the equation $\frac{f[A,\alpha]}{A} = x$, corresponding to $\tilde{\sigma}$ decreasing with x (i.e. with s), behaves as

$$A(x,\alpha) \sim x^{\varepsilon(\alpha)}, \quad \text{where} \quad \varepsilon(\alpha) \ll 1 \quad \text{at} \quad \alpha \gtrsim 1 \quad \Rightarrow \quad \tilde{\sigma} \propto \frac{1}{s}$$

$\Rightarrow$ $L \sim R_{xy}^4$, i.e. the Hausdorff dimension of a light-quark trajectory is equal to 4 $\Rightarrow$ Trajectories of light quarks are similar to branched polymers.

An application to the YM thermodynamics at $T < T_c$

A two-component YM vacuum:

Soft stochastic fields with $|p| \lesssim 1/\lambda \Rightarrow$ the minimal-area law;

Hard fluctuations with $|p| \gtrsim 1/\lambda$ yield string excitations.

$\Rightarrow$ A gluon-chain model (N. Isgur & J. Paton, '85; J. Greensite & C. Thorn, '02; G. 't Hooft, '03; E. Shuryak & J.-F. Liao, '06).

At $T \to T_c$, the gluon-chain model can describe 1-st or 2-nd order deconfinement phase transitions.

An application to the YM thermodynamics at $T < T_c$

Various links of the gluon chain are color-independent $\Rightarrow$ large $S = N_c^{L/a}$ necessary for the phase transition.

To form the chain, its end-point jumps from one gluon to another, performing a random walk from Q to $\bar{Q}$.

A random walker is attached to Q along the chain:

$$\mathcal{Z}(R,T) = \sum_n \int_0^\infty ds\, P_n(s,R)\, \mathrm{e}^{-\frac{\sigma L}{T} + \frac{L}{a}\cdot \ln N_c},$$

where

$$P_n(s,R) = \frac{\mathrm{e}^{-\frac{R^2+(\beta n)^2}{4s}}}{(4\pi s)^2}.$$

For a Brownian random walk,

$$L = \frac{s}{a}.$$

An application to the YM thermodynamics at $T < T_c$

The effective string tension and the critical temperature:

$$\sigma(T) = \sigma - \frac{T}{R}\ln\frac{\mathcal{Z}(R,T)}{\mathcal{Z}(R,T_0)}\bigg|_{R\to\infty} =$$

$$= \sigma + \frac{T}{\sqrt{a}}\left[\sqrt{\frac{\sigma}{T} - \frac{\ln N_c}{a}} - \sqrt{\frac{\sigma}{T_0} - \frac{\ln N_c}{a}}\right]$$

$$\Rightarrow \quad T_c\Big|_{N_c>1} = \frac{\sigma a}{\ln N_c}.$$

The critical behavior

$$\sigma(T) \sim (T_c - T)^{1/2} \quad \text{at} \quad T \to T_c, \quad \text{i.e.} \quad \nu = 1/2$$

implies 2-nd order mean-field phase transition $\Rightarrow$ only $N_c = 2$ applies $\Rightarrow$ for $a = 0.21\,\mathrm{fm}$, the lattice value $T_c = 304\,\mathrm{MeV}$ is reproduced (D.A., S. Domdey, H.-J. Pirner, '07).

The temperature, below which valence gluons cannot be considered static: $T_0 = T_c/(\ln N_c + 1)$.

An application to the YM thermodynamics at $T < T_c$

While a Brownian random walk has Hausdorff dimension 2, branched polymers have Hausdorff dimension 4 $\Rightarrow$

$$L = \frac{s^2}{a^3} \quad \Rightarrow \quad V_T(R) = \gamma(T) \cdot R^{4/3},$$

with the weak 1-st order phase transition as in SU(3) YM:

$$\gamma(T) \sim (T_c - T)^{1/3} \quad \text{at} \quad T \to T_c, \;\; \text{i.e.} \;\; \nu = 1/3.$$

For $N_c = 1$, both phase transitions become 2-nd order, with the 2D-Ising universality class:

$$\sigma(T) \sim \gamma(T) \sim (T_c - T) \quad \text{at} \quad T \to T_c, \;\; \text{i.e.} \;\; \nu = 1.$$

This limiting case is equivalent to the models, which are ignorant of the Svetitsky–Yaffe conjecture.

Further developments

Accounting for excitations of the $q\bar{q}$ pairs in the condensate:

Assuming that the radial-excitation energies are given by the Regge formula

$$E_n = \sqrt{\pi\sigma(4n+3)}$$

$\Rightarrow$ a factor determining an increase of the area of the string world sheet.

Using this factor in the quark effective action $\Rightarrow$

corrections to the constituent mass yield a primary contribution to E_n:

$$\delta m_n \to \sqrt{\pi\sigma n} \quad \text{for} \quad n \gg 1.$$

The contribution stemming from the elongation of the string is secondary:

$$L_n \sim n^{1/4}.$$

The $(n=1)$ correction to the constituent quark mass is

$$\delta m_1 \simeq 26\,\mathrm{MeV}.$$

Further developments

The Hausdorff dimension of quark trajectories decreases down to $4/3 \Rightarrow$

the trajectories are still fractal, but less than Brownian random walks.

Conclusions

A Wilson loop interpolating between the area law at large distances and the "area-squared" law at small distances yields a linear decrease of the effective string tension of a light quark with the Schwinger proper time $\Rightarrow$

- The lower bound of 460 MeV for the constituent quark mass;

- The Hausdorff dimension of typical light-quark trajectories is equal to 4 (cf. branched polymers).

A gluon-chain model based on Brownian random walks yields a mean-field deconfinement phase transition ($\nu = 1/2$).

Changing to branched polymers $\Rightarrow$ weak 1-st order ($\nu = 1/3$), like in SU(3) Yang-Mills.

Conclusions

Excitations of the $q\bar{q}$ bound states in the chiral condensate enlarge the constituent quark mass: $\delta m_1 \simeq 26\,\mathrm{MeV}$, while $\delta m_n = \sqrt{\pi\sigma n}$ for $n \gg 1 \Rightarrow$ the Hausdorff dimension decreases down to $4/3$.

Outlook: to describe the lattice result (F. Karsch and M. Lütgemeier, '98): $T_\chi = T_c$ for quarks in the fundamental representation, while $T_\chi \simeq 8\,T_c$ for quarks in the adjoint representation.