Fluctuations in quenched spin-glasses and coarsening systems: are they similar ?

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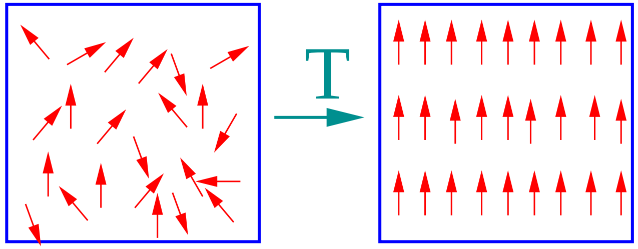
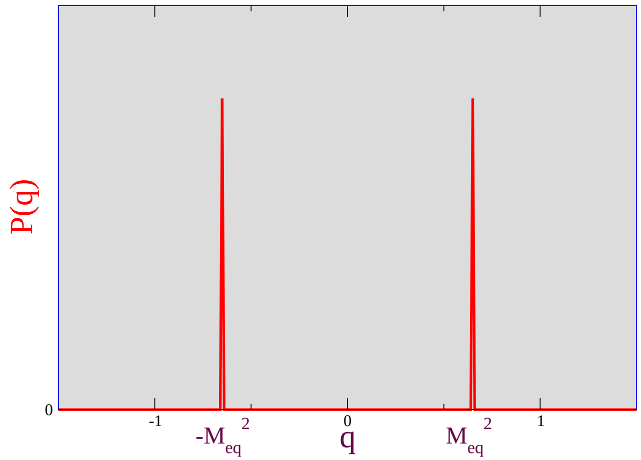
- Conclusions

Equilibrium - Statics

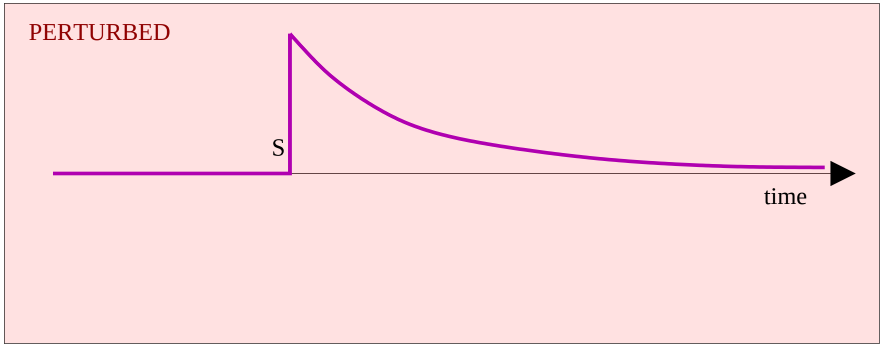


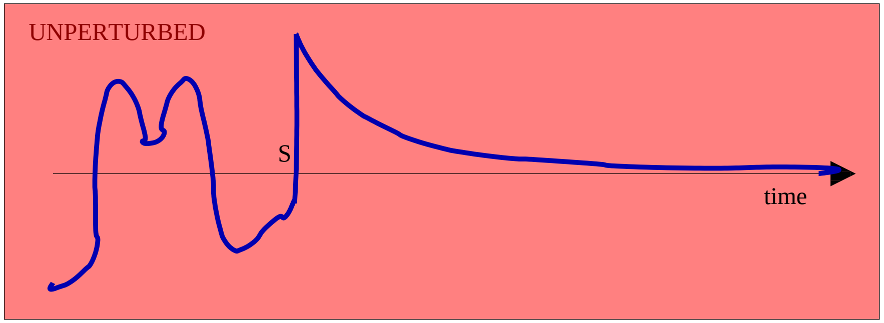
$$q^{\alpha\beta} = (1/N)\sum_{i=1}^{N} \sigma_i^{\alpha}\sigma_i^{\beta} \qquad \textit{Overlap}$$



Quasi-Equilibrium Dynamics

$$R_i(t,s) = \frac{\partial \langle \sigma_i(t) \rangle}{\partial h_i(s)} \qquad\qquad h \to 0$$





Quasi-Equilibrium Dynamics: FDT

- $$C_i(t,s) = \langle \sigma_i(t)\sigma_i(s)\rangle - \langle \sigma_i(t)\rangle\langle \sigma_i(s)\rangle$$

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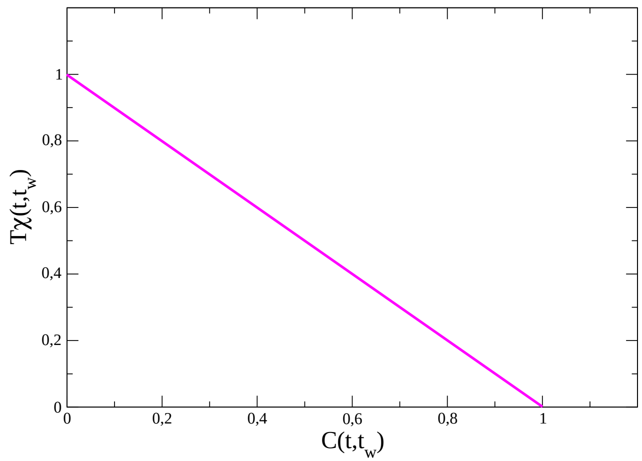
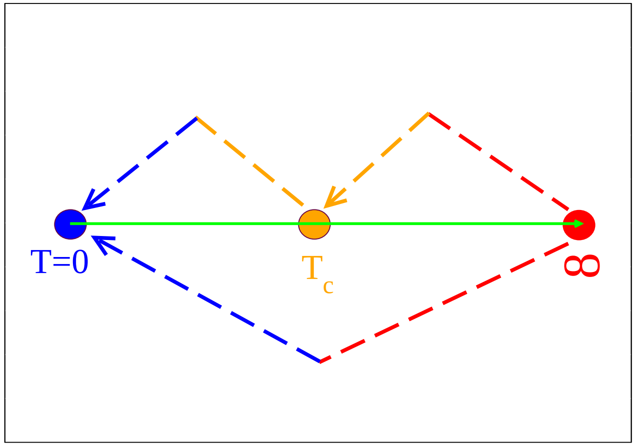
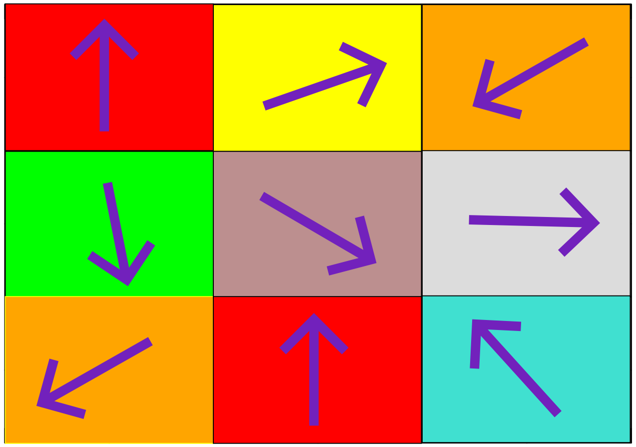
- $$\chi_i = f(C_i)\,; \qquad f \;\; is \;\; linear\,; \qquad \chi_i(t,s) = \int_s^t R_i(t,z)dz$$



Far from Equilibrium: Quenches

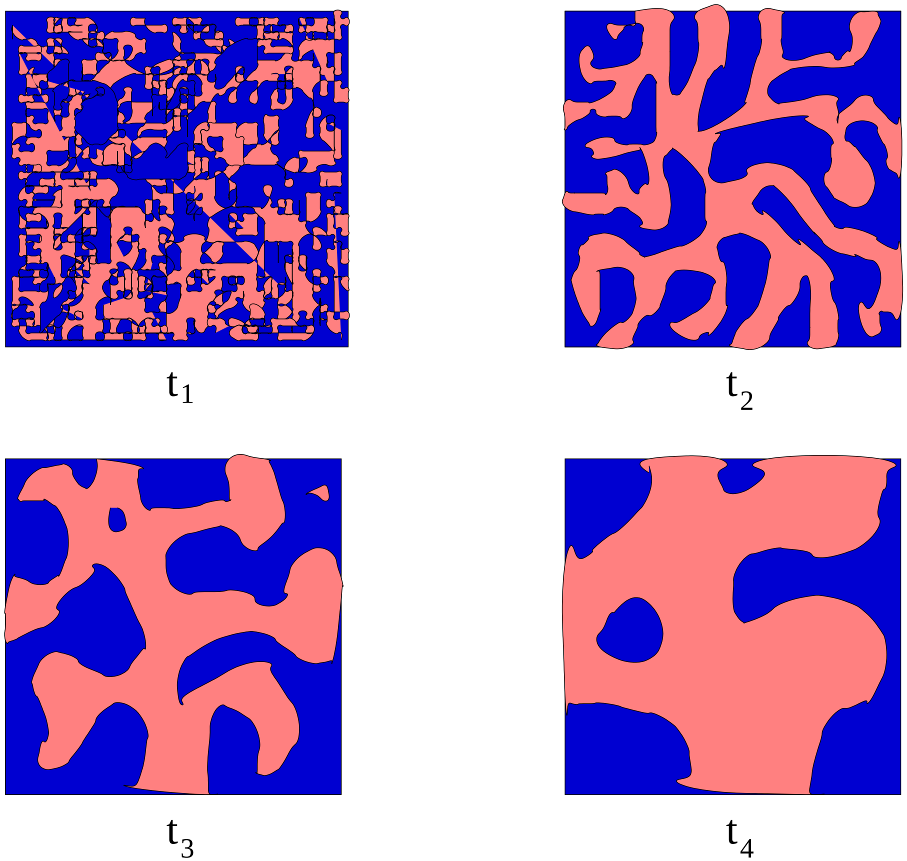




Far from Equilibrium-Growth of order

Size of *domains* $L(t)$ is growing in time.

$$L(t) \simeq t^{1/z} \qquad usually$$



Far from Equilibrium - Scaling

$$C_i(t,s) = L(s)^{bz} f_C[L(t)/L(s)]$$

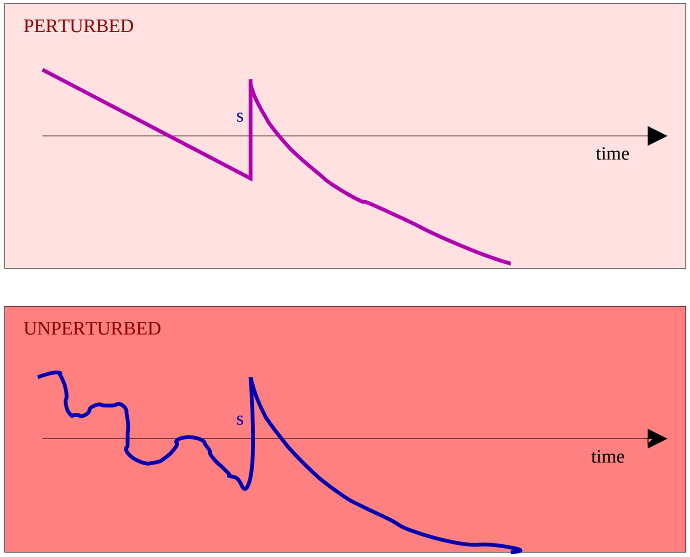
$$\chi_i(t,s) = L(s)^{az} f_\chi[L(t)/L(s)]$$

Analogous to (static)equilibrium scaling, i.e.
$g(r,T) = \xi(T)^{-(d-2+\eta)} f_g[r/\xi(T)]$

Using $L(t) \simeq t^{1/z} \Rightarrow$

$$C_i(t,s) = s^b \tilde{f}_C[t/s]\,; \qquad \chi_i(t,s) = s^a \tilde{f}_\chi[t/s]$$

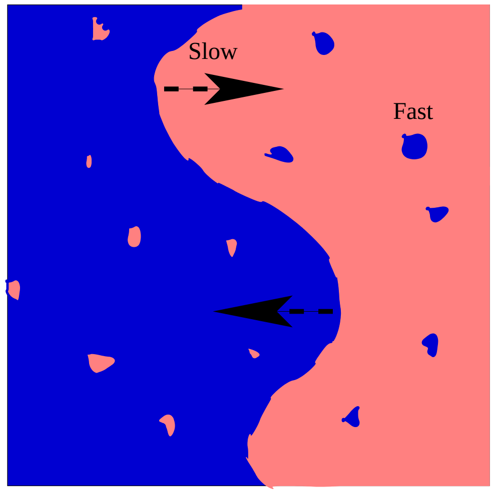
Far-from-Equilibrium: FDT

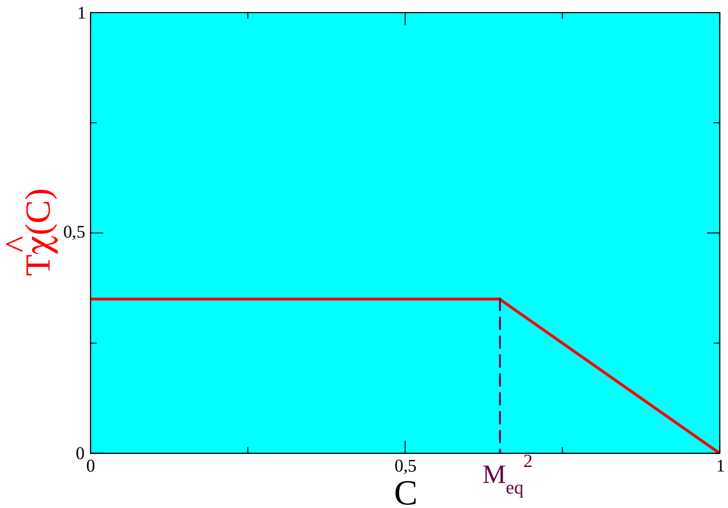


$$\chi_i = g(C_i, D_i)\,; \qquad D_i \ is\ another\ correlation\ function^a$$

[a] L.F. Cugliandolo, J. Kurchan and G. Parisi, J.Phys.I France 4, 1641 (1994); E.Lippiello, F.Corberi, M.Zannetti, Phys. Rev. E 71, 036104 (2005); M.Baiesi, C.Maes, and B.Wynants, Phys. Rev. Lett. 103, 010602 (2009).

Far-from-Equilibrium-2 components





Classification of systems (T_{eff})

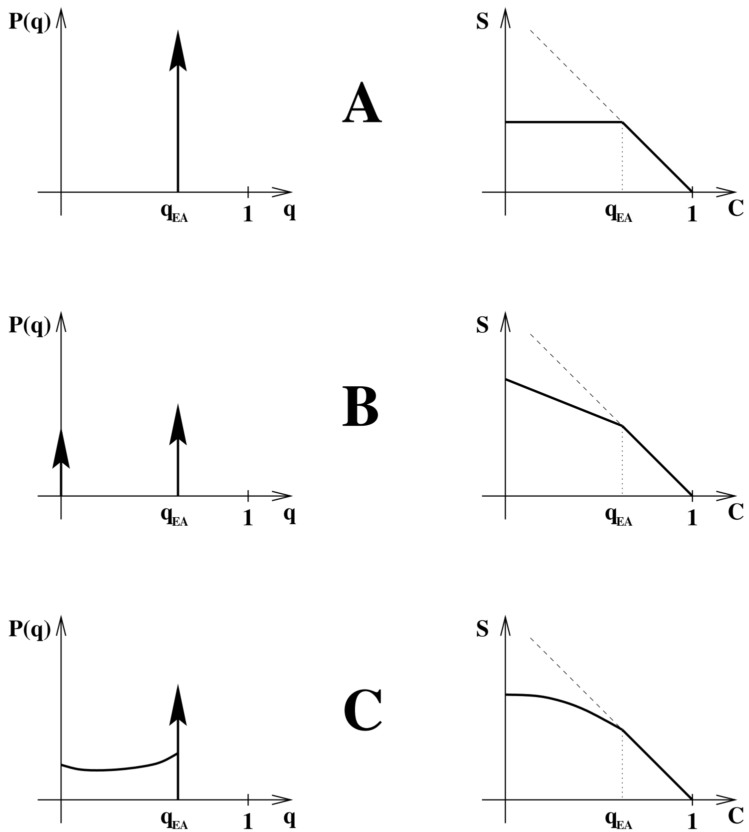


Figure 10: A possible model classification based on the function $S(C)$. The big arrows represent delta functions.

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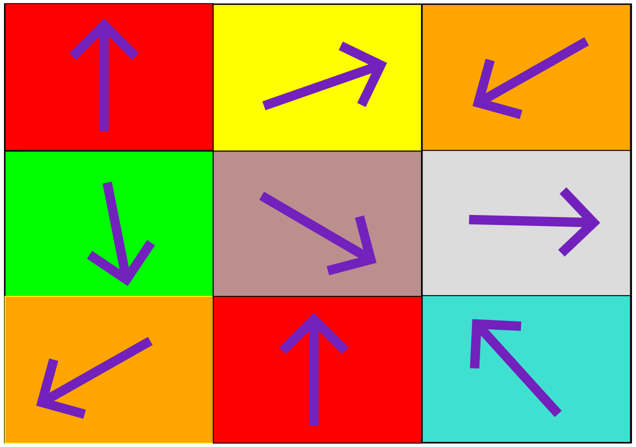
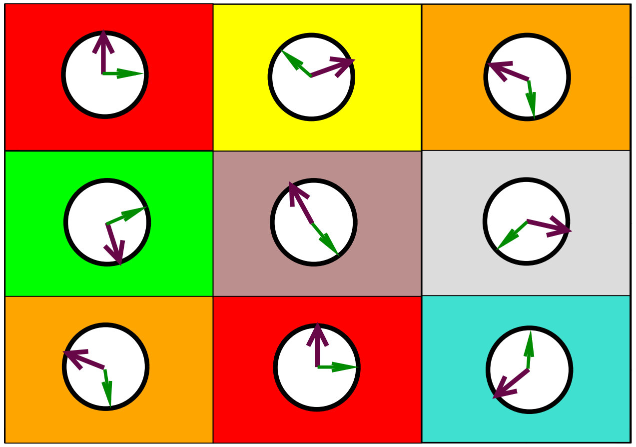
Where are finite dimensional SG and real glasses ?

t-reparametrization invariance-TRI



$O(N)$ symmetry



$t \to h(t)\,; \quad C \to C'$

TRI should hold true for glassy systems with $T_{eff} < \infty$ but not for coarsening with $T_{eff} = \infty$.

Fluctuating parts

$$\widehat{C}_i(t,s) = [\sigma_i(t) - \langle\sigma_i(t)\rangle][\sigma_i(s) - \langle\sigma_i(s)\rangle]$$

or $\widehat{C}_i(t,s) = \sum_{box\,around\,i} \widehat{C}_i(t,s)$

Define properly also $\widehat{\chi}_i(t,s)$.

$$V_{ij}^{CC}(t,s) = \langle \widehat{C}_i(t,s)\widehat{C}_j(t,s)\rangle - \langle \widehat{C}_i(t,s)\rangle\langle \widehat{C}_j(t,s)\rangle$$

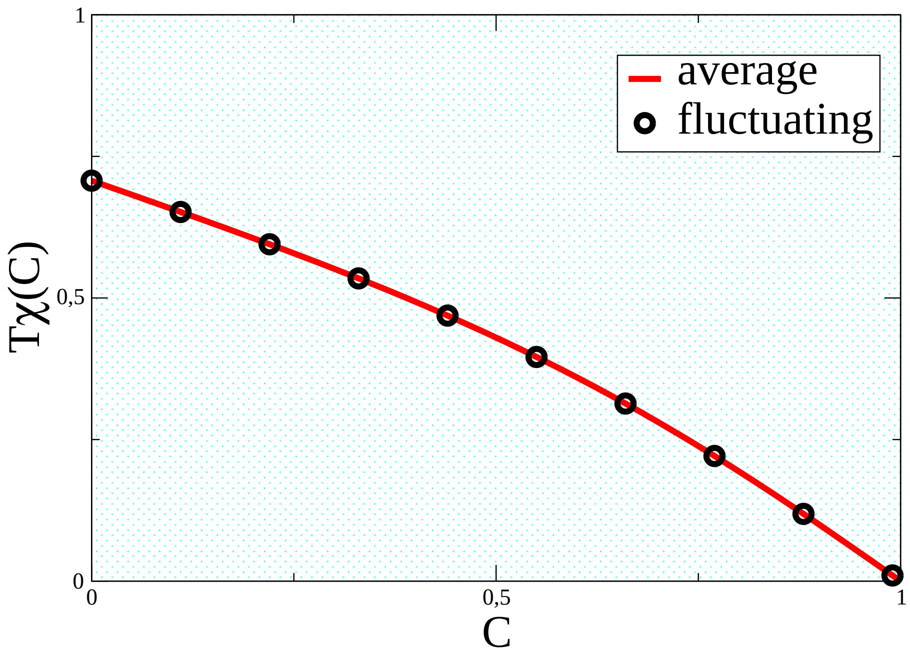
$$V_{k=0}^{CC}(t,s) = \sum_{ij} V_{ij}^{CC}(t,s)$$

and similarly for $V^{\chi\chi}$ and $V^{C\chi}$.
Generic variance is denoted V^{XY} onwards.

Transformations of $\widehat{C}_i$,$\widehat{\chi}_i$ under TRI

In a glassy system with $T_{eff} < \infty$, $\widehat{C}_i$ and $\widehat{\chi}_i$ transform *in the same way* when $t \to h(t)$.

Physical meaning: all sort of fluctuations of $\widehat{\chi}_i$ are accounted for by the fluctuations of the local clock $\widehat{C}_i$.



Transformations of V^{XY} under TRI

If TRI holds also all the moments of $\widehat{C}$ and $\widehat{\chi}$ must transform *in the same way*.

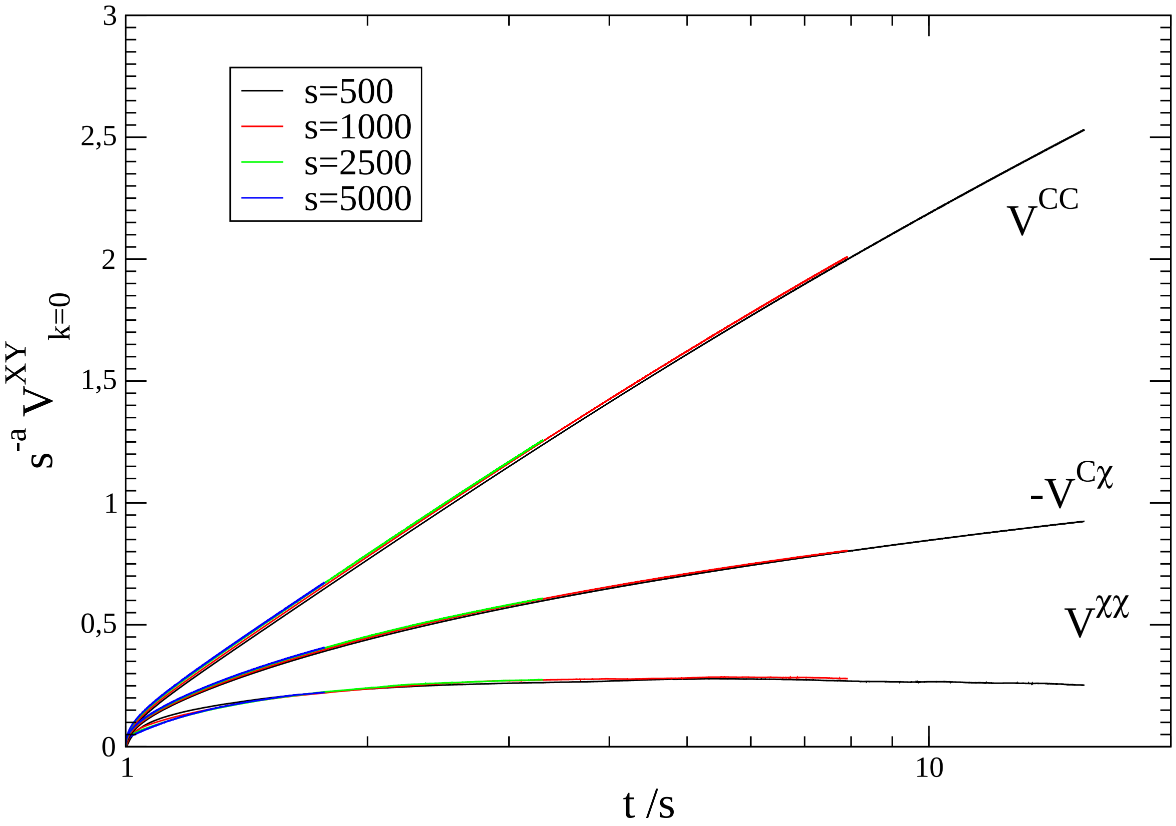
If scaling is obeyed in the form:

$$V^{XY}(t,s) = s^{a_{XY}} f_{XY}[t/s]$$

one must have the same exponent and behavior of f_{XY}.

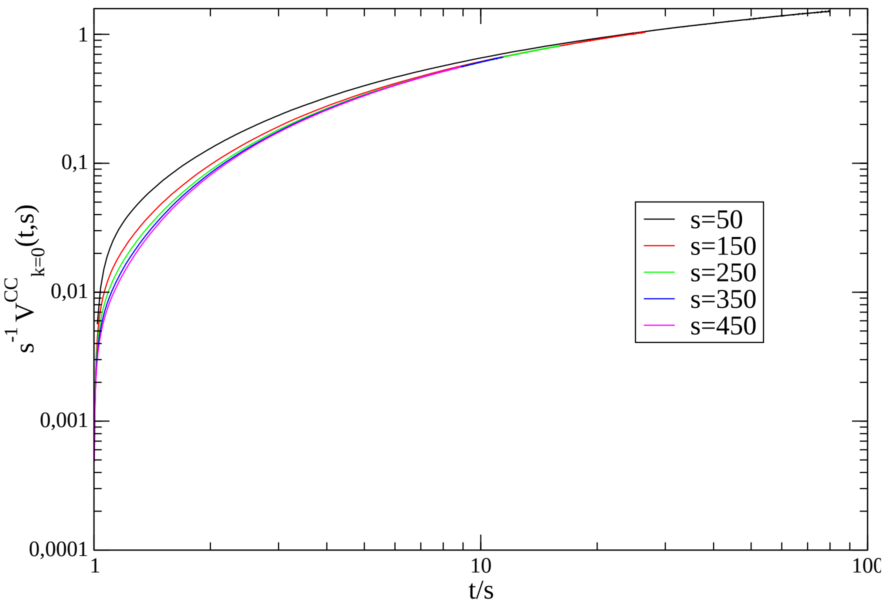
Strong prediction !

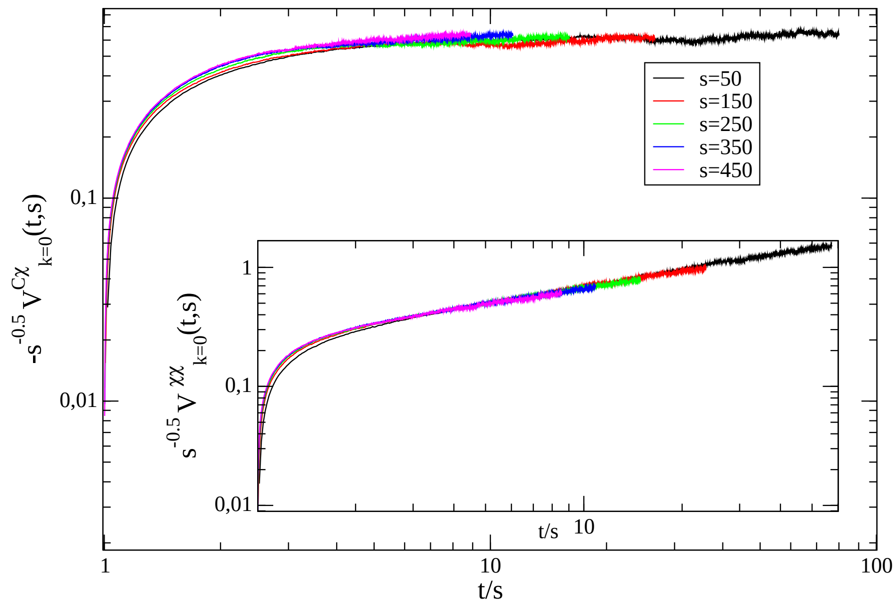
Numerical results SG 3d



$a = 0.3$ for all V^{XY}

Numerical results Ising 2d





[a]

[a] F.Corberi,E.Lippiello,A.Sarracino,M.Zannetti,J.Stat.Mech.(2010) P04003

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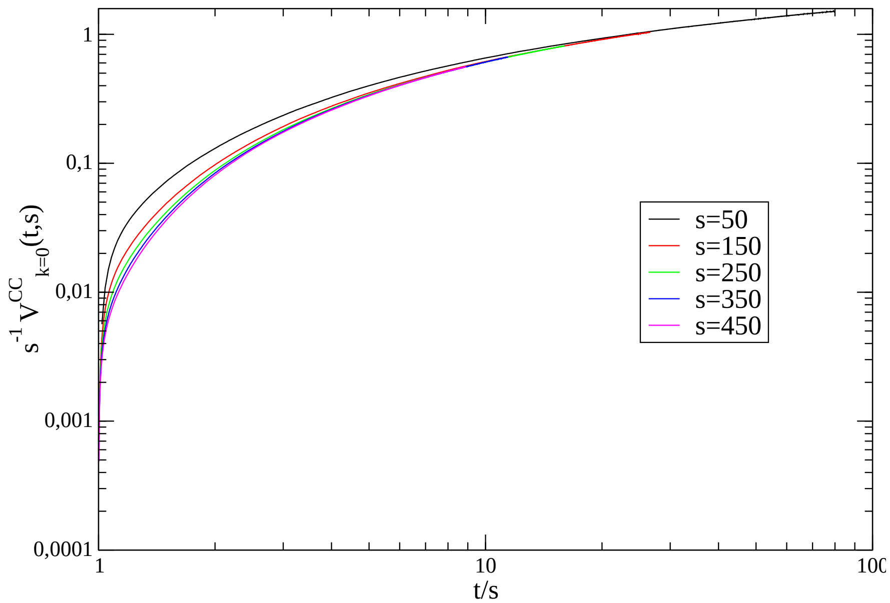

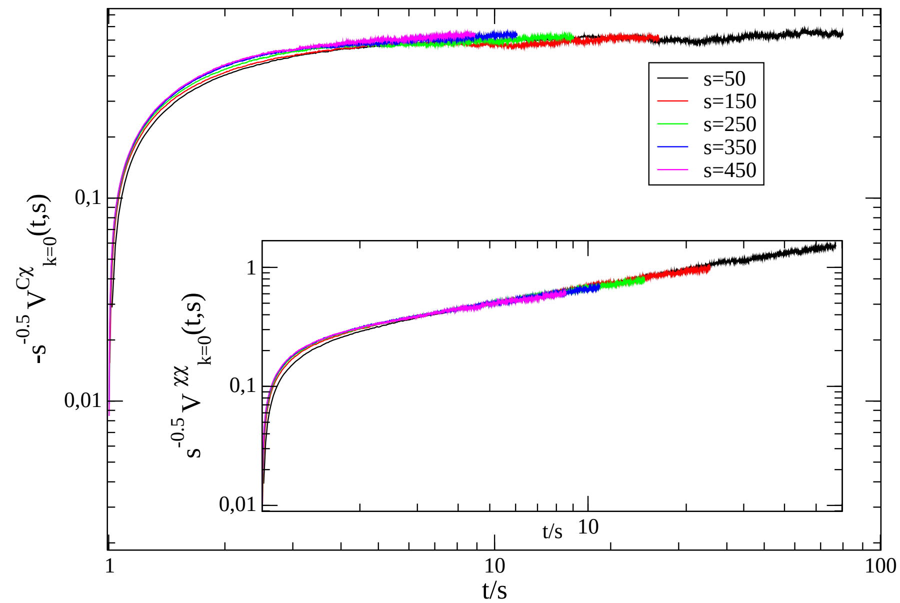

[a]

- Data are consistent with TRI being obeyed in 3d EA spin-glass but not in Ising model in $d = 2, 3$.

[a] F.Corberi,E.Lippiello,A.Sarracino,M.Zannetti,J.Stat.Mech.(2010) P04003

Restricted averages

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- For a variance it is:

$$V^{\chi\chi} = \sum_{\widehat{C}} V_{\widehat{C}}^{\chi\chi} P(\widehat{C}) + \mathcal{V}^{\chi\chi},$$

where

$V_{\widehat{C}}^{\chi\chi} = \langle \widehat{\chi}\widehat{\chi} \rangle_{\widehat{C}} - \langle \widehat{\chi} \rangle_{\widehat{C}} \langle \widehat{\chi} \rangle_{\widehat{C}}$ (*restricted covariance* ($\widehat{C}$ fixed))

$\mathcal{V}^{\chi\chi} = \sum_{\widehat{C}} \chi_{\widehat{C}} \chi_{\widehat{C}} P(\widehat{C}) - \sum_{\widehat{C}} \chi_{\widehat{C}} P(\widehat{C}) \sum_{\widehat{C}} \chi_{\widehat{C}} P(\widehat{C})$ (covariance of the *restricted* averages $\chi_{\widehat{C}}$ as $\widehat{C}$ is varied.)

Restricted averages and TRI

- Physical meaning: all sort of fluctuations of $\widehat{\chi}_i$ are accounted for by the fluctuations of the local clock $\widehat{C}_i$.

---

[a] $V^{\chi\chi} = \sum_{\widehat{C}} V^{\chi\chi}_{\widehat{C}} P(\widehat{C}) + \mathcal{V}^{\chi\chi}$

Restricted averages and TRI

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- Hence one expects [a]

$$\sum_{\widehat{C}} V_{\widehat{C}}^{\chi\chi} P(\widehat{C}) \ll \mathcal{V}^{\chi\chi},$$

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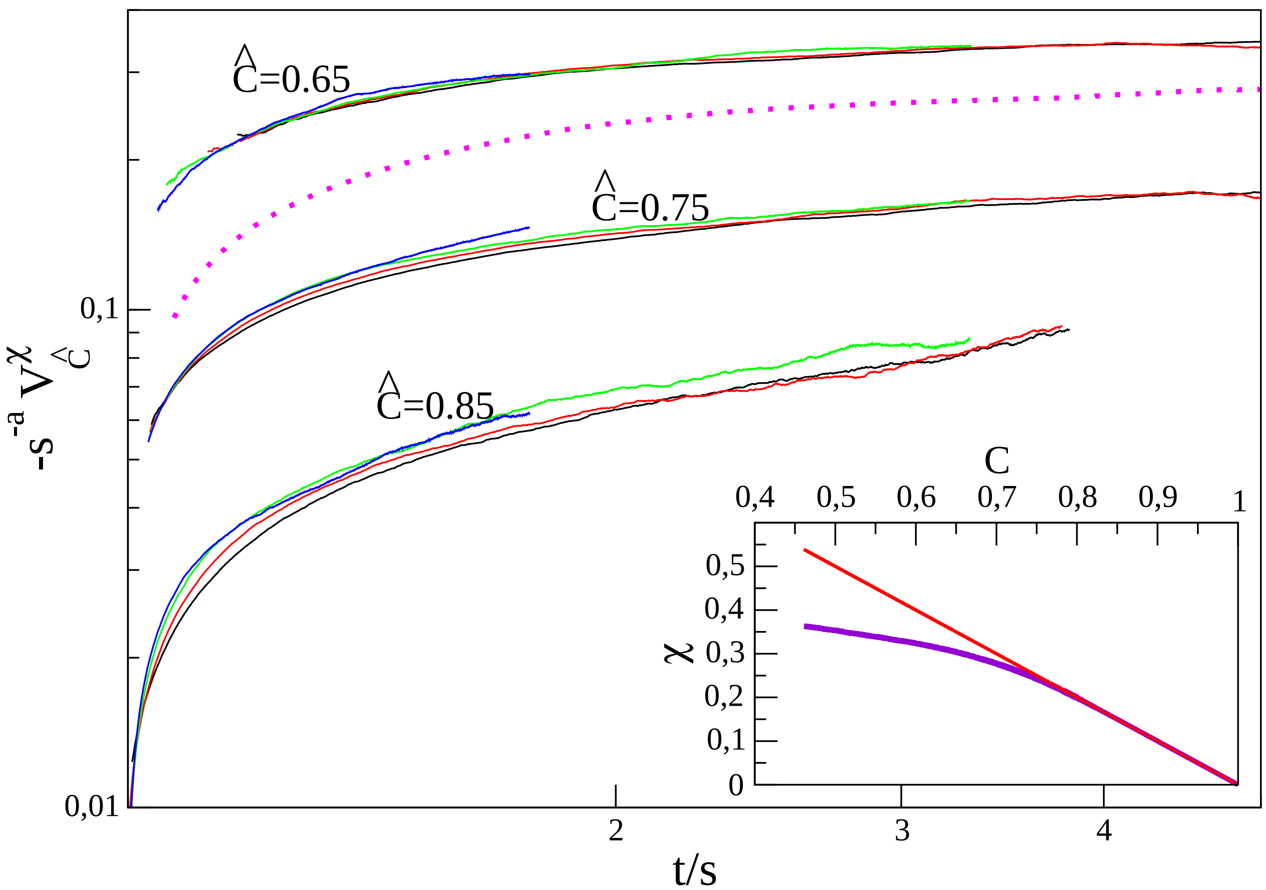
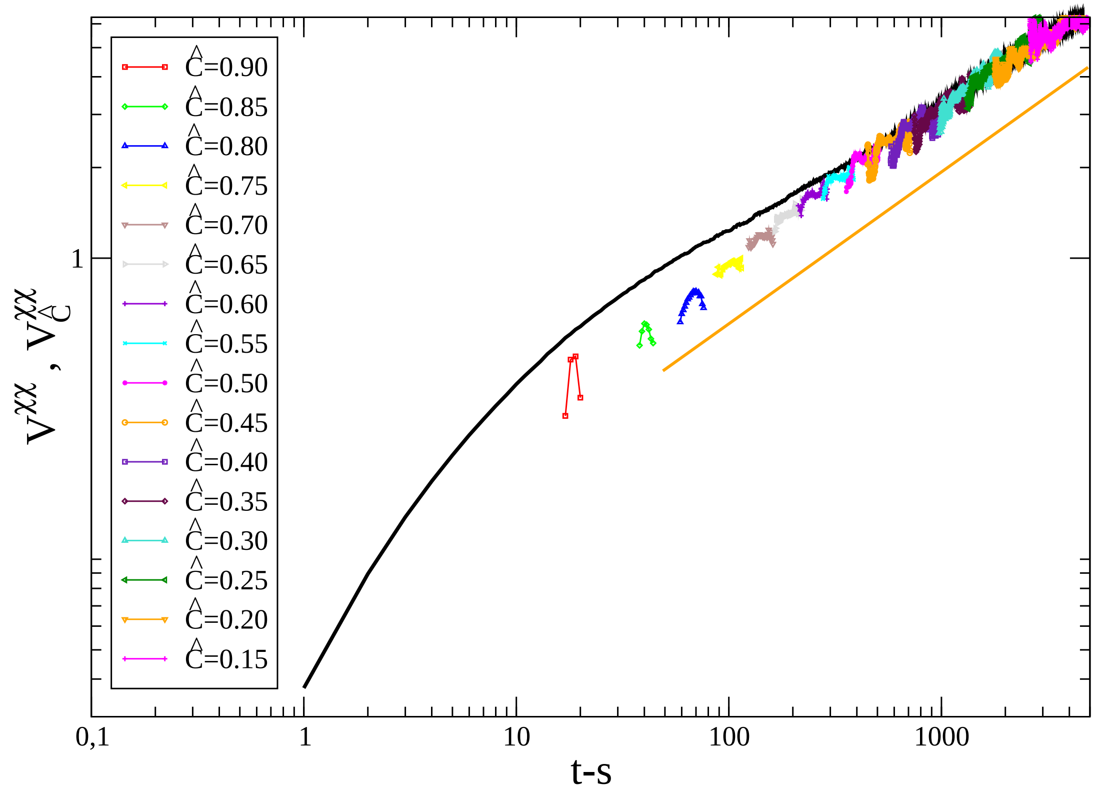
Restricted averages:numerics SG-3d



$$\sum_{\widehat{C}} V^{XY}_{\widehat{C}} P(\widehat{C}) \ll \mathcal{V}^{XY}$$

Restricted averages:numerics Ising-2d



$$\sum_{\widehat{C}} V^{XY}_{\widehat{C}} P(\widehat{C}) \gg \mathcal{V}^{XY}$$ [a]

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[a] $$V^{XY} = \sum_{\widehat{C}} V^{XY}_{\widehat{C}} P(\widehat{C}) + \mathcal{V}^{XY}$$

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- Need for analytic results.

Thank you !