Confinement by dual superconductivity: an update.

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BASED ON

1)A.Di Giacomo, L.Lepori and F.Pucci, Homotopy, monopoles and 't Hooft tensor in QCD with generic gauge group.
JHEP **0810**, 096 (2008)

2) C.Bonati, A.Di Giacomo, L.Lepori, F.Pucci, Monopoles, abelian projection and gauge invariance.
Phys. Rev. D **81**, 085022 (2010)

3) C. Bonati, A. Di Giacomo, M. D'Elia, Detecting monopoles on the lattice.
Phys. Rev. D **81**, 085022 (2010)

4) C. Bonati, G. Cossu, M. D'Elia, A. Di Giacomo, Toward a well defined monopole creation operator.
PoS LATTICE2010:271(2010) [arXiv:1010.5428 [hep-lat]]

5) C. Bonati, G. Cossu, M. D'Elia, A. Di Giacomo. In preparation.

MONOPOLE CONDENSATION AND CONFINEMENT.

- CONFINEMENT BY DUAL SUPERCONDUCTIVITY OF THE VACUUM. ['t Hooft '75, Mandelstam '75].
- CONDENSATION OF MAGNETIC CHARGES (MONOPOLES).
- CHROMO-ELECTRIC FLUX TUBES

LATTICE

TWO MAIN APPROACHES

- I. DETECT MONOPOLES IN LATTICE CONFIGURATIONS. ASSUME MONOPOLE DOMINANCE. ABSTRACT AN EFFECTIVE ACTION ($L_{eff} = -\mu^2\Phi^2 - \frac{\lambda}{2}\Phi^4 + ...$).
READ CONDENSATION FROM THE EFFECTIVE ACTION.
[Kanazawa 92, ITEP]
- II. DEFINE AN ORDER PARAMETER $\langle\mu\rangle$, THE *vev* OF A GAUGE INVARIANT OPERATOR μ CARRYING MAGNETIC CHARGE.
$\langle\mu\rangle \neq 0 \rightarrow$ HIGGS BREAKING OF MAGNETIC $U(1)$, DUAL SUPERCONDUCTIVITY ,CONFINEMENT.
IN DECONFINED PHASE $\langle\mu\rangle = 0$. [Pisa, Bari, ITEP]
- I WILL REVIEW THE STATUS OF BOTH.

APPROACH I $_1$

- MONOPOLES ARE $U(1)$ OBJECTS [Coleman 85]. BELONG TO A $U(1)$ SUBGROUP OF GAUGE GROUP (ABELIAN PROJECTION).
  IN THE 'tHOOFT-POLYAKOV MONOPOLE $U(1)$ IDENTIFIED BY $\langle \Phi \rangle$. IN QCD NO HIGGS.
  'tHOOFT[81] : PHYSICS INDEPENDENT ON THE CHOICE OF THE $U(1)$. ? ?
- NUMBER AND LOCATION OF MONOPOLES STRONGLY DEPEND ON THE CHOICE OF THE PROJECTION. GAUGE INVARIANCE?
- MOST OF THE WORK IN MAXIMAL ABELIAN GAUGE. BETTER MONOPOLE DOMINANCE. RESULTS UNSATISFACTORY.

WE HAVE SHOWN THAT

- MONOPOLES ARE GAUGE-INVARIANT. THEY ARE VIOLATIONS OF NON ABELIAN BIANCHI IDENTITIES.
- EACH MONOPOLE IDENTIFIES A PRIVILEGED DIRECTION IN COLOR SPACE, WHICH IS DIAGONAL IN MAXIMAL-ABELIAN GAUGE.
- DETECTION BY THE USUAL METHOD IS PROJECTION DEPENDENT. IN MAXIMAL ABELIAN GAUGE ALL MONOPOLES ARE DETECTED. MONOPOLES IN MAX.ABEL ARE THE TRUE MONOPOLES.
- THE ANALYSIS SHOULD BE REPEATED RELAXING MONOPOLE DOMINANCE HYPOTHESIS, EXCEPT IN THE VICINITY OF THE DECONFINING TRANSITION, WHERE MONOPOLES ARE EXPECTED TO DOMINATE.

APPROACH II

DEFINING THE ORDER PARAMETER.

- $U(1)$ G.T. CONTINUUM ($\vec{A}^0_\perp$ FIELD OF A MONOPOLE, $\vec{\nabla} \cdot \vec{A}^0_\perp = 0$).

$$\mu(\vec{x}, t) = \exp\left(i \int d^3y \vec{E}_\perp(\vec{y}, t) \cdot \vec{A}^0_\perp(\vec{y} - \vec{x})\right)$$

$$\mu(\vec{x}, t)|\vec{A}_\perp(\vec{z}, t)\rangle = |\vec{A}_\perp(\vec{z}, t) + \vec{A}^0_\perp(\vec{z} - \vec{x})\rangle$$

$$\exp(ipa)|x\rangle = |x + a\rangle$$

- $$\langle \mu(\beta) \rangle = \frac{Z(\beta(S + \Delta S))}{Z(\beta S)}$$

$$\Delta S = \int d^3y \vec{E}_\perp(\vec{y}, t) \cdot \vec{A}^0_\perp(\vec{y} - \vec{x})$$

$$\rho(\beta) \equiv \frac{\partial \ln\langle \mu(\beta) \rangle}{\partial \beta} = \langle S \rangle_S - \langle S + \Delta S \rangle_{S+\Delta S}$$

$$\langle\mu(\beta)\rangle = \exp\left(\int_0^\beta \rho(\beta')d\beta'\right)$$

- ON THE LATTICE

$$\begin{aligned} S &= \Sigma_{n,\mu<\nu} Re(1-\Pi_{\mu\nu}(n)) \\ S+\Delta S &= \Sigma_{n,\mu<\nu} Re(1-\Pi'_{\mu\nu}(n)) \end{aligned}$$

$\Pi_{\mu\nu}(n)$ PLAQUETTE IN THE PLANE $\mu\nu$

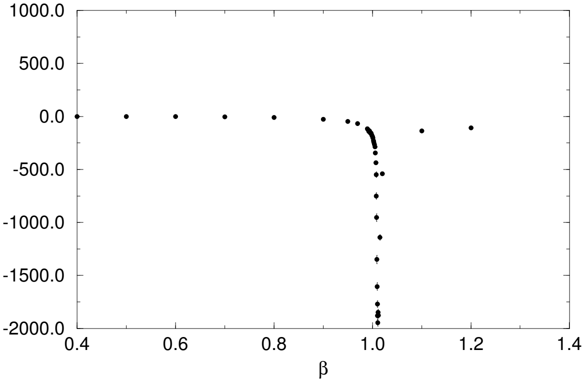
- $\Pi'_{0,n}(t,\vec{y}) = \Pi_{0,n}(t,\vec{y})\exp\left(-iA^0_n(\vec{x}-\vec{y})\right)$ $n_0 = t$ $\vec{n} = \vec{y}$
  $\Pi'_{\mu\nu}(n) = \Pi_{\mu\nu}(n))$ OTHERWISE.
- IN TERMS OF STATES
  $Z(\beta(S+\Delta S)) = \langle\mu_{out}|0_{in}\rangle$,$Z(\beta S) = \langle 0_{out}|0_{in}\rangle = \langle\mu_{out}|\mu_{in}\rangle$

  $\langle\mu\rangle = \frac{\langle\mu_{out}|0_{in}\rangle}{(\langle 0_{out}|0_{in}\rangle)^{\frac{1}{2}}(\langle\mu_{out}|\mu_{in}\rangle)^{\frac{1}{2}}}$

  $\langle\mu\rangle$ IS A PROBABILITY AMPLITUDE, AS SHOULD A GOOD ORDER PARAMETER.

- (FROELICH, MARCHETTI '87,CIRIGLIANO, PAFFUTI '99) THEOREM : THE PHASE TRANSITION AT $\beta \approx 1.01$ IN $U(1)$ LATTICE GAUGE THEORY IS A TRANSITION SUPERCONDUCTOR$\rightarrow$ NORMAL: AN ORDER PARAMETER EXISTS, THE *vev* OF A DIRAC-LIKE, GAUGE INVARIANT MONOPOLE FIELD.
- (A.D.G., G. PAFFUTI '97) THEOREM : THE ORDER PARAMETER DISCUSSED ABOVE IS EQUAL TO THAT OF FROELICH et al.
- MEASURE ρ NUMERICALLY. EXPECT $[\langle\mu(\beta)\rangle = \exp\left(\int_0^\beta \rho(\beta')d\beta'\right)]$.
  $\beta < \beta_c$ ρ FINITE AS $V \rightarrow \infty$ OR $\langle\mu\rangle \neq 0$
  $\beta \gg \beta_c$ $\rho \propto -|C_1|L_s + c_2$ OR $\langle\mu\rangle \rightarrow 0$
  $\beta \approx \beta_c$ ρ HAS A SHARP DIVERGING NEGATIVE PEAK.
  $\rho \approx L_s^{\frac{1}{\nu}}\Phi(\tau L_s^{\frac{1}{\nu}})$. $\tau \equiv (1 - \frac{T}{T_c})$.

FIG.1: ρ $U(1)$ NUMERICAL DETERMINATION



A.D.G., G. PAFFUTI PHYS.REV. D 56, 6816 (1997)
$V \to \infty$ OK. SCALING COMPATIBLE WITH FIRST ORDER.

$\langle\mu\rangle$ FOR NON-ABELIAN GAUGE THEORIES$_1$

- CONSIDER $SU(2)$ FOR SIMPLICITY. VALID FOR ANY GAUGE GROUP.
- [A.D.G., L. MONTESI, G.PAFFUTI, B. LUCINI 2000]
  FIX THE GAUGE AND PLAY THE SAME GAME AS FOR $U(1)$ ON THE $U(1)$ DIAGONAL SUBGROUP.
  CHOICE OF THE SUBGROUP [ABELIAN PROJECTION] IRRELEVANT (A THEOREM).
- AS DONE FOR $U(1)$

$$\begin{aligned}
\langle\mu(\beta)\rangle &= \frac{Z(\beta(S+\Delta S))}{Z(\beta S)} \\
S &= \Sigma_{n,\mu<\nu} Re(1-\Pi_{\mu\nu}(n)) \\
S+\Delta S &= \Sigma_{n,\mu<\nu} Re(1-\Pi'_{\mu\nu}(n))
\end{aligned}$$

["NAIVE" DEFINITION]

$$\begin{aligned}\Pi_{\mu\nu}(n) &= Tr\left[U_\mu(n)U_\nu(n+\hat{\mu})U^\dagger_\mu(n+\hat{\nu})U^\dagger(n)\right]\\ \Pi'_{i0}(t,\vec{n}) &= Tr[U_i(t,\vec{n})U_0(t,\vec{n}+\hat{i})U^\dagger_i(t+1,\vec{n})M_i(\vec{n}+\hat{i})\\ &\quad U^\dagger(t,\vec{n})]\\ M_i(\vec{n}) &= \exp\left(iA^0_i(\vec{x}-\vec{n})\hat{\Phi}\right)\end{aligned}$$

- $Z(\beta(S+\Delta S)) = \langle \mu_{out}|0_{in}\rangle$, $Z(\beta S) = \langle 0_{out}|0_{in}\rangle$

$$\langle \mu \rangle = \frac{\langle \mu_{out}|0_{in}\rangle}{\langle 0_{out}|0_{in}\rangle}$$

- CORRECT DEFINITION [C.BONATI et al. IN PREPARATION]

$$\langle \mu \rangle = \frac{\langle \mu_{out}|0_{in}\rangle}{\langle 0_{out}|0_{in}\rangle^{\frac{1}{2}}\langle \mu_{out}|\mu_{in}\rangle^{\frac{1}{2}}}$$

HISTORY_1

- FIRST PAPERS, NAIVE DEFINITION : PURE GAUGE $SU(2)$ $SU(3)$ [A.D.G., B.LUCINI, L. MONTESI, G.PAFFUTI Phys.Rev.D61 034503,034504,2000], RATHER SMALL LATTICES AND STATISTICS. CONSISTENT WITH EXPECTATION WITHIN ERRORS.
- INDEPENDENCE ON ABELIAN PROJECTION [J.CARMONA,M.D'ELIA, A.D.G., B.LUCINI,G.PAFFUTI Phys.Rev.D64,114507 2001]
- FULL ($N_f = 2$) QCD [J.CARMONA,L.DEL DEBBIO,M.D'ELIA,A.D.G.,B.LUCINI,G.PAFFUTI, Phys.Rev.D66011503 2002]

HISTORY$_2$

- 2-d ISING MODEL [J.CARMONA,A.D.G. Phys.Lett.B485, 126 2000]; 3-d XY MODEL [G.DI CECIO, A.D.G., G.PAFFUTI,M.TRIGIANTE Nucl.Phys. B489,739 (1997); 3-d HEISENBERG MODEL[A.D.G., D.MARTELLI,G.PAFFUTI Phys.Rev.D60 094511 1997]
- CONFINEMENT: G(2) GROUP CASE. [G. COSSU, M.DELIA A. D.G, B. LUCINI,C.PICA, PoS LAT2007:296,2007]. G(2): NO CENTER VORTICES, DOES IT CONFINE? QUALITATIVELY CORRECT BEHAVIOR FOR ρ SUPERIMPOSED TO A NEGATIVE DIVERGING BACKGROUND AS V$\rightarrow\infty$. $\langle\mu\rangle = 0$ EVERYWHERE: NO ORDER PARAMETER. MORE CAREFUL ANALYSIS FOR $SU(2)$, $SU(3)$ SHOWS THE SAME PROBLEM.[G.COSSU, M. D'ELIA, A. D.G, B. LUCINI,C.PICA, UNPUBLISHED, J.GREENSITE and B.LUCINI Phys.Rev. D78 085004 2008]

- NAIVE ORDER PARAMETER

$$\langle\mu\rangle = \frac{Z(\beta(S+\Delta S))}{Z(\beta S)} = \frac{\langle\mu_{out}|0_{in}\rangle}{\langle 0_{out}|0_{in}\rangle} = \exp\left(\int_0^{\beta} \rho(\beta')d\beta'\right)$$

$$\rho = \langle S\rangle_S - \langle S+\Delta S\rangle_{S+\Delta S}$$

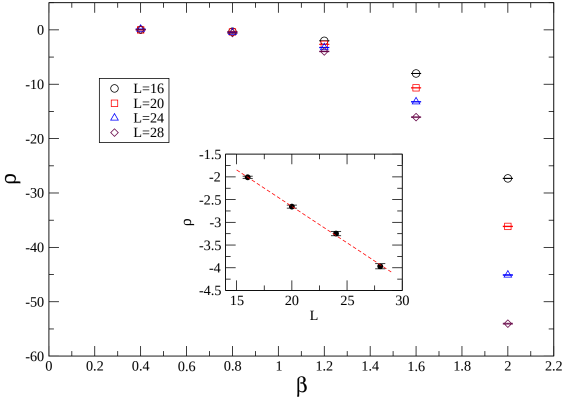
- CORRECT ORDER PARAMETER $\langle\bar{\mu}\rangle$

$$\begin{aligned}\langle\bar{\mu}\rangle &= \frac{\langle\mu_{out}|0_{in}\rangle}{\langle 0_{out}|0_{in}\rangle^{\frac{1}{2}}\langle\mu_{out}|\mu_{in}\rangle^{\frac{1}{2}}} = \frac{Z(\beta(S+\Delta S))}{Z(\beta S)^{\frac{1}{2}} Z(\beta(S+\bar{\Delta} S)^{\frac{1}{2}}} \\ &= \exp\left(\int_0^{\beta} \bar{\rho}(\beta')d\beta'\right)\end{aligned}$$

$$\bar{\rho} = \frac{1}{2}\langle S\rangle_S + \frac{1}{2}\langle S+\bar{\Delta}S\rangle_{S+\bar{\Delta}S} - \langle S+\Delta S\rangle_{S+\Delta S}$$

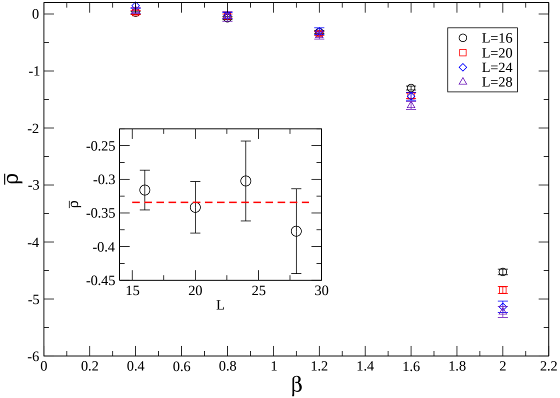
- $S + \bar{\Delta} S = \Sigma_{n,\mu<\nu} Re(1 - \bar{\Pi}_{\mu\nu}(n))$
- $\bar{\Pi}_{i0}(t,\vec{n}) = Tr[U_i(t,\vec{n})U_0(t,\vec{n}+\hat{i})U_i^\dagger(t+1,\vec{n})M_i(\vec{n}+\hat{i})$
  $U_0^\dagger(t,\vec{n})M_i^\dagger(\vec{n}+\hat{i})]\ n_0 = t$
  $\bar{\Pi}_{\mu\nu}(n) = \Pi_{\mu\nu}(n)$ OTHERWISE: $M_i(\vec{n}) = \exp\left(iA_i^0(\vec{x}-\vec{n})\hat{\Phi}\right)$
- FOR $U(1)$ THEORY EVERYTHING COMMUTES, M_i AND $M_i^\dagger$ CANCEL, $\bar{\Delta} S = 0$ AND THE NAIVE DEFINITION COINCIDES WITH THE CORRECT ONE.
- STRONG COUPLING EXPANSION OF ρ , $\bar{\rho}$ AT SMALL β'S (CONFINED). LOWEST ORDER IS THE CUBE ($O(\beta^5)$)
  $\rho \propto \beta^5 \sum_{\vec{n}ij}(B_{ij}(\vec{n}) - 1)$ $\quad \bar{\rho} \propto -\frac{1}{2}\beta^5 \sum_{\vec{n}ij}(B_{ij}(\vec{n}) - 1)^2$
  $(B_{ij}(\vec{n}) - 1) \propto_{n\to\infty} -\frac{1}{n^2}$
  ρ DIVERGES AS $-V^{\frac{1}{3}}$ $\quad \bar{\rho}$ STAYS FINITE.

NUMERICAL RESULTS : ρ NAIVE



ρ AT LOW $\beta \propto L$ IN AGREEMENT WITH STRONG COUPLING.

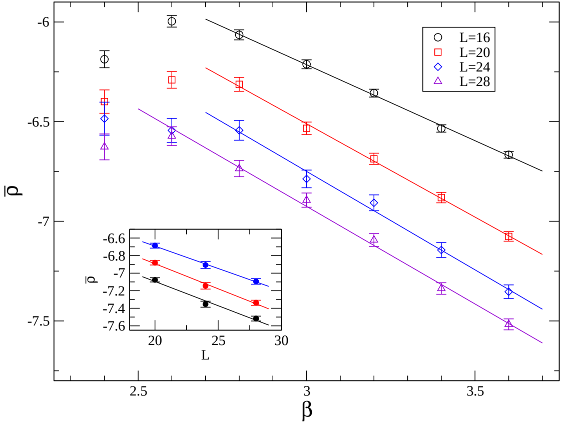
NUMERICAL RESULTS : $\bar{\rho}$ AT LOW β



CORRECT ORDER PARAMETER.

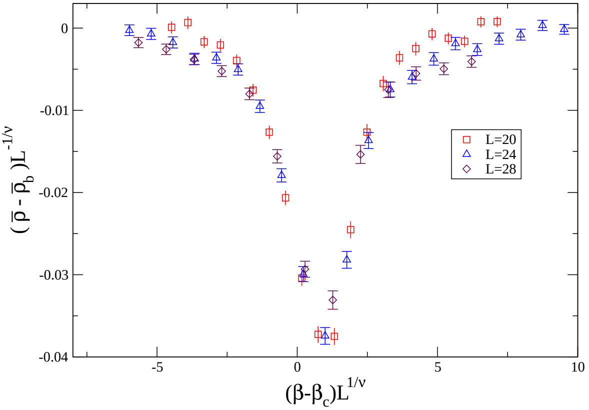
$\bar{\rho}$ STAYS CONSTANT AS $L \to \infty$ OR $\langle \bar{\mu} \rangle \neq 0$ (CONFINEMENT).

NUMERICAL RESULTS : $\bar{\rho}$ AT LARGE BETA



$\bar{\rho} \approx -|c|L + d$ AT LARGE β'S. $\langle \bar{\mu} \rangle = 0$ (DECONFINED)

NUMERICAL RESULTS : $\bar{\rho}$ AT $\beta \approx \beta_c$. SCALING



SCALING WITH 3-d ISING CRITICAL INDEXES.

$\rho \approx L_s^{\frac{1}{\nu}} \Phi(\tau L_s^{\frac{1}{\nu}})$.

MONOPOLE AND N.A.B.I.

- A.B.I.
  $\partial_\mu \tilde{F}_{\mu\nu} = j_\nu \;,\; \tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma} F_{\rho\sigma}$

  $\vec{\nabla}\vec{b} = j_0 \qquad \rightarrow [\; Q_M = \int d^3x \vec{\nabla}\vec{b} = \int_S d\vec{S}\vec{b}]$

  $\vec{\nabla} \wedge \vec{E} - \partial_0 \vec{B} = \vec{j}$

- N.A.B.I.
  $D_\mu \tilde{G}_{\mu\nu} = J_\nu \qquad \rightarrow D_\nu J_\nu = 0 \qquad \tilde{G}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma} G_{\rho\sigma}$

  GAUGE INVARIANT CONTENT: DIAGONALIZE.
  UNITARY GAUGE $\equiv$ MAX.ABELIAN GAUGE.
  $[J_\mu, J_\nu] = 0$ (COLEMAN- HAAG THEOREM).

- r INDEPENDENT EQUATIONS (r = RANK OF THE GROUP)

- PROJECT ON THE (DIAGONAL) FUNDAMENTAL WEIGHTS Φ^a_I $(a = 1..r)$: $Tr(\Phi^a_I D_\mu \tilde{G}_{\mu\nu}) = Tr(Tr(\Phi^a_I J_\nu)$
- $[\vec{H}, E_{\vec{\alpha}}] = \vec{\alpha} E_{\vec{\alpha}}$, $[E_{\vec{\alpha}}, E_\beta] = N_{\vec{\alpha}\vec{\beta}} E_{\vec{\alpha}+\vec{\beta}}$, $[E_{\vec{\alpha}}, E_{-\vec{\alpha}}] = \vec{H}\vec{\alpha}$, $[H^i, H^j] = 0$ $Tr(\Phi^a_1 H^i) = \delta_{ia}$
- CAN ALSO PROJECT ON $\Phi^a_U = U^\dagger(x)\Phi^a_I U(x)$, $U(x)$ A GENERIC GAUGE TRANSFORMATION.
- THEOREM [B.DG.L.P.] THE RESULT ARE THE ABELIAN BIANCHI IDENTITIES

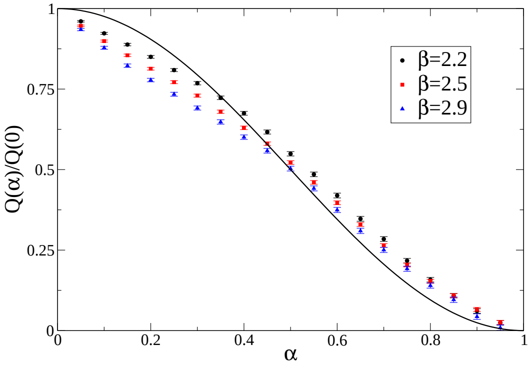
$$\partial_\mu \tilde{F}^{aU}_{\mu\nu} = Tr(\Phi^a_U J_\nu) \qquad (1)$$

WITH $F^{aU}_{\mu\nu}$ THE ABELIAN DIAGONAL FIELD IN THE GENERIC ABELIAN PROJECTION IN WHICH Φ^a_U IS DIAGONAL. $U = I$,(MAX.AB.) IS A SPECIAL CASE.

- FAR REACHING CONSEQUENCES!
- CONSIDER $SU(2)$ FOR SIMPLICITY [r =1].

- THE M_0 TERM IN THE MULTIPOLE EXPANSION AT LARGE DISTANCES IS ABELIAN, OBEYS ABELIAN EQUATIONS OF MOTION AND IS THE FIELD OF A MONOPOLE OF CHARGE $Q_M = \frac{m}{2g}$. [Coleman 85]
  $\rightarrow$ IN THE MAX.ab. GAUGE $\vec{H} = Q_M \frac{\hat{r}}{r^2} \frac{\sigma_3}{2}$,
  $\vec{H} \approx \vec{h}^I \frac{\sigma_3}{2}$, $Q^I = \int_S d\vec{S} \vec{h}^I = Q_M$
  IN THE MAX.AB. GAUGE THE MAGNETIC CHARGE IS Q_M !
- IN THE GENERIC ABELIAN PROJECTION $j_0^U = Tr(\Phi_U J_0) = Tr(\Phi_I U J_0 U^\dagger) = \cos(\alpha) j_0^I$. THE ABELIAN MAGNETIC CHARGE IS GENERICALLY DIFFERENT FROM THE MONOPOLE CHARGE.
- $[j_0^I(\vec{x}), \mu(\vec{y})] \approx Q\delta(\vec{x} - \vec{y})\mu(\vec{x})$
  $[j_0^U(\vec{x}), \mu(\vec{y})] \approx Q\cos(\alpha)\delta(\vec{x} - \vec{y})\mu(\vec{x})$
  μ IS MAGNETICALLY CHARGED IN ALL ABELIAN PROJECTIONS.

MAGNETIC CHARGE IS MAXIMAL IN MAX.AB. GAUGE: THE RADIAL FREE FIELD AT INFINITY IS DIAGONAL. IN OTHER GAUGES IT CAN GO PARTLY OFF DIAGONAL.



SAMPLE OF CONFIGURATIONS. NUMBER OF MONOPOLES OBSERVED IN A CONTINUOUS SEQUENCE OF ABELIAN PROJECTIONS FROM MAX. AB. ($\alpha = 0$) TO LANDAU ($\alpha = 1$) C. Bonati, A. Di Giacomo, M. D'Elia, Phys. Rev. D **81**, 085022 (2010)

CONCLUSIONS AND OUTLOOK

- WE HAVE SHOWN THAT MONOPOLES ARE GAUGE INVARIANT OBJECTS. DETECTION ON THE LATTICE IS PROJECTION DEPENDENT. IN MAX. ABELIAN DETECTION FAITHFUL.
- WE HAVE CORRECTED BY A FIRST-PRINCIPLE ARGUMENT A PRE-EXISTING DEFINITION OF THE ORDER PARAMETER FOR DUAL SUPERCONDUCTIVITY IN QCD, THUS ELIMINATING INCONSISTENCIES . THE CORRECT ORDER PARAMETER SHOWS THAT CONFINED PHASE IS SUPERCONDUCTING, DECONFINED IS NORMAL. A STRONG EVIDENCE FOR DUAL SUPERCONDUCTIVITY AS A MECHANISM OF CONFINEMENT.
- NEXT STEPS : G_2 GAUGE THEORY, FULL QCD, 3-d XY....