Dressed Wilson loops as dual condensates in response to magnetic fields

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Introduction

- confinement in gauge theories
  - Polyakov loops: zero expectation value
  - Wilson loops: area law
- contact to chiral symmetry
  - define dressed Wilson loops using the chiral condensate
- contact to external fields
  - Fourier transform in the external field

$\rightarrow$ confirm area law

$\rightarrow$ recover conventional Wilson loops

$\rightarrow$ no additive renormalization due to dressing

$\rightarrow$ make Wilson loops accessible in diagrammatic approaches/model calculations

Main idea

[arXiv:1104.5664]

- consider planar Wilson loops of arbitrary geometry, with area S
- apply external (abelian) constant field B
- closed loops in e.g. chiral condensate (with probe mass m) receive factor of e^{iBS}
- Fourier transform in B at fixed S to obtain dressed Wilson loop
- conventional Wilson loops are recovered as $m \to \infty$

Analogy with dressed Polyakov loops

[Bilgici, Bruckmann, Gattringer, Hagen '08]

- consider closed (possibly winding) loops, with winding number q
- apply phase boundary conditions φ
- winding loops in e.g. chiral condensate (with probe mass m) receive factor of $e^{i\varphi q}$
- Fourier transform in φ at fixed q to obtain dressed Polyakov loop
- conventional Polyakov loop is recovered as $m \to \infty$

Magnetic field in finite volume

- $\mathbf{B} = (0, 0, B)$ constant, abelian, in the z direction
  [talk by F. Negro]
- vector potential $A_\nu = (0, Bx, 0, 0)$
  to get $\partial_x A_y - \partial_y A_x = B$
- for any closed loop in $x - y$ plane, with area S:

$$W(S) \to W(S)\, e^{i \oint_C A_\nu \mathrm{d}x_\nu} = W(S)\, e^{i \iint \mathbf{B} \mathrm{d}\sigma} = W(S)\, e^{iBS}$$

- quantization due to finite volume + bc.

$$L_x L_y \cdot B = 2\pi b \qquad\qquad b \in \mathbb{Z}$$

['t Hooft '79; Al-Hashimi, Wiese '09; D'Elia, Negro '11]

(electric charge is set to 1)

Magnetic field on the lattice

- multiply gluon links U_ν with $u_\nu = e^{iaA_\nu} \in U(1)$

$$
\begin{aligned}
u_y(n) &= e^{ia^2 B n_x} & & \\
u_x(n) &= 1 & & n \neq N_x - 1 \\
u_x(N_x - 1, n_y, n_z, n_t) &= e^{-ia^2 B N_x n_y} & & \\
u_\nu(n) &= 1 & & \nu \neq x, y
\end{aligned}
$$

- quantized flux and area on the lattice:

$$
\begin{aligned}
a^2 N_x N_y \cdot B &= 2\pi b & & 0 \leq b \leq N_x N_y \\
S/a^2 &= s & & 0 \leq s \leq N_x N_y
\end{aligned}
$$

Definition

- quark condensate in external magnetic field

$$\Sigma_b = \frac{1}{L_x L_y} \left\langle \mathrm{tr} \frac{1}{D_b + m} \right\rangle$$

- gauge invariance $\Rightarrow$ contains all closed loops:

$$\Sigma_b = \ldots \cdot 1 + \ldots \langle \text{plaquette} \rangle e^{ib} + \ldots \langle W \Big|_{s=2} \rangle e^{2ib} + \ldots$$

- dual condensate through discrete Fourier trafo:

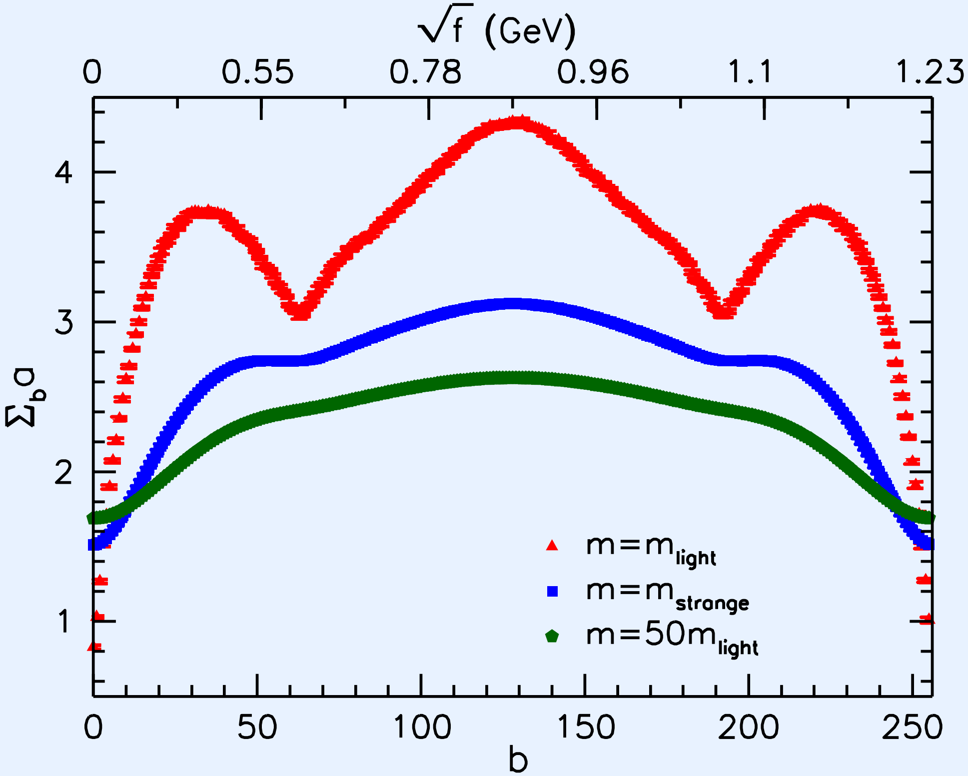
$$\tilde{\Sigma}(s) \equiv \sum_b e^{-isb} \Sigma_b$$

picks loops of fixed area s $\equiv$ dressed Wilson loops

- remarks: - partially quanched approach
  - all magnetic fields for Fourier trafo
  - 2D Dirac operator D_b is used

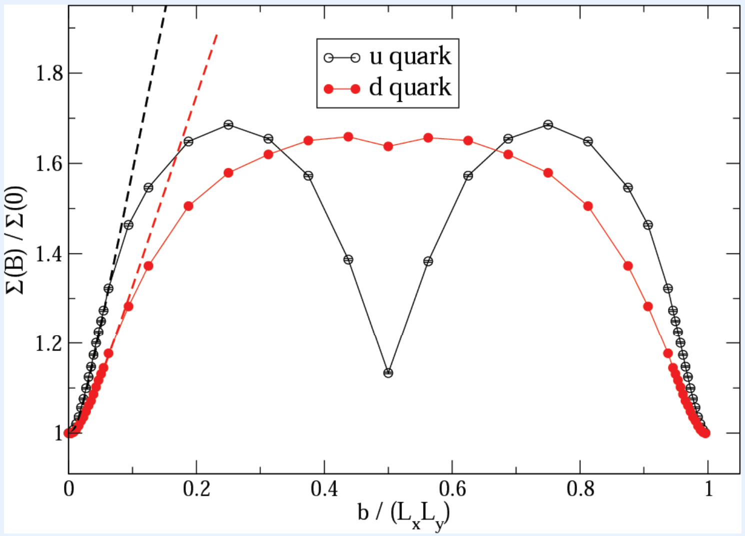
Lattice results - Σ_b

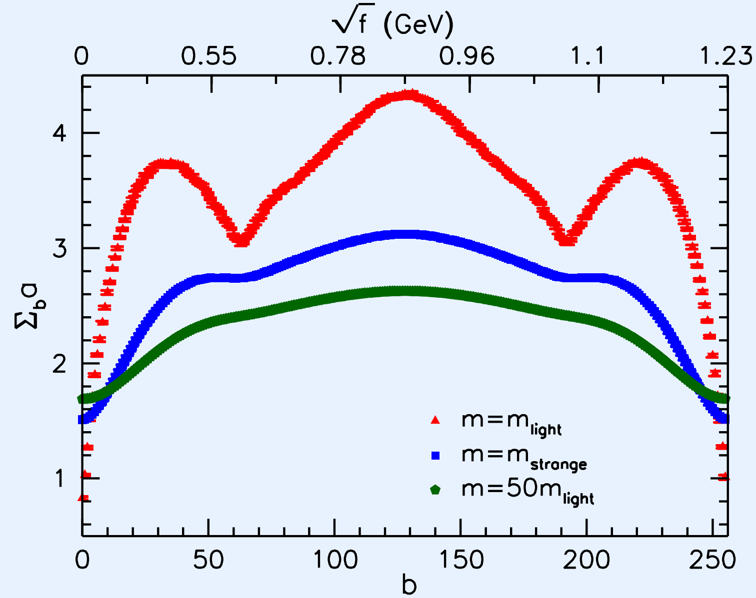
- $N_f = 2{+}1$ stout smeared staggered fermions with physical pion mass on $16^3 \times 4$ lattice
- using only 5 configurations



- remarks: - Σ_b grows with b, then saturates
  - heavy quarks wash out the effect

Comparison to dynamical case





dynamical B

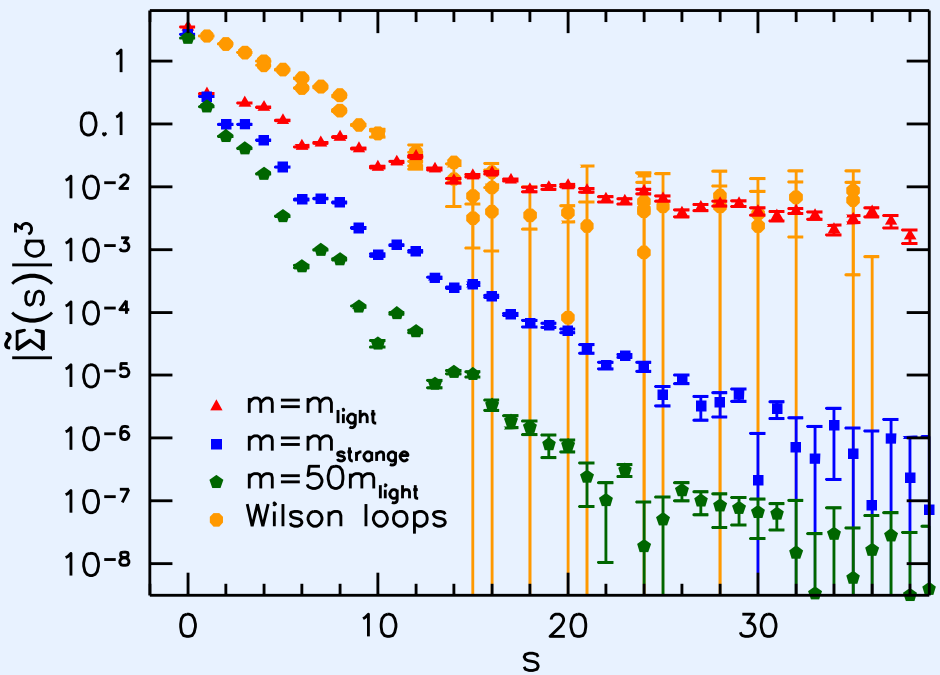
[talk by F. Negro]

partial quenched B

- deviations between 'valence' and 'total' large at $b > N_x N_y / 8$

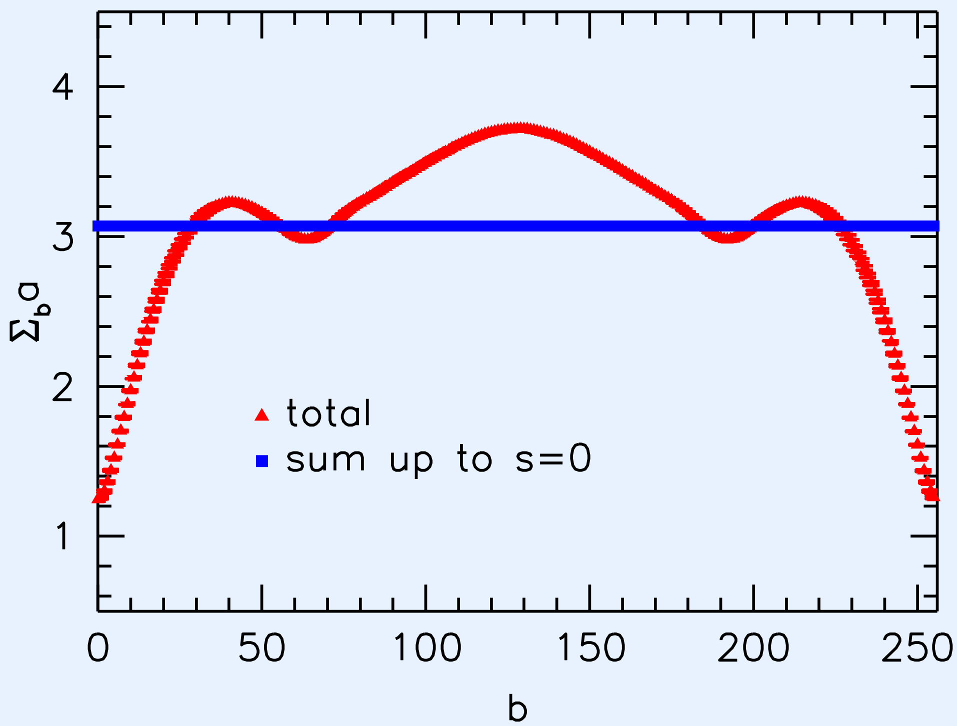
Lattice results - $\tilde{\Sigma}(s)$

- dressed Wilson loops as function of area s
- compare to conventional Wilson loops of size $r_x \times r_y$ such that $r_x r_y = s$

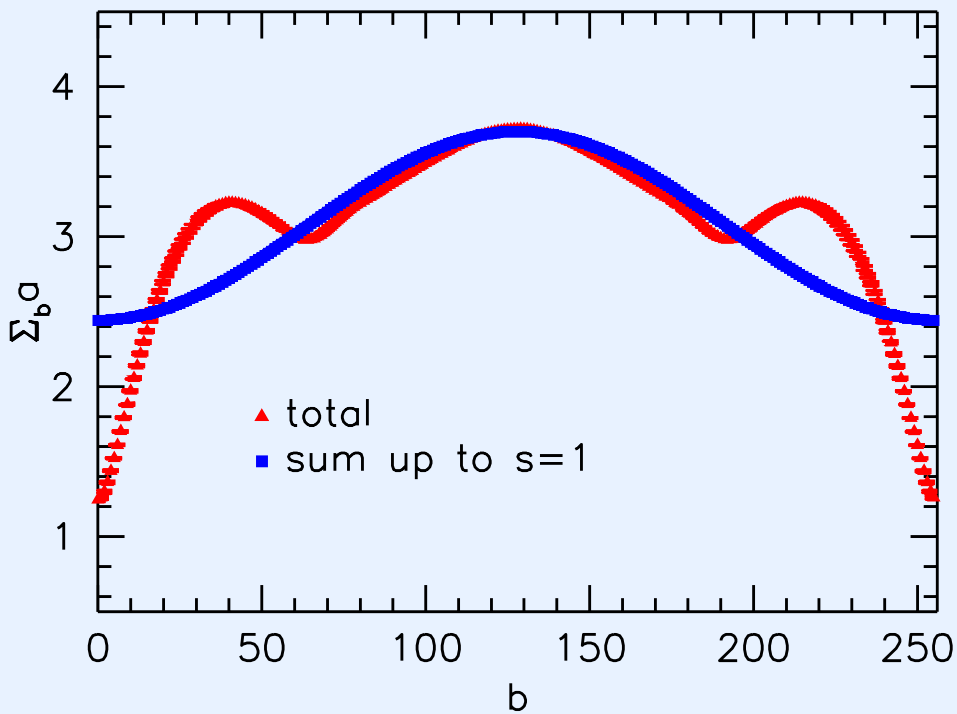


- remarks: - decays with area
  - modulations?
  - smaller errors due to self-averaging

Decomposition of Σ



Decomposition of Σ



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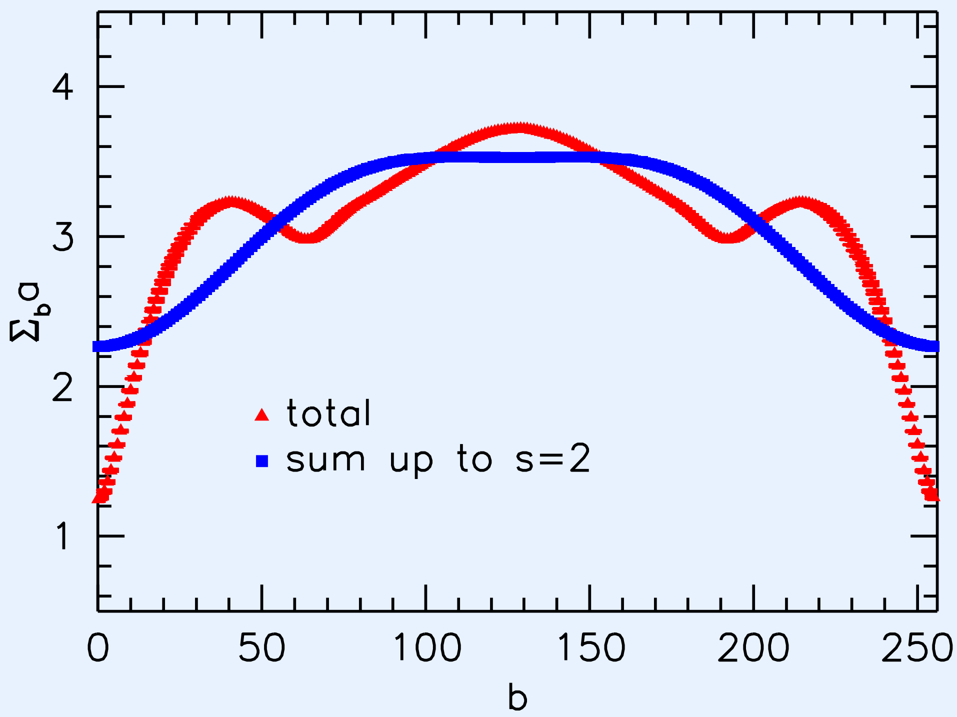
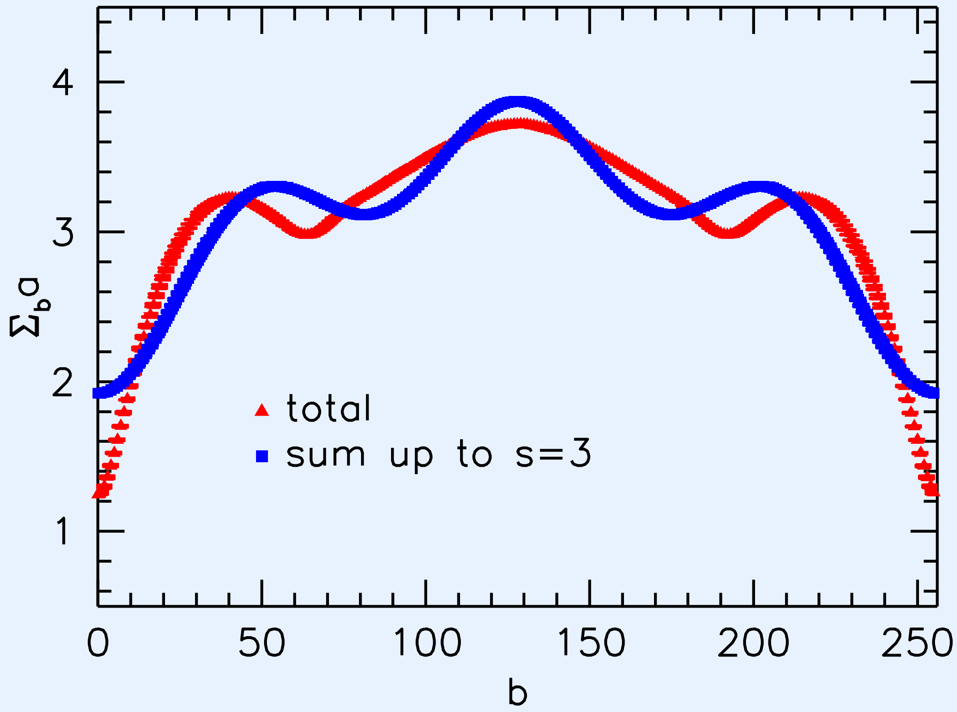


Decomposition of Σ



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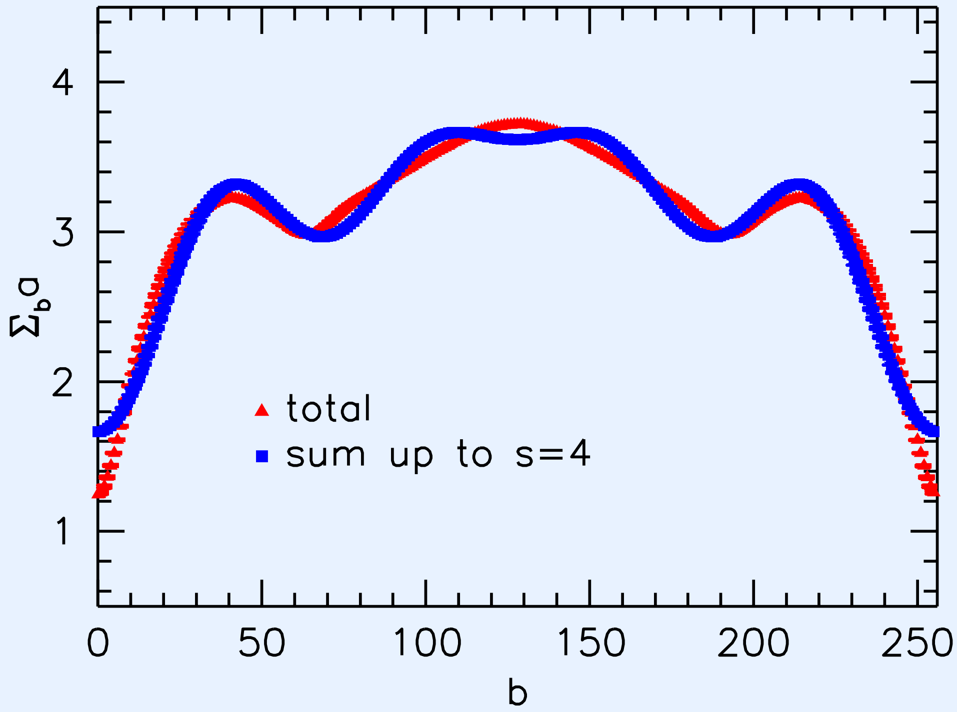

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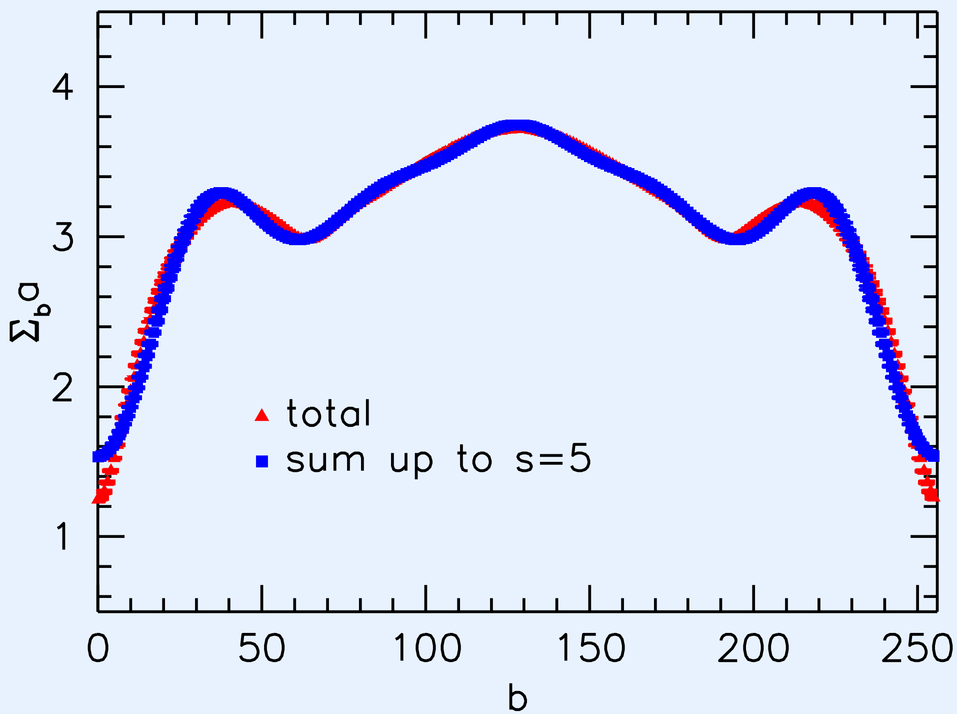
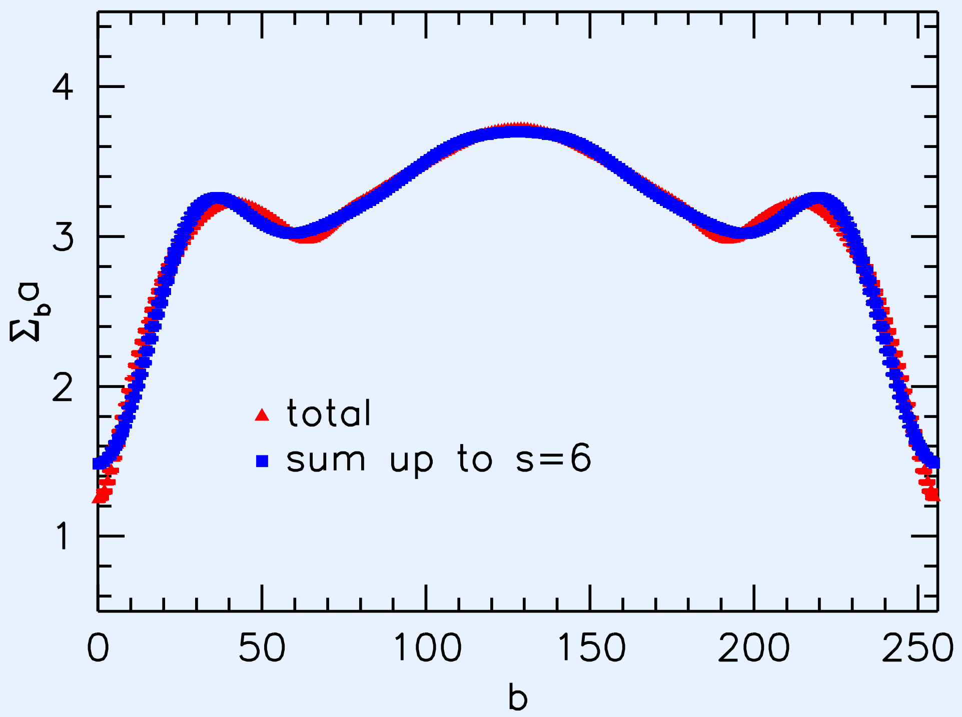
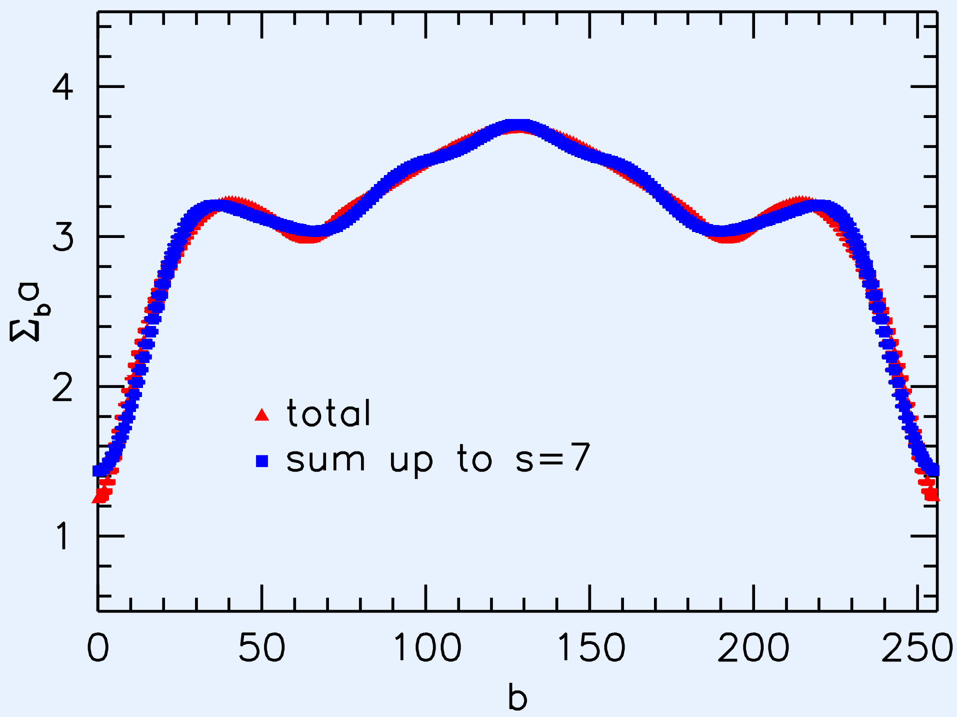


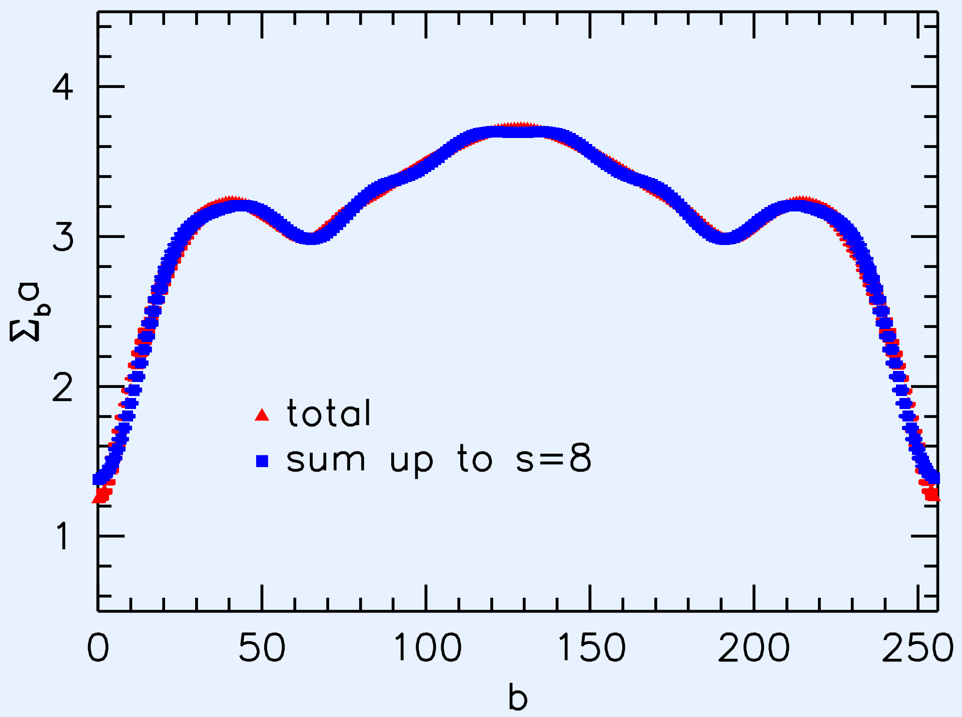
Decomposition of Σ



Decomposition of Σ



Decomposition of Σ



Mechanism - mass suppression

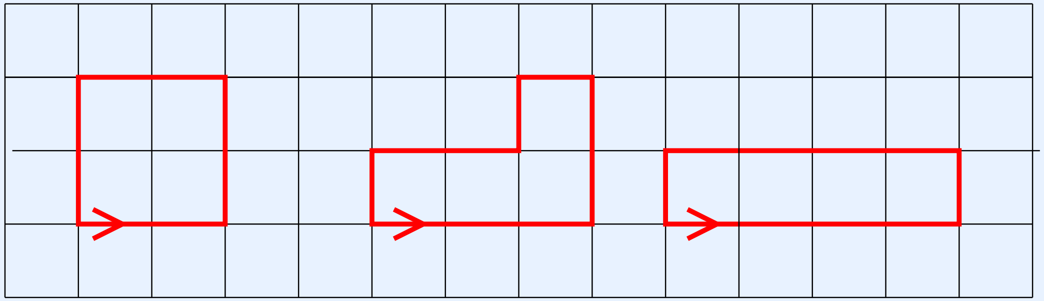
- geometric series for the condensate:

$$\mathrm{tr}\,\frac{1}{D_b + m} = \frac{1}{m} \sum_{\ell\,\text{even}} \frac{\mathrm{tr}\,D_b^\ell}{m^\ell}$$

  with $\ell = L/a$ length of loop

  $\Rightarrow$ large mass suppresses long loops

- remark: no disconnected loops due to tr
- loops with $S = 4a^2$:

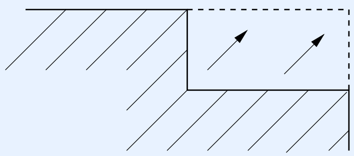


circumference $\ell = 8$ circumference $\ell = 10$

- minimal circumference $\ell_{\text{min}}(s)$ for each area s

Mechanism - entropy of loops

- ideal lattice loop: maximal area with fixed circumference
  $\rightarrow$ squares (not circles)



- large ℓ is suppressed, but # of loops larger?
- number of loops $F(s, \ell)$ with recursive algorithm
  $\rightarrow$ asymptotics for given s, large $\ell \gg s$:

$$F(s, \ell) \approx 4^{\ell}/\ell^2$$

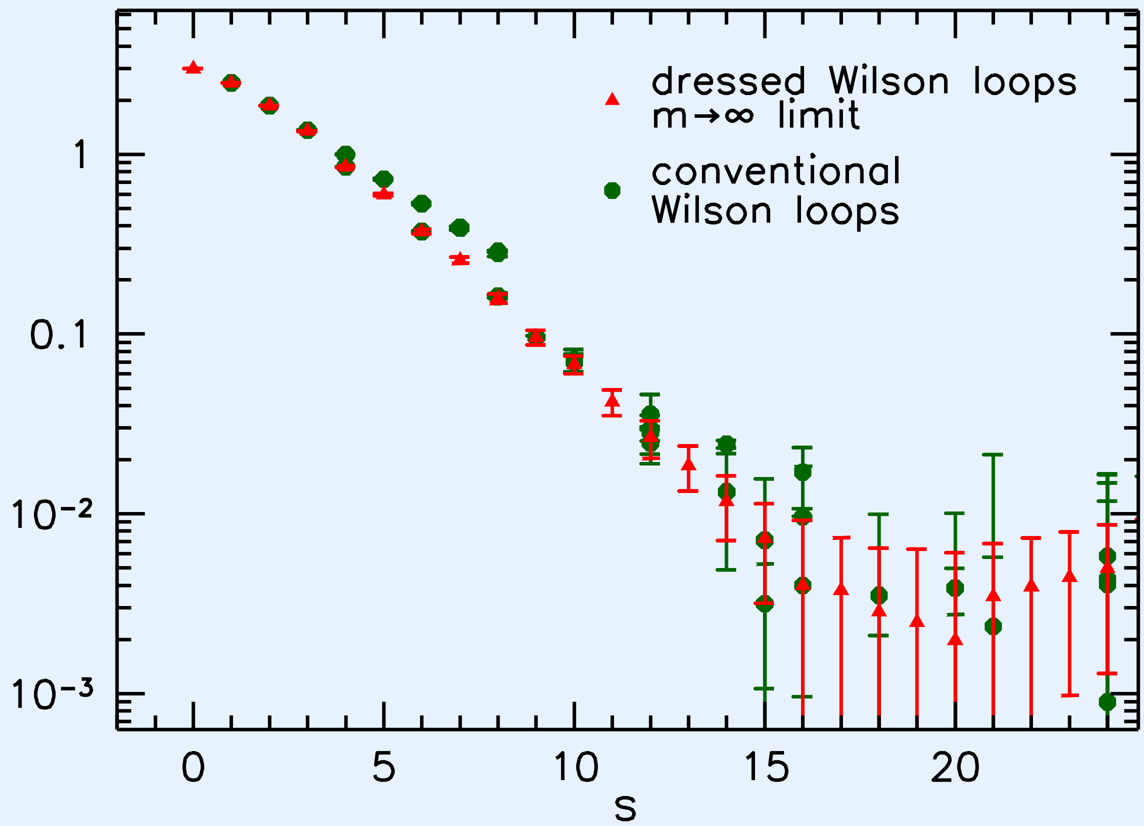
  $\Rightarrow$ entropy is always exceeded by $m^{-\ell}$ if m is large enough (i.e. $m > 4$)
- for $m \rightarrow \infty$ dressed Wilson loop $\tilde{\Sigma}(s)$ contains only loops with $\ell = \ell_{\text{min}}(s)$

Lattice results - $\lim_{m\to\infty}\tilde{\Sigma}(s)$

- large m limit of dressed Wilson loops:

$$m\tilde{\Sigma}(s)\cdot\frac{m^{\ell_{\rm min}(s)}}{F(s,\ell_{\rm min}(s))}\to\langle W(s)\rangle$$

for square-like Wilson loops $W(s)$



Mechanism - IR and UV behavior

- expectation: fuzziness with width $\sim 1/m$
  $\Rightarrow$ less sensitive to lattice spacing
- spectral representation

$$\Sigma_b = \frac{1}{L_x L_y} \left\langle \mathrm{tr} \frac{1}{D_b + m} \right\rangle = \frac{1}{L_x L_y} \left\langle \sum_{\lambda_{b,i} > 0} \frac{2m}{\lambda_{b,i}^2 + m^2} \right\rangle$$

  $\rightarrow$ dominated by IR modes up to $\lambda \simeq m$
  (lost for conventional Wilson loop as $m \rightarrow \infty$)
- renormalization:
  - Fourier trafo removes additive renormalization
  - multiplicative divergence cancels in $m \cdot \Sigma$

  $\Rightarrow$ $m\tilde{\Sigma}$ has a meaningful continuum limit

Summary and outlook

- chiral condensate plus magnetic fields to describe confinement
- large probe mass suppresses long loops
  $\Rightarrow \lim_{m \to \infty} m\tilde{\Sigma}(s) \propto W(s)$
- dressed Wilson loops: - IR dominance
  - better renormalization
- meaning of $\tilde{\Sigma}(s)$ at finite mass?
- applicability beyond lattice?