Lattice Planar QED in external magnetic field

P.Cea[1],L.Cosmai[2], **Pietro Giudice[3]**, A.Papa[4]

paolo.cea@ba.infn.it, leonardo.cosmai@ba.infn.it, p.giudice@swan.ac.uk, papa@cs.infn.it

[1]Dipartimento di Fisica dell'Universita' di Bari, Italy; INFN, Sezione di Bari, Italy

[2]INFN, Sezione di Bari, Italy

[3]Department of Physics, College of Science, Swansea University, UK

[4]Dipartimento di Fisica dell'Universita' della Calabria and
INFN, Gruppo collegato di Cosenza, Italy

SMFT 2011, BARI, Italy, 22/September/2011

Introduction (I)

- The vacuum structure of lattice gauge theories can be understood probing it by an external background field $\vec{A}^{\mathrm{ext}}$
- This can be done defining on the lattice a gauge invariant effective action $\Gamma(\vec{A}^{\mathrm{ext}})$ by using the Schrodinger Functional (SF)
  [P. Cea, L. Cosmai, Phys. Rev. D60 (1999) 094506. [hep-lat/9903005]]
- The Euclidean SF in Yang-Mills theories without matter is defined by:
  $Z[A^f, A^i] = \langle A^f | e^{-HT} \mathcal{P} | A^i \rangle$
- NOTE: it is the propagation kernel for going from some field configuration A^i at time $x_4 = 0$ to some other configuration A^f at $x_4 = T$
- The lattice SF is given by $Z[U^f, U^i] = \int DU e^{-S}$
- *S* is the Wilson action modified to take in account the boundaries:

$$U(x)_{x_4=0} = U^i, \quad U(x)_{x_4=T} = U^f$$

Introduction (II)

- We define the lattice effective action for a background field $\vec{A}^{\text{ext}}$:

$$\Gamma(\vec{A}^{\text{ext}}) = -\frac{1}{T}\ln\left(\frac{\tilde{Z}[U^{\text{ext}}]}{\tilde{Z}[0]}\right), \quad \text{where } \tilde{Z}[U^{\text{ext}}] = Z[U^{\text{ext}}, U^{\text{ext}}]$$

- $\Gamma(\vec{A}^{\text{ext}})$ turns out to be invariant under lattice gauge transformation of the external link U^{ext}
- Since in this definition $U^f = U^i$, we have periodic condition in the time direction and the lattice action is now the familiar Wilson action
- It is possible to show that: $\Gamma(\vec{A}^{\text{ext}}) \to E_0(\vec{A}^{\text{ext}}) - E_0(\vec{0})$, [when $T \to \infty$] $E_0(\vec{A}^{\text{ext}})$ is the vacuum energy in presence of the external background
- Therefore $\Gamma(\vec{A}^{\text{ext}})$ is the lattice gauge invariant effective action for the background field $\vec{A}^{\text{ext}}$
- In other words to study a theory with an external background field we have to simulate on the lattice the ”standard” action (without any external field) but introducing proper constraints

U(1) in a uniform external magnetic field

- We impose spatial and temporal boundary conditions
- We constrain the spatial lattice links belonging to a fixed time slice to:
  $U_1^{\rm ext}(\vec{x}) = 1$ and $U_2^{\rm ext}(\vec{x}) = \cos(gHx_1) + i\sin(gHx_1)$ ($x_4 = 0$)
- The same constraints are imposed at the spatial boundaries of the other time slices (fluctuations over the background field vanish at infinity)
- The temporal links are not constrained because this is coeherent with the definition of the correct thermal partition functional
- Because the lattice has the topology of a torus, the magnetic field turns out to be quantized: $$a^2 gH = \frac{2\pi}{L_t} n_{\rm ext} \,, \quad (n_{\rm ext} = 0, 1, \dots)$$
- A different approach to introduce the external magnetic field: [J.Alexandre, K.Farakos, S.J.Hands, G.Koutsoumbas, S.E.Morrison, Phys.Rev. D64 (2001) 034502 [hep-lat/0101011]]

QED in 3d

- The continuum Lagrangian density describing QED3 is given in Minkowski metric by: $\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + \overline{\psi}_i i D_\mu \gamma^\mu \psi_i - m_0 \overline{\psi}_i \psi_i$
- ψ_i ($i = 1, \ldots, N_f$) are 4-component spinors
- QED3 is a super-renormalizable theory, dim[e]=+1/2

- A convenient representation for the γ_μ is the reducible 4×4 representation of the Dirac algebra in three dimensions:

$$\gamma^0 = \begin{pmatrix} \sigma_3 & 0 \\ 0 & -\sigma_3 \end{pmatrix} , \quad \gamma^1 = \begin{pmatrix} i\sigma_1 & 0 \\ 0 & -i\sigma_1 \end{pmatrix} , \quad \gamma^2 = \begin{pmatrix} i\sigma_2 & 0 \\ 0 & -i\sigma_2 \end{pmatrix}$$

- We define also two more matrices anticommuting with them:

$$\gamma^3 = i \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} , \quad \gamma^5 = i \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

QED in 3d

- The massless theory will therefore be invariant under the chiral transformations: $\psi \to e^{i\alpha\gamma^3}\psi\ , \qquad \psi \to e^{i\beta\gamma^5}\psi$
- If we write the 4-component spinor as 2-component spinors: $\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$
- Then the mass term becomes: $m\overline{\psi}\psi = m\psi_1^\dagger\sigma_3\psi_1 - m\psi_2^\dagger\sigma_3\psi_2$
- Since in three dimensions the parity transformation reads:

$$\psi_1(x_0, x_1, x_2) \to \sigma_1\psi_2(x_0, -x_1, x_2)$$
$$\psi_2(x_0, x_1, x_2) \to \sigma_1\psi_1(x_0, -x_1, x_2)$$

- Then $m\overline{\psi}\psi$ is parity conserving

Our model

- We want to study QED3 with $N_f = 2$ flavours of 4-component fermions using the staggered fermion approach
- We need to simulate *N=1* staggered fermions fields $\chi, \bar{\chi}$ with the Euclidean action:

$$S = S_G + \sum_{i=1}^{N} \sum_{n,k} \overline{\chi}_i(n) M_{n,k} \chi_i(k)$$

- The fermion matrix is given by ($\eta_\nu(n) = (-1)^{n_1 + \ldots + n_{\nu-1}}$):

$$M_{n,k}[U] = \sum_{\nu=1,2,3} \frac{\eta_\nu(n)}{2} \left\{ [U_\nu(n)]\delta_{k,n+\hat{\nu}} - [U_\nu^\dagger(k)]\delta_{k,n-\hat{\nu}} \right\} + m\ \delta_{n,k}$$

- We choose the compact formulation of QED:

$$S_G[U] = \beta \sum_{n,\mu<\nu} \left[1 - \frac{1}{2} \left(U_{\mu\nu}(n) + U_{\mu\nu}^\dagger(n) \right) \right]$$

- $\beta = 1/(e^2 a)$
- The introduction of the fermions in the theory does not change anything about the way we introduce the external field $\vec{A}^{\text{ext}}$

Dynamical symmetry breaking

- It is a general result that a constant magnetic field leads to the generation of a fermion dynamical mass: ”magnetic catalysis” [P.Cea, L.Tedesco, J.Phys.G {26} (2000) 411 [hep-th/9909029]] [V.P.Gusynin, V.A.Miransky, I.A.Shovkovy, Phys.Rev.Lett. {73} (1994) 3499-3502 [hep-ph/9405262]]
- It is possible to evaluate the chiral condensate in the one-loop approximation [P.Cea, [arXiv:1101.5703 [cond-mat.mes-hall]]]:

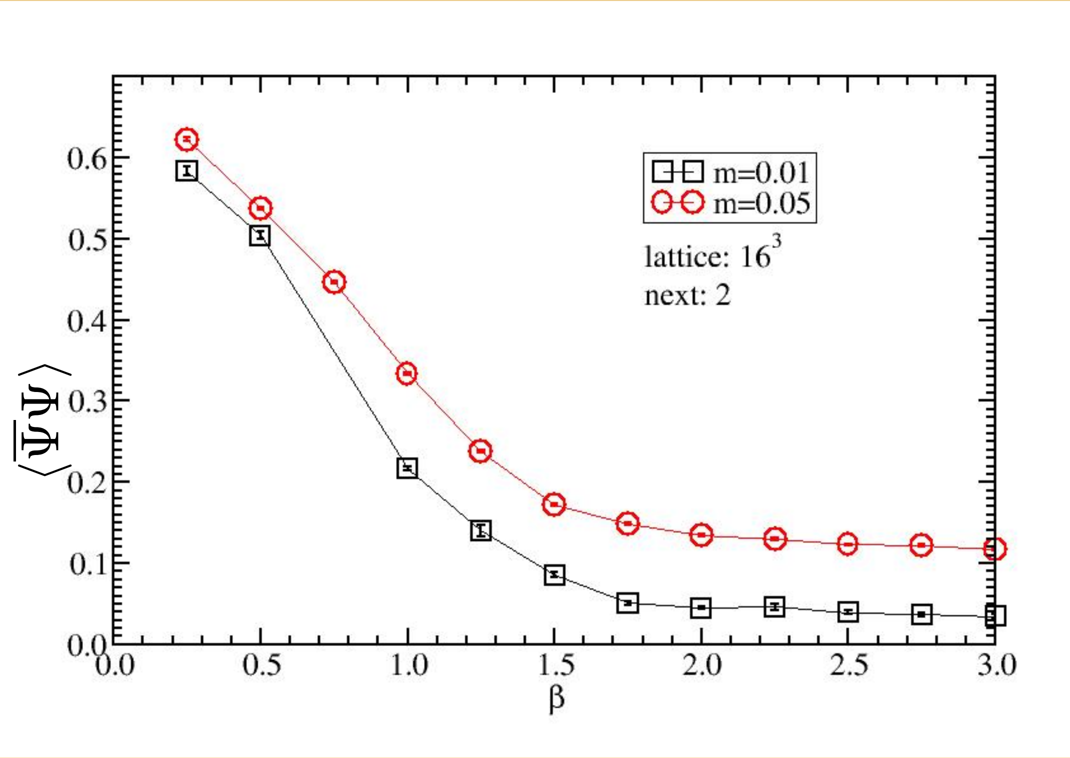
$$\langle \overline{\Psi}\Psi \rangle \; = \; -2\, N_f \, |m| c^2 \, \frac{\hbar c e H}{2\pi} \sum_{n=1}^{\infty} \frac{1}{\sqrt{2n\hbar c e H + m^2 c^4}} \qquad \Delta_0 = mc^2$$

- After regularization of the integral and for $\frac{\Delta_0}{\sqrt{H(T)}} \ll \sqrt{2\hbar v_F^2 e/c} \approx 420K \times k_B$

$$\langle \overline{\Psi}\Psi \rangle \; \simeq \; - \frac{\hbar c e H}{2\pi} N_f \frac{\zeta(\frac{1}{2})}{\sqrt{\pi}} \frac{\Delta_0}{\sqrt{\frac{\hbar c e H}{2\pi}}} \qquad \Rightarrow \frac{\langle \overline{\Psi}\Psi \rangle}{\frac{eH}{2\pi}}, \quad \frac{m}{\sqrt{\frac{eH}{2\pi}}} \qquad [cgs]$$

Numerical results (I)

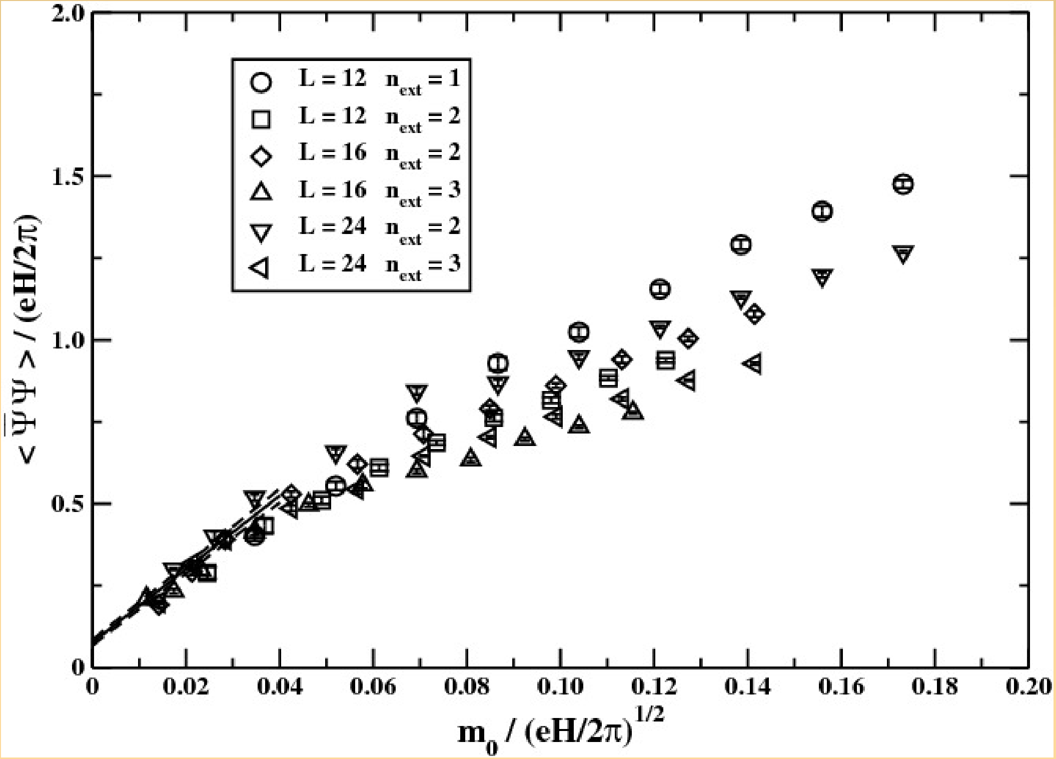
- The choice of β is based on [R.Fiore, P.Giudice, D.Giuliano, D.Marmottini, A.Papa, P.Sodano, Phys.Rev.{D72 } (2005) 094508]



- Only one value: 2.0
- In progress: 2.5

Numerical results (II)

- Simulations with β=2.0, L=12,16,24; next=1,2,3; m=0.005-0.05;



- $x = \frac{m_0}{\sqrt{eH/2\pi}}$
- Scaling law for $x \lesssim 0.04$
- In the chiral limit ($x \to 0$):

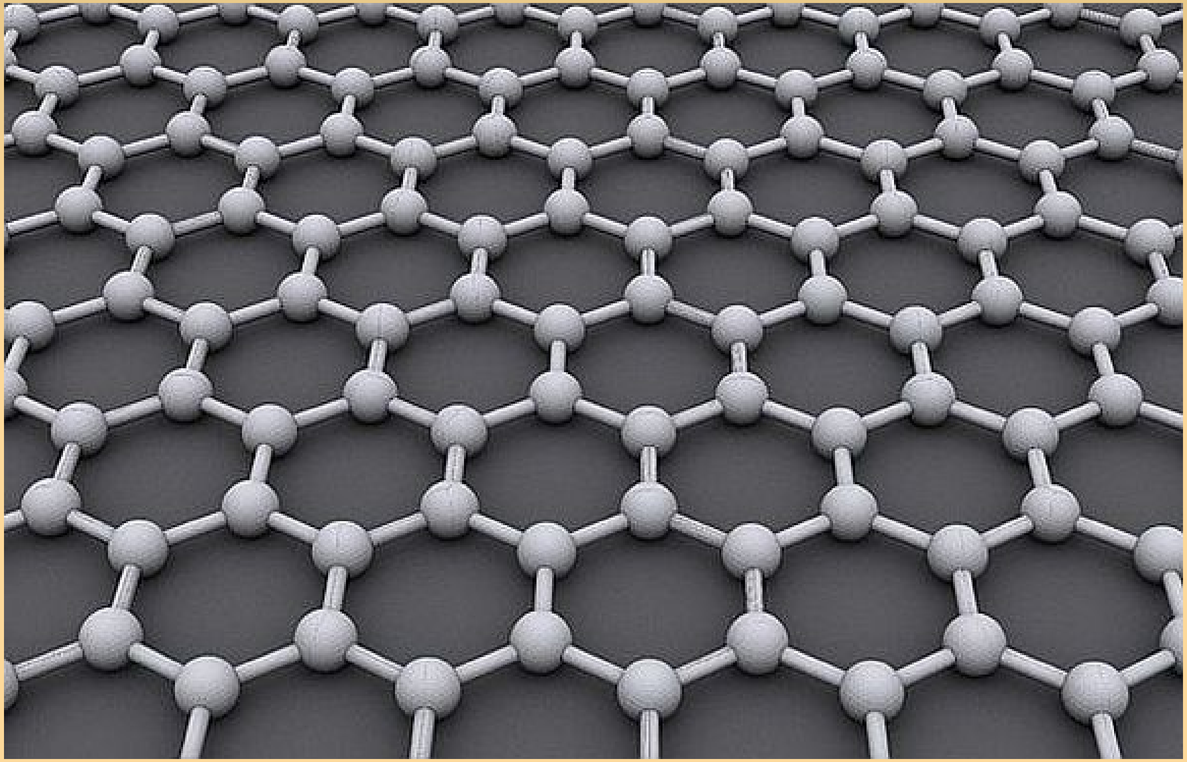
$$\frac{\langle\overline{\Psi}\Psi\rangle}{\frac{eH}{2\pi}} = a_0 + a_1 x$$

- $$a_0 = 0.07668 \pm 0.00930$$
  $$a_1 = 11.20 \pm 0.48$$

$$\langle\overline{\Psi}\Psi\rangle = \frac{\hbar c e H}{2\pi}(0.07668 \pm 0.00930)$$

Graphene, introduction

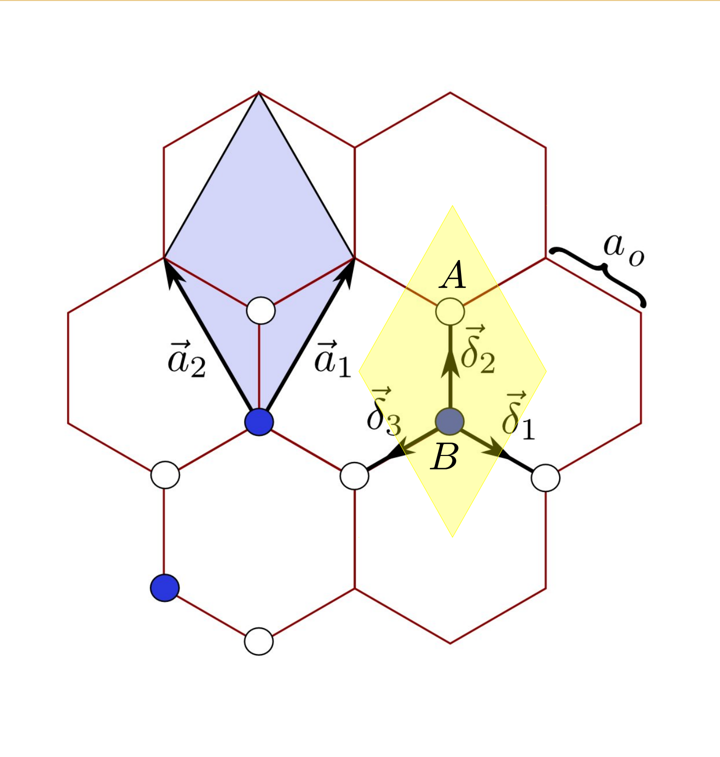
- Graphene is a honeycomb (hexagonal) lattice made of carbon atoms:



- It has one valence electron per atomic site
- It is a semi-metal or zero-gap semiconductor
- $v_F \approx 1.0 \times 10^8 cm/s$
- $a_0 \approx 1.42 \times 10^{-8} cm$

Graphene, real space lattice

- The theory of graphene was first explored by
  [P.R.Wallace,"The Band Theory of Graphite", Phys.Rev.71 (9) (1947) 622]



[N.M.R.Peres,Rev.Mod.Phys.82:2673-2700,2010]

• The unit cell is a rhombus and contains two atoms A and B (yellow shadow)
• The Bravais lattice is triangular

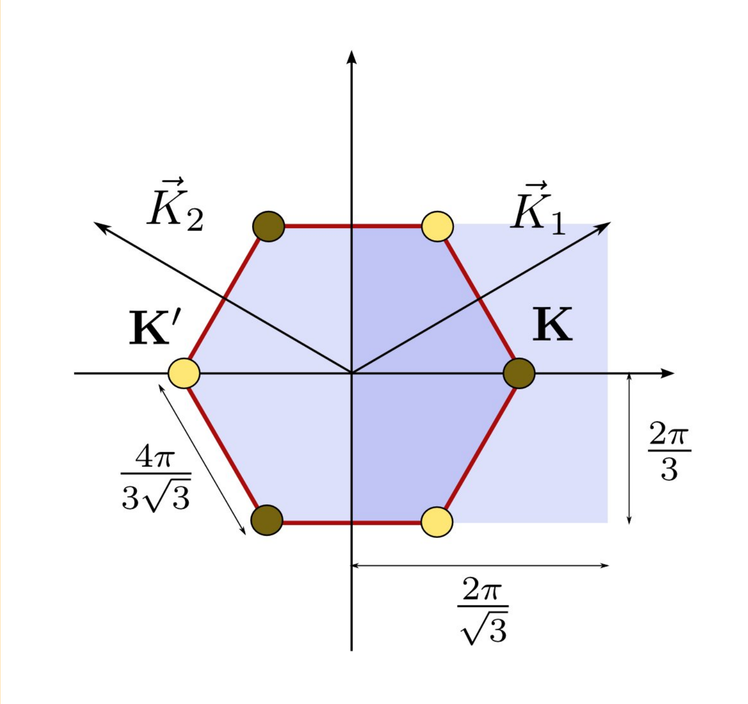
•The hexagonal lattice is made of two interpenetrating triangular Bravais lattices
•The basis vectors are: $\vec{a}_1, \vec{a}_2$

• $\vec{A}(n_1, n_2) = n_1\vec{a}_1 + n_2\vec{a}_2$

• $\vec{B}(m_1, m_2) = m_1\vec{a}_1 + m_2\vec{a}_2 + \vec{\delta}_2$

Graphene, reciprocal lattice

- The Brillouin zone is an hexagon
- The reciprocal lattice basis vectors are: $\vec{K}_1, \vec{K}_2; \quad [\vec{K}_i \vec{a}_j = 2\pi\delta_{ij}]$
- There are two special, non-equivalent (i.e. not connected by a reciprocal lattice vector) corners of the BZ, termed K and K'



[N.M.R.Peres,Rev.Mod.Phys.82:2673-2700,2010]

Tight-binding model (I)

- If we conside only the nearest-neighbor interaction between electrons

$$H = -t \sum_{\vec{A},i} \left[U^\dagger(\vec{A}) V(\vec{A}+\vec{\alpha}_i) + V^\dagger(\vec{A}+\vec{\alpha}_i) U(\vec{A}) \right]$$

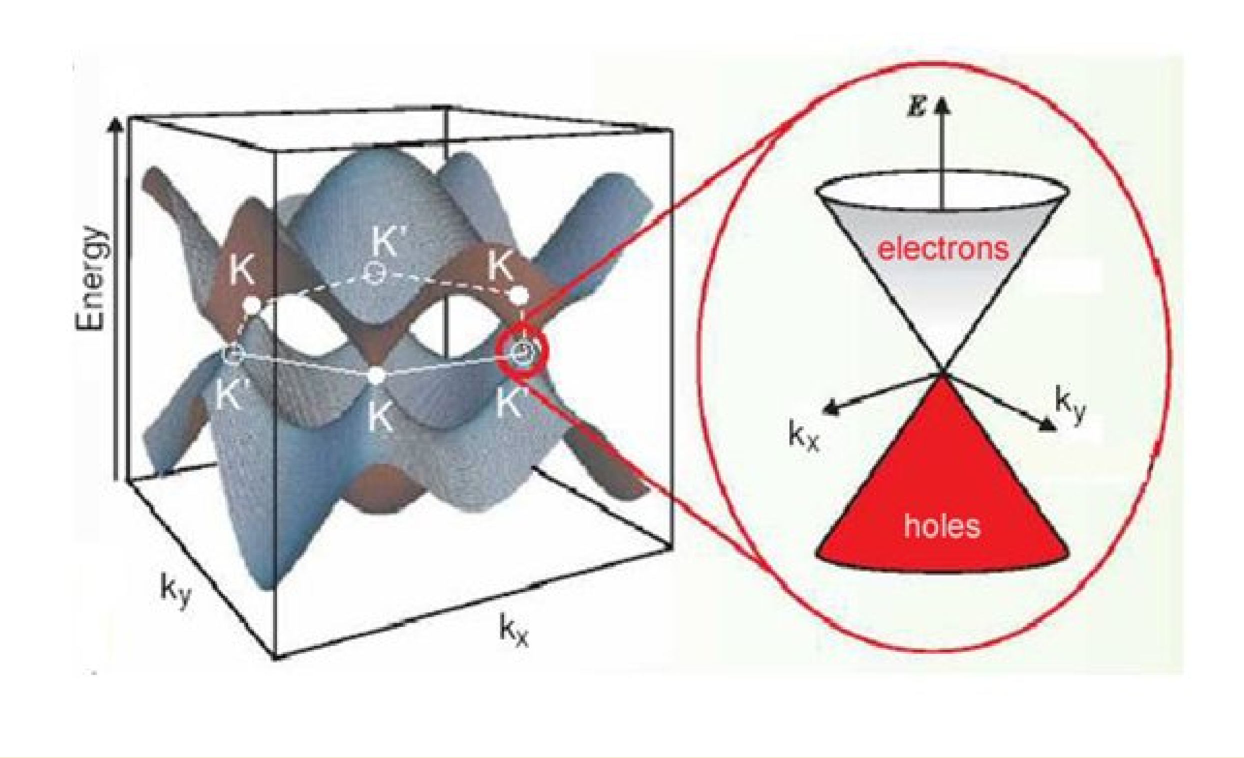
- The hopping parameter (related to the probability amplitude for electron transfer between neighboring sites) is $t \approx 2.7eV$
- Diagonalising H, we get the energy eigenvalues:

$$E(k_1, k_2) = \pm t \sqrt{1 + 4\cos(\sqrt{3}k_1 a_0/2)\cos(k_2 a_0/2) + 4\cos^2(k_2 a_0/2)}$$

- There are two bands, one at negative energies (hole/valence band) and the other at positive ones (a particle/conduction band)
- $E(\vec{k}) \to 0$, at the corners of the BZ (where also the Fermi energy lies: a finite number of Fermi points is quite unusual!)

Tight-binding model (II)

- In the continuum limit, $a_0 \to 0$ (low energy) only the electron states near K and K' partecipate in the dynamics and the energy dispersion relation is linear: $E(\vec{k}) = \pm v_F \hbar |\vec{k}|$, where $v_F = 3ta_0/2$
- Correspondly, $H = v_F \vec{\sigma}\vec{k}$: a field theory of 2 massless Dirac spinors in 2 dimensions [G.W.Semenoff, Phys.Rev.Lett. 53 (1984) (26) 5449]

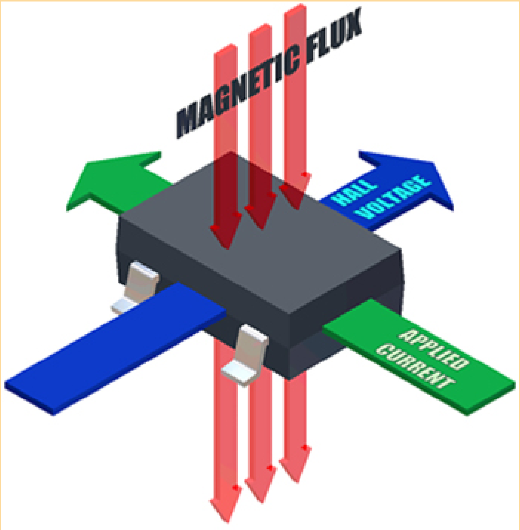


- electrons and holes are called Dirac fermions
- the six corners of the BZ are called the Dirac points
- "valley" degeneracy in the spectrum because K and K'

Graphene in a magnetic field

- If H is applied perpendicularly to a conventional 2d electron gas, we have the Landau levels: $E_n = \hbar\,\omega_c(n+1/2)$, where $\omega_c = eH/mc$
- Every LL has a degeneracy density: $g = eH/hc$
- In graphene, because the relativistic massless dispersion relation, we have non-equidistant Landau levels: $E_n = \mathrm{sign(n)}\sqrt{2\hbar eH|n|\frac{v_F^2}{c}},\quad n = 0, \pm 1, \ldots$

Hall Effect (HE):



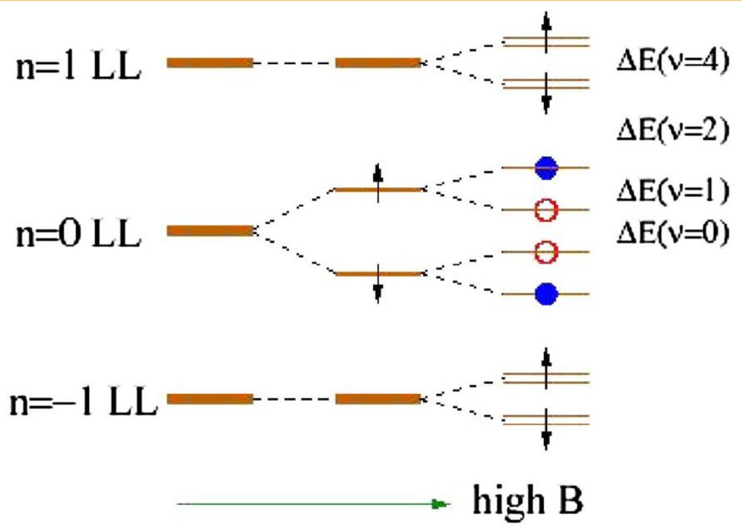
- Conductance: $\sigma_{xy} = \frac{e^2}{h}\nu$, ν filling factor
- Integer QHE: $\nu = 0, \pm 1, \pm 2, \ldots$
- In Graphene, anomalous QHE (because $E = 0$):

$$\nu = \pm 4\left(N + \frac{1}{2}\right) = \pm 2, \pm 6, \pm 10, \ldots$$

(the factor 4 because spin and valley degeneracy)

A new discovery (I)

- In a very strong magnetic field (up to 45T) a new set of QH states at filling $\nu = 0, \pm 1, \pm 4$ [Y.Zhang et al, Phys.Rev.Lett. {96}, 136806 (2006)]
- This implies that the 4-fold degeneracy is now lifted:



- n=0 degeneracy: fully lifted
- n=1 degeneracy: partially lifted

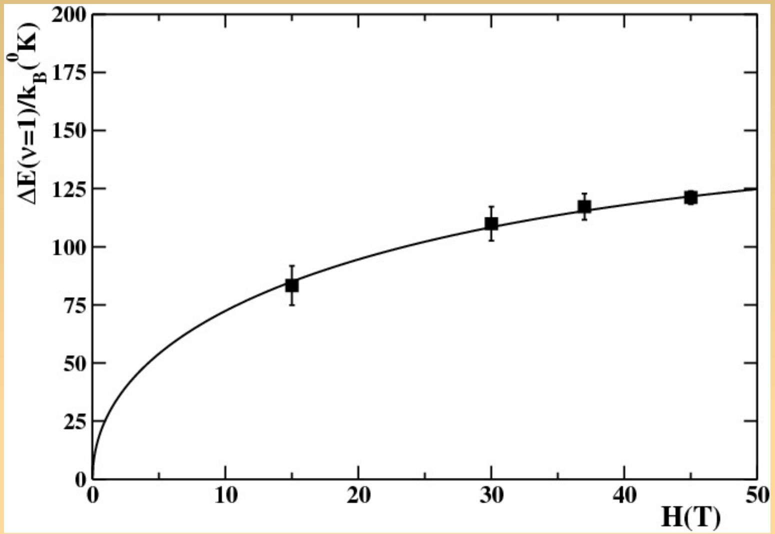
- What is the ORIGIN of the lifting of these degeneracies ?
  - (Zeeman) spin splitting
  - Valley symmetry breaking and GAP formation

- $\nu = 0, \pm 4$ it is belived that they are spin states

- $\nu = \pm 1$ is therefore related to the valley symmetry breaking and consequently to the generation of a GAP $\Delta_0(H)$

A new discovery (II)

[Z.Jiang et al, Phys.Rev.Lett. {99}, 106802 (2007)]



- We fitted these data by (using $\Delta_0(H) \propto \sqrt{H}$):

$$\Delta E(\nu = 1) = 2\left(\Delta_0(H) - \frac{g}{2}\mu_B H\right)$$

- We get: $\Delta_0(H) = (13.57 \pm 0.28)K \times k_B\sqrt{H}$

- It is belived that the generation of the gap is driven by the electron-electron interaction (in a magnetic field) [V.N.Kotov et al, arXiv:1012.3484]
- In this picture: $\Delta_0(H) \approx \frac{e^2}{\epsilon}\sqrt{\frac{eH}{\hbar c}} \approx 163K \times k_B\sqrt{H}$
- [P.Cea, [arXiv:1101.5703 [cond-mat.mes-hall]]] shows that, in the graphene, a dynamical gap is energetically convenient and $\Delta_0(H) \propto \sqrt{H}$
- Moreover, the proposal is that the GAP is generated by spontaneous symmetry breaking

Our approach (I)

- We think that it is possible to use our QED3 result to estimate correctly the value of the GAP
- Usually QED3 is not used in the graphene context because:
  - fermions 2d, photons 3d (3d coulomb interaction)
  - relativistic invariance is broken (at m=0: fermions v_F, photons c)
- How we circunvent these problems:
  - We think that the Coulomb interaction can be neglected for our purpose: in fact in 2d we would have: $\Delta_0 \propto e^2 \ln(H)$ but, at posteriori, we see that $\Delta_0 \propto \sqrt{H}$: so in such a way it is not important that we consider 2d or 3d
  - We get the relevant result with the substitution: $c \to v_F^2/c$

Our approach (II)

- Combining:

$$\langle\overline{\Psi}\Psi\rangle \simeq -\frac{\hbar c e H}{2\pi} N_f \frac{\zeta(\frac{1}{2})}{\sqrt{\pi}} \frac{\Delta_0}{\sqrt{\frac{\hbar c e H}{2\pi}}}$$ and $$\langle\overline{\Psi}\Psi\rangle = \frac{\hbar c e H}{2\pi}(0.07668 \pm 0.00930)$$

- We get: $$\Delta_0 \simeq -\frac{\sqrt{\pi}}{N_f\zeta(\frac{1}{2})}\sqrt{\frac{\hbar c e H}{2\pi}}(0.07668 \pm 0.00930)\,.$$

- To restore the correct asymmetry between fermions and photons:

$$\Delta_0 \simeq -\frac{\sqrt{\pi}}{N_f\zeta(\frac{1}{2})}\sqrt{\frac{\hbar \frac{v_F^2}{c} e H}{2\pi}}(0.07668 \pm 0.00930)$$

- Finally: $\Delta_0(H) = (5.52 \pm 0.67)\,\mathrm{K} \times k_B\,\sqrt{H(T)} \ll 420\,\mathrm{K} \times k_B\,\sqrt{H(T)}$

  compare with the experimental value: $\Delta_0(H) = (13.57 \pm 0.28) K \times k_B \sqrt{H}$

- Is it good? 1) 1-loop formula; 2) small GAP hypothesis; 3) v_F error

Conclusion

- We have verified that in QED3, in the chiral limit, with a magnetic background, there is a dynamical symmetry breaking and:

$$\langle \overline{\Psi}\Psi \rangle = \frac{\hbar c e H}{2\pi} (0.07668 \pm 0.00930)$$

- We applied our numerical result to determine the value of the GAP that explains the observed new quantum Hall states for n=0 Landau level under very strong magnetic field:

$$\Delta_0(H) = (5.52 \pm 0.67)\,\mathrm{K} \times k_B \sqrt{H(T)}$$