Nonreciprocal wave propagation in a nonlinear system

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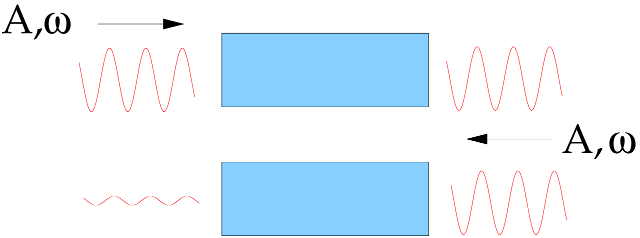
Introduction

- Control of wave propagation
- "Wave diode"
- Simple model: Discrete Nonlinear Scrödinger (DNLS)
- Applications: BEC, photonic/phononic lattices etc.

S.L., G. Casati, Phys. Rev. Lett. 106, 164101 (2011)

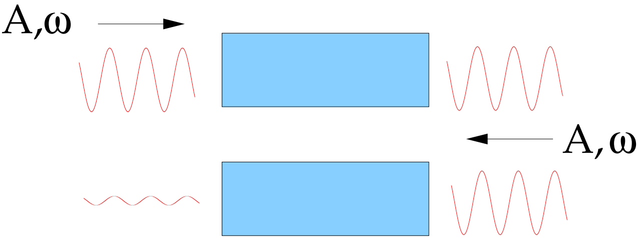
The Reciprocity theorem



Lord Rayleigh *"The theory of sound"*:

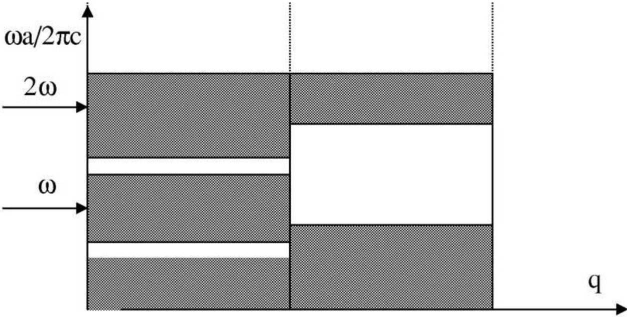
> *Let A and B be two points ... between which are situated obstacles of any kind. Than a sound originating at A is perceived at B with the same intensity as that with which an equal sound originating at B would be perceived at A. In acoustics ... in consequence of the not insignificant value of the wavelength in comparison with the dimension of ordinary obstacles the reciprocal relation is of considerable interest*

The (ideal) “wave diode”



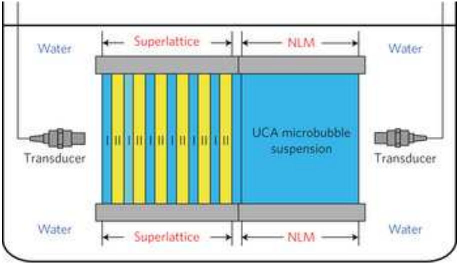
To violate the *reciprocity theorem* (without breaking time-reversal) both asymmetry *and* nonlinearity are necessary !

Frequency doublers



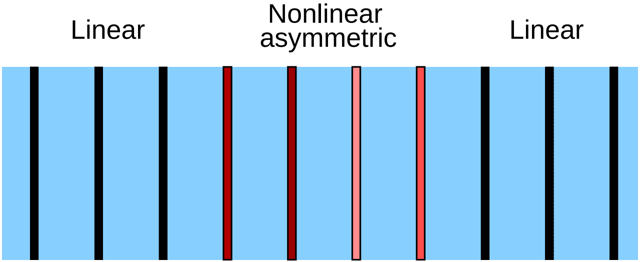
[Konotop and Kuzmiak, PRB (2002)]

Acoustic rectifier



[Liang et al. Nature Materials (2010)]

Layered photonic or phononic crystal



For linear propagation perpendicular to the layers:

$$\cos k(d_1 + d_2) = \cos(\frac{\omega d_1}{c_1}) \cos(\frac{\omega d_2}{c_2}) - \frac{1}{2}(\frac{c_1}{c_2} + \frac{c_2}{c_1}) \sin(\frac{\omega d_1}{c_1}) \sin(\frac{\omega d_2}{c_2})$$

DNLS approximation

- Thin layers $d_1 \ll d_2$: "Kronig-Penney model"
- Approximate dispersion for high-frequency bands: $\omega(k) = \omega_0 \pm 2C \cos kd$ (single band approx.)
- Defective layers
- Kerr nonlinearity
- Rescale units, band center at $\omega = 0$

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Altogether:

$$i\dot{\phi}_n = V_n\phi_n - \phi_{n+1} - \phi_{n-1} + \alpha_n|\phi_n|^2\phi_n$$

Conservation of energy and norm, no harmonics.

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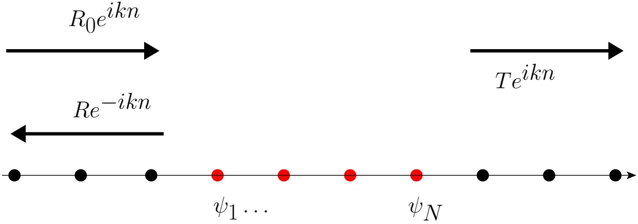
Transmission problem

Stationary DNLS, $\phi_n = \psi_n e^{-i\omega t}$, $V_n \neq 0$ and $\alpha_n \neq$ for $1 \leq n \leq N$

$$\omega\psi_n = V_n\psi_n - \psi_{n+1} - \psi_{n-1} + \alpha_n|\psi_n|^2\psi_n$$



$$\omega = -2\cos k, \qquad 0 \leq k \leq \pi$$

Transmission problem

Look for complex solutions such that:

$$\psi_n = \begin{cases} R_0 e^{ikn} + R e^{-ikn} & n \leq 1 \\ T e^{ikn} & n \geq N \end{cases}$$

- ψ_n complex, current $J = 2|T|^2 \sin k$
- For $-k$: $(V_n, \alpha_n) \longrightarrow (V_{N-n+1}, \alpha_{N-n+1})$ ("flip the sample")
- For $\alpha_n = 0$: reciprocity for any V_n
- To break the mirror symmetry: $V_n \neq V_{N-n+1}$ and/or $\alpha_n \neq \alpha_{N-n+1}$

Reduction to nonlinear map

Let $u_n = \psi_n$ and $v_n = \psi_{n+1}$. Back iterating from $u_N = T\,\mathsf{exp}(ikN)$, $v_N = T\,\mathsf{exp}(ik(N+1))$

$$u_{n-1} = -v_n + (V_n - \omega + \alpha_n |u_n|^2)u_n, \quad v_{n-1} = u_n$$

Map is area preserving.
For given T and k

$$R_0 = \frac{\mathsf{exp}(-ik)u_0 - v_0}{\mathsf{exp}(-ik) - \mathsf{exp}(ik)}, \quad R = \frac{\mathsf{exp}(ik)u_0 - v_0}{\mathsf{exp}(ik) - \mathsf{exp}(-ik)}$$

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$$R_0 = \frac{\exp(-ik)u_0 - v_0}{\exp(-ik) - \exp(ik)}, \quad R = \frac{\exp(ik)u_0 - v_0}{\exp(ik) - \exp(-ik)}$$

Transmission coefficient

$$t(k, |T|^2) = \frac{|T|^2}{|R_0|^2}$$

The simplest case: the dimer $N = 2$

For $k > 0$:

$$t = \left| \frac{e^{ik} - e^{-ik}}{1 + (\nu - e^{ik})(e^{ik} - \delta)} \right|^2$$

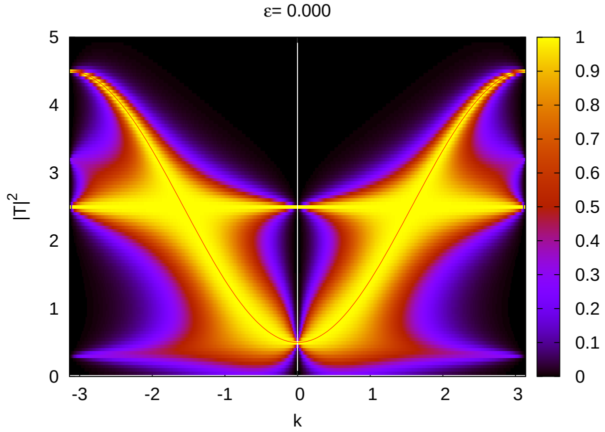
$$\delta = V_2 - \omega + \alpha_2 T^2, \quad \nu = V_1 - \omega + \alpha_1 T^2 [1 - 2\delta \cos k + \delta^2].$$

For $k < 0$: exchange the subscripts 1 and 2
Symmetric case ($V_{1,2} = V_0, \alpha_{1,2} = \alpha$): two nonlinear resonances

$$V_0 + \alpha T^2 = 0 \qquad (V_0 < 0)$$
$$V_0 + \alpha T^2 = \omega \qquad (V_0 < \omega)$$

The dimer $N = 2$: reciprocity breaking

$$V_{1,2} = -2.5(1 \pm \varepsilon) \qquad \alpha_{1,2} = 1$$



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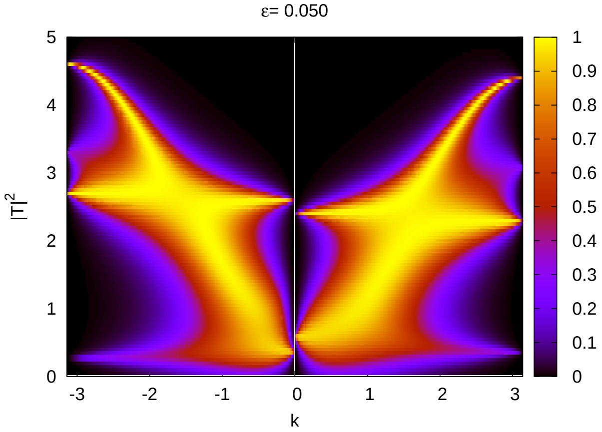
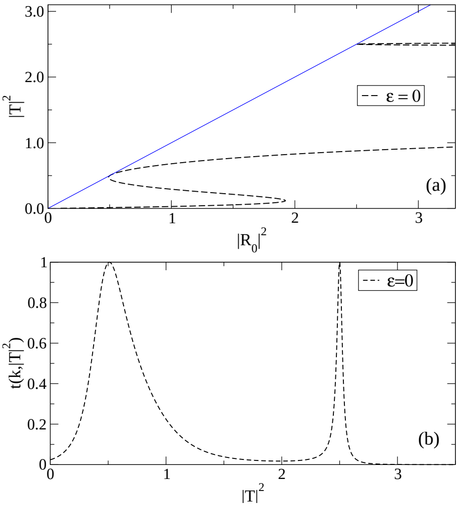


The dimer $N = 2$: transmission curves



$V_0 = -2.5, \alpha = 1, |k| = 0.1, \varepsilon = 0.05$

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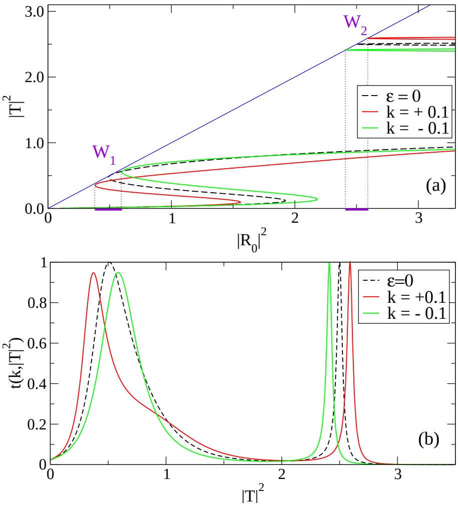

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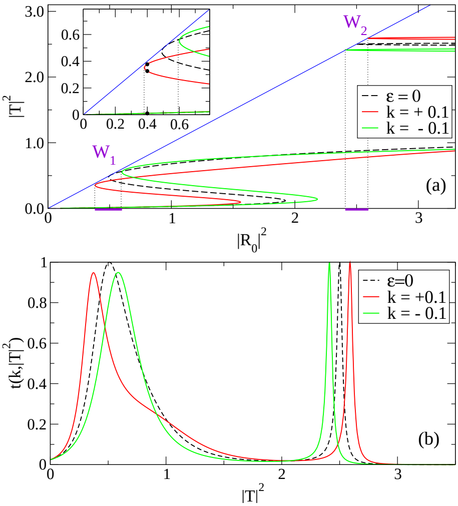

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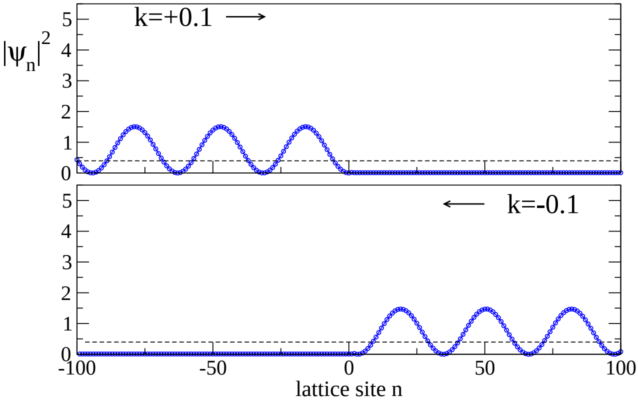
The dimer $N = 2$: multistability

In the window W_1



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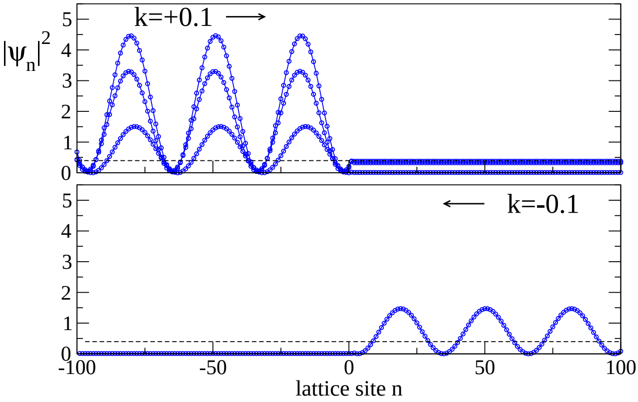
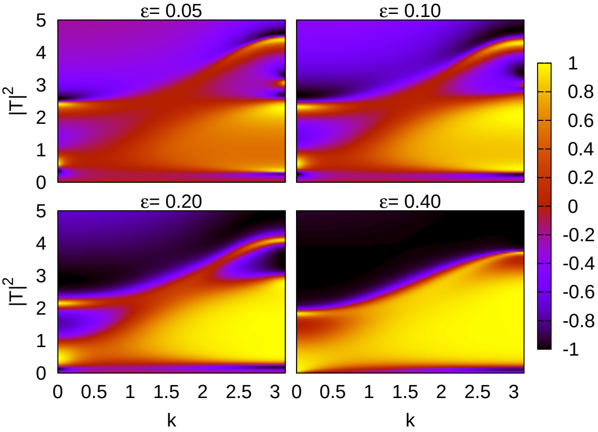


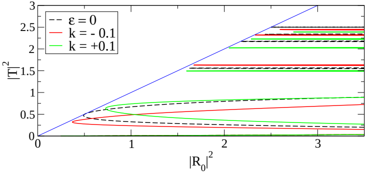
$V_0 = -2.5, \alpha = 1, |k| = 0.1, \varepsilon = 0.05$

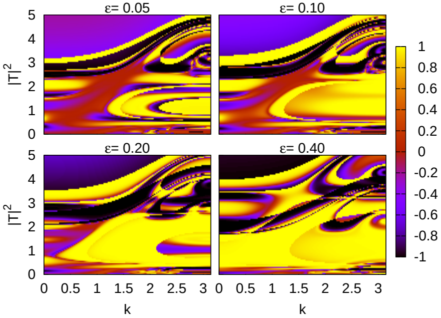
Rectification factor

$$f \;=\; \frac{t(k,|T|^2)-t(-k,|T|^2)}{t(k,|T|^2)+t(-k,|T|^2)}$$



Several layers



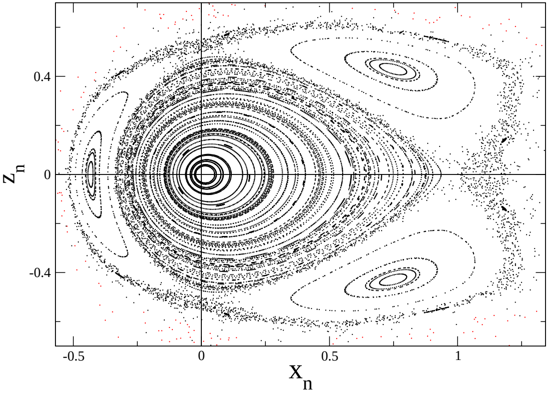


$$N = 4, V_n = -2.5[1 + \varepsilon(1 - 2(n-1)/(N-1)], \alpha = 1$$

Consequence of the mixed phase-space of the map!

2D transfer map

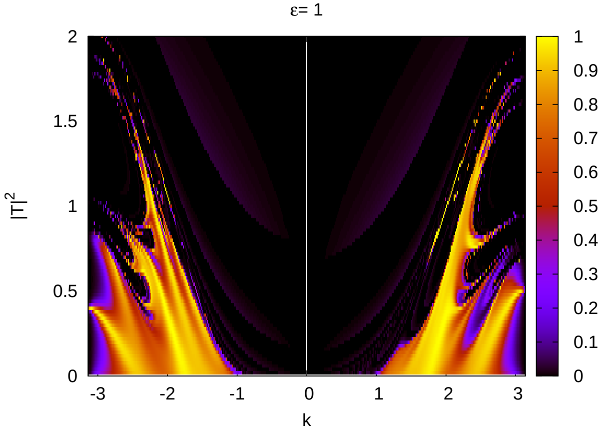
$$x_n = u_n^* v_n + u_n v_n^*, \quad z_n = |u_n|^2 - |v_n|^2$$



$V_0 = -2.5, \alpha = 1, k = 0.1, \varepsilon = 0,$ 100values of T^2

Nonlinear Anderson model (in progress)

$N = 10$; V_n random in $[-\varepsilon, \varepsilon]$ $\alpha_{1,2} = 1$



Dynamical stability (in progress)

Nonstandard linear stability problem: integrate out the ψ_n, $n \leq 0$ and $n > N$ the problem is equivalent to the following boundary conditions

$$\psi_0(t) = F_0(t) - i \int_0^t G(t-s)\psi_1(s)ds$$
$$\psi_{N+1}(t) = F_{N+1}(t) - i \int_0^t G(t-s)\psi_N(s)ds$$

Memory term:

$$G(t) \equiv G_{0,0} = \frac{J_1(2t)}{t}$$

From Hamiltonian problem to *driven, dissipative*

Dynamical stability

- Let $\phi_n = (\psi_n + \chi_n) e^{-i\omega t}$, $|\chi_n|$ "small"

$$\chi_n \; = \; A_n \, \mathsf{exp}(i\lambda t) + B_n^* \, \mathsf{exp}(-i\lambda^* t)$$

  instability for $Im\lambda < 0$.
- Linear integro-differential equations for χ_n
- The system is thus a *driven, dissipative* and the type of bifurcations scenario may change.
- "Chaotic wave scattering" (already for $N = 2$?) ....

Wavepacket transmission

Numerical simulation on a finite lattice $|n| < M$

$$i\dot{\phi}_n = V_n\phi_n - \phi_{n+1} - \phi_{n-1} + \alpha_n|\phi_n|^2\phi_n$$

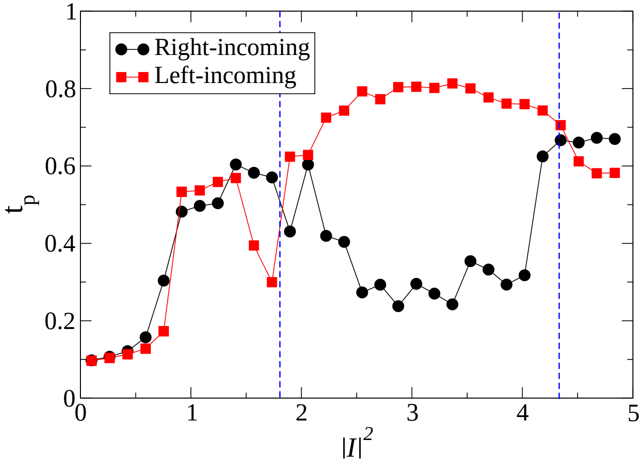
Initial condition: Gaussian

$$\phi_n(0) \;=\; I \exp\left[-\frac{(n-n_0)^2}{w} + ik_0 n\right]$$

Transmission coefficient (for $n_0 < 0$)

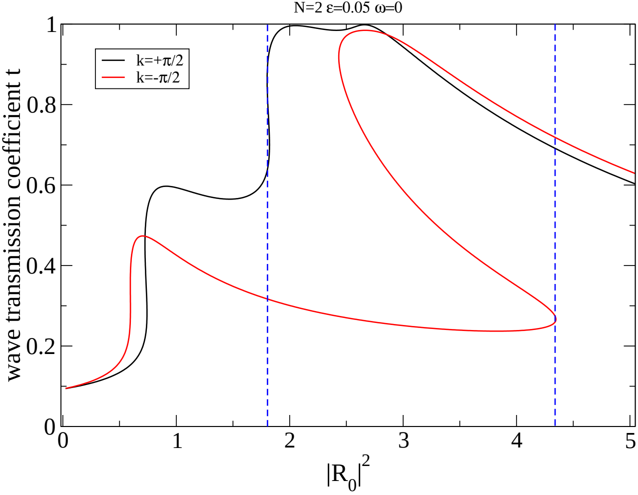
$$t_p = \frac{\sum_{n>N} |\phi_n(t_{fin})|^2}{\sum_{n<0} |\phi_n(0)|^2}$$

Wavepacket transmission



$$V_0 = -2.5, |k_0| = 1.57, \varepsilon = 0.05, M = 500, n_0 = \pm 250, w = 10^4$$

Wavepacket transmission



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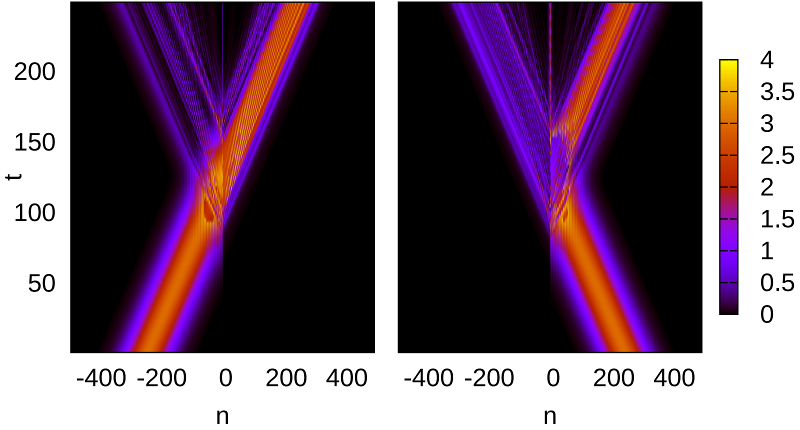

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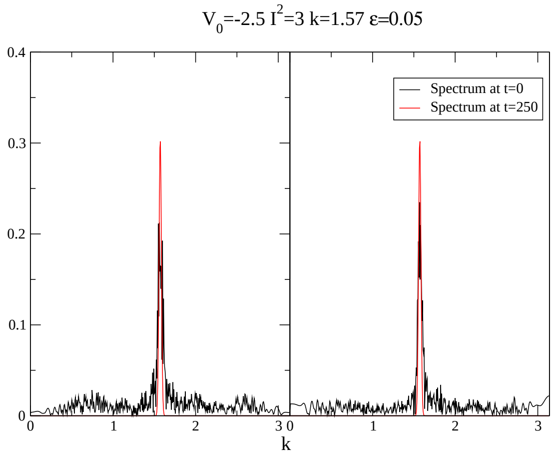

... my 15 minutes of fame

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