The Delta-Statistics of Uncoventional Quarkonium-like Resonances

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Outline

Unconventional Hadrons

Random Matrix Theory

Statistical Analysis

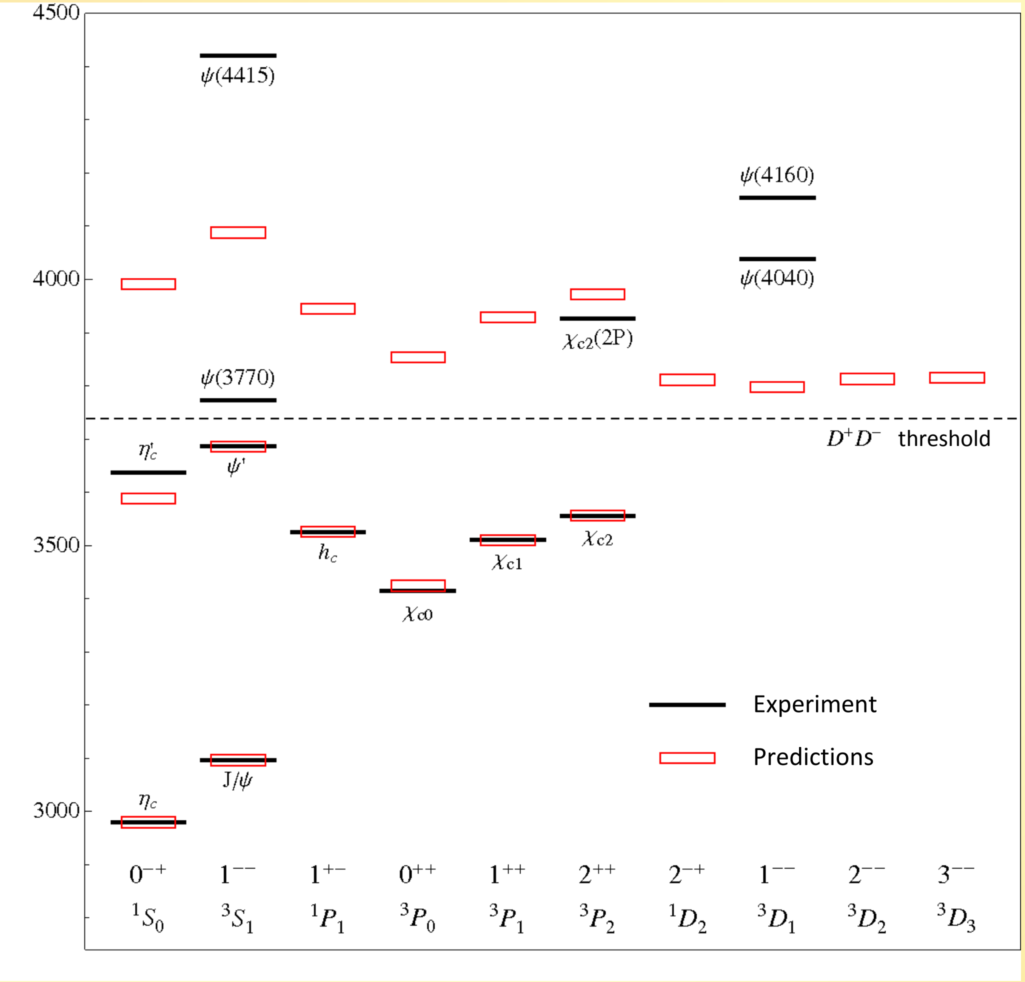
Conclusions

E. N. M. Cirillo, M. Mori and A. D. Polosa,
"The Delta-Statistics of Unconventional Quarkonium-like Resonances",
arXiv:1106.4497 [hep-ph].

Standard Quarkonia

Charmonia and bottomonia spectroscopy provide good tests for QCD (NRQCD, lattice, …).

Good agreement between theory and experiments, expecially below open-flavour threshold



Newly discovered standard charmonia

$\eta_c(2S)$	(2002)
$h_c(1P)$	(2004)
$\chi_{2c}(2P)$	(2005)

Unconventional Charmonia: *X(3872)*

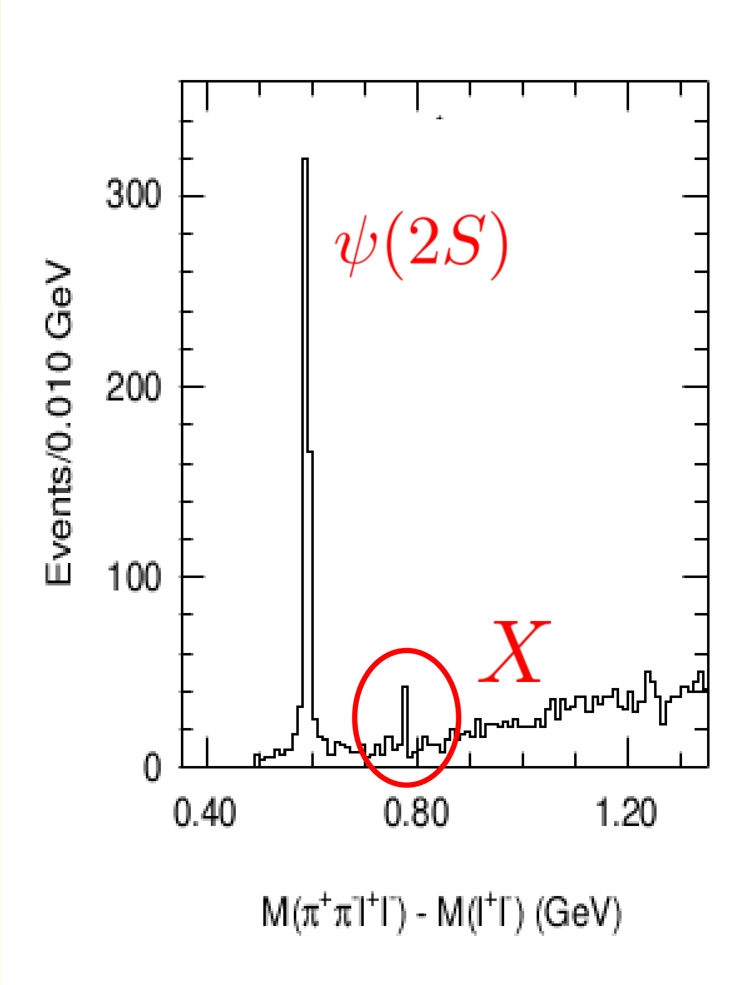
First exotic charmonium state discovered (Belle, 2003)

Total width smaller than 3 MeV!

Strong isospin violation:

$$\frac{BR\,(X(3872) \to J/\psi\,\omega)}{BR\,(X(3872) \to J/\psi\,\rho)} = 0.8 \pm 0.3$$

Quantum numbers: $1^{++}\,/\,2^{-+}$



Mass of the resonance very close to the *DD** threshold...

$$M(X(3872)) - M(D) - M(D^*) = -0.16 \pm 0.33\,\mathrm{MeV}$$

Unconventional Charmonia: *Y(4260)*

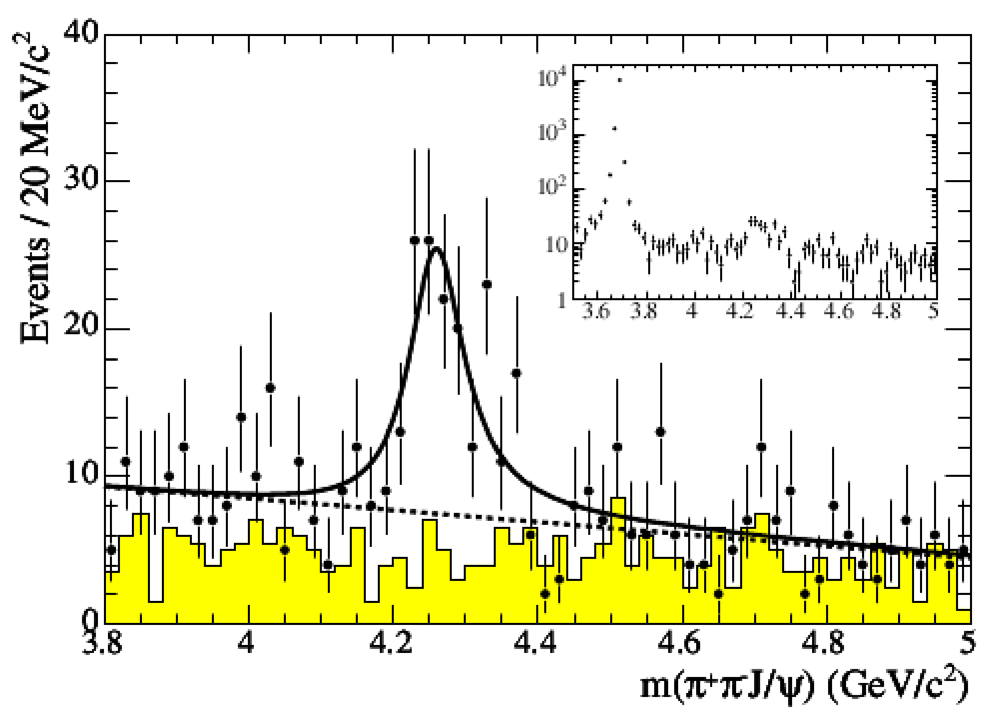
In Initial State Radiation processes the effective center-of-mass energy is lower than the sum of the initial e^+e^- energies: radiated photons can substract energy before the collisions occurs.

Lots of new 1^{--} resonances discovered!

Y(4260) state (BaBar, 2005)

Its mass is well above the open charm threshold, but:

$$\frac{BR(Y(4260) \to D\bar{D})}{BR(Y(4260) \to J/\psi\pi^+\pi^-)} < 1$$



Unconventional Charmonia: summary

State	m (MeV)	Γ (MeV)	J^{PC}	Process (mode)	Experiment (#σ)	Year	Status
$X(3872)$	3871.52 ± 0.20	1.3 ± 0.6	$1^{++}/2^{-+}$	$B\to K(\pi^+\pi^- J/\psi)$	Belle [85, 86] (12.8), BABAR [87] (8.6)	2003	OK
		(<2.2)		$p\bar{p}\to(\pi^+\pi^- J/\psi)+\ldots$	CDF [88–90] (np), DØ [91] (5.2)		
				$B\to K(\omega J/\psi)$	Belle [92] (4.3), BABAR [93] (4.0)		
				$B\to K(D^{*0}\bar{D}^0)$	Belle [94, 95] (6.4), BABAR [96] (4.9)		
				$B\to K(\gamma J/\psi)$	Belle [92] (4.0), BABAR [97, 98] (3.6)		
				$B\to K(\gamma\psi(2S))$	BABAR [98] (3.5), Belle [99] (0.4)		
$X(3915)$	3915.6 ± 3.1	28 ± 10	$0/2^{?+}$	$B\to K(\omega J/\psi)$	Belle [100] (8.1), BABAR [101] (19)	2004	OK
				$e^+e^-\to e^+e^-(\omega J/\psi)$	Belle [102] (7.7)		
$X(3940)$	3942^{+9}_{-8}	37^{+27}_{-17}	$?^{?+}$	$e^+e^-\to J/\psi(D\bar{D}^*)$	Belle [103] (6.0)	2007	NC!
				$e^+e^-\to J/\psi\ (\ldots)$	Belle [54] (5.0)		
$G(3900)$	3943 ± 21	52 ± 11	1^{--}	$e^+e^-\to\gamma(D\bar{D})$	BABAR [27] (np), Belle [21] (np)	2007	OK
$Y(4008)$	4008^{+121}_{-49}	226 ± 97	1^{--}	$e^+e^-\to\gamma(\pi^+\pi^- J/\psi)$	Belle [104] (7.4)	2007	NC!
$Z_1(4050)^+$	4051^{+24}_{-43}	82^{+51}_{-55}	?	$B\to K(\pi^+\chi_{c1}(1P))$	Belle [105] (5.0)	2008	NC!
$Y(4140)$	4143.4 ± 3.0	15^{+11}_{-7}	$?^{?+}$	$B\to K(\phi J/\psi)$	CDF [106, 107] (5.0)	2009	NC!
$X(4160)$	4156^{+29}_{-25}	139^{+113}_{-65}	$?^{?+}$	$e^+e^-\to J/\psi(D\bar{D}^*)$	Belle [103] (5.5)	2007	NC!
$Z_2(4250)^+$	4248^{+185}_{-45}	177^{+321}_{-72}	?	$B\to K(\pi^+\chi_{c1}(1P))$	Belle [105] (5.0)	2008	NC!
$Y(4260)$	4263 ± 5	108 ± 14	1^{--}	$e^+e^-\to\gamma(\pi^+\pi^- J/\psi)$	BABAR [108, 109] (8.0)	2005	OK
					CLEO [110] (5.4)		
					Belle [104] (15)		
				$e^+e^-\to(\pi^+\pi^- J/\psi)$	CLEO [111] (11)		
				$e^+e^-\to(\pi^0\pi^0 J/\psi)$	CLEO [111] (5.1)		
$Y(4274)$	$4274.4^{+8.4}_{-6.7}$	32^{+22}_{-15}	$?^{?+}$	$B\to K(\phi J/\psi)$	CDF [107] (3.1)	2010	NC!
$X(4350)$	$4350.6^{+4.6}_{-5.1}$	$13.3^{+18.4}_{-10.0}$	$0,2^{++}$	$e^+e^-\to e^+e^-(\phi J/\psi)$	Belle [112] (3.2)	2009	NC!
$Y(4360)$	4353 ± 11	96 ± 42	1^{--}	$e^+e^-\to\gamma(\pi^+\pi^-\psi(2S))$	BABAR [113] (np), Belle [114] (8.0)	2007	OK
$Z(4430)^+$	4443^{+24}_{-18}	107^{+113}_{-71}	?	$B\to K(\pi^+\psi(2S))$	Belle [115, 116] (6.4)	2007	NC!
$X(4630)$	4634^{+9}_{-11}	92^{+41}_{-32}	1^{--}	$e^+e^-\to\gamma(\Lambda_c^+\Lambda_c^-)$	Belle [25] (8.2)	2007	NC!
$Y(4660)$	4664 ± 12	48 ± 15	1^{--}	$e^+e^-\to\gamma(\pi^+\pi^-\psi(2S))$	Belle [114] (5.8)	2007	NC!

Most studied

ISR states

Charged!

Unconfirmed!

Table from: “Heavy quarkonium: progress, puzzles, and opportunities”, Eur. Phys. J. C71, 1534 (2011). [arXiv:1010.5827 [hep-ph]]

Unconventional Charmonia: interpretations

Meson molecule: two charmed mesons held together by pion exchange. Small bound energy.

$$X(3872) \sim [D^0 \bar{D}^{0*}] \sim [c\bar{u}][\bar{c}u]$$

Not all the states lie on thresholds!

Tetraquarks: diquark-antidiquark bound states.
Interaction via gluon exchange, strongly bound state

$$X(3872) \sim [cu][\bar{c}\bar{u}]$$

States proliferation! Where are the charged partners?

Hybrids mesons: quark-antiquark-gluon bound states.

$$Y(4260) \sim [c\bar{c}g]$$

Charged states?

Glueballs: gluon bound states $[gg]$, $[ggg]$, $\ldots$

Random Matrix Theory: I

The Hamiltonian of these exotic states is unknown.
Can we say something about some *general features* of the Hamiltonian by looking at the experimental spectroscopy in its entirety?

The problem was first faced by E. Wigner in the '50s: what kind of information about nuclei can be extracted from the analysis of the neutron spectroscopy data?

Answer: Hamiltonians belonging to the same universality class, which is determinated by the symmetries of the systems, actually share some features!

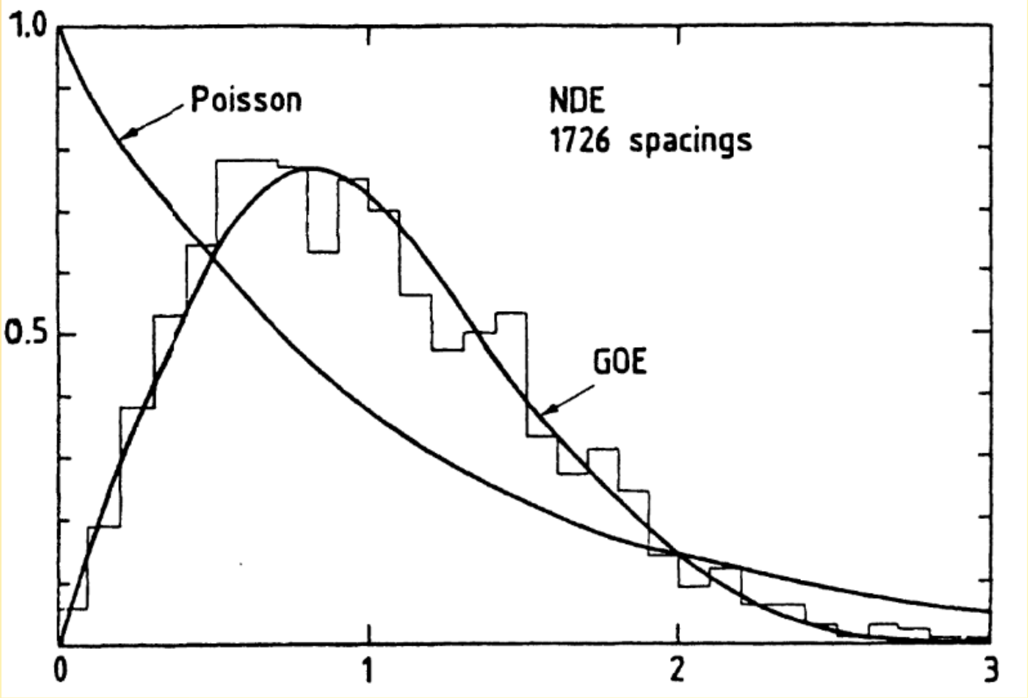
> (Quasi)energies of quantum systems behave locally like the eigenvalues of large random matrices extracted by an ensemble of the same universality class of the system.

First conjectured by Bohigas, Giannoni, Schmidt in 1984.

Random Matrix Theory: II

Universality Classes:

- Integrable Hamiltonians with two or more degrees of freedom: locally uncorrelated levels (Poisson process). Since the spacings distribuition is exponential, levels tend to cluster.
- Non integrabile (chaotic) Hamiltonians: level repulsion. The spacings distribution goes to zero for small spacings (e.g. Wigner distribution).
- Different Hamiltonian symmetries imply different level repulsion degrees.



$$P(s) = e^{-s}$$ (exponential)

$$W_\beta(s) = As^\beta e^{-Bs^2}$$ (Wigner)

On the left: level spacings in the Nuclear Data Ensemble, compared with exponential and Wigner distribution with $\beta = 1\,,\ A = \frac{\pi}{2}$ and $B = \frac{\pi}{4}$.

Random Matrix Theory: III

Extracting the local behaviour of a RM spectrum: start with a given matrix ensemble, for example a Gaussian Ensemble of order N matrices:

$$A_{ij}\ ,\ 1 \leq i,j \leq N, \qquad P(A) \sim \exp\left(-\mathrm{tr} A^2\right)$$

Eigenvalues distribution:

$$P_\beta(x_1,\ldots,x_N) \sim \left|\prod_{i\neq j}(x_i - x_j)^\beta\right| \exp\Big(-\sum_i x_i^2\Big)$$

β	Ensemble		Physical System
0	Diagonal Ensemble		Integrabile
1	GOE		Chaotic (T-invariant)
2	GUE		Chaotic (non T-invariant)
4	GSE		Chaotic (T-invariant, Kramers deg.)

Random Matrix Theory: IV

Calculate the "correlation functions":

$$R_\beta^{(m)}(x_1,\ldots,x_m) = \frac{N!}{(N-m)!}\int \mathrm{d}x_{m+1}\ldots\int \mathrm{d}x_N P_\beta(x_1,\ldots,x_N)$$

The correlation functions have well-definite $N \to \infty$ limits if expressed in terms of properly rescaled variables:

$$\lim_{N\to\infty} \alpha^m R_\beta^{(m)}(\alpha x_1,\ldots,\alpha x_m) \equiv X_\beta^{(m)}(x_1,\ldots,x_m)\;, \quad \alpha = \sqrt{N/2\pi}$$

The $X_\beta^{(m)}$ functions are universal, i.e. they do not depend from the chosen matrix ensemble, but only from the universality class: these quantities can be exactly calculated, for example in the Gaussian Ensembles.

Delta Statistics

The Dyson-Mehta Δ_3 statistic is defined as the mean quadratic deviation between the eigenvalues cumulative and the best line fitting it in a given interval:

$$\Delta_3(s) = \min_{A,B} \int_{-s/2}^{s/2} \Big(C(t) - At - B \Big)^2 \mathrm{d}t$$

Its ensemble average is calculated in terms of the two point correlation function:

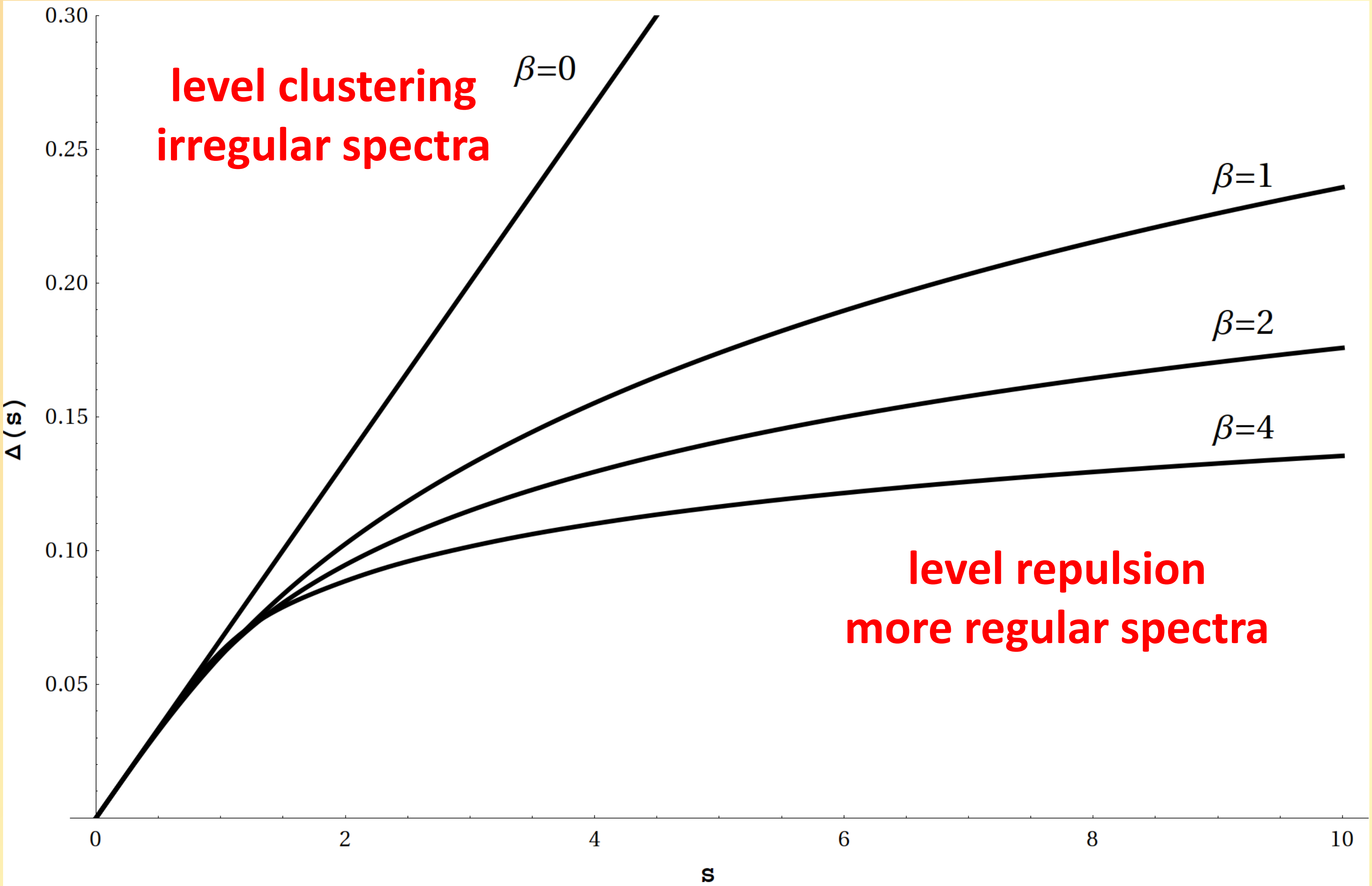
$$X_\beta^{(2)}(x,y) = 1 - Y_\beta(|x-y|)$$

$$\langle \Delta_3(s) \rangle_\beta = \frac{1}{15 s^4} \int_0^s (s-u)^3 (2s^2 - 9su - 3u^2) \Big(\frac{1}{2}\delta(u) - Y_\beta(u) \Big)\, \mathrm{d}u$$

Uncorrelated (Poisson) levels: $\langle \Delta_3(s) \rangle = s/15 \qquad (Y_0(u) = 0)$

GOE, GUE, GSE levels: $\langle \Delta_3(s) \rangle \sim \ln s + \mathcal{O}(s^{-1})$

Delta Statistics: mean values



Uniform spacings give a costant $\langle \Delta_3(s) \rangle \sim 0.08$ for $s \geq 1$

Analysis: guidelines

$\mathcal{E}$	{3943, 4008, 4263, 4353, 4634, 4664} MeV (6 levels)
$\mathcal{E}'$	{3943, 4008, 4263, 4353, 4660.7} MeV (5 levels)
$\mathcal{S}$	{3097, 3686, 3773, 4039, 4153, 4421} MeV (6 levels)

X(4630) and Y(4660) exotic resonances have compatibile masses and widths: study of the 5 ISR 1^{--} states resulted from the merging of these two resonances in a single Y_B state.

Do the ISR 1^{--} exotic resonances (5 or 6 states) behave like levels of some unknown Hamiltonian? What can be said about this Hamiltonian?

Comparison with the 1^{--} standard charmonia (6 states).

Have these series definite statistical properties? Comparison between Poissonian Ensemble and GOE (i.e. non-chaotic versus chaotic behaviour).

(merging proposed in: G. Cotugno *et al*., ''*Charmed Baryonium*'', Phys. Rev. Lett. **104**, 132005 (2010).)

Analysis: techniques

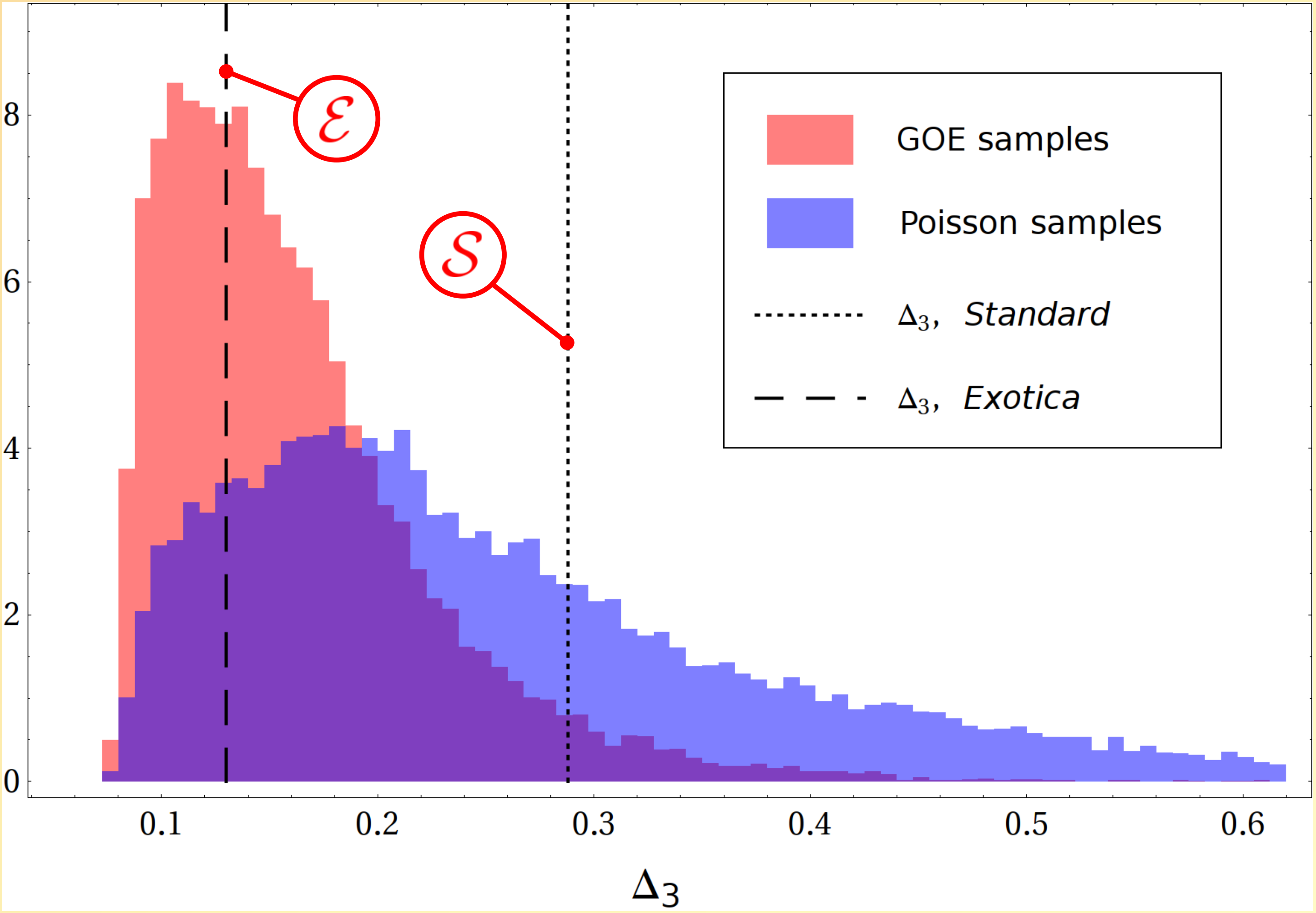
1) Rescaling to **unit mean spacing**. Only linear rescaling has been used (no particular unfolding).
2) Very small level numbers cause **large finite-size effects**; furthermore, most of the statistics used are not Gaussian-distribuited. **Monte Carlo samples** are needed!

Poissonian samples: 120000 indipendent numbers (uniform in [0,1]) grouped in 5 or 6-level series and rescaled as the experimental data.

GOE samples: 30 series of eigenvalues extracted from 4000x4000 GOE matrices, grouped and rescaled as the experimental data.

Experimental series must be compared with samples of the same cardinality!

Analysis: Delta-Statistics



The two exotica series $\mathcal{E}$ and $\mathcal{E}'$ are fully compatibile with both GOE and Poisson samples.

The $\mathcal{S}$ series is hardly compatibile with GOE distribution (p-value about 5%).

Analysis: Lambda statistics I

The Delta statistic is related to long-range correlations; a new variabile can be built averaging the Delta calculated on sub-intervals in the series:

$$\Delta(x,y) = \min_{A,B}\frac{1}{y}\int_x^{x+y}\Big(C(t) - At - B\Big)^2 \, \mathrm{d}t$$

$$\Lambda(y,w) = \frac{1}{2L+2w-y}\int_{-L-w}^{L+w-y}\Delta(x,y)\,\mathrm{d}x\ , \quad 0 \le y \le 2L+2w$$

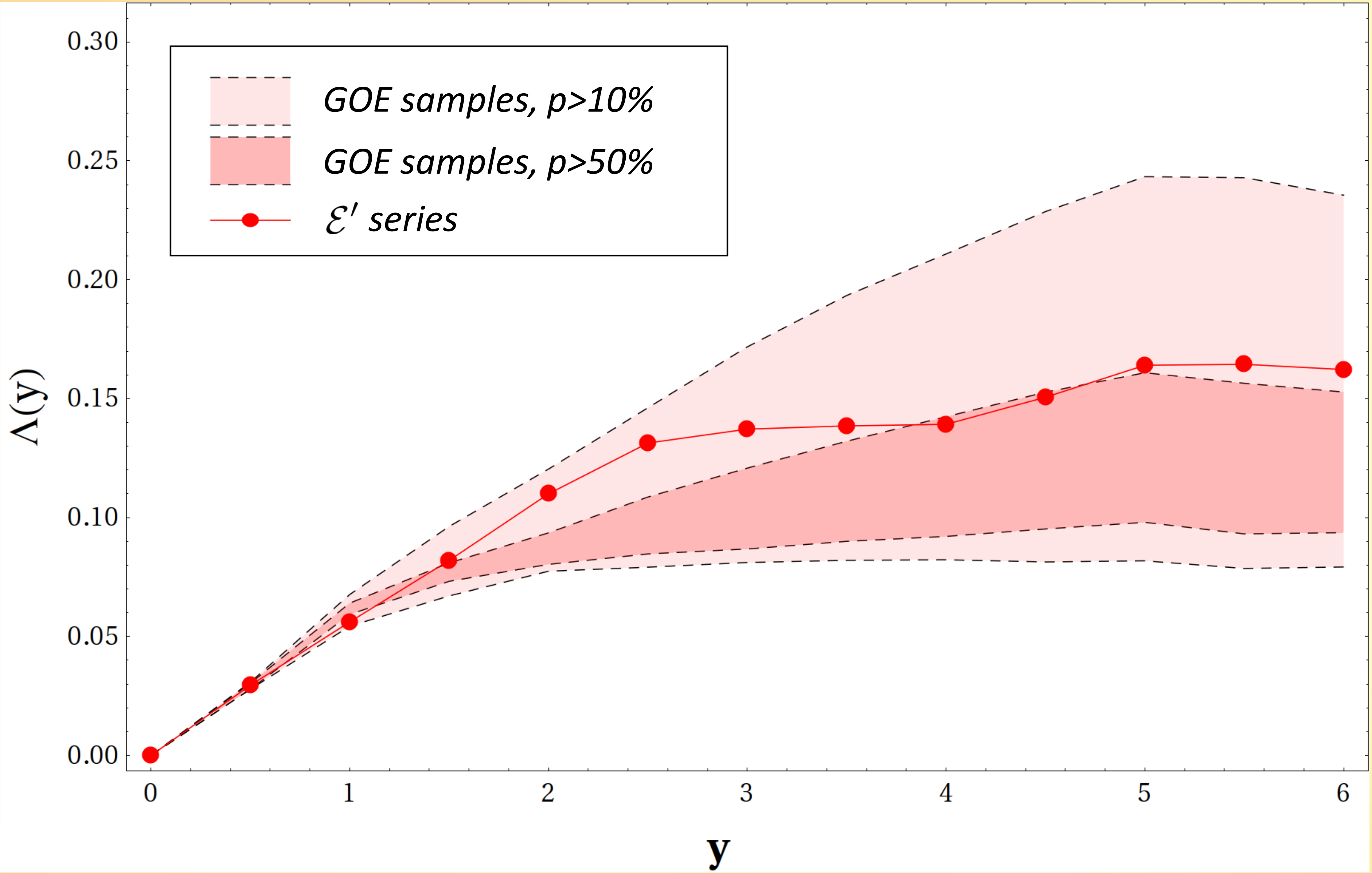
(first level in $-L$, last level in L, number of levels $= 2L+1$)

We choose $w = 1$ in order to minimize finite-size effects.
Distribuitions sampled only for integer or semi-integer y values.

$y \sim 2L$: similar to Δ_3, long range correlation

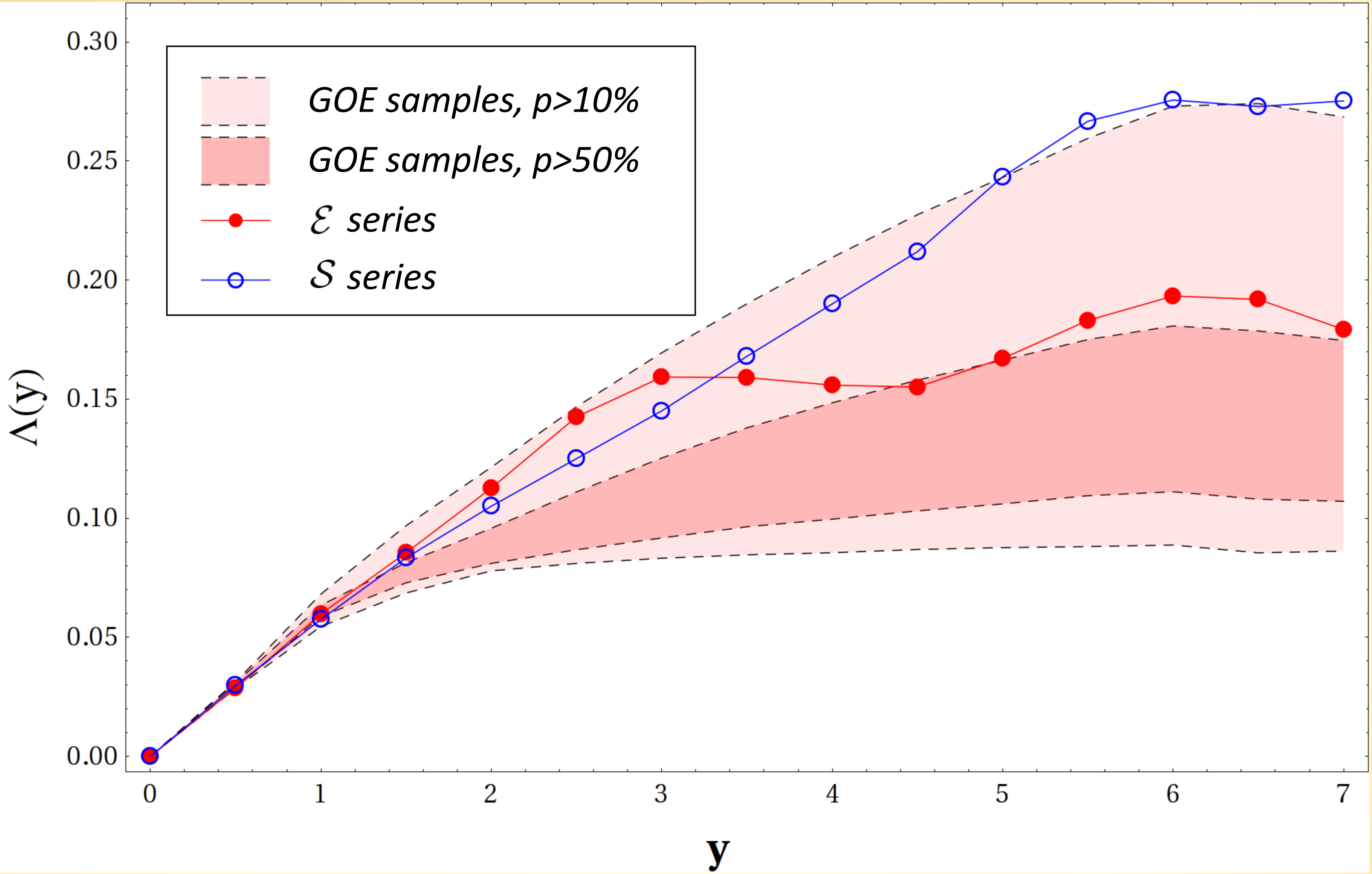
$y = 2 \div 3$: short range correlation (clustering)

Analysis: Lambda statistics II



The edges of the coloured areas are, for fixed *y*, 50% and 90% probability intervals.

Analysis: Lambda statistics III



The edges of the coloured areas are, for fixed *y*, 50% and 90% probability intervals.

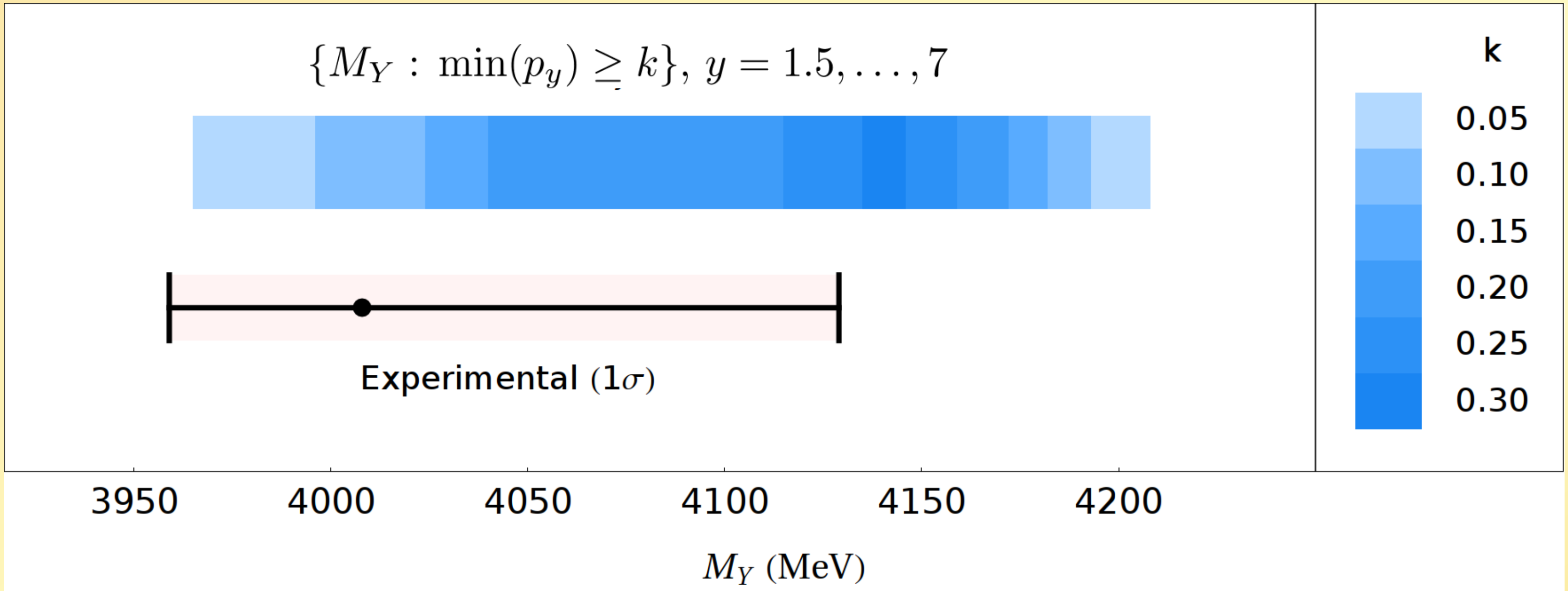
Y(4008) mass

$M_Y = 4008^{+121}_{-49}\,\mathrm{MeV}$ Great uncertainty! Mean spacing: $144\,\mathrm{MeV}$

While keeping fixed the others resonances, we vary M_Y to find the best fit with the GOE hypothesis: maximize the minimum p-value obtained for $y = 1.5,\ 2,\ \ldots,\ 7$ ($y = 0.5$ and $y = 1$ are not reliable)

Best fit: $M_Y \in [4135, 4146]\,\mathrm{MeV}$

GOE compatibility ($p_y > 0.1$): $M_Y \in [3996, 4193]\,\mathrm{MeV}$



Conclusions

Although we have only 5/6 levels, some conclusions can be actually drawn!

Good compatibility of both $\mathcal{E}$ and $\mathcal{E}'$ series with the GOE ensemble, expecially increasing the *Y(4008)* mass. This can be an indication of an underlying multiquark, chaotic Hamiltonian...

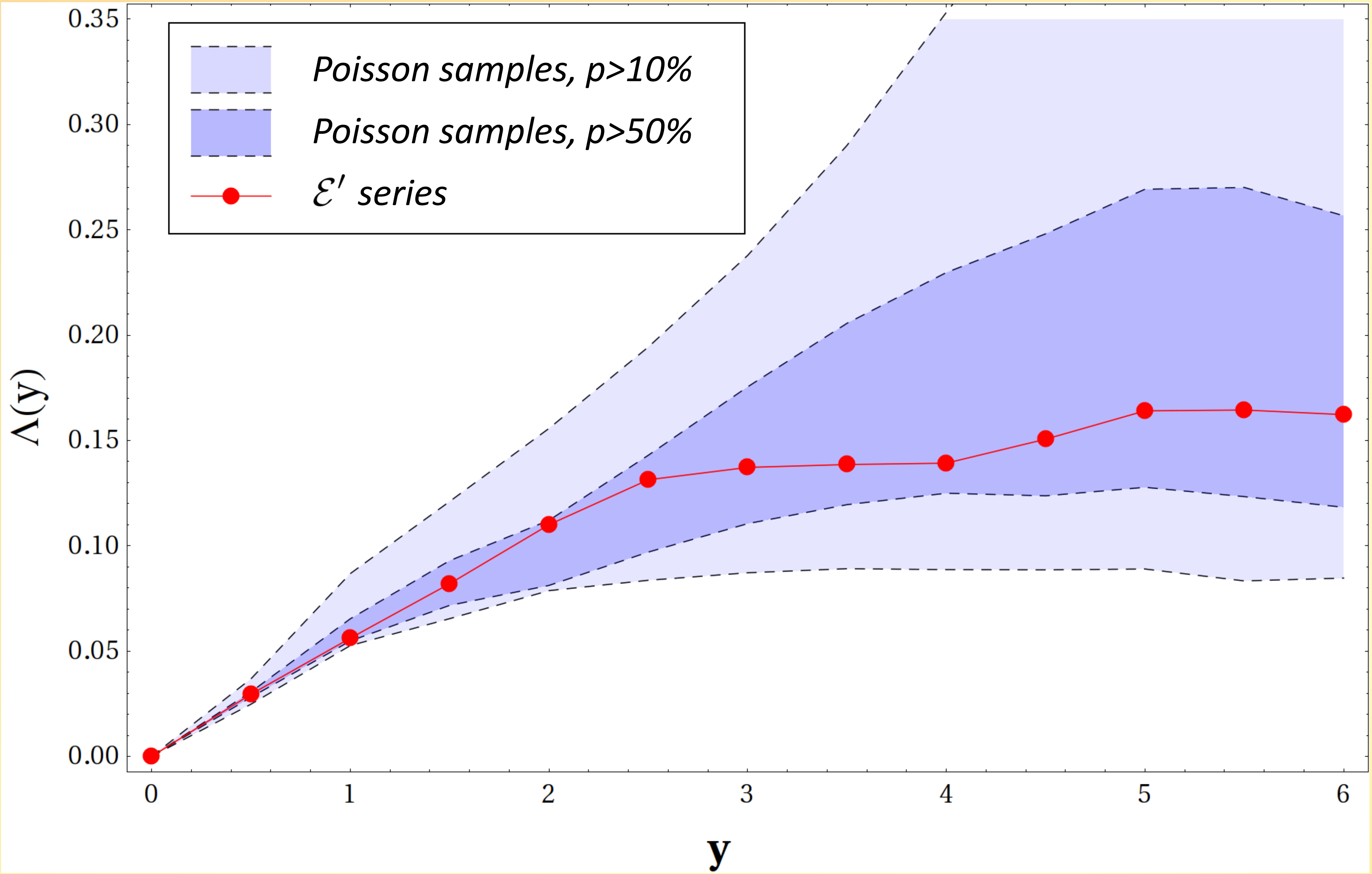
On the other hand, the standard charmonia ($\mathcal{S}$ series) seem not compatibile with the GOE.

What about the levels obtained by the current models?

The quality of the analysis will increase as more resonances are found (confirms of known resonances are welcome, too...).

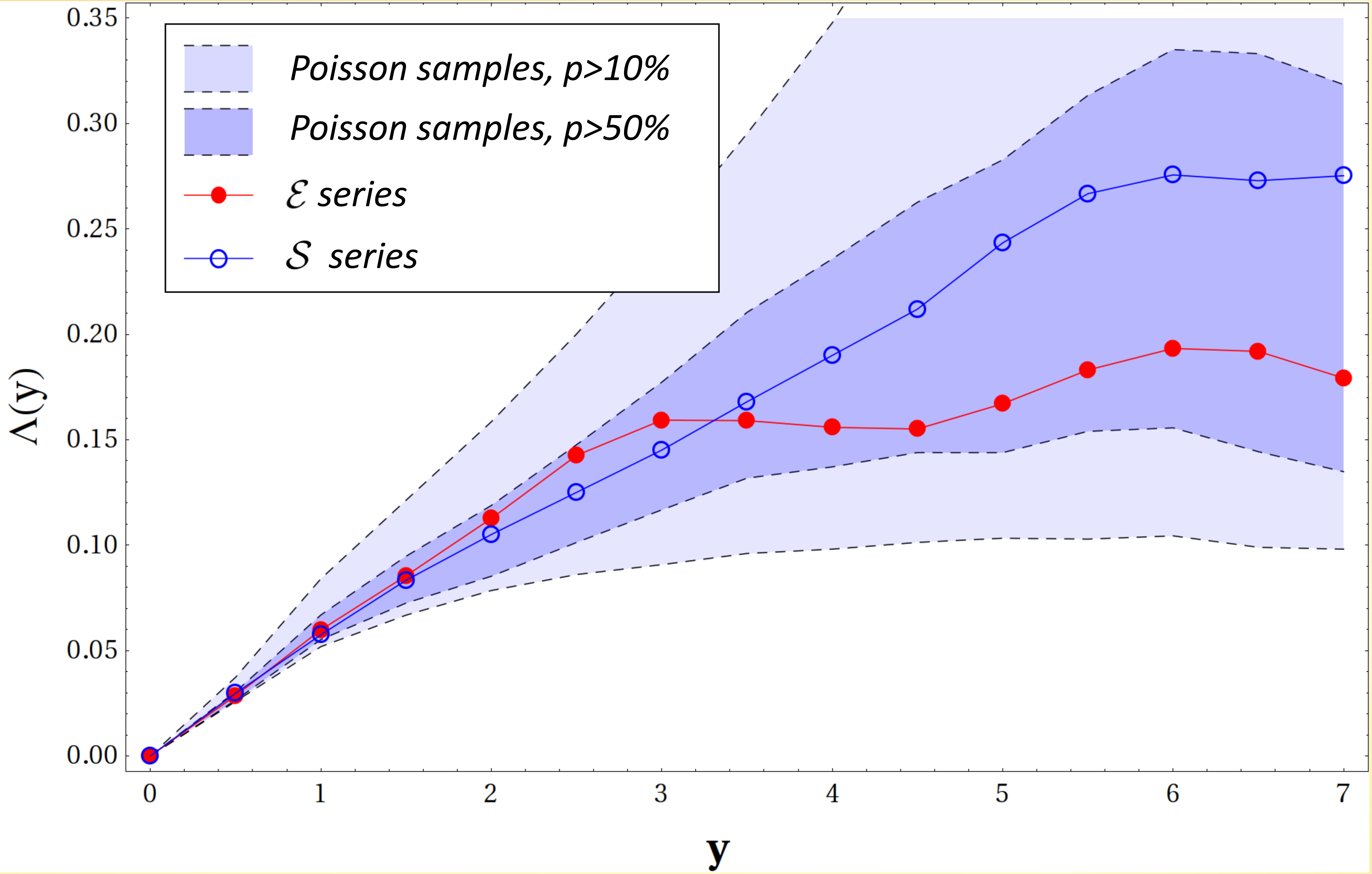
Backup

Analysis: Lambda statistics IIb



The edges of the coloured areas are, for fixed *y*, 50% and 90% probability intervals.

Analysis: Lambda statistics IIIb



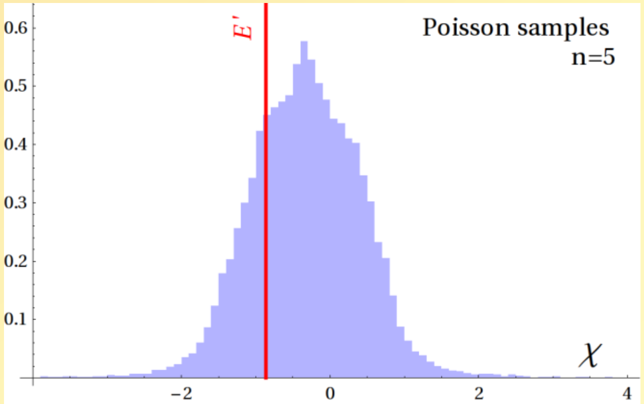
The edges of the coloured areas are, for fixed *y*, 50% and 90% probability intervals.

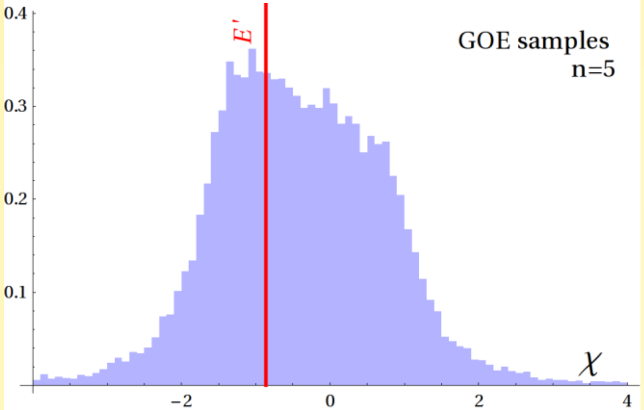
Correlation between adjacent spacings

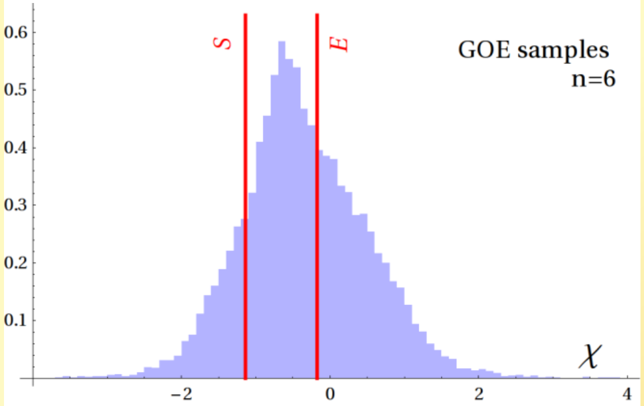
The correlation between adjacent spacings is defined as:

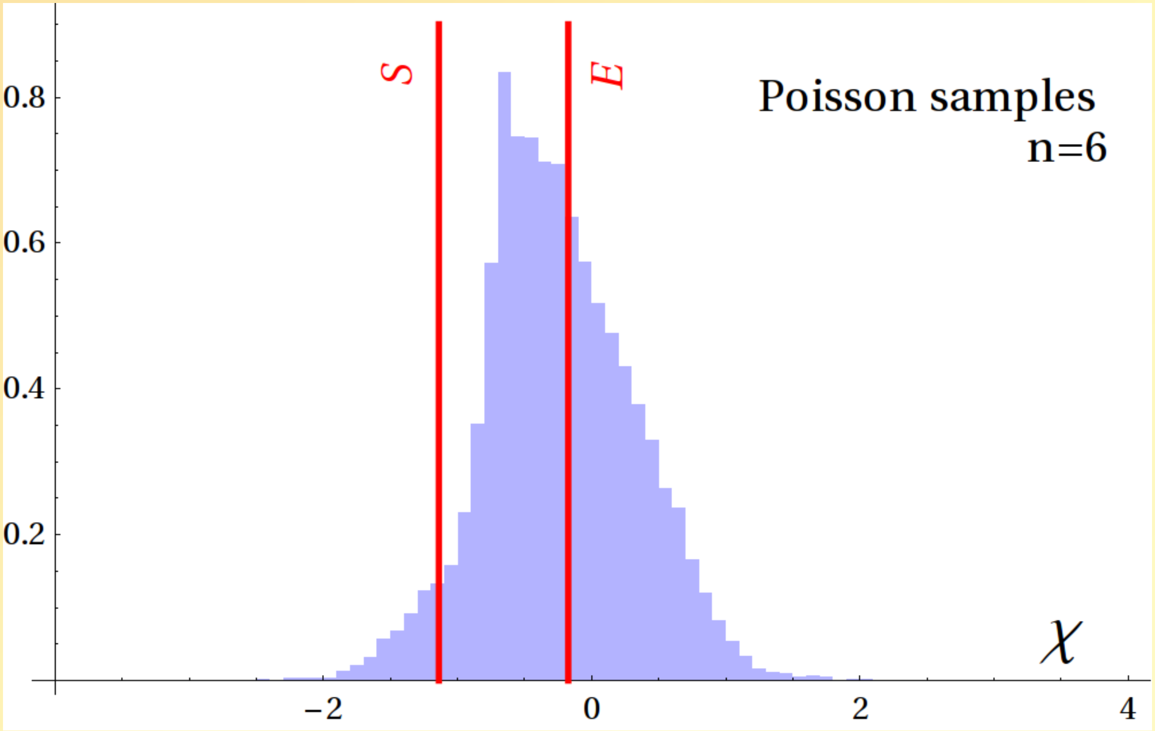
$$\chi = \frac{\langle s_i s_{i+1} \rangle - \langle s_i \rangle \langle s_{i+1} \rangle}{\langle s_i^2 \rangle - \langle s_i \rangle^2} = \frac{\langle s_i s_{i+1} \rangle - 1}{\langle s_i^2 \rangle - 1}$$

where the angled brackets indicate the mean on the sampled spacings. For the standard charmonia series one obtains a small p-value for the Poisson hypothesis, approximatively 8.5%.









P-values

The compatibility between a sample and a distribution is usually quantified with probability intervals, with related p-values. We thus define, for a given probability density $f(x)$:

$$\int_{-\infty}^{+\infty} f(x)\,\mathrm{d}x = 1 \qquad\qquad \int_a^b f(x)\,\mathrm{d}x = q \qquad (a < b)$$

$[a, b]$ is then said to be a q-probability interval. Given an experimental value $\bar{x}$, the p-value $p(\bar{x})$ is the probability of extracting a value with a probability density less than $f(\bar{x})$ (tails' total area).

$$p(\bar{x}) = \int_{\mathcal{D}} f(x)\,\mathrm{d}x$$

$$\mathcal{D} = \{x : f(x) < f(\bar{x})\}$$