QCD thermodynamics and the large-N limit – A review

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Outline

1. Physical motivation
2. The large-N limit
3. Lattice QCD
4. Equation of state in $D = 3 + 1$ dimensions
5. Equation of state in $D = 2 + 1$ dimensions
6. Conclusions

Based on:

- M.P., Phys. Rev. Lett. **103** (2009) 232001
- M. Caselle *et al.*, JHEP 1106 (2011) 142
- M. Caselle *et al.*, in preparation

See also the talk by A. Mykkänen

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The physical problem - I

- Due to asymptotic freedom in non-Abelian gauge theories **[Gross and Wilczek, 1973; Politzer, 1973]**, hadronic matter is expected to undergo a change of state to a deconfined phase at sufficiently high temperatures or densities **[Cabibbo and Parisi, 1975; Collins and Perry, 1974]**.
- Extensive experimental investigation through heavy ion collisions since the Eighties: first at AGS (BNL) and SPS (CERN), then at RHIC (BNL) and more recently at LHC (CERN)
- Experimental evidence from SPS, RHIC and LHC: a ‘A new state of matter’ has been created **[Heinz and Jacob, 2000, Arsene *et al.*, 2004; Back *et al.*, 2004; Adcox *et al.*, 2004; Adams *et al.*, 2005; Aad *et al.*, 2010; Aamodt *et al.*, 2010; Chatrchyan *et al.*, 2011]**

The physical problem - II

- The plasma behaves as an almost ideal fluid **[Kolb and Heinz, 2003]** ('The most perfect liquid observed in Nature')
- Further experiments at LHC, FAIR, NICA and J-PARC to provide a more detailed picture
- However, the theoretical understanding of the QCD plasma **[Rischke, 2003; Shuryak, 2008]** is still incomplete . . .

Theoretical approaches - I

- Relativistic fluidodynamics is a successful phenomenological description **[Kolb, Heinz, Huovinen, Eskola and Tuominen, 2001]**—see also **[Romatschke, 2009]** for an introductory review—, but is not derived from QCD first principles
- The perturbative approach in thermal gauge theory has a non-trivial mathematical structure, involving odd powers of the coupling **[Kapusta, 1979]**, as well as contributions from diagrams involving arbitrarily large numbers of loops **[Linde, 1980; Gross, Pisarski and Yaffe, 1980]** . . .
- . . . and shows poor convergence at the temperatures probed in experiments **[Kajantie, Laine, Rummukainen and Schröder, 2002]**
- The long-wavelength modes of the plasma are strongly coupled even at high temperature **[Blaizot, 2011]**
- Dimensional reduction **[Ginsparg, 1980; Appelquist and Pisarski, 1981]** to EQCD and MQCD **[Braaten and Nieto, 1995]**, hard-thermal loop resummations **[Blaizot and Iancu, 2002]**, and other effective theory approaches **[Kraemmer and Rebhan, 2004]**

Theoretical approaches - II

- Analytical progress in strongly interacting gauge theories: the AdS/CFT conjecture **[Maldacena, 1997]** and related theories as possible models for the non-perturbative features of QCD, including spectral **[Erdmenger, Evans, Kirsch and Threlfall, 2007]** and thermal properties **[Gubser and Karch, 2009; Verschelde and Zakharov, 2011]**
- In the large-N limit, the Maldacena conjecture relates a strongly interacting gauge theory to the classical limit of a gravity model

Theoretical approaches - III

- Numerical approach: Computer simulations of QCD regularized on a lattice allow first-principle, non-perturbative studies of the finite-temperature plasma
- The lattice determination of equilibrium thermodynamic properties in $\mathrm{SU}(3)$ gauge theory is regarded as a solved problem **[Boyd *et al.*, 1996; S. Borsányi *et al.*, 2011]**
- In recent years, finite-temperature lattice QCD has steadily progressed towards parameters corresponding to the physical point **[Karsch *et al.*, 2000; Ali Khan *et al.*, 2001; Aoki *et al.*, 2005; Bernard *et al.*, 2006; Cheng *et al.*, 2007; Bazavov *et al.*, 2009; S. Borsányi *et al.*, 2010]**
- Simulations at finite μ must cope with a NP-hard **[Troyer and Wiese, 2004]** sign problem **[de Forcrand and Philipsen, 2002; D'Elia and Lombardo, 2002; Allton *et al.*, 2002; Fodor and Katz, 2004; Cea, Cosmai, D'Elia, Manneschi and Papa, 2009]**
- Related lattice studies of the thermal properties of gauge theories in the large-N limit: **[Lucini, Teper and Wenger, 2003]**, **[Bringoltz and Teper, 2005]** and **[Datta and Gupta, 2010]**

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The old perspective: QCD at large N

- 't Hooft proposed to use $1/N$ (N being the number of colors) as an expansion parameter **['t Hooft, 1974]**
- Generically, a large-N limit can be interpreted as a 'classical limit'; identification of coherent states and construction of a classical Hamiltonian **[Yaffe, 1982]**
- In the large-N limit at fixed 't Hooft coupling $\lambda = g^2 N$ and fixed number of flavors N_f, certain non-trivial non-perturbative features of QCD can be easily explained in terms of combinatorics **[Witten, 1979; Manohar, 1998]**
- Planar diagrams' dominance
- Formal connection to string theory: loop expansion in Riemann surfaces for closed string theory with coupling constant $g_{\text{string}} \sim 1/N$ **[Aharony, Gubser, Maldacena, Ooguri and Oz, 1999; Mateos, 2007]**

$$\mathcal{A} = \sum_{G=0}^{\infty} N^{2-2G} \sum_{n=0}^{\infty} c_{G,n} \lambda^n$$

- Interesting implications from strong coupling expansions on the lattice **[Langelage and Philipsen, 2010]**

The AdS/CFT correspondence

- Maldacena conjectured that the large-N limit of the maximally supersymmetric $\mathcal{N} = 4$ supersymmetric YM (SYM) theory in four dimensions is dual to type IIB string theory in a $AdS_5 \times S^5$ space **[Maldacena, 1997]**

$$ds^2 = \frac{r^2}{R^2}\left(-dt^2 + d\mathbf{x}^2\right) + \frac{R^2}{r^2}dr^2 + R^2 d\Omega_5^2$$

- The conjecture arises from the observation that the low-energy dynamics of open strings ending on a stack of N D3 branes in $AdS_5 \times S^5$ can be described in terms of $\mathcal{N} = 4$ SYM
- Geometric interpretation: There exists a correspondence of symmetries in the two theories
- A highly non-trivial correspondence, linking the strongly coupled regime of field theory to the weak-coupling limit of a gravity model
- Identification of the generating functional of connected Green's functions in the gauge theory with the minimum of the supergravity action with given boundary conditions: correlation functions of gauge theory operators from perturbative calculations in the gravity theory **[Gubser, Klebanov and Polyakov, 1998]**
- A stringy realization of the holographic principle: the description of dynamics within a volume of space is "encoded on the boundary" **['t Hooft, 1993; Susskind, 1995]**—see also **[Bousso, 2002]** for a review
- The large-N limit of the $\mathcal{N} = 4$ SYM theory exhibits a phase transition which can be related to the thermodynamics of AdS black holes **[Witten, 1998]**

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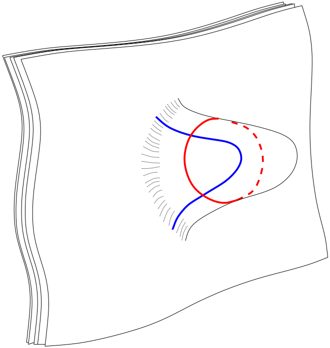

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  - $\mathcal{R}$-symmetry in the gauge theory is $SU(4) \sim SO(6)$ symmetry of S^5
  - The conformal invariance group in the gauge theory is isomorphic to $SO(2, 4)$, the symmetry group of AdS_5
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$$
\begin{aligned}
g^2 &= 4\pi g_s \\
g^2 N &= \frac{R^4}{l_s^4}
\end{aligned}
$$

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Non-perturbative predictions for QCD-like theories from holographic models

- ‘Top-down’ approach: break some symmetries of the $\mathcal{N} = 4$ theory explicitly, add fundamental matter fields to the gauge theory by including new branes in the string theory **[Bertolini, Di Vecchia, Frau, Lerda, and Marotta, 2001; Graña and Polchinski, 2001; Karch and Katz, 2002]** to get a non-trivial hadron sector with ‘mesons’ and χSB **[Erdmenger, Evans, Kirsch and Threlfall, 2007]**
- Derivation of hydrodynamic and thermodynamic properties for a strongly interacting system from gauge/gravity duality **[Policastro, Son and Starinets, 2001]**—see also **[Son and Starinets, 2007; Mateos, 2007; Gubser and Karch, 2009]** and references therein
- ‘Bottom-up’ approach: construct a 5D gravitational background reproducing the main features of QCD **[Polchinski and Strassler, 2001; Erlich, Katz, Son and Stephanov, 2005; Da Rold and Pomarol, 2005; Karch, Katz, Son and Stephanov, 2006]**
- Hard-wall *versus* soft-wall AdS/QCD, and related thermodynamic features **[Herzog, 2007]**

Improved holographic QCD model – I

- Kiritsis and collaborators **[Gürsoy, Kiritsis, Mazzanti and Nitti, 2008]** proposed an AdS/QCD model based on a 5D Einstein-dilaton gravity theory, with the fifth direction dual to the energy scale of the $\mathrm{SU}(N)$ gauge theory
- Field content on the gravity side: metric (dual to the $\mathrm{SU}(N)$ energy-momentum tensor), the dilaton (dual to the trace of F^2) and the axion (dual to the trace of $F\tilde{F}$)
- Gravity action:

$$S_{IHQCD} = -M_P^3 N^2 \int d^5x \sqrt{g} \left[R - \frac{4}{3}(\partial\Phi)^2 + V(\lambda) \right] + 2M_P^3 N^2 \int_{\partial M} d^4x \sqrt{h}\, K$$

- Φ is the dilaton field, $\lambda = \exp(\Phi)$ is identified with the running 't Hooft coupling of the dual $\mathrm{SU}(N)$ YM theory
- The effective five-dimensional Newton constant $G_5 = 1/\left(16\pi M_P^3 N^2\right)$ becomes small in the large-N limit

Improved holographic QCD model – II

- Dilaton potential $V(\lambda)$ defined by requiring asymptotic freedom with a logarithmically running coupling in the UV and linear confinement in the IR of the gauge theory; a possible *Ansatz* is:

$$V(\lambda) = \frac{12}{\ell^2}\left[1 + V_0\lambda + V_1\lambda^{4/3}\sqrt{\log\left(1 + V_2\lambda^{4/3} + V_3\lambda^2\right)}\right],$$

  where ℓ is the AdS scale (overall normalization), and two free parameters are fixed by imposing that the dual model reproduces the first two coefficients of the $\mathrm{SU}(N)$ β-function
- Gauge/gravity duality expected to hold in the large-N limit only, because calculations in the gravity model neglect string interactions which can become important above a scale $M_P N^{2/3} \simeq 2.5$ GeV in $\mathrm{SU}(3)$
- First-order transition from a thermal-graviton- to a black-hole-dominated regime in the 5D gravity theory dual to the $\mathrm{SU}(N)$ deconfinement transition
- The model successfully reproduces the main non-perturbative spectral and thermodynamical features of the $\mathrm{SU}(3)$ YM theory
- Can also be used to derive predictions for observables such as the plasma bulk viscosity, drag force and jet quenching parameter **[Gürsoy, Kiritsis, Michalogiorgakis and Nitti, 2009]**

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Lattice QCD: The basics

- Discretize a finite hypervolume in Euclidean spacetime by a regular grid with finite spacing a
- Transcribe gauge and fermion d.o.f. to lattice elements, build lattice observables
- Discretization of the continuum gauge action with the Wilson lattice action **[Wilson, 1974]**:

$$S = \beta \sum_{\square} \left(1 - \frac{1}{N} \mathrm{Re\,Tr}\, U_{\square} \right), \quad \text{with: } \beta = \frac{2N}{g_0^2}$$

- A gauge-invariant, non-perturbative regularization
- Amenable to numerical simulation: Sample configuration space according to a statistical weight proportional to $\exp(-S)$
- Physical results recovered by extrapolation to the continuum limit $a \to 0$

Thermodynamics on the lattice

- Thermal averages from simulations on a lattice with compactified Euclidean time direction, with $T = 1/(aN_\tau)$
- Pressure $p(T)$ via the 'integral method' **[Engels *et al.*, 1990]**:

$$\begin{aligned} p &= T\frac{\partial}{\partial V}\log\mathcal{Z} \simeq \frac{T}{V}\log\mathcal{Z} = \frac{1}{a^4 N_s^3 N_\tau}\int_{\beta_0}^{\beta} d\beta' \frac{\partial \log \mathcal{Z}}{\partial \beta'} \\ &= \frac{6}{a^4}\int_{\beta_0}^{\beta} d\beta' \left(\langle U_\square\rangle_T - \langle U_\square\rangle_0\right) \end{aligned}$$

Thermodynamics on the lattice

- Other equilibrium thermodynamic observables obtained from indirect measurements
  - Trace of the stress tensor $\Delta = \epsilon - 3p$:

$$\Delta = T^5 \frac{\partial}{\partial T} \frac{p}{T^4} = \frac{6}{a^4} \frac{\partial \beta}{\partial \log a} \left(\langle U_\square \rangle_0 - \langle U_\square \rangle_T \right)$$

  - Energy density:

$$\epsilon = \frac{T^2}{V} \frac{\partial}{\partial T} \log \mathcal{Z} = \Delta + 3p$$

  - Entropy density:

$$s = \frac{S}{V} = \frac{\epsilon - f}{T} = \frac{\Delta + 4p}{T}$$

Simulation details

- Lattice sizes $N_s^{D-1} \times N_\tau$, with N_s from 16 to 64, and N_τ from 5 to 12
- Simulation algorithm: heat-bath **[Kennedy and Pendleton, 1985]** for $\mathrm{SU}(2)$ subgroups **[Cabibbo and Marinari, 1982]** and full-$\mathrm{SU}(N)$ overrelaxation **[Kiskis, Narayanan and Neuberger, 2003; Dürr, 2004; de Forcrand and Jahn, 2005]**
- Cross-check with $T = 0$ simulations run using the Chroma suite **[Edwards and Joó, 2004]**
- Physical scale for $\mathrm{SU}(3)$ in 4D determined from r_0 **[Necco and Sommer, 2001]**
- Physical scale for $\mathrm{SU}(N > 3)$ in 4D determined from the string tension σ **[Lucini, Teper and Wenger, 2004; Lucini and Teper, 2001; Del Debbio, Panagopoulos, Rossi and Vicari, 2001]** in combination with the 3-loop lattice β-function **[Allés, Feo and Panagopoulos, 1997; Allton, Teper and Trivini, 2008]** in the mean-field improved lattice scheme **[Parisi, 1980; Lepage and Mackenzie, 1993]**
- Physical scale for $\mathrm{SU}(N)$ in 3D determined from lattice computations of σ **[Liddle and Teper, 2008]**

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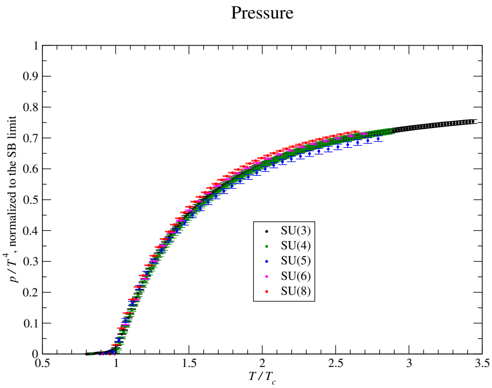
Improved holographic QCD model *vs.* lattice data



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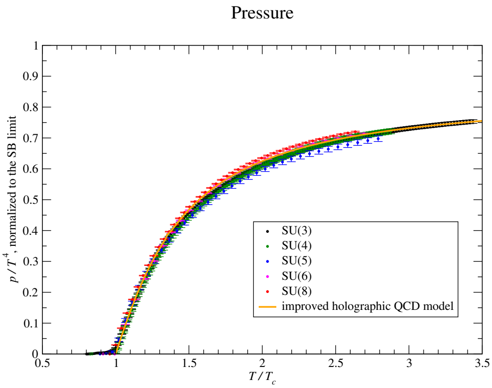

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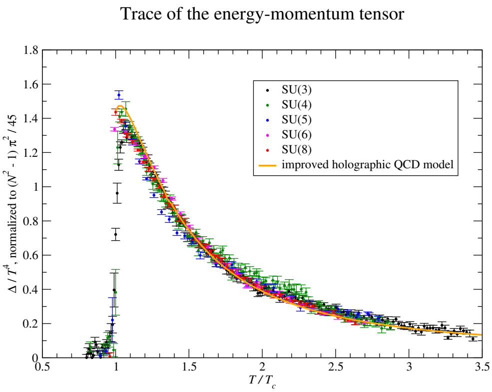

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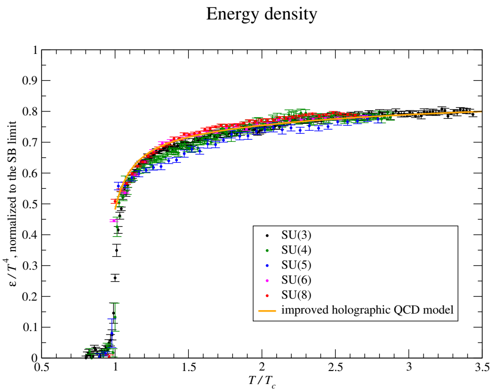

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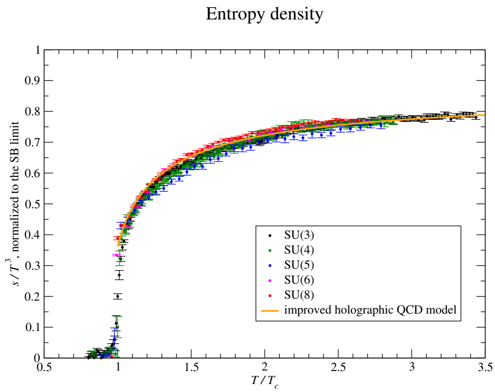
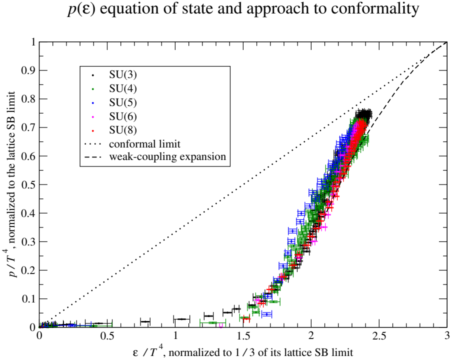


AdS/CFT *vs.* lattice data in a 'quasi-conformal' regime

For $T \simeq 3T_c$, the lattice results reveal that the deconfined plasma, while still strongly interacting and far from the Stefan-Boltzmann limit, approaches a scale-invariant regime . . .

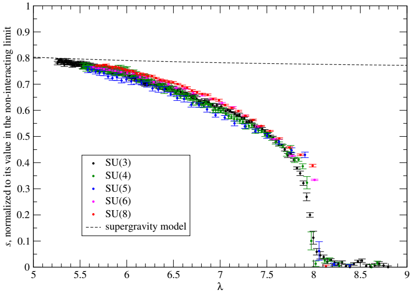


AdS/CFT *vs.* lattice data in a 'quasi-conformal' regime

...in which the entropy density is comparable with the supergravity prediction for $\mathcal{N} = 4$ SYM **[Gubser, Klebanov and Tseytlin, 1998]**

$$\frac{s}{s_0} = \frac{3}{4} + \frac{45}{32}\zeta(3)(2\lambda)^{-3/2} + \dots$$

Entropy density *vs.* 't Hooft coupling



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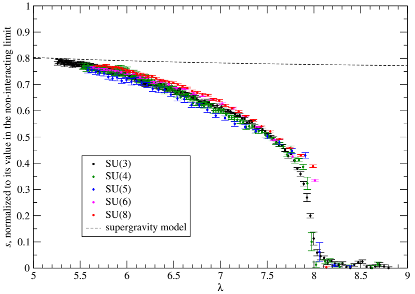
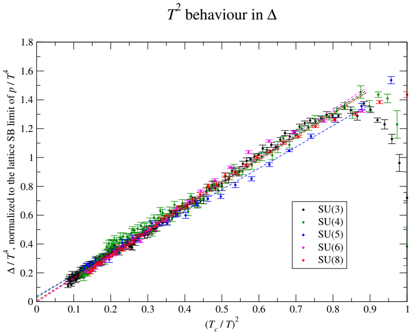


Note that a comparison of $\mathcal{N} = 4$ SYM and full-QCD lattice results for the drag force on heavy quarks also yields $\lambda \simeq 5.5$ **[Gubser, 2006]**

T^2 contributions to the trace anomaly?

The trace anomaly reveals a characteristic T^2-behavior, possibly of non-perturbative origin **[Megías, Ruiz Arriola and Salcedo, 2003; Pisarski, 2006; Andreev, 2007]**

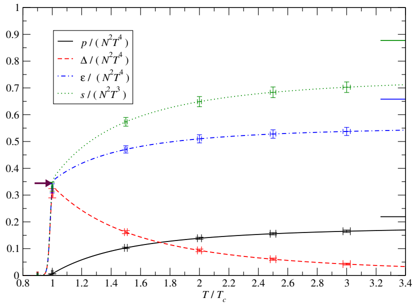


Extrapolation to $N \to \infty$

Based on the parametrization **[Bazavov *et al.*, 2009]**:

$$\frac{\Delta}{T^4} = \frac{\pi^2}{45}(N^2-1)\cdot\left(1-\left\{1+\exp\left[\frac{(T/T_c)-f_1}{f_2}\right]\right\}^{-2}\right)\left(f_3\frac{T_c^2}{T^2}+f_4\frac{T_c^4}{T^4}\right)$$

Extrapolation to the large-N limit



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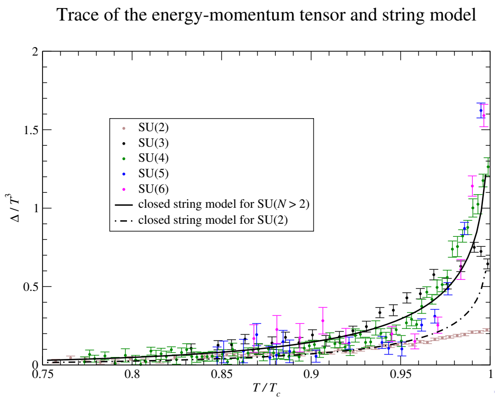
The confining phase of Yang-Mills theories in $D = 2 + 1$ dimensions

For $T < T_c$, the equation of state is essentially independent of N for all $\mathrm{SU}(N \geq 3)$, and can be described by a gas of massive, non-interacting glueballs, with spectral density modelled by a closed bosonic string **[Isgur and Paton, 1985]**

$$\tilde{\rho}_D(m) = 2\frac{(D-2)^{D-1}}{m}\left(\frac{\pi T_H}{3m}\right)^{D-1} e^{m/T_H}$$



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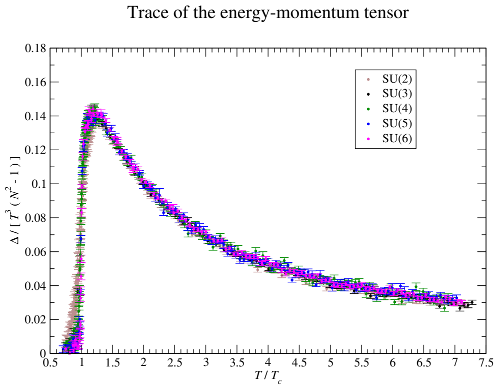
The deconfined phase of Yang-Mills theories in $D = 2 + 1$ dimensions

Similarly to the $D = 3 + 1$ case, in the deconfined phase the equation of state scales proportionally to $N^2 - 1$ . . .



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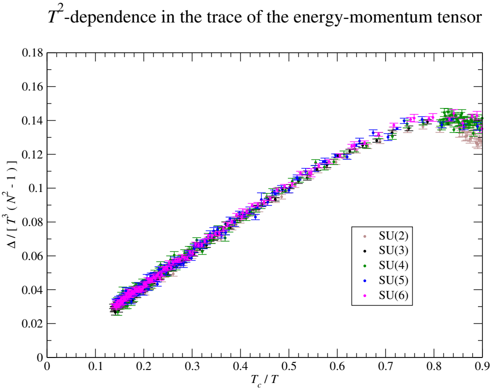
…and the trace of the energy-momentum tensor appears to be dominated by contributions proportional to T^2



Outline

1. Physical motivation
2. The large-N limit
3. Lattice QCD
4. Equation of state in $D = 3 + 1$ dimensions
5. Equation of state in $D = 2 + 1$ dimensions
6. **Conclusions**

Conclusions

- In the deconfined phase, the equation of state of non-supersymmetric Yang-Mills theories appears to be nearly exactly proportional to $N^2 - 1$; this holds both in both $D = 3 + 1$ and $D = 2 + 1$ dimensions
- The IHQCD model provides a *quantitative* description of the results for the $D = 3 + 1$ case
- For the $D = 3 + 1$ case, the bulk thermodynamic quantities in a nearly conformal, yet strongly coupled regime near $T \sim 3T_c$ can be compared with holographic predictions for $\mathcal{N} = 4$ SYM
- Both in $D = 3 + 1$ and $D = 2 + 1$ dimensions, in the deconfined phase Δ exhibits a characteristic T^2-dependence
- In the confining phase, the equation of state of YM theories in $D = 2 + 1$ is described by a gas of massive, non-interacting glueballs (with multiplicities independent of N—except for the $N = 2$ case), whose spectral density can be modelled by a bosonic string model