Constraints on the QCD phase diagram from imaginary chemical potential

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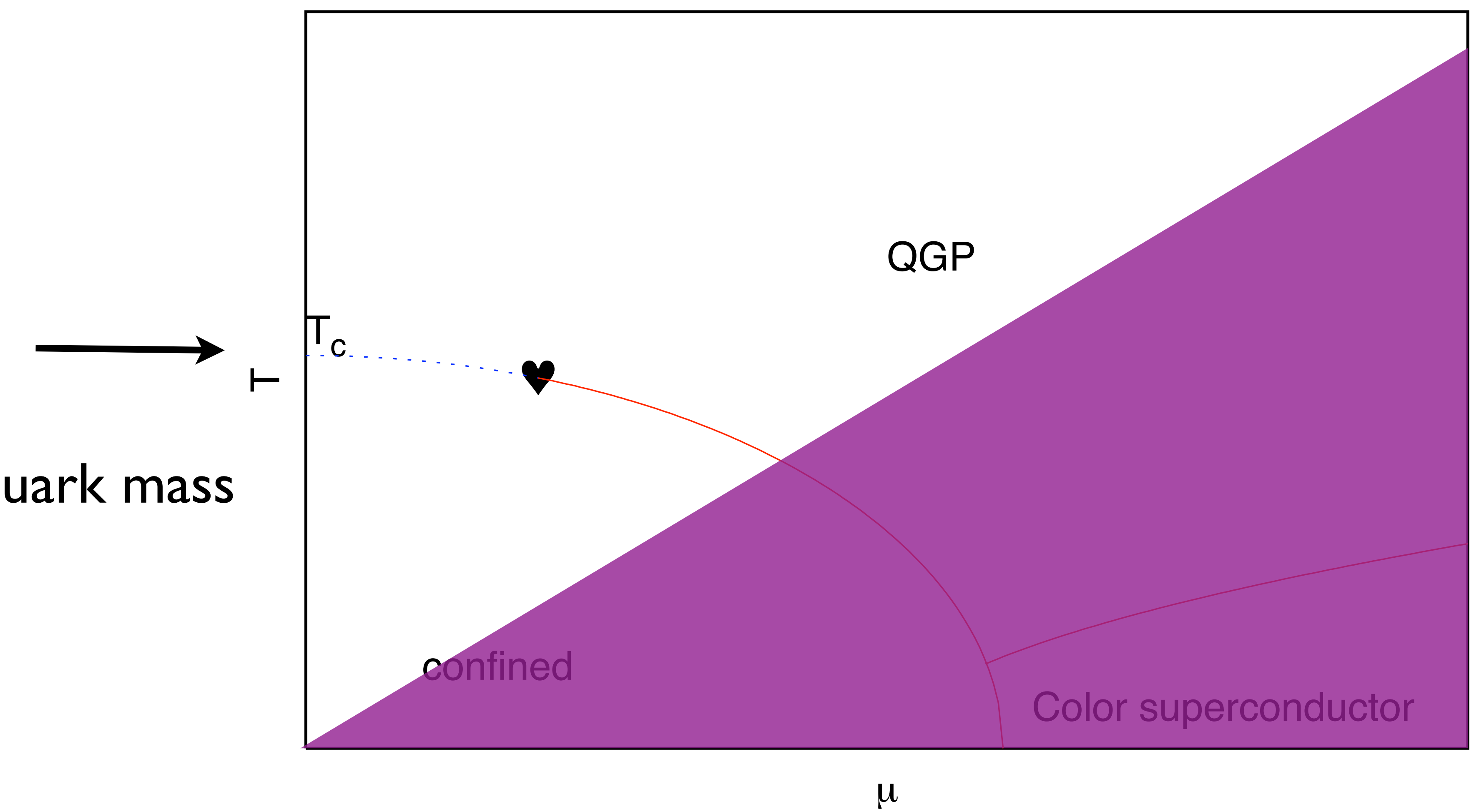
- Introduction: summary on QCD phase diagram
- Taking imaginary μ more seriously
- Triple, critical and tri-critical structures at $\mu = i\frac{\pi T}{3}$
- Implications for the QCD phase diagram

The (lattice) calculable region of the phase diagram

order of p.t.
at zero density
depends on Nf, quark mass

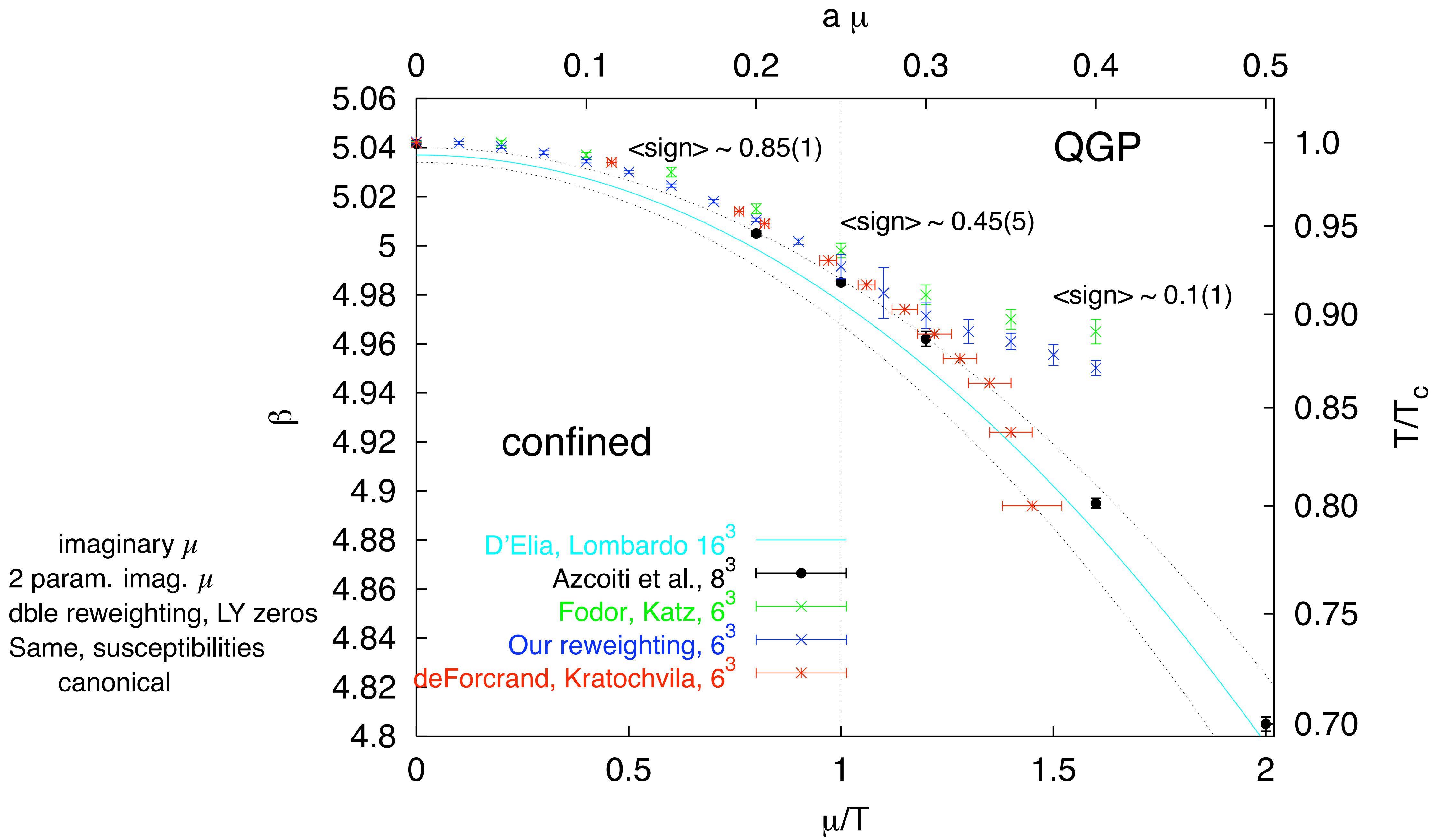


- Sign problem prohibits direct simulation, circumvented by approximate methods: reweigthing, Taylor expansion, imaginary chem. pot., need $\mu/T \lesssim 1 \;\; (\mu = \mu_B/3)$
- Upper region: equation of state, screening masses, quark number susceptibilities etc. under control
- Here: phase diagram itself, so far based on models, most difficult!

Comparing approaches: the critical line

de Forcrand, Kratochvila LAT 05

$N_t = 4, N_f = 4$; same actions (unimproved staggered), same mass



Agreement for $\mu/T \lesssim 1$

Pseudo-critical temperature

$$\frac{T_c(\mu)}{T_c(0)} = 1 - \kappa(N_f, m_q)\left(\frac{\mu}{T}\right)^2 + \dots$$

- Curvature of crit. line from Taylor expansion
  2+1 flavours, Nt=4, 8 improved staggered
- Extrapolation to chiral limit assuming
  O(4),O(2) scaling of magn. EoS
- $\kappa(\bar{\psi}\psi) = 0.059(2)(4)$

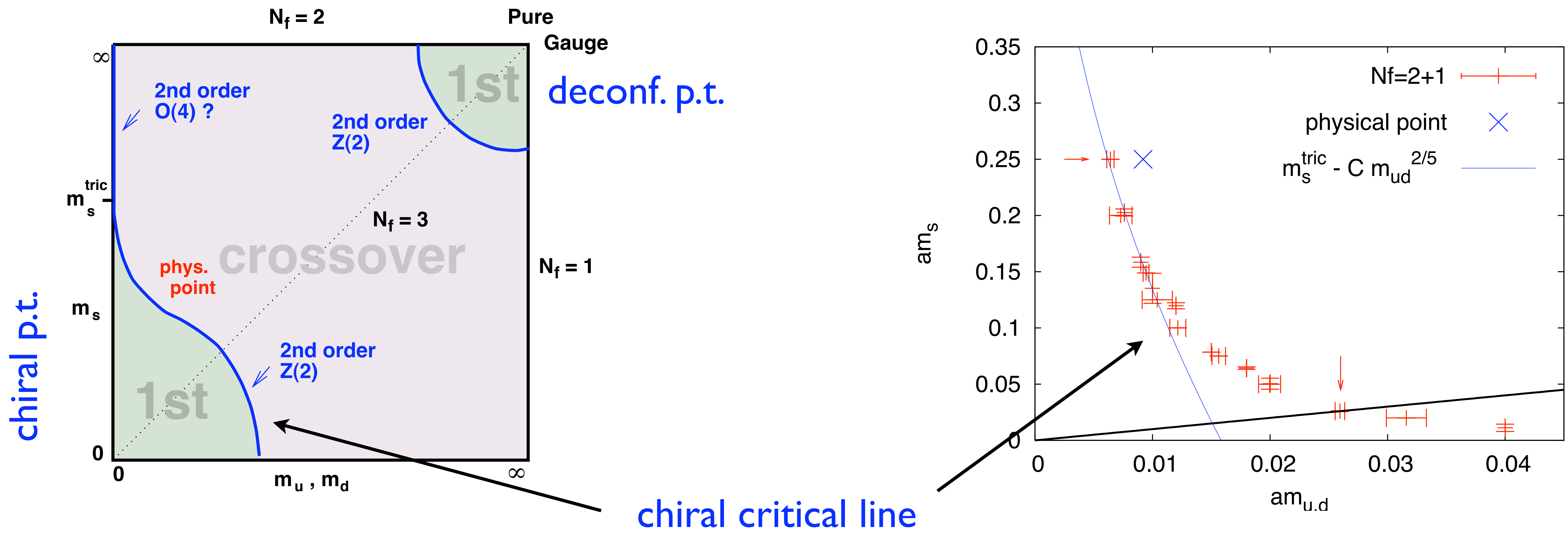
hotQCD 11

Endrödi et al. 11

- Curvature of crit. line from Taylor expansion
  2+1 flavours, Nt=6,8,10 improved staggered
- Observables $\bar{\psi}\psi_r, \chi_s$
- Continuum extrapolation:

$\kappa^{(\bar{\psi}\psi_r)} = 0.0066(20)$ $\kappa^{(\chi_s/T^2)} = 0.0089(14)$

Hard part: order of p.t., arbitrary quark masses $\mu = 0$



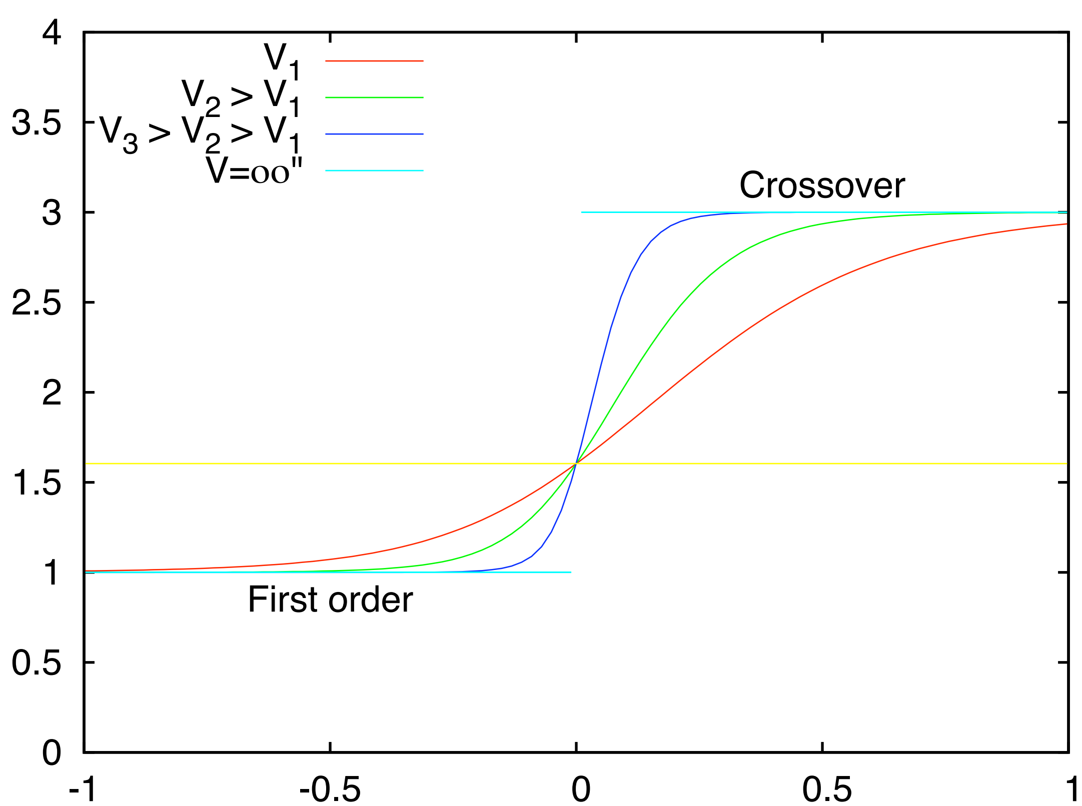
- physical point: crossover in the continuum — Aoki et al 06
- chiral critical line on $N_t = 4, a \sim 0.3$ fm — de Forcrand, O.P. 07
- consistent with tri-critical point at $m_{u,d} = 0, m_s^{\rm tric} \sim 2.8T$
- But: $N_f = 2$ chiral O(4) vs. 1st still open — Di Giacomo et al 05, Kogut, Sinclair 07
  $U_A(1)$ anomaly! — Chandrasekharan, Mehta 07, RBC-BI 09

How to identify the order of the phase transition

$$B_4(\bar\psi\psi) \equiv \frac{\langle(\delta\bar\psi\psi)^4\rangle}{\langle(\delta\bar\psi\psi)^2\rangle^2} \xrightarrow{V\to\infty} \begin{cases} 1.604 & \text{3d Ising} \\ 1 & \text{first-order} \\ 3 & \text{crossover} \end{cases}$$

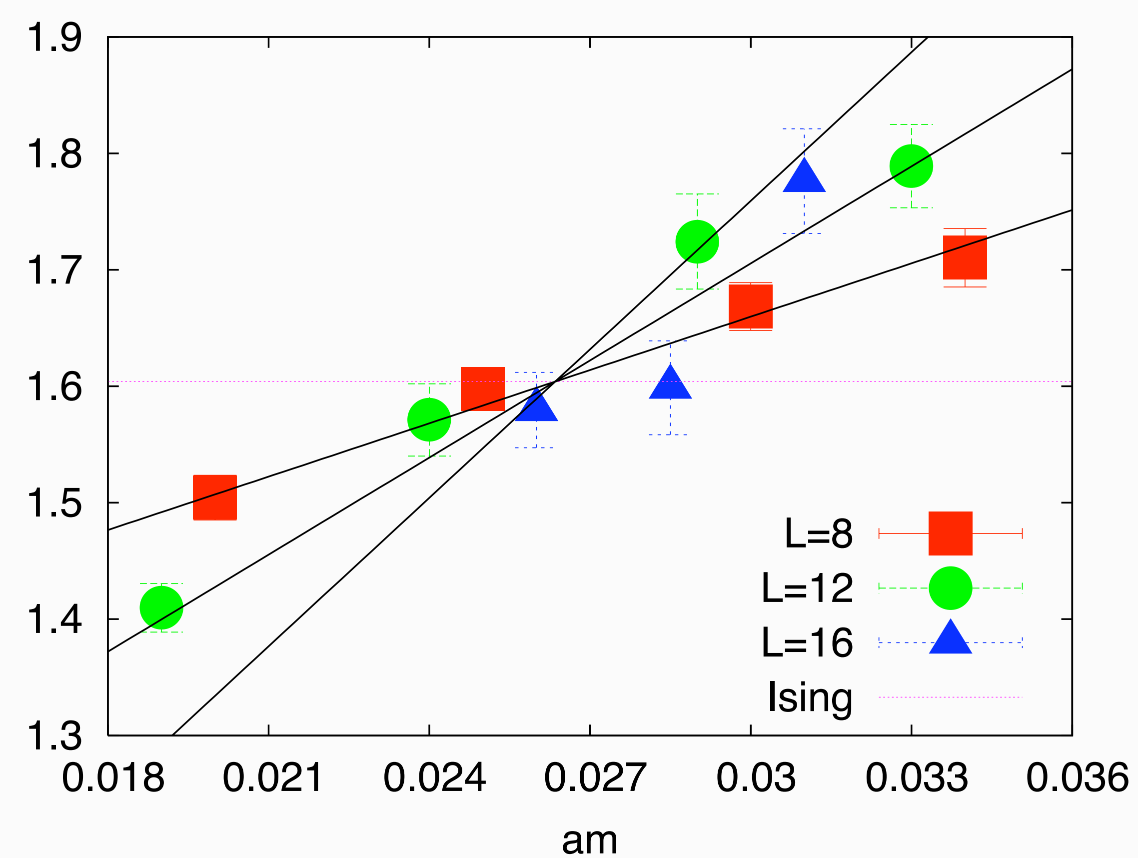
$\mu = 0$: $\quad B_4(m,L) = 1.604 + bL^{1/\nu}(m - m_0^c), \quad \nu = 0.63$

B_4

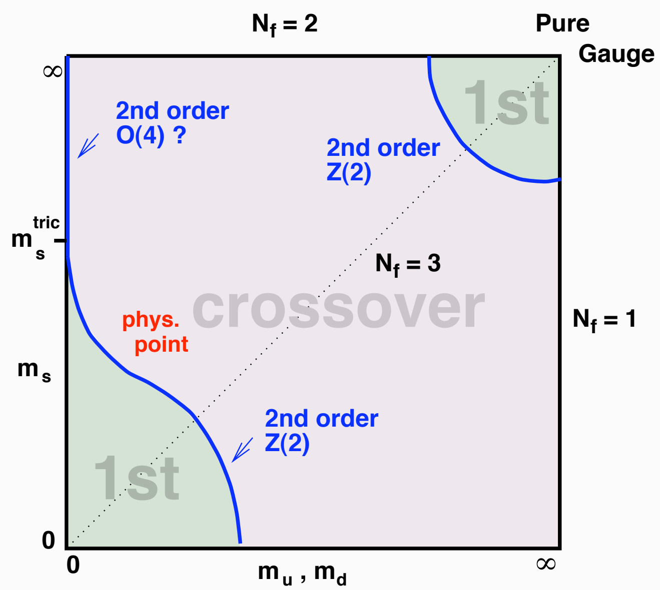


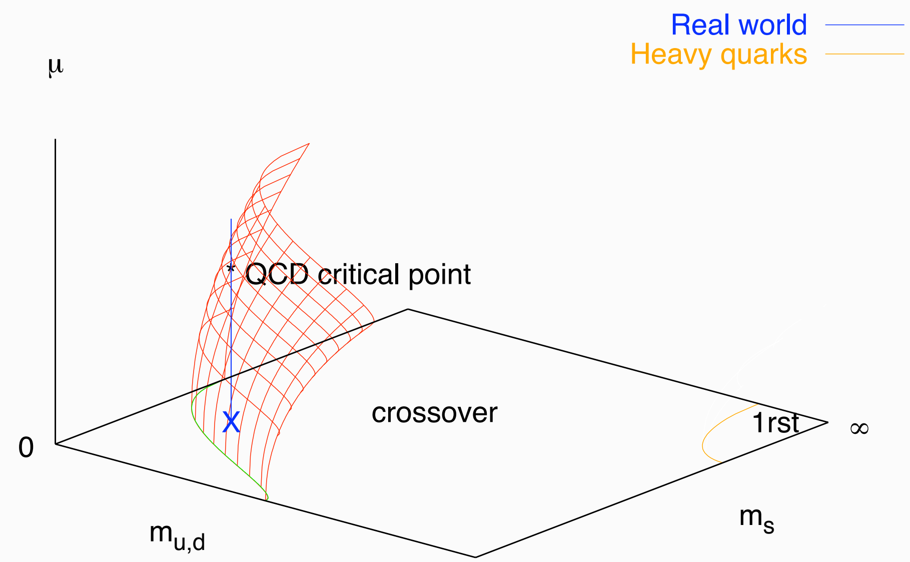
$x - x_c$

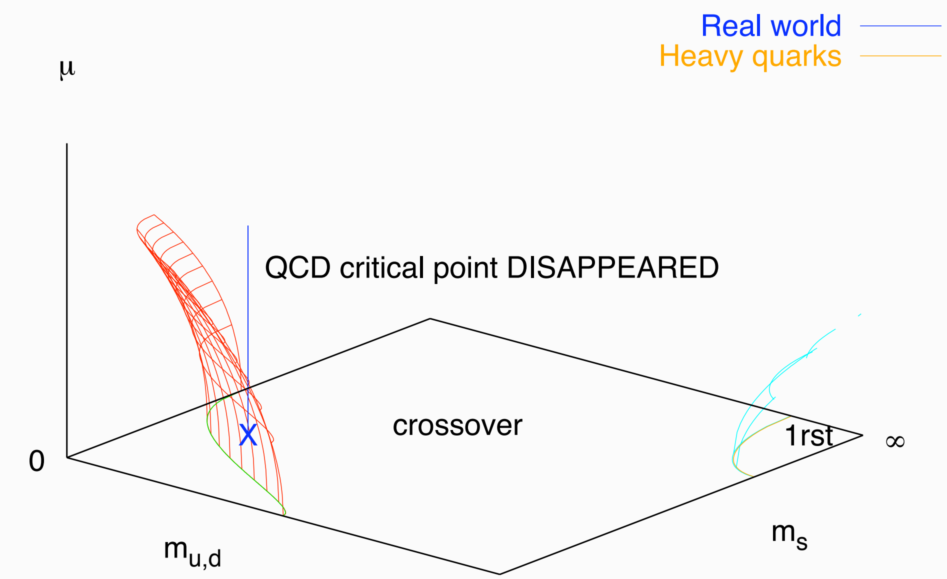
parameter along phase boundary, $T = T_c(x)$



Finite density: chiral critical line ⟶ critical surface





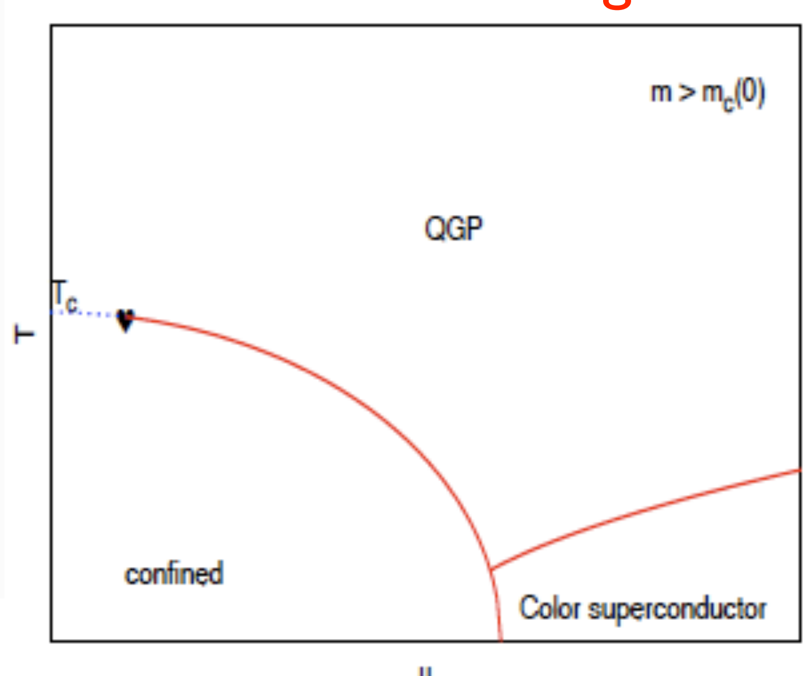


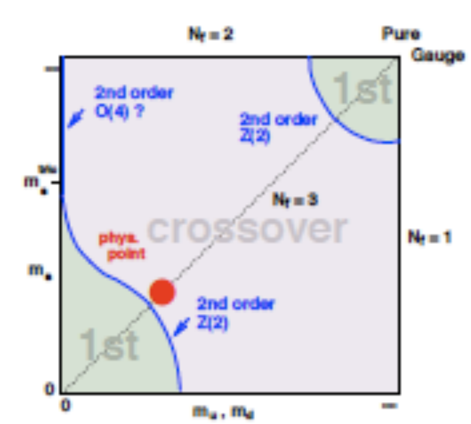
$$\frac{m_c(\mu)}{m_c(0)} = 1 + \sum_{k=1} \mathbf{c_k} \left(\frac{\mu}{\pi T}\right)^{2k}$$

$c_1 > 0$ $c_1 < 0$

Standard scenario

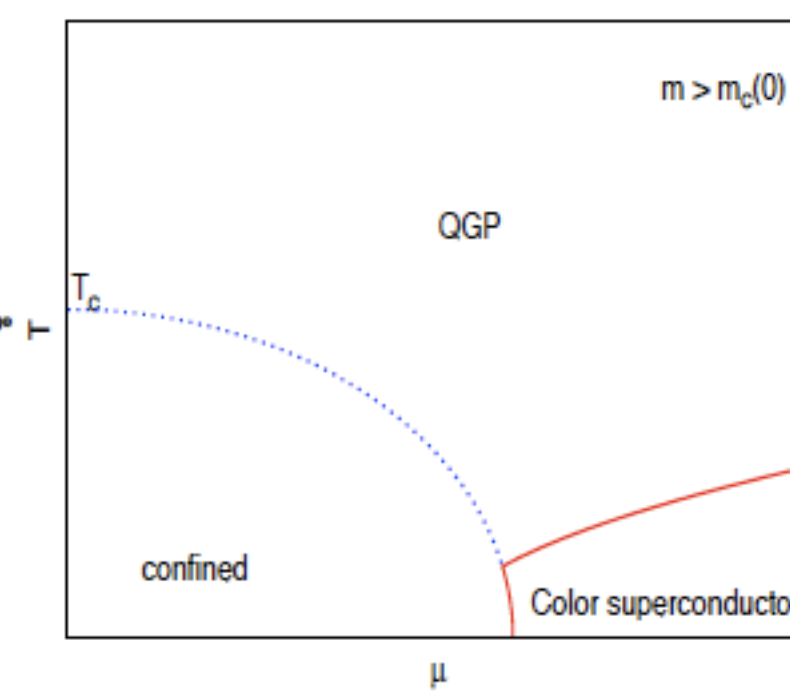
transition strengthens



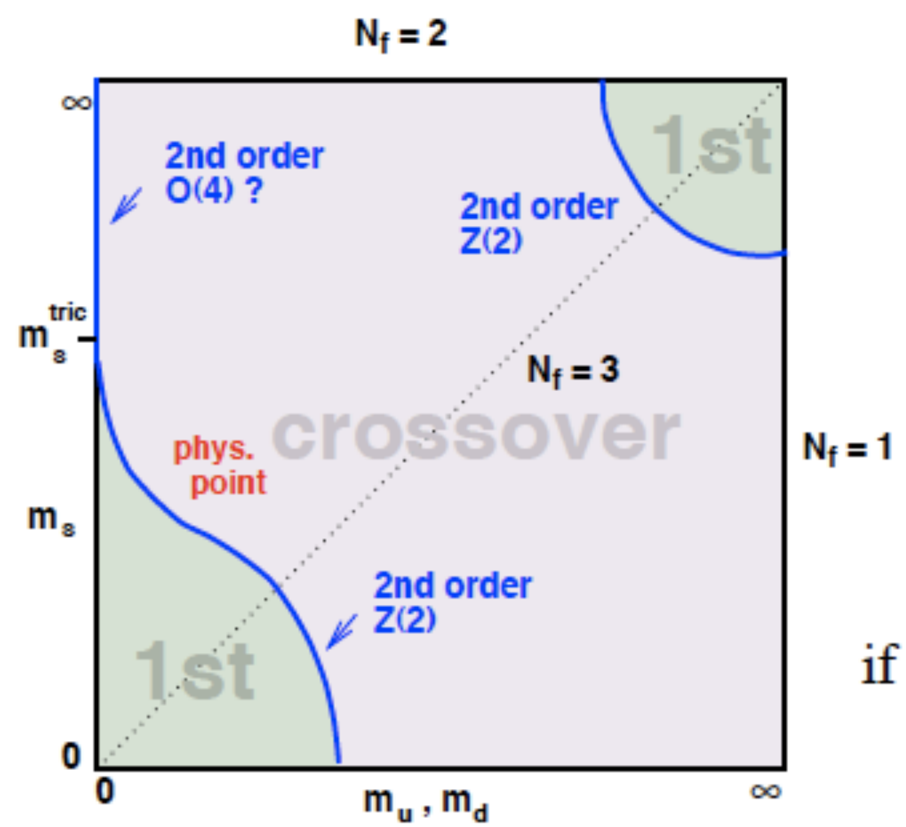


Exotic scenario

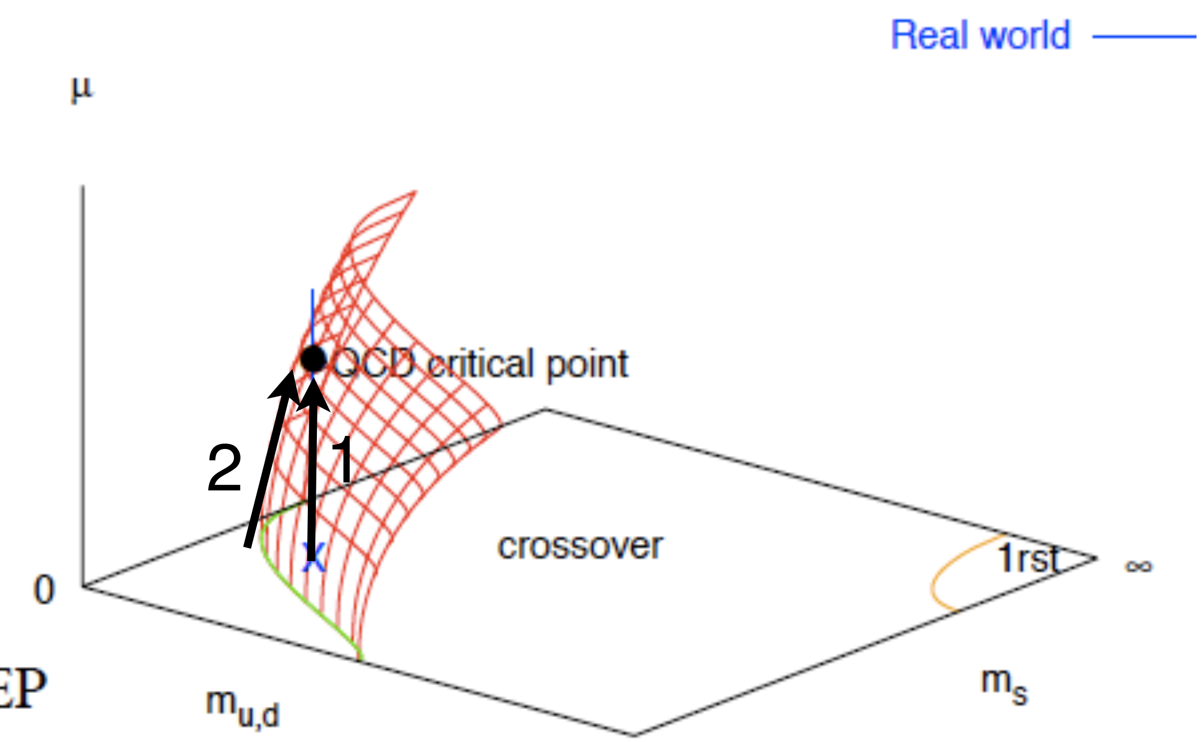
transition weakens



Much harder: is there a QCD critical point?



if *chiral* CEP



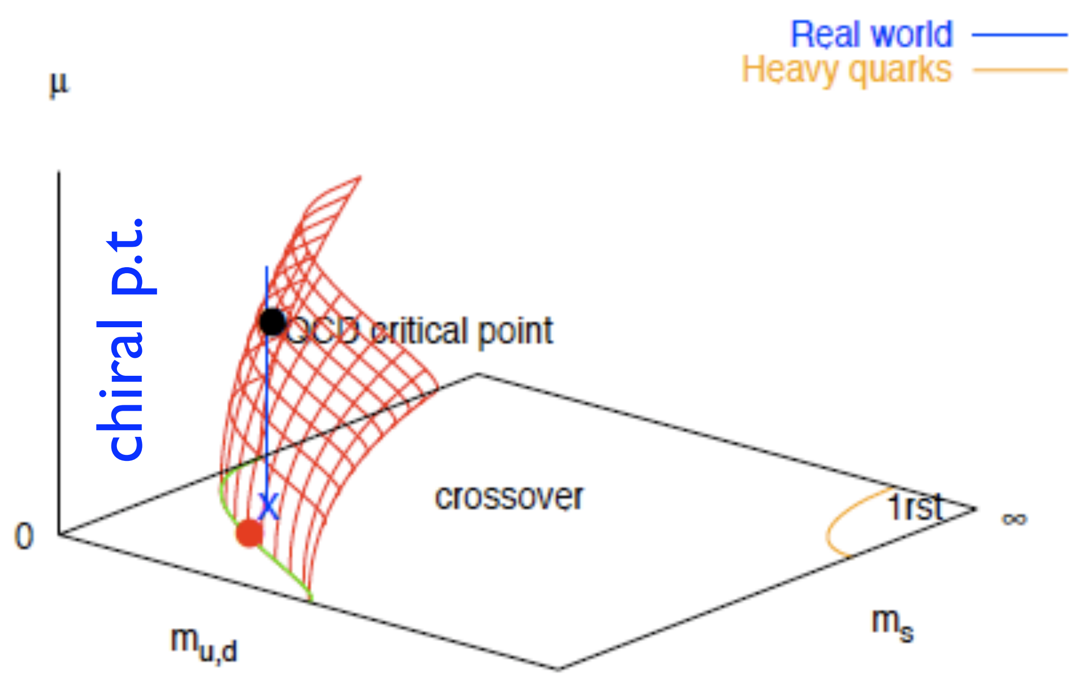
Two strategies:

1 follow vertical line: $m = m_{\rm phys}$, turn on μ

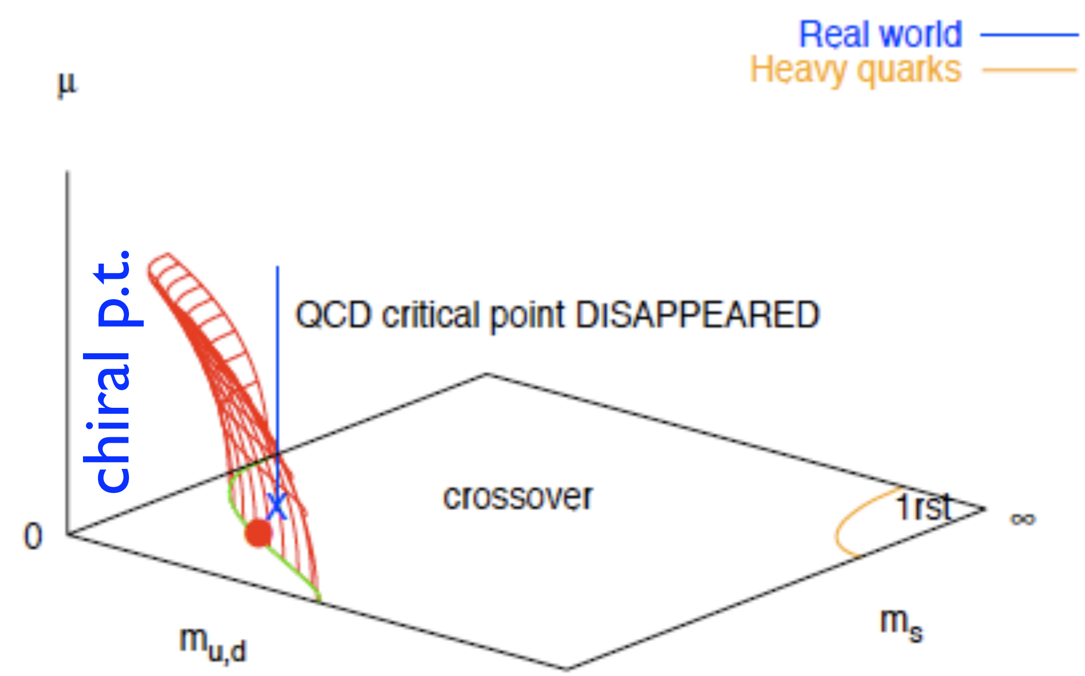
crit. point from reweighting
Fodor,Katz, systematics?

2 follow critical surface: $m = m_{\rm crit}(\mu)$

$\mu \neq 0$, conservative: follow chiral critical line ➡ surface



$c_1 > 0$



$c_1 < 0$

$$\frac{m_c(\mu)}{m_c(0)} = 1 + \sum_{k=1} c_k \left(\frac{\mu}{\pi T}\right)^{2k}$$

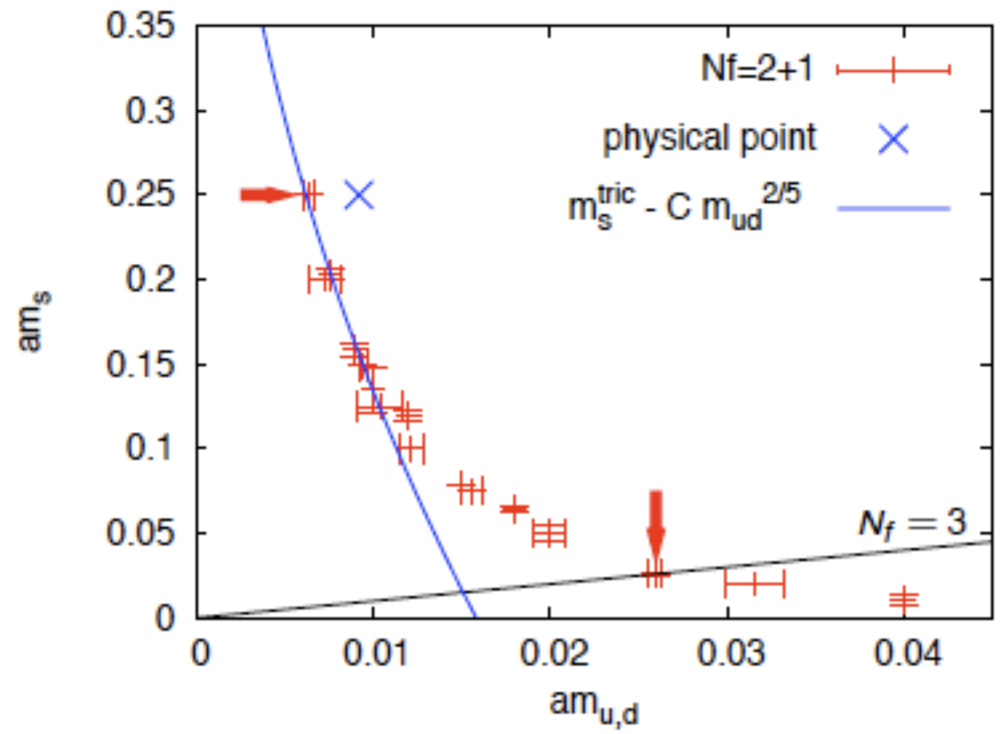
1. Tune quark mass(es) to $m_c(0)$: 2nd order transition at $\mu = 0, T = T_c$
   known universality class: 3d Ising

2. Measure derivatives $\frac{d^k m_c}{d\mu^{2k}}|_{\mu=0}$:

   Turn on imaginary μ and measure $\frac{m_c(\mu)}{m_c(0)}$

de Forcrand, O.P. 08,09

Curvature of the chiral critical surface



$N_f = 3$

$N_f = 2+1,\ m_s = m_s^{\text{phys}}$

consistent $8^3 \times 4$ and $12^3 \times 4$, $\sim 5 \times 10^6$ traj.

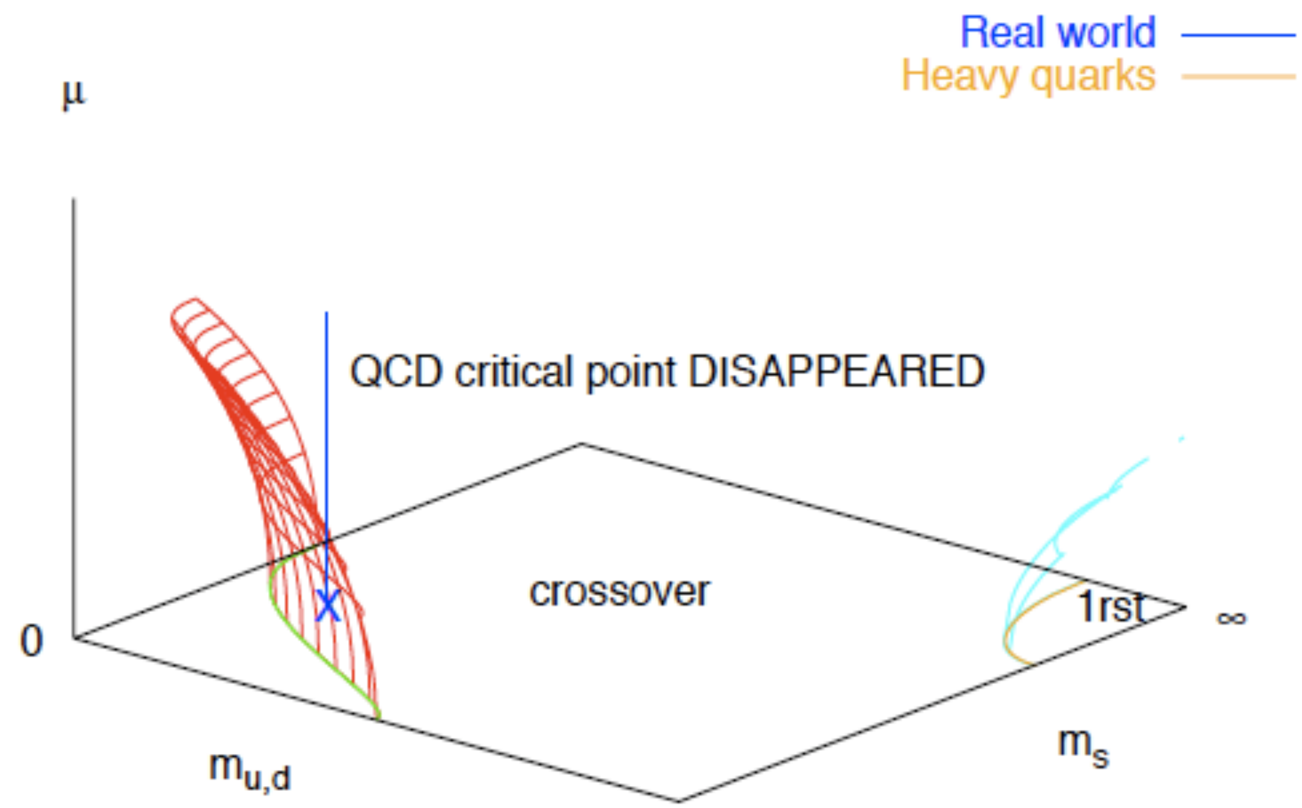
$$\frac{m_c(\mu)}{m_c(0)} = 1 - 3.3(3)\left(\frac{\mu}{\pi T}\right)^2 \underbrace{- 47(20)\left(\frac{\mu}{\pi T}\right)^4}_{\text{8th derivative of P}} - \ldots$$

$16^3 \times 4$, Grid computing, $\sim 10^6$ traj.

$$\frac{m_c^{u,d}(\mu)}{m_c^{u,d}(0)} = 1 - 39(8)\left(\frac{\mu}{\pi T}\right)^2 - \ldots$$

de Forcrand, O.P. 08,09

On coarse lattice exotic scenario: no chiral critical point at small density



Weakening of p.t. with chemical potential also for:

- -Heavy quarks — de Forcrand, Kim, Takaishi 05
- -Light quarks with finite isospin density — Kogut, Sinclair 07
- -Electroweak phase transition with finite lepton density — Gynther 03

QCD at complex μ: general properties

$$Z(V,\mu,T) = \mathrm{Tr}\left(\mathrm{e}^{-(\hat{H}-\mu\hat{Q})/T}\right); \quad \mu = \mu_r + \mathrm{i}\mu_i; \quad \bar{\mu} = \mu/T$$

exact symmetries: μ-reflection and μ_i-periodicity Roberge, Weiss

$$\boxed{Z(\bar{\mu}) = Z(-\bar{\mu}), \qquad Z(\bar{\mu}_r,\bar{\mu}_i) = Z(\bar{\mu}_r,\bar{\mu}_i + 2\pi/N_c)}$$

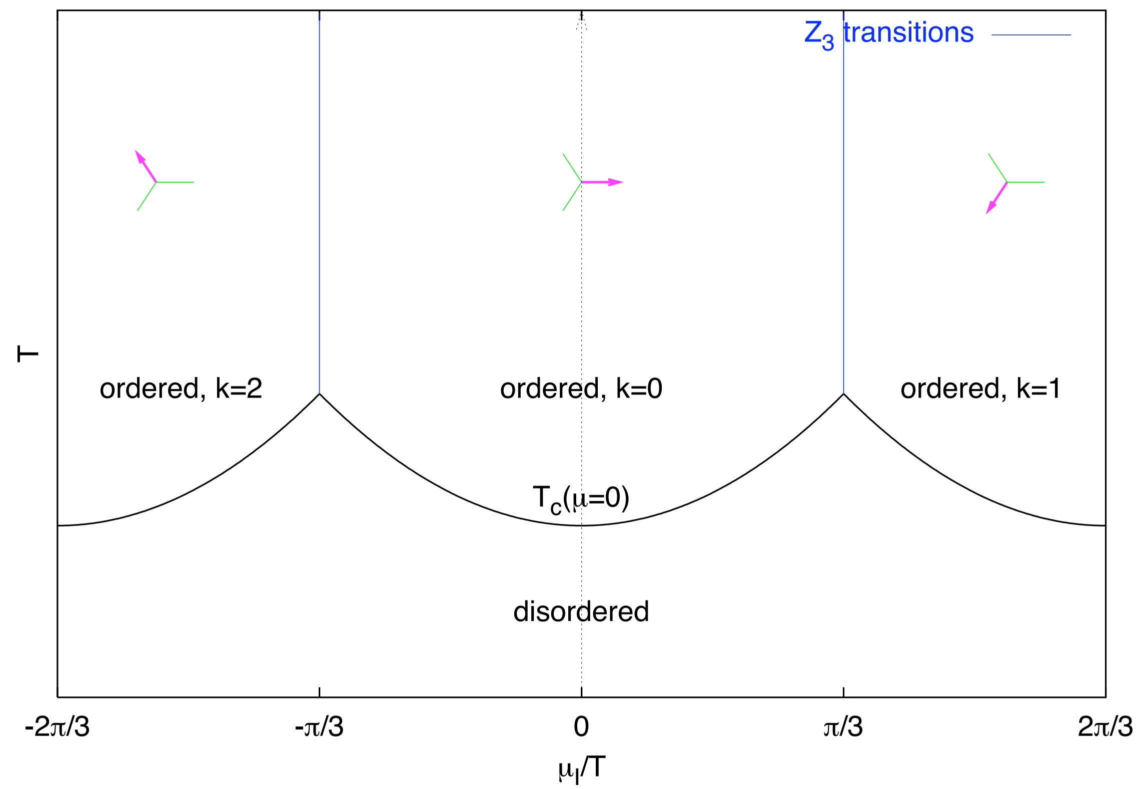
Imaginary μ phase diagram:

Z(3)-transitions: $\bar{\mu}_i^c = \frac{2\pi}{3}\left(n+\frac{1}{2}\right)$

1rst order for high T, crossover for low T

analytic continuation within:

$|\mu|/T \leq \pi/3 \Rightarrow \mu_B \lesssim 550\mathrm{MeV}$



So far: $\langle O \rangle = \sum_n^N c_n \bar{\mu}_i^{2n} \Rightarrow \mu_i \longrightarrow -\mathrm{i}\mu_i$

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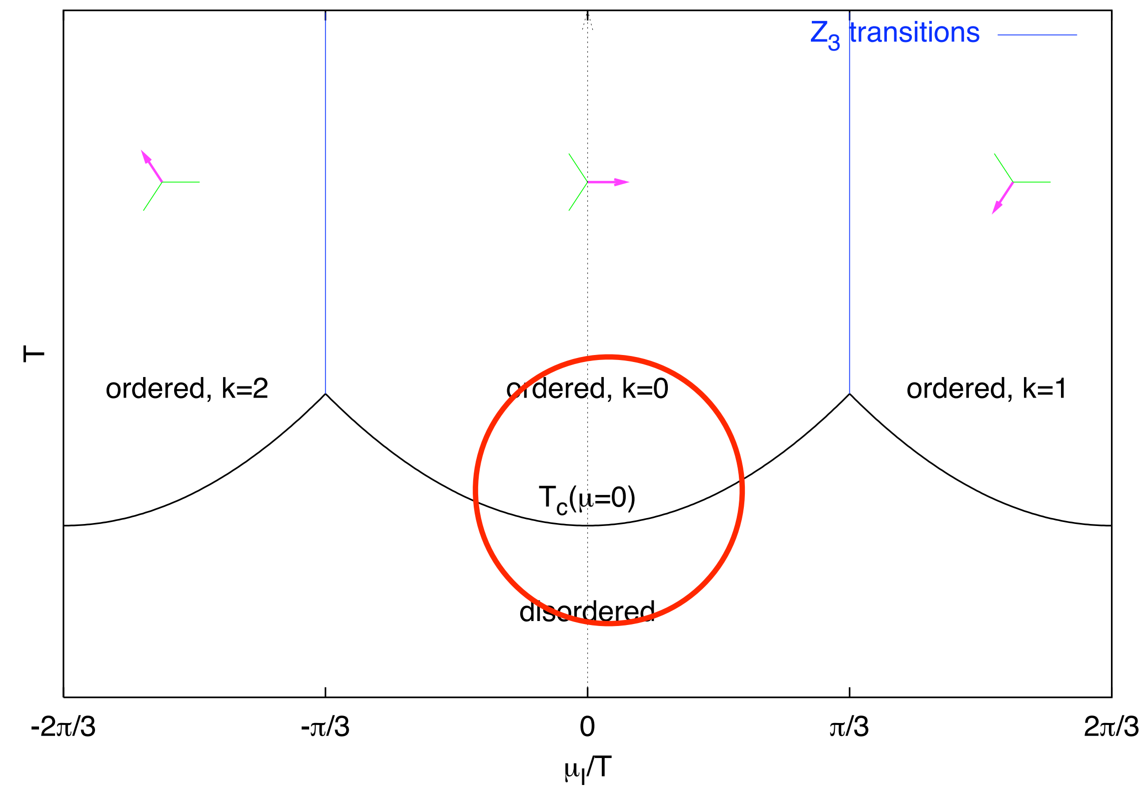

So far: $\langle O \rangle = \sum_n^N c_n \bar{\mu}_i^{2n} \Rightarrow \mu_i \longrightarrow -\mathrm{i}\mu_i$ chiral/deconf. transition

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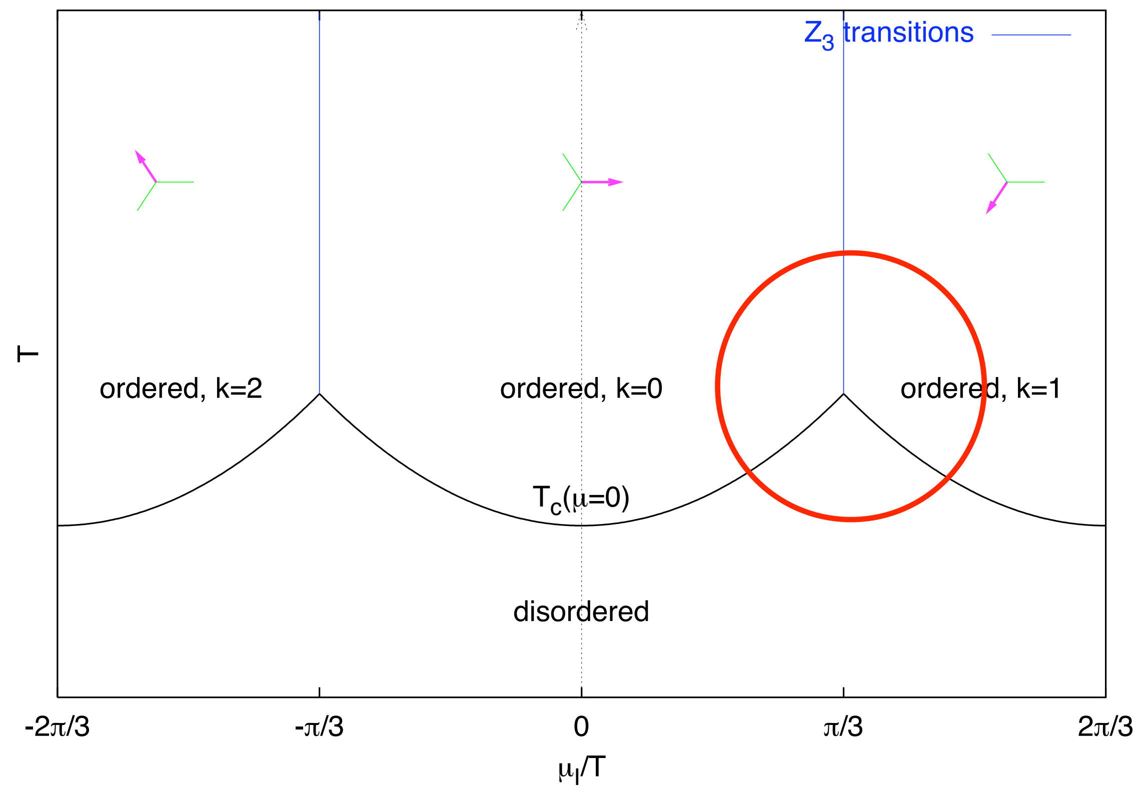

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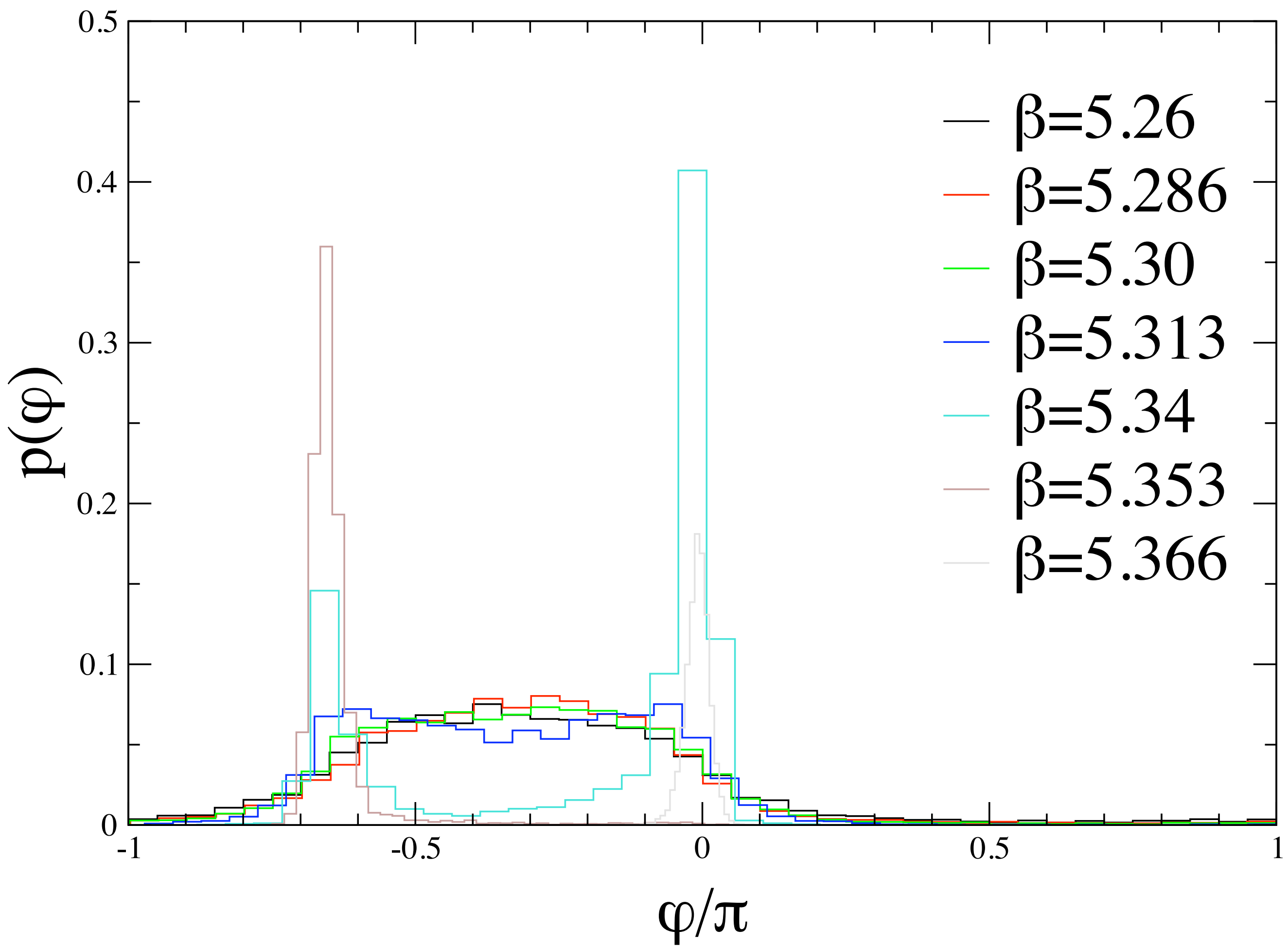
Now: endpoint of Z(N) transition

The Z(3) transition numerically

Nf=2: de Forcrand, O.P. 02

Nf=4: D'Elia, Lombardo 03

Sectors characterised by phase of Polyakov loop: $\langle L(x)\rangle = |\langle L(x)\rangle|\, \mathrm{e}^{i\varphi}$



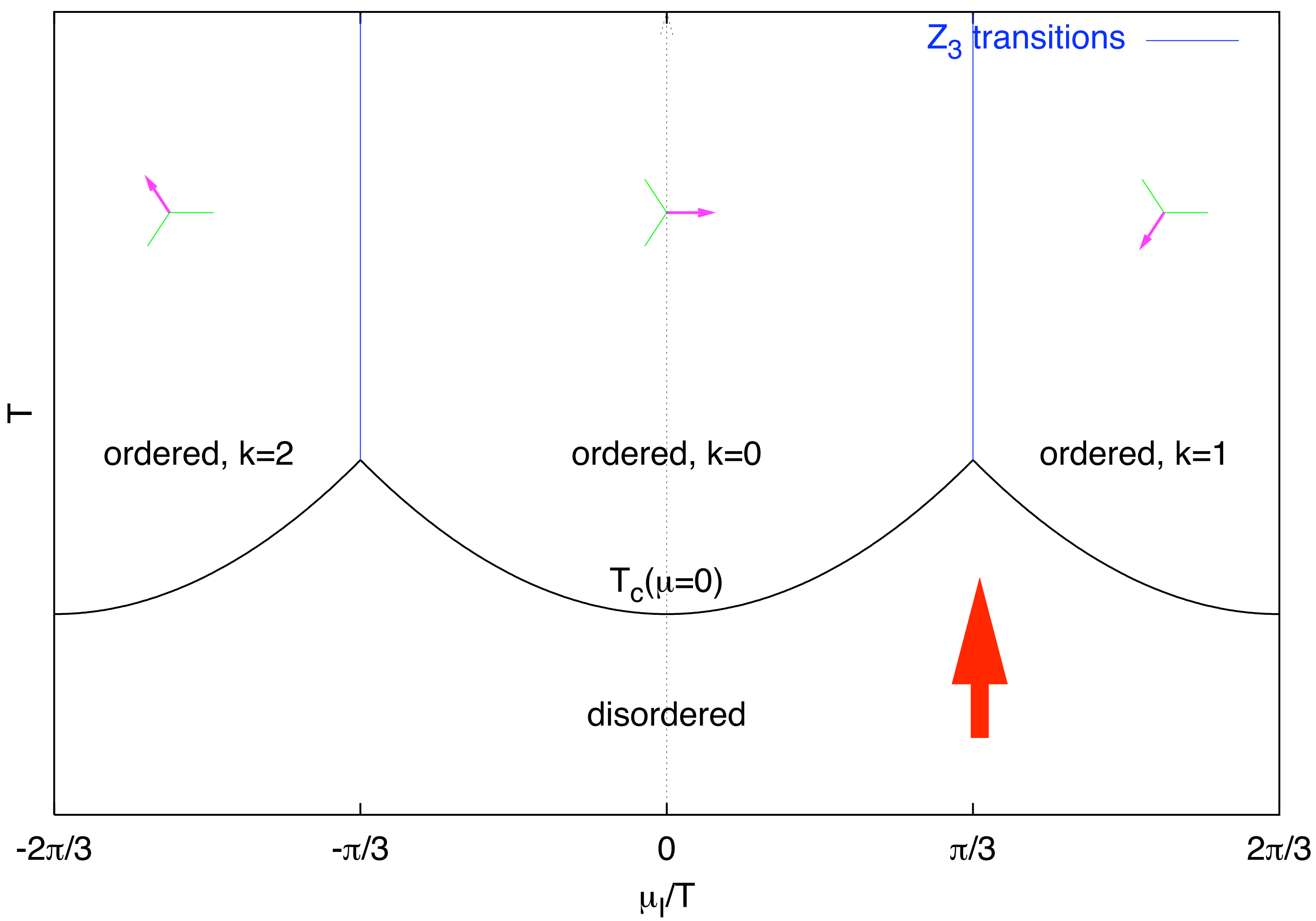
Low T: crossover High T: first order p.t.

The nature of the Z(3) end points

Nf=4: D'Elia, Di Renzo, Lombardo 07 Nf=2: D'Elia, Sanfilippo 09 Here: Nf=3

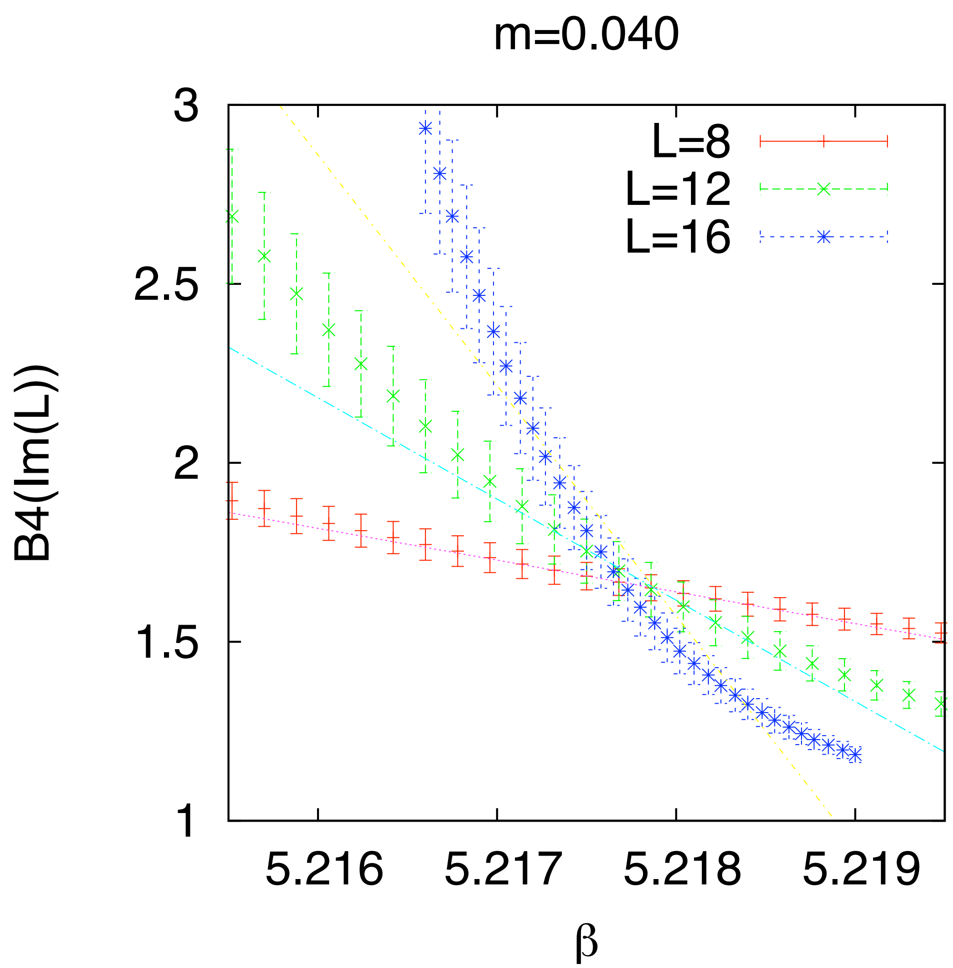
Strategy: fix $\frac{\mu_i}{T} = \frac{\pi}{3}, \pi$, measure Im(L), order parameter at $\frac{\mu_i}{T} = \pi$

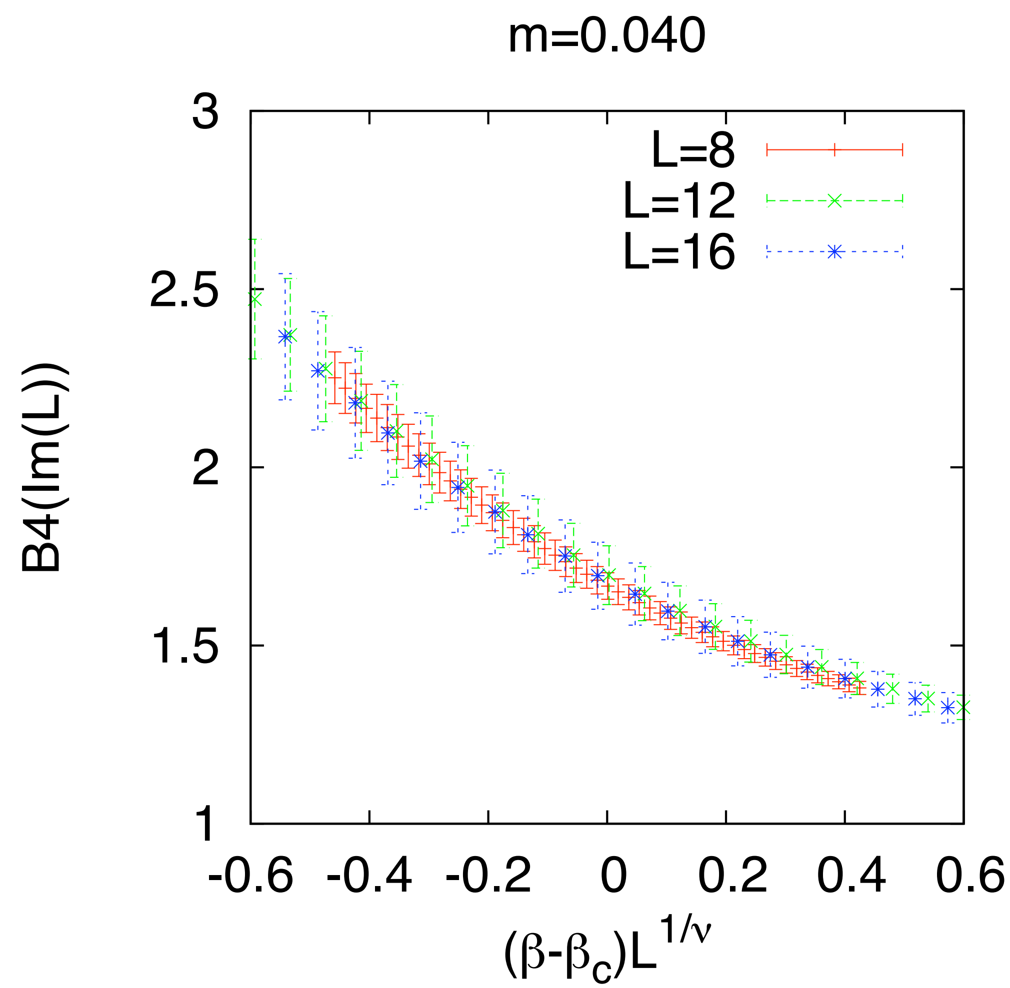
determine order of Z(3) branch/end point as function of m



$$B_4 = \frac{\langle \delta \mathrm{Im}(L)^4 \rangle}{\langle \delta \mathrm{Im}(L)^2 \rangle^2}$$

Results:



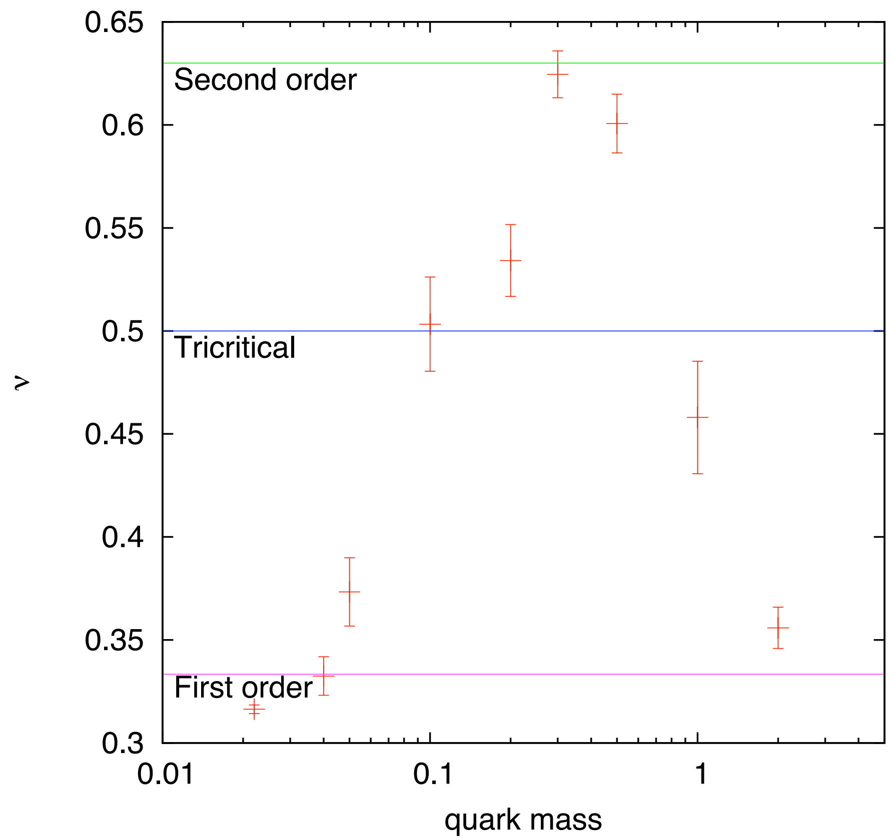


$\nu = 0.33$

$$B_4(\beta, L) = B_4(\beta_c, \infty) + C_1(\beta - \beta_c)L^{1/\nu} + C_2(\beta - \beta_c)^2 L^{2/\nu} \ldots$$

B4 at intersection has large finite size corrections (well known), ν more stable

$\nu = 0.33, 0.5, 0.63$ for 1st order, tri-critical, 3d Ising scaling



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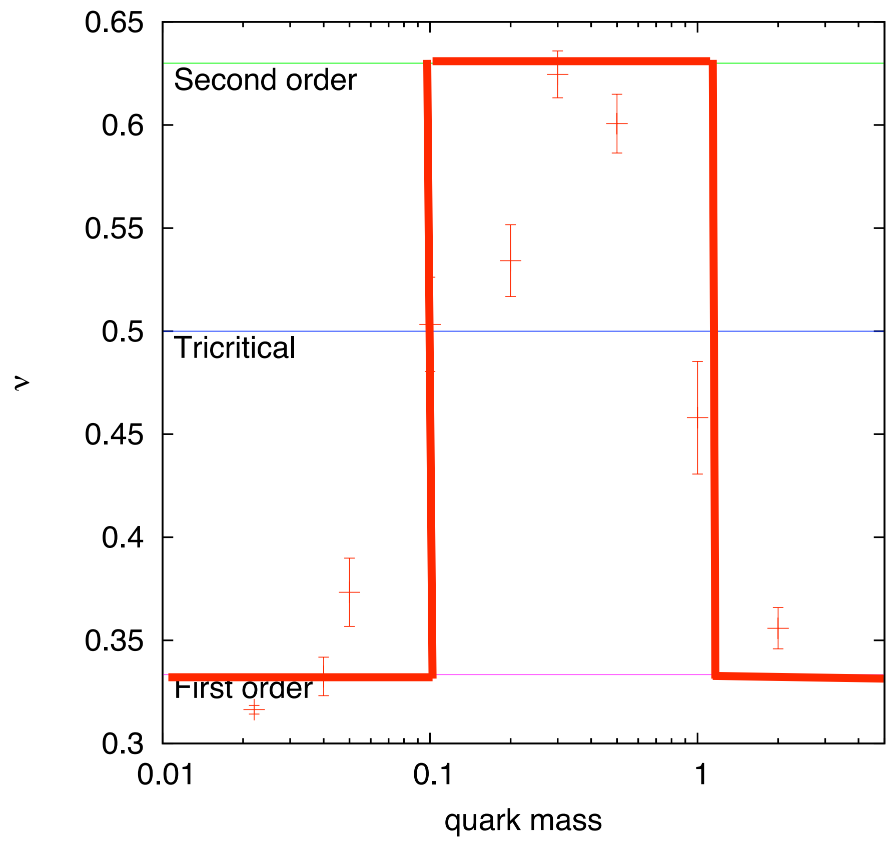
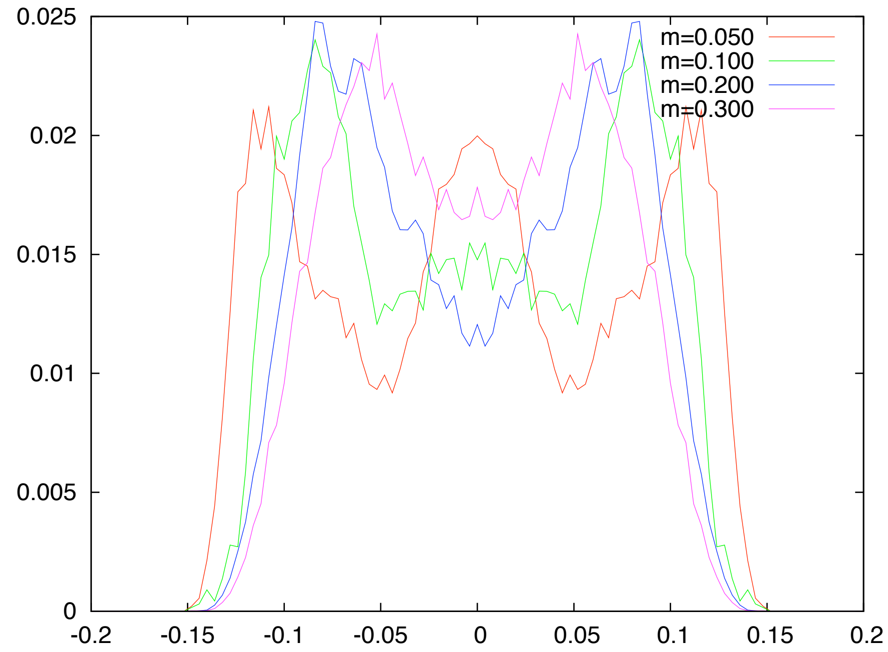


On infinite volume, this becomes a step function, smoothness due to finite L

Details of RW-point: distribution of Im(L)

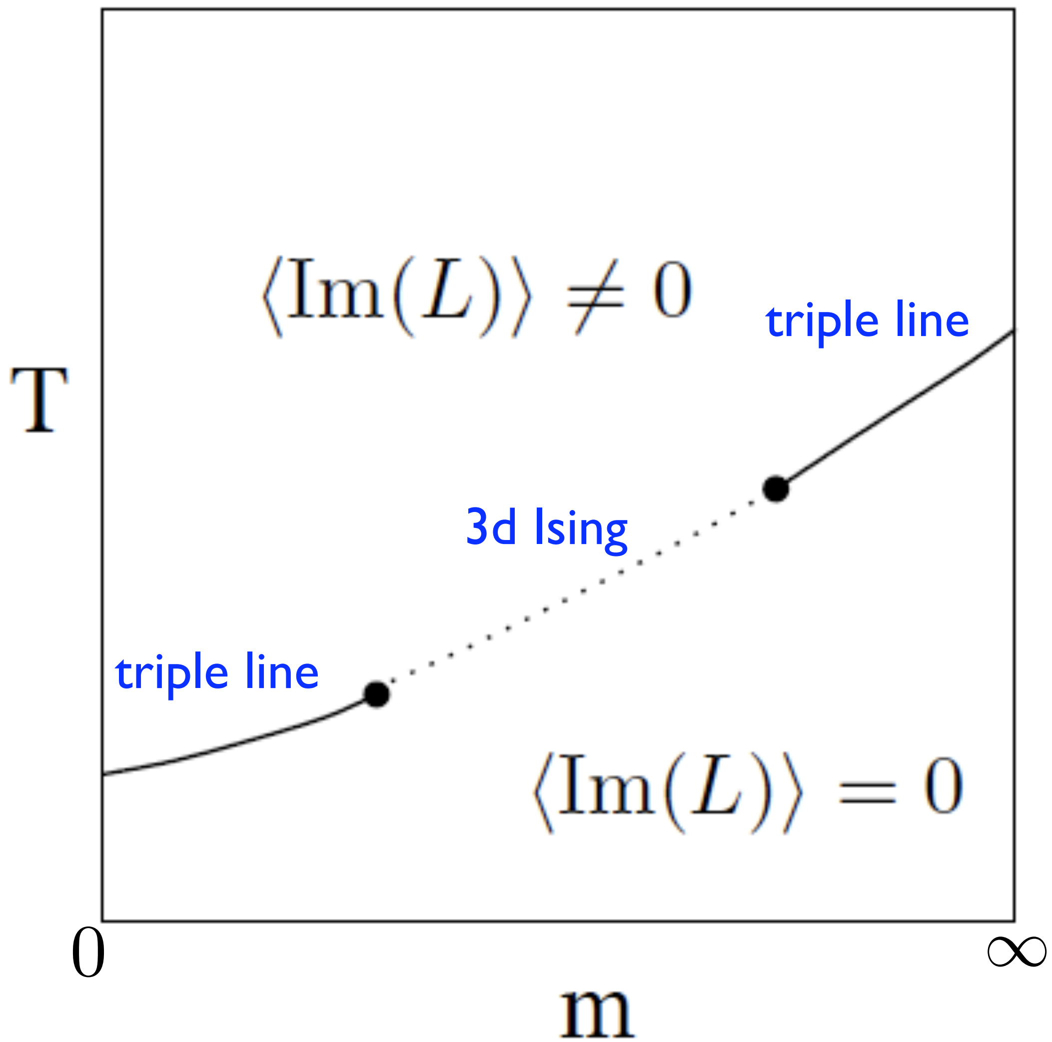


Small+large masses: three-state coexistence — triple point

Intermediate masses: middle peak disappears — Ising distribtion in magn. direction

→ tri-critical point in between!

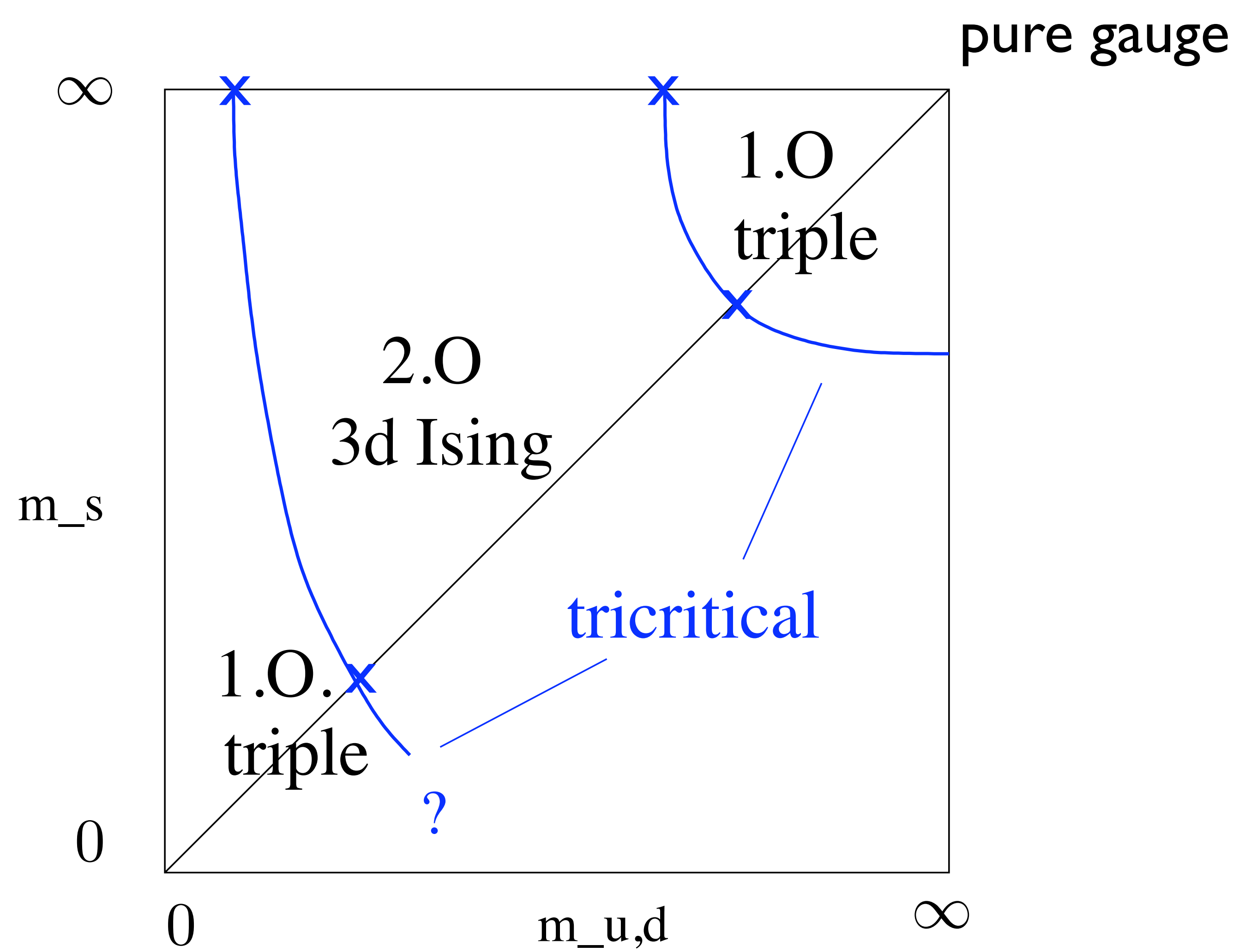
Phase diagram at $\mu = i\frac{\pi T}{3}$



Nf=2, light and intermediate masses: 1st and 3d Ising behaviour D'Elia, Sanfilippo 09

Generalisation: nature of the Z(3) endpoint for Nf=2+1

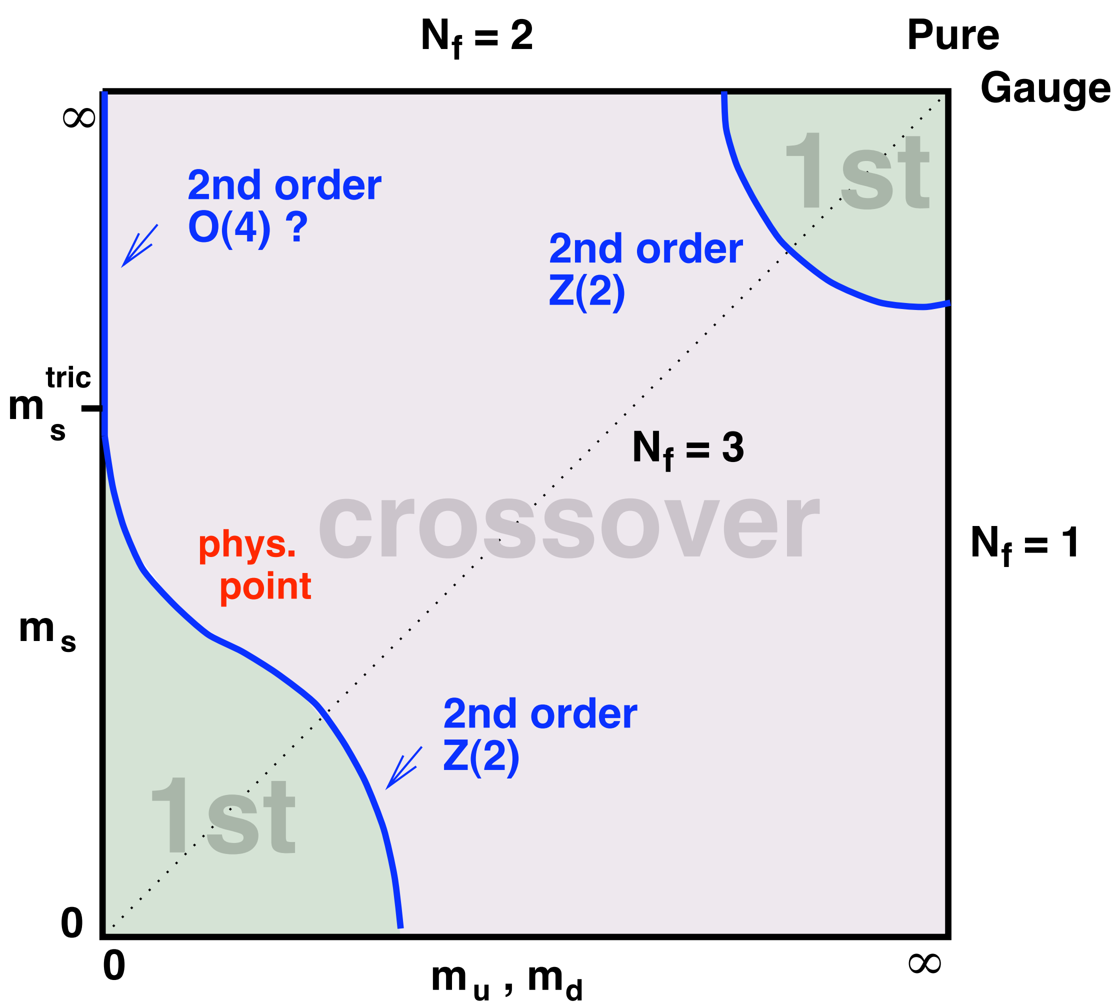
x known tric. points

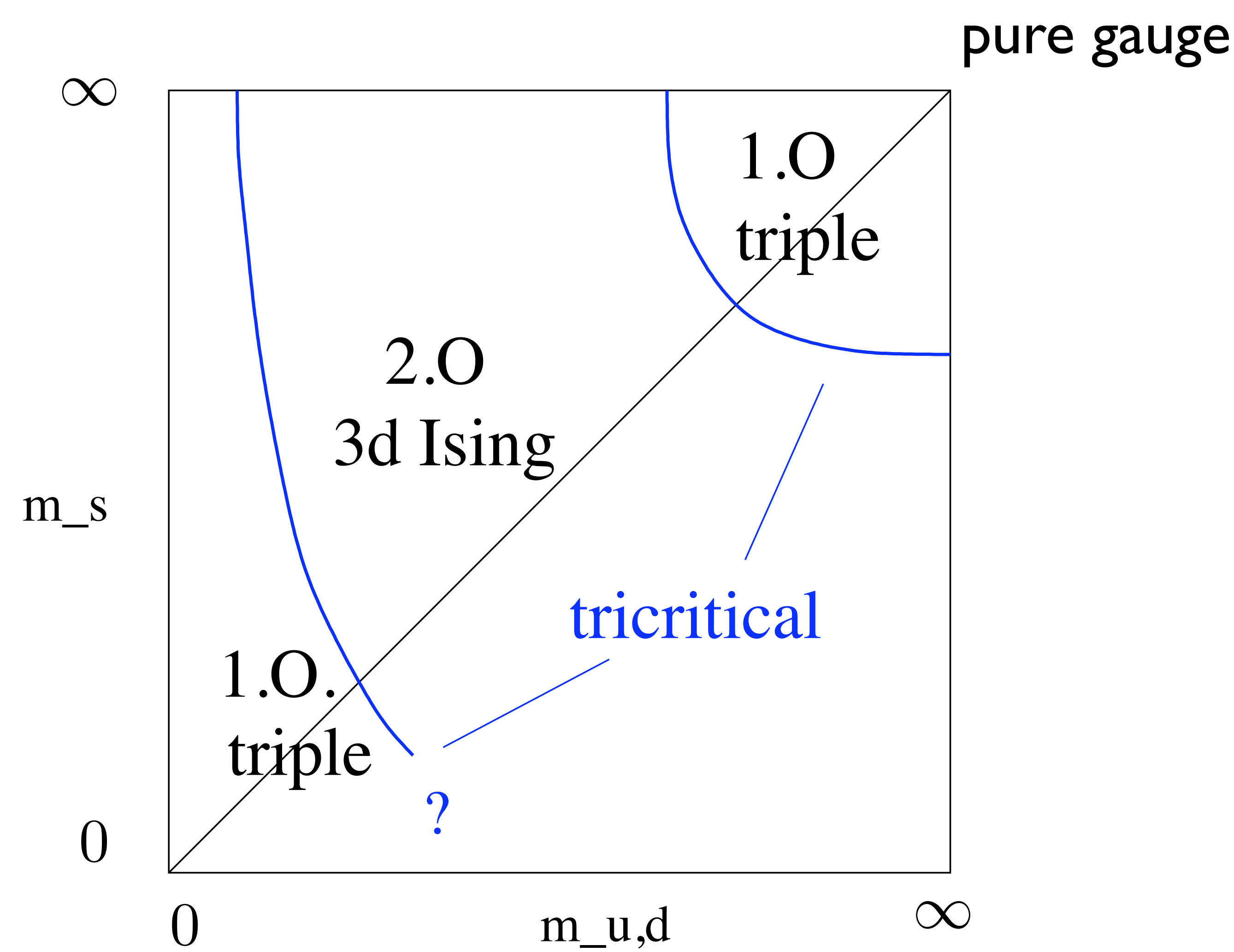


-Diagram computable with standard Monte Carlo, continuum limit feasible!

-Benchmarks for PNJL, chiral models etc.

Connection between zero and imaginary μ



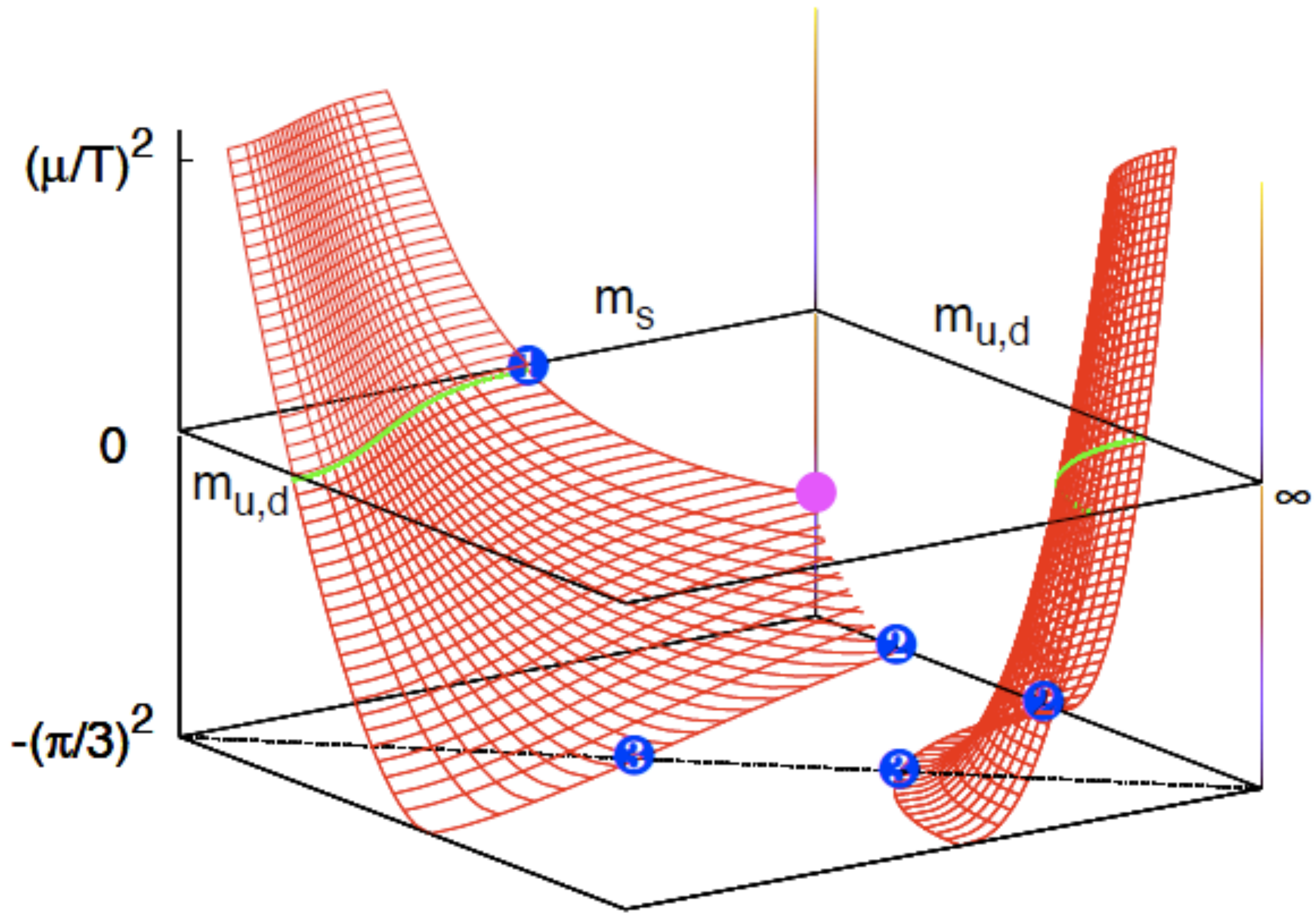


$$\mu = 0$$

$$\mu = i\frac{\pi T}{3}$$

-Connection computable with standard Monte Carlo!

3d, imaginary chemical potential included:

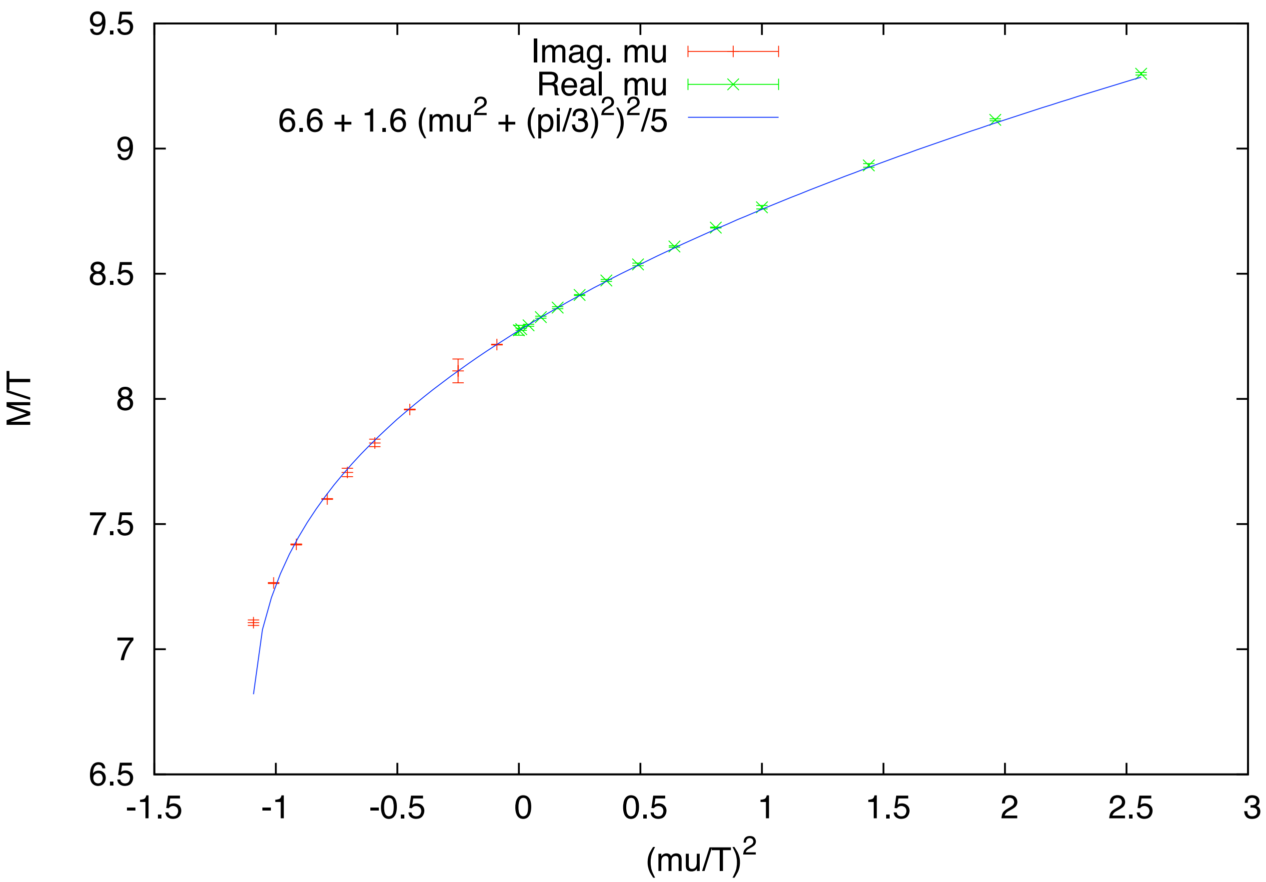


Heavy quarks: 3d 3-state Potts and strong coupling

small μ/T: sign problem mild, doable for real μ!

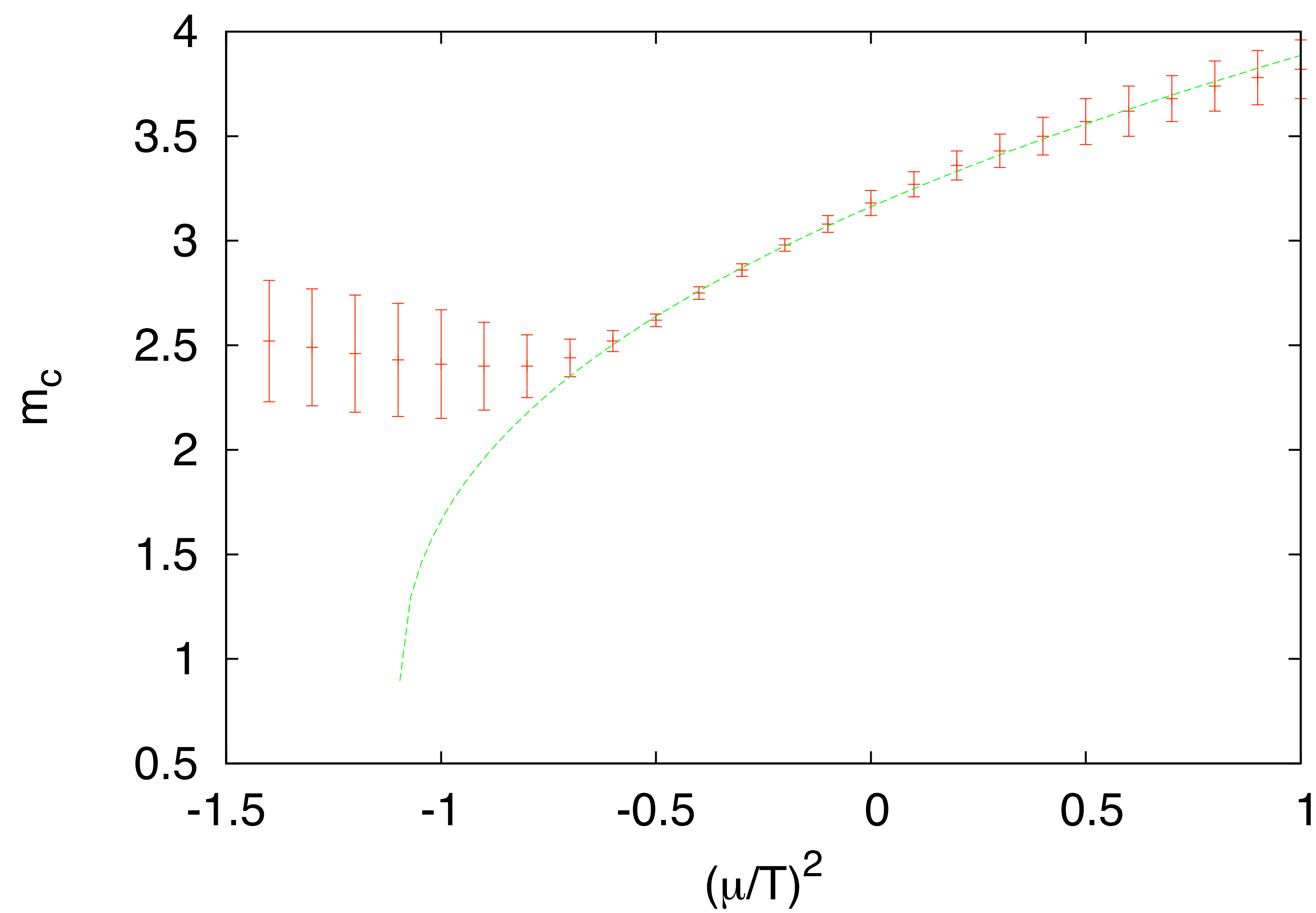
de Forcrand, Kim, Kratochvila, Takaishi

Potts:



QCD, Nt=1, strong coupling series:

Langelage, O.P. 09



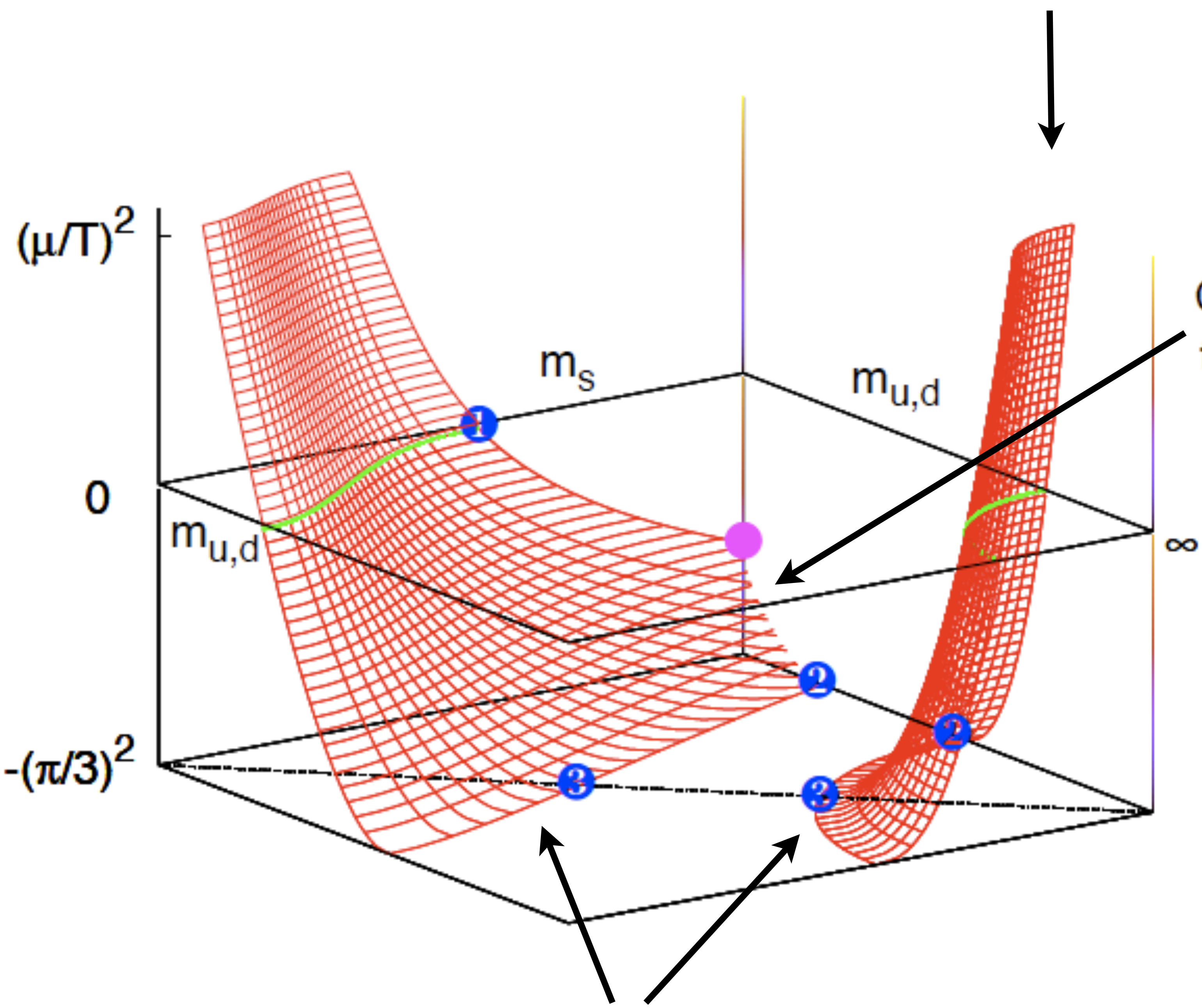
tri-critical scaling:

$$\frac{m_c}{T}(\mu^2) = \frac{m_{tric}}{T} + K\left[\left(\frac{\pi}{3}\right)^2 + \left(\frac{\mu}{T}\right)^2\right]^{2/5}$$

exponent universal

Deconfinement critical surface: tric. scaling!

shape determined by tric. scaling!

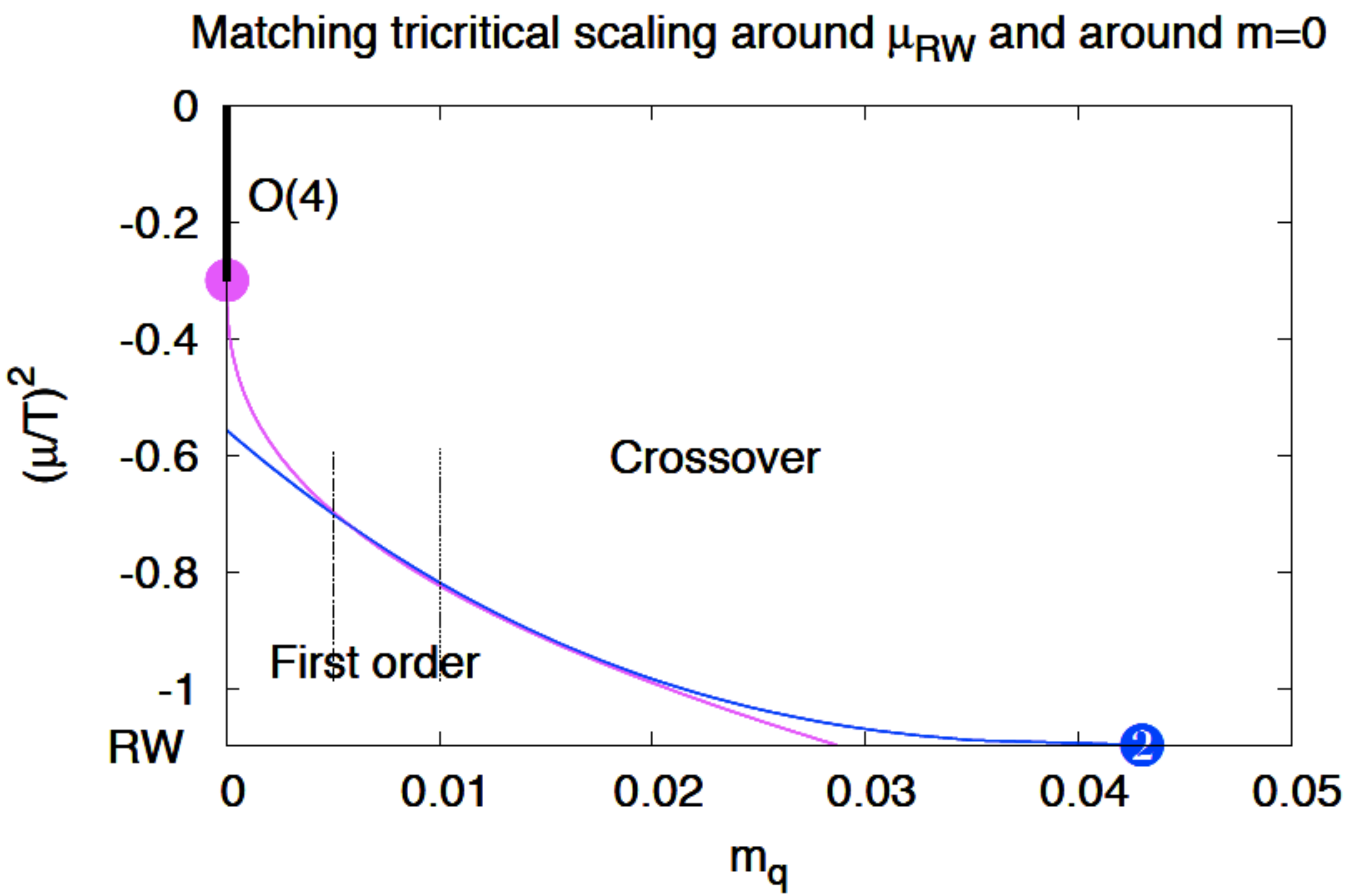


Can tricritical scaling constrain the phase diagram in the $N_f = 2$ backplane?

tricritical lines!

The Nf=2 backplane

in progress with Bonati, D'Elia, de Forcrand, Sanfilippo



Two tricritical points joined by a critical (Ising) line

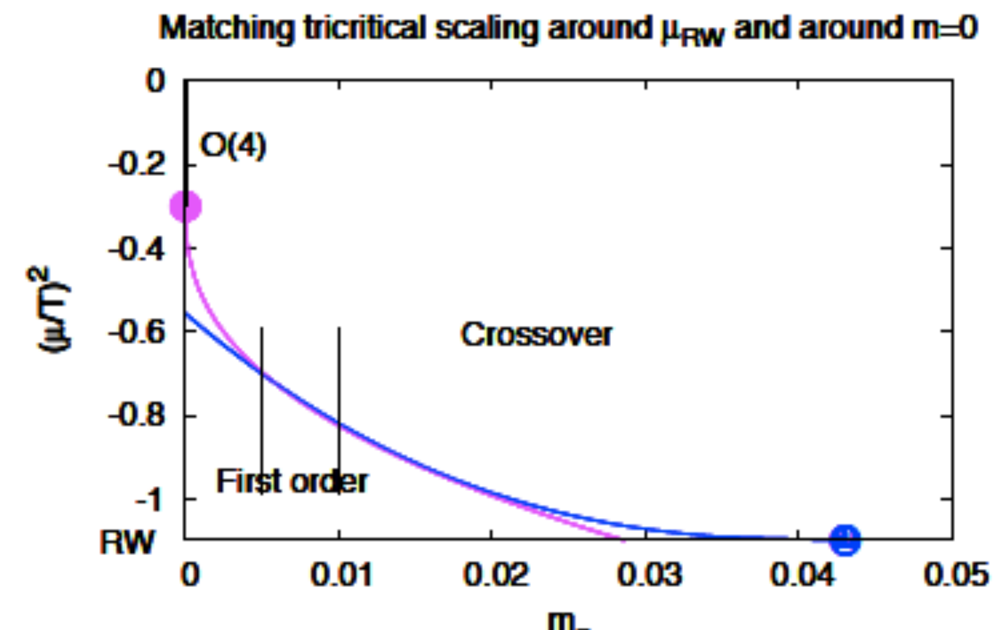
One tricritical point known – where is the other?

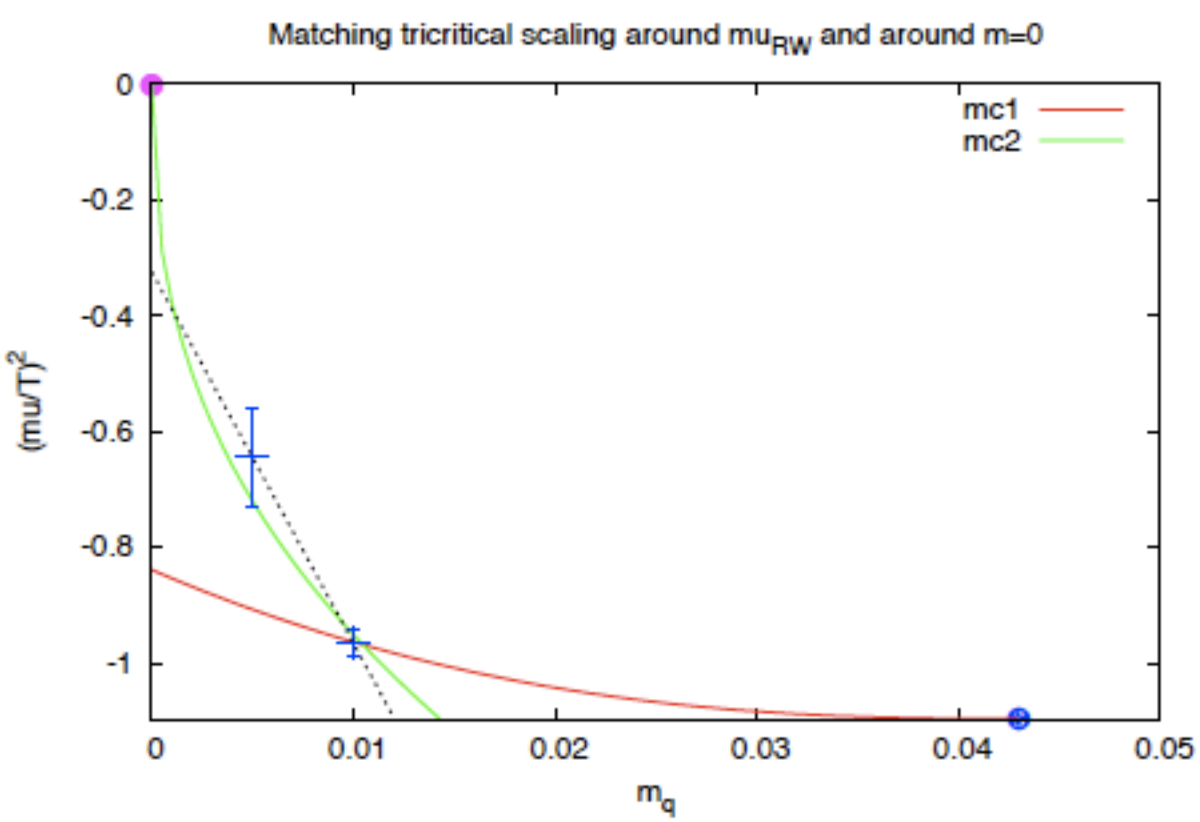
Preliminary results

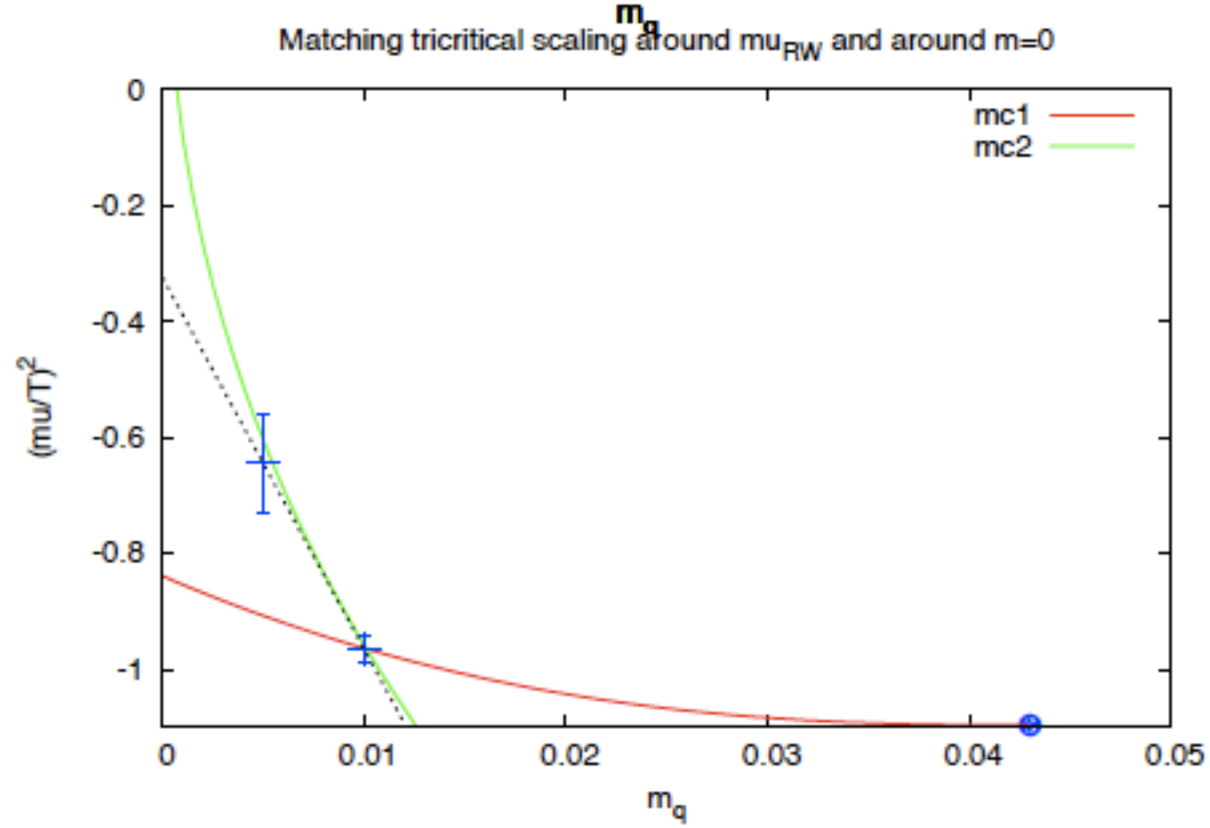
Not as expected:

Smooth matching very difficult

Need more points on critical line







Convexity $\Rightarrow (\mu/T)^2 \in [-0.3, 0]$ or $(\mu/T)^2 > 0$, ie. 1rst-order at $\mu = 0$

Conclusions

- For lattices with a~0.3 fm no chiral critical point for $\mu/T \lesssim 1$
- CEP scenario not yet clear: exploring uncharted territory!
- Z(3) transition at imaginary chem. pot. connects with chiral/deconf. transition
- Curvature of deconfinement critical surface determined by tri-critical scaling!
- Check if same holds for chiral critical surface,
  consequences for Nf=2 at zero density!