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QUARK MATTER WITH AXIAL CHEMICAL POTENTIAL

Bari, *2011年　09月 23日*

Outline

- Axial Chemical Potential: motivations
- The Model
- Phase Diagram with an Axial Chemical Potential
  .) Chiral Symmetry Restoration
  .) Deconfinement
  .) Critical Endpoint
- Conclusions and Outlook

Motivations

Why am I interested to QCD with axial chemical potential?

Chirality in QGP

Chiral density (*chirality*):

$$N_5 = N_R - N_L$$

Imbalance between left- and right-handed quarks

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Integrated Ward Identity: connecting chirality to topological charge

$$\Delta N_5 = 2Q_W$$

$$Q_W = \frac{g^2}{32\pi^2} \int d^4x \; F^a_{\mu\nu} \tilde{F}^{\mu\nu}_a = \Delta N_{CS}$$

Change of chirality due to interaction with nonperturbative gluon configurations with a nonvanishing Winding Number

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Change of chirality due to interaction with nonperturbative gluon configurations with a nonvanishing Winding Number

Moore and Tassler, **JHEP 1102 (2011) 105**

At high temperature, we expect copious production of *gluon* configurations with nonvanishing winding number (*strong –i.e. QCD- sphaleron*)

↓

Chirality can be produced in the high temperature phase of QCD

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Change of chirality due to interaction with nonperturbative gluon configurations with a nonvanishing Winding Number

Chirality can be produced in the high temperature phase of QCD

Simplest way to treat quark matter with chirality in effective models:

$$\mu_5 \Leftrightarrow N_5$$

Chiral chemical potential, conjugated to chiral density

$$+\ \mu_5 \bar{q}\gamma^0\gamma^5 q$$

Chiral density operator added to the Lagrangian density

Chirality in QGP

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Change of chirality due to interaction with nonperturbative gluon configurations with a nonvanishing Winding Number

Chirality can be produced in the high temperature phase of QCD

$$\mu_5 \Leftrightarrow N_5$$

$$+\ \mu_5 \bar{q}\gamma^0\gamma^5 q$$

Chiral chemical potential

$$\mu \Leftrightarrow N$$

$$+\ \mu \bar{q}\gamma^0 q$$

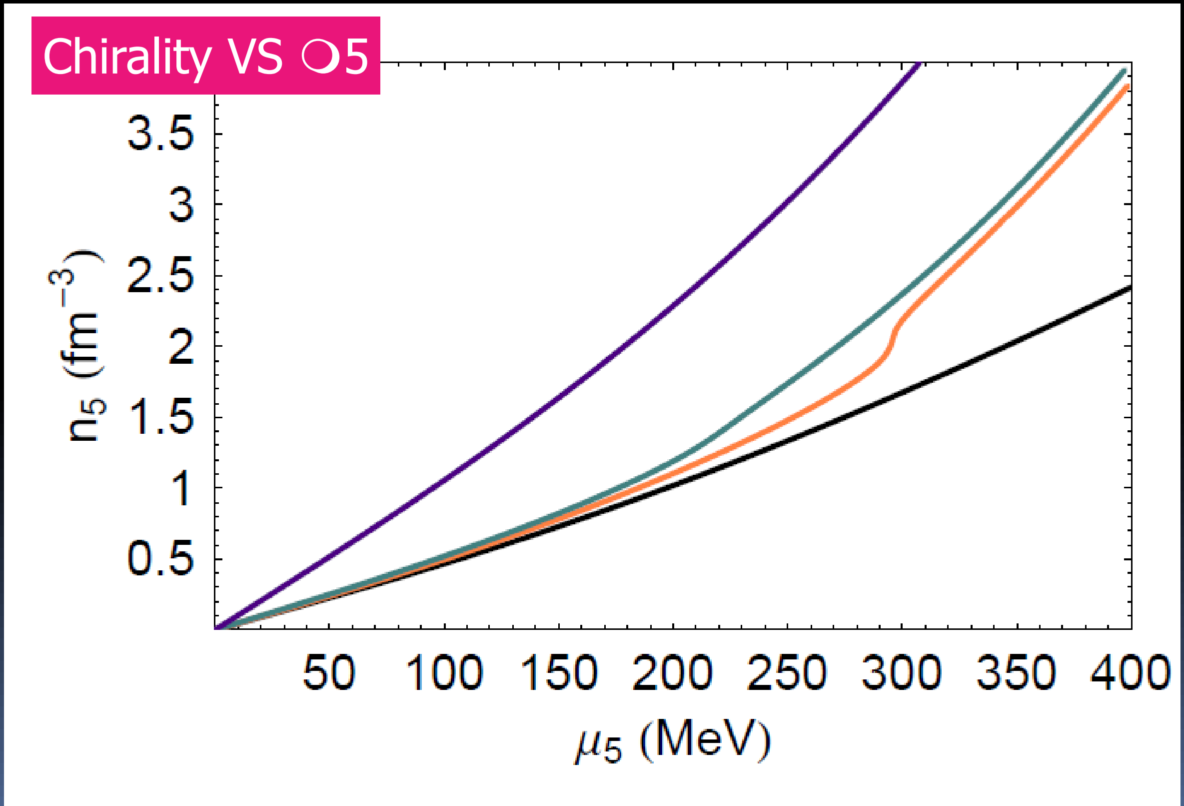
Baryon chemical potential

Chirality in QGP

$$\mu_5 \Leftrightarrow N_5$$

$$+\ \mu_5 \bar{q}\gamma^0\gamma^5 q$$

We take into account:
.) χSB
.) Effective Confinement



Chirality in QGP

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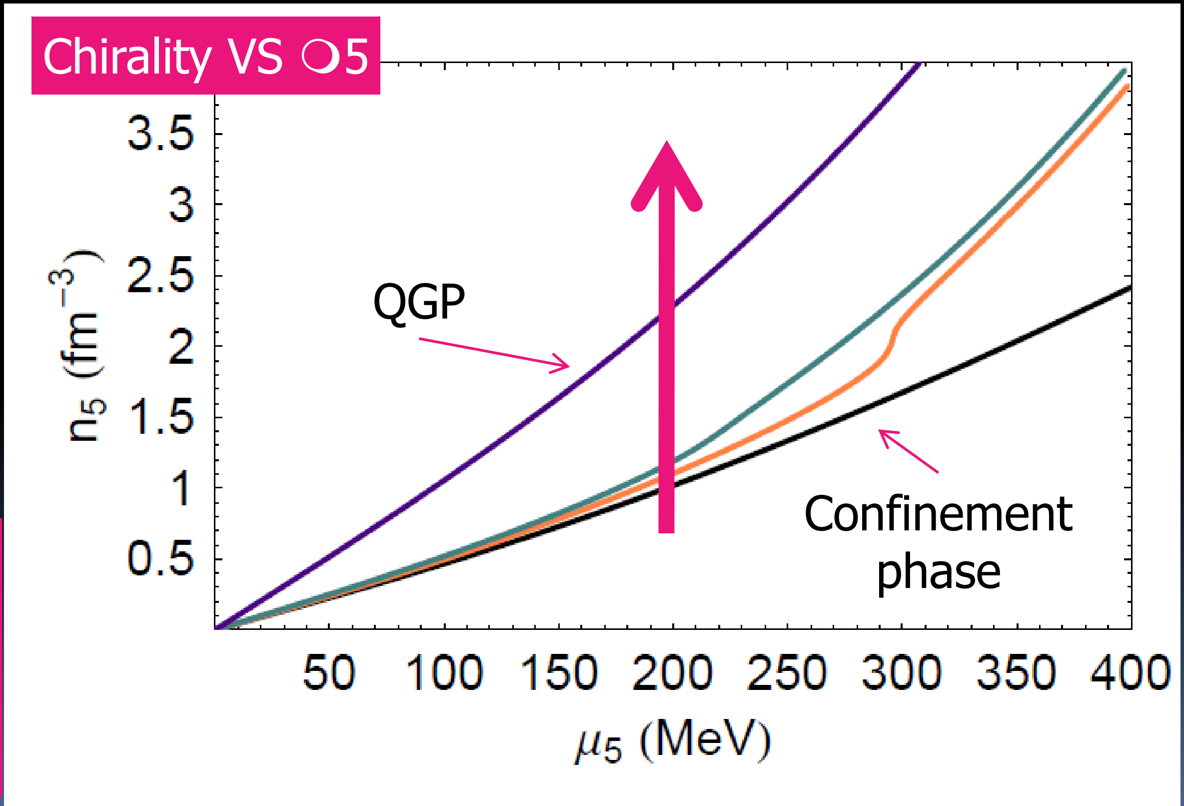
We take into account:
✍️) ♍SB
.) Effective Confinement

Crossing the critical temperature: liberation of chirality (or, of topological charge)



Remarks

Remark 1

We are aware that ❍5 is not a true chemical potential:
.) chiral condensate mixes L and R components, thus making N5 a non-conserved quantity.
.) temporal fluctuations of the topological charge.

Treat ❍5 as a mere mathematical artifact.

It can be considered as a true chemical potential when time scale is shorter than inverse of topological transition rate, See A.Yamamoto, 2011.

The nonconservation causes are mitigated in the QGP phase:
.) effective quark mass drastically diminishes around the critical temperature.

Sign-free theory

Remarks

Remark 2

Not affected by the **sign problem**, see for a nice explanation:
K. Fukushima *et al*, **Phys.Rev. D78 (2008) 074033**

Grand Canonical Ensembles with a chiral chemical potential can be simulated on the Lattice with Nc=3, see for example:
A. Yamamoto, **arXiv:1105.0385 [hep-lat]** (Chiral Magnetic Effect)
arXiv:1106.5479 [hep-lat] (Dirac Operator)

Another Motivation

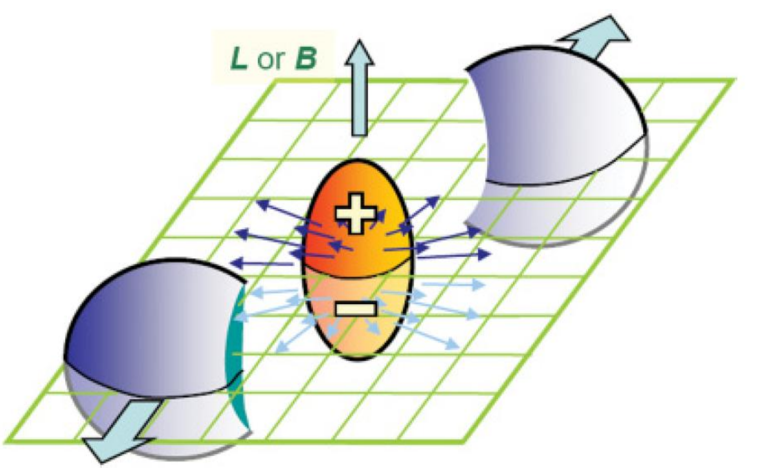
Results from the sign-free theory can be mapped to results in Nc=3 QCD with a finite baryon chemical potential

Relevant for RHIC and LHC phenomenology

Remarks

Remark 3

Relevant for the HICs phenomenology (Chiral Magnetic Effect):
D. Kharzeev et al., **Nucl.Phys. A803 (2008) 227**
K. Fukushima *et al*, **Phys.Rev. D78 (2008) 074033**
M. Chernodub *et al.*, **Phys.Rev. D80 (2009) 054503**
M. R. *et al.*, **Phys.Rev. D81 (2010) 114031**



Strong magnetic field → Charge separation ← Chirality imbalance

Magnetic fields in HICs

Kharzeev et al., **Nucl.Phys. A803 (2008) 227**
V. Skokov et al., **Int. J. Mod. Phys. A 24 (2009) 5925**
Voronyuk et al., **arXiv:1103.4239[nucl-th]**

Some References

References

In **QGP context**, to mimic *chirality change* induced by *instantons* and *strong sphalerons* in Quark-Gluon-Plasma:
K. Fukushima *et al*, **Phys.Rev. D78 (2008) 074033**
K. Fukushima, R. Gatto and M. R., **Phys.Rev. D81 (2010) 114031**
M. Chernodub and A. Nedelin, **Phys.Rev. D83 (2011) 105008**
M. R., **arXiv:1103.6186 [hep-ph]**
A. Yamamoto, **arXiv:1105.0385 [hep-lat]**
arXiv:1106.5479 [hep-lat]
C. A. Ballon Bayona et al., **arXiv:1104.2291[hep-th]**

References

In **several non-QGP contexts**:
L. D. McLerran *et al*, **Phys.Rev. D43 (1991) 2027**
Nielsen and Ninomiya, **Phys.Lett. B130 (1983) 389**
A. N. Sisakian *et al*, **hep-th/9806047**
M. Joyce *et al.*, **Phys.Rev. D53 (1996) 2958**

The Microscopic Model

Description of the model I use in my calculations.

M.R., **Phys. Rev. D84 (2011)**

The Model

Nambu and Jona-Lasinio, **Phys. Rev. 122 (1961)**
M. Frasca, **arXiv:1105.5274 [hep-ph]**

NJL Model with the Polyakov Loop

$$\mathcal{L} = \bar{q}\left(i\gamma^\mu D_\mu - m\right)q + G\left[\left(\bar{q}q\right)^2 + \left(i\bar{q}\gamma_5\boldsymbol{\tau}q\right)^2\right]$$

$$G = g\left[1 - \alpha_1 L L^\dagger - \alpha_2(L^3 + (L^\dagger)^3)\right]$$

Polyakov Loop:
sensitive to confinement – deconfinement transition

$$L = \frac{1}{3}\mathrm{Tr}_c \exp\left(i\beta\lambda_a A_4^a\right)$$

In the model:
A_4 is background field

K. Fukushima, **Phys.Lett. B591 (2004) 277-284**
W. Weise *et al.*, **Phys.Rev. D73 (2006) 014019**
M. Yahiro *et al.*, **Phys.Rev. D82 (2010) 076003**

M.R., **Phys. Rev. D84 (2011)**

The Model

NJL Model with the Polyakov Loop

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$$G = g\left[1 - \alpha_1 L L^\dagger - \alpha_2 (L^3 + (L^\dagger)^3)\right]$$

Coupling to quarks via:
.) Coupling constant
.) Covariant derivative

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K. Fukushima, **Phys.Lett. B591 (2004) 277-284**
W. Weise *et al.*, **Phys.Rev. D73 (2006) 014019**
M. Yahiro *et al.*, **Phys.Rev. D82 (2010) 076003**

Coupling dependent on L inspired by:
K. Kondo, **Phys.Rev. D82 (2010) 065024**

M.R., **Phys. Rev. D84 (2011)**

The 1-loop TP

$$\mathcal{L} = \bar{q}\left(i\gamma^\mu D_\mu - m\right)q + G\left[\left(\bar{q}q\right)^2 + \left(i\bar{q}\gamma_5\boldsymbol{\tau}q\right)^2\right]$$

M.R., **Phys. Rev. D84 (2011)**

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Add a chiral chemical potential

M.R., **Phys. Rev. D84 (2011)**

The 1-loop TP

$$\mathcal{L} = \bar{q}\left(i\gamma^\mu D_\mu - m\right)q + G\left[(\bar{q}q)^2 + (i\bar{q}\gamma_5\boldsymbol{\tau}q)^2\right] + \mu_5\bar{q}\gamma^0\gamma^5 q$$

One-loop Thermodynamic Potential

$$V = \mathcal{U}(L, L^\dagger, T) + \frac{\sigma^2}{G} - N_c N_f \sum_{s=\pm1}\int\frac{d^3\boldsymbol{p}}{(2\pi)^3}\omega_s - \frac{N_c N_f}{\beta}\sum_{s=\pm1}\int\frac{d^3\boldsymbol{p}}{(2\pi)^3}\log\left(F_+F_-\right)$$

Minimization of V leads to physical values of
.) • (chiral condensate)
.) L

$$F_- = 1 + 3Le^{-\beta(\omega_s-\mu)} + 3L^\dagger e^{-2\beta(\omega_s-\mu)} + e^{-3\beta(\omega_s-\mu)}$$

$$F_+ = 1 + 3L^\dagger e^{-\beta(\omega_s+\mu)} + 3Le^{-2\beta(\omega_s+\mu)} + e^{-3\beta(\omega_s+\mu)}$$

1- and 2-quark states suppression in the confinement phase

Statistically confining distribution functions

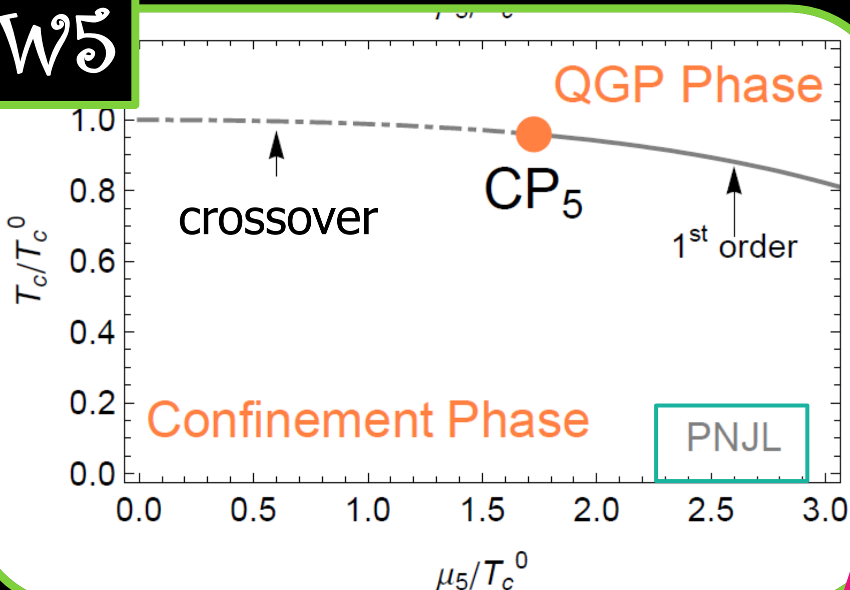
Phase Diagram of the Model

Deconfinement and Chiral symmetry restoration

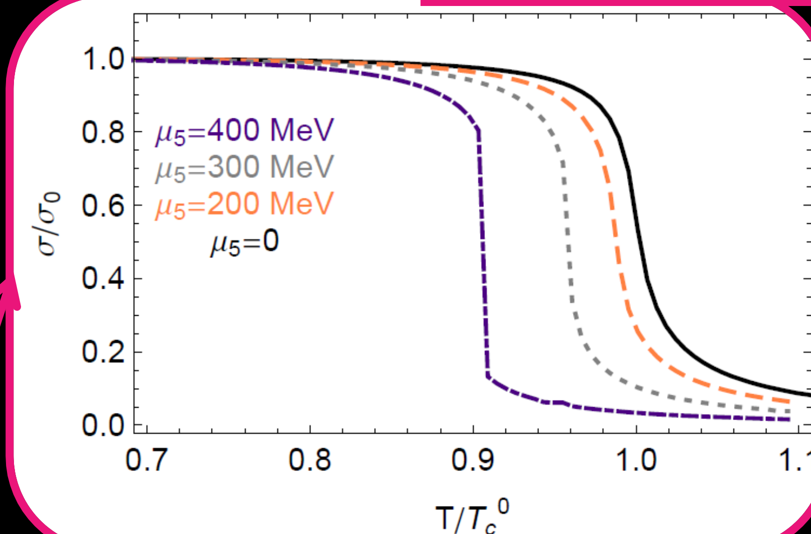
M.R., **Phys. Rev. D84 (2011)**

Phase Diagram: Results

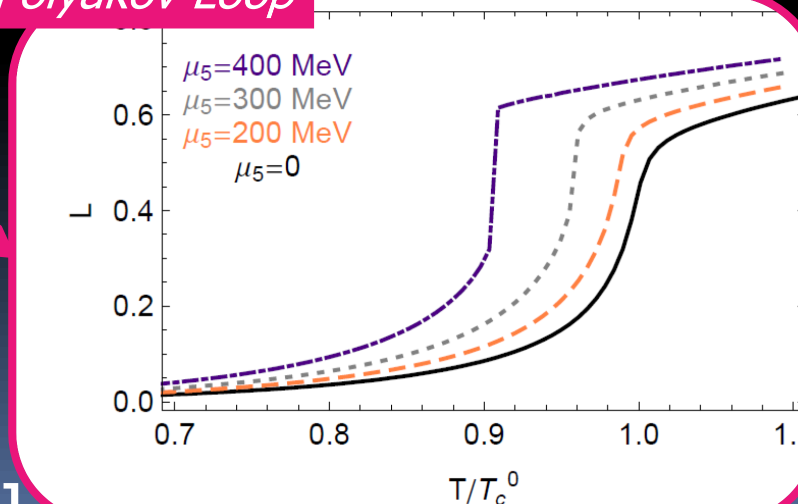
W5



Chiral Condensate



Polyakov Loop



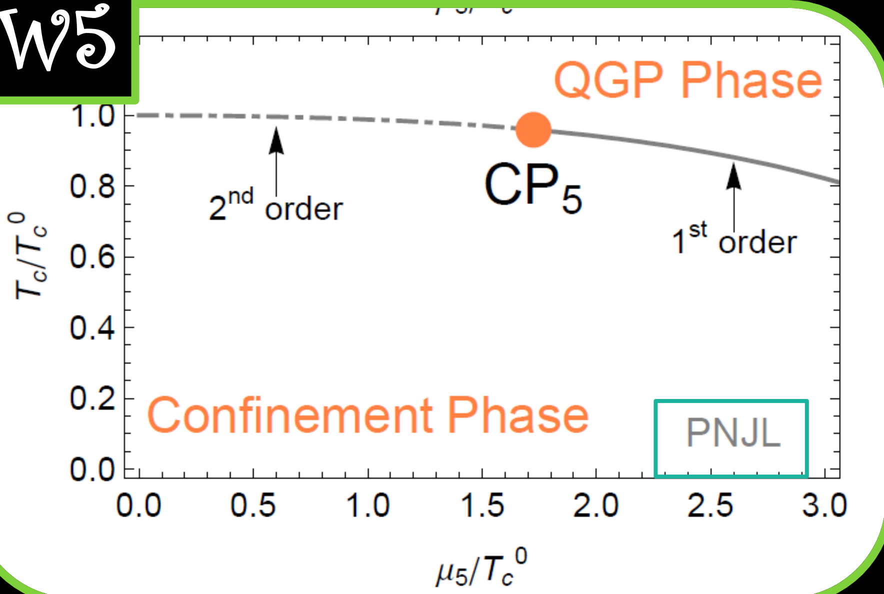
Deconfinement and ♍S Restoration are <u>entangled</u> for any value of ○5

This is different from what is found at real chemical potential, see:
M. Yahiro *et al.*, **arXiv:1104.2394 [hep-ph]**

M.R., **Phys. Rev. D84 (2011)**

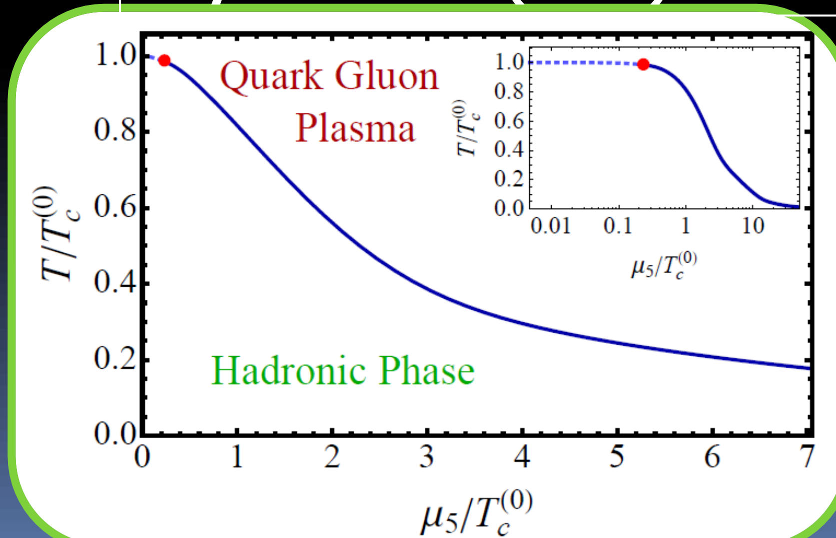
Phase Diagram

W5



PNJL vs PQM

M. Chernodub and A. Nedelin,
Phys.Rev. D83 (2011) 105008

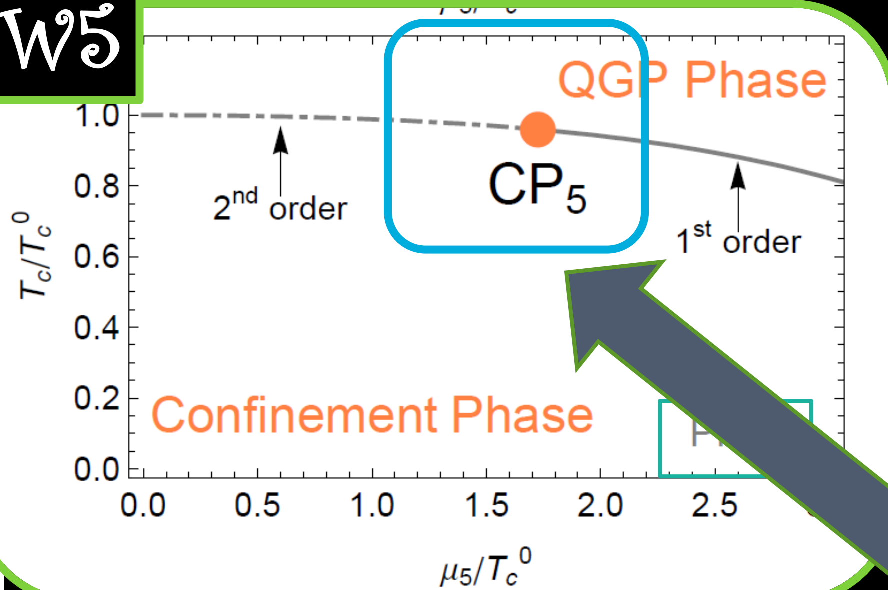


Comparison with previous results
QM model (without vacuum fluctuations)

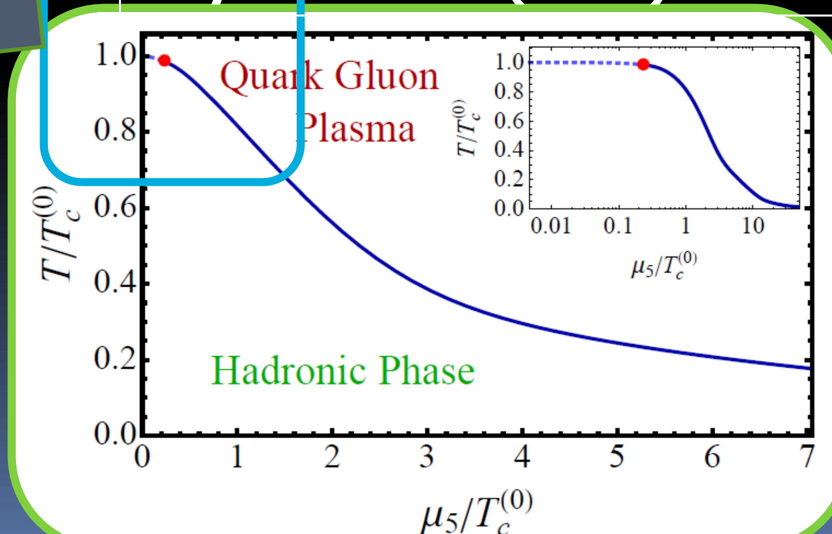
M.R., **Phys. Rev. D84 (2011)**

Phase Diagram

W5



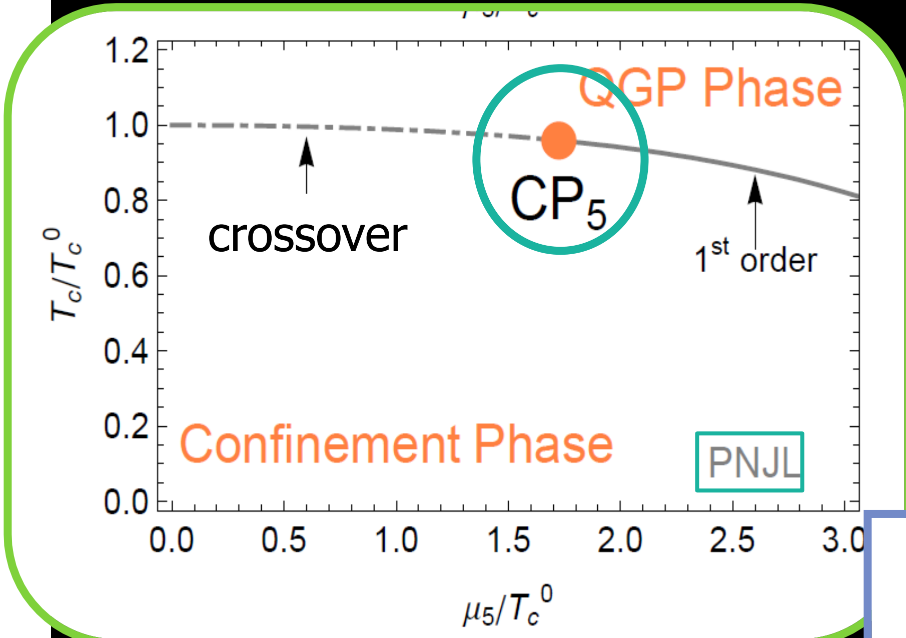
M. Chernodub and A. Nedelin,
Phys.Rev. D83 (2011) 105008



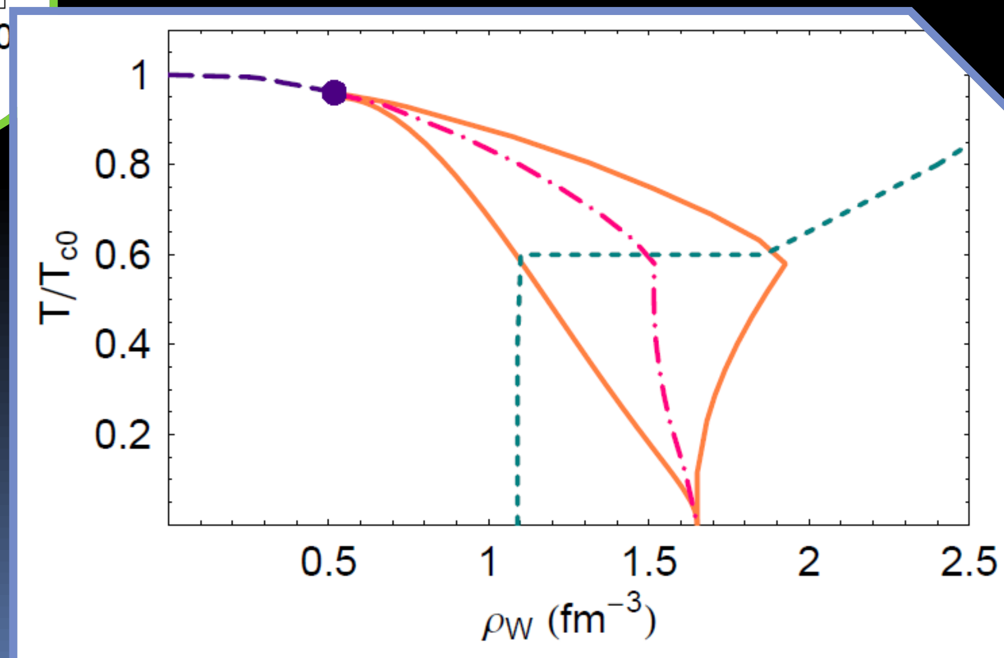
Comparison with previous results
QM model (without vacuum fluctuations)

R. Gatto and M.R.: WiP

Phase Diagram

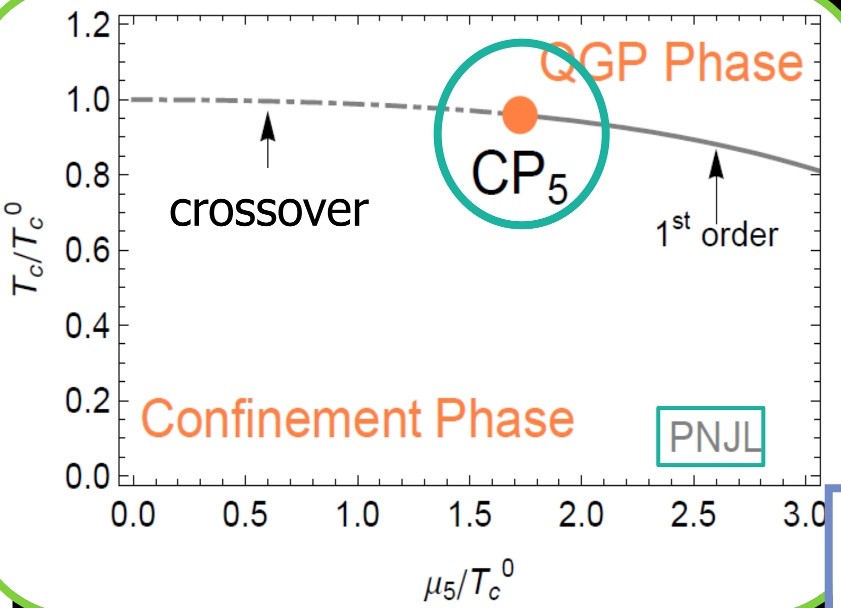


From chiral chemical potential to topological charge density



R. Gatto and M.R.: WiP

Phase Diagram



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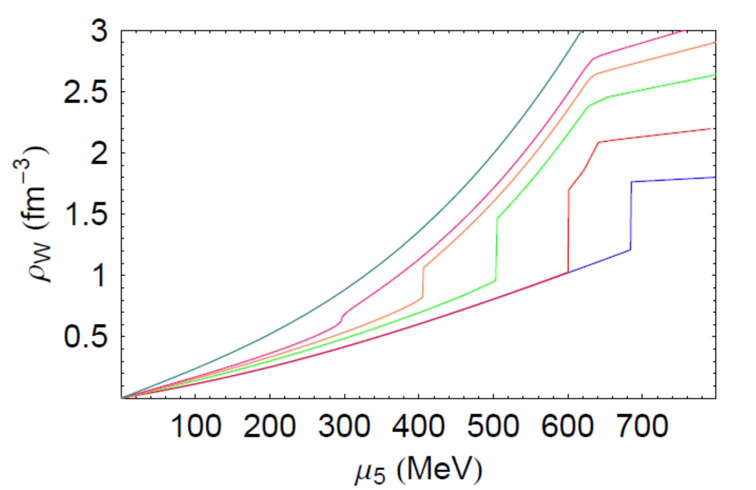
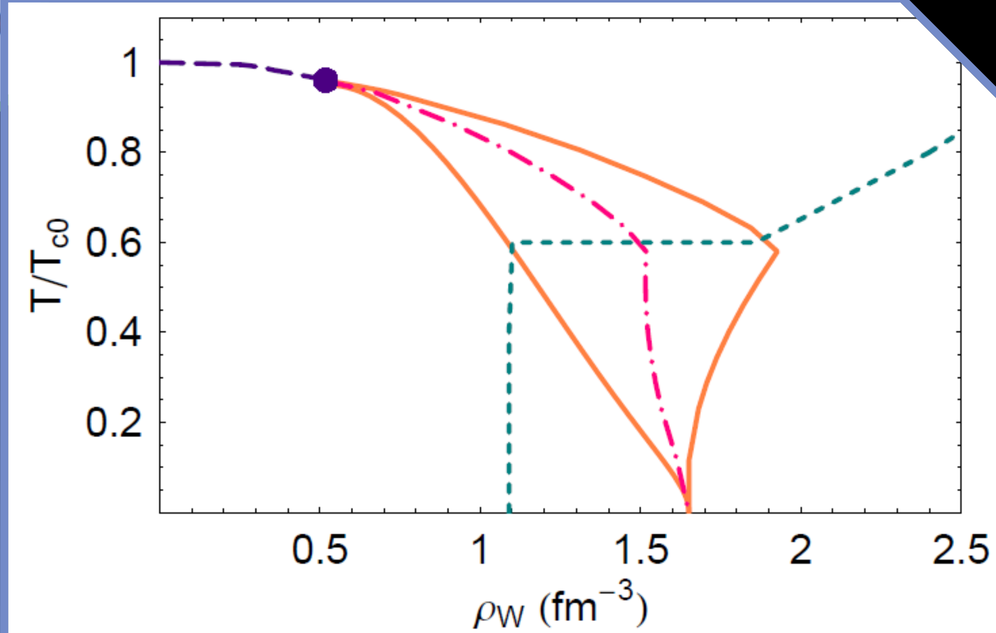
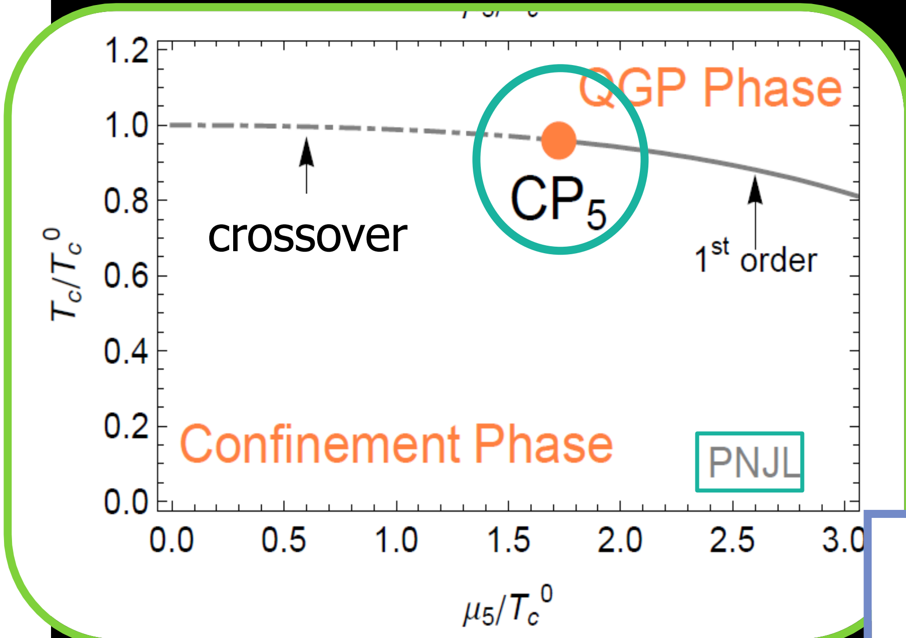




R. Gatto and M.R.: WiP

Phase Diagram



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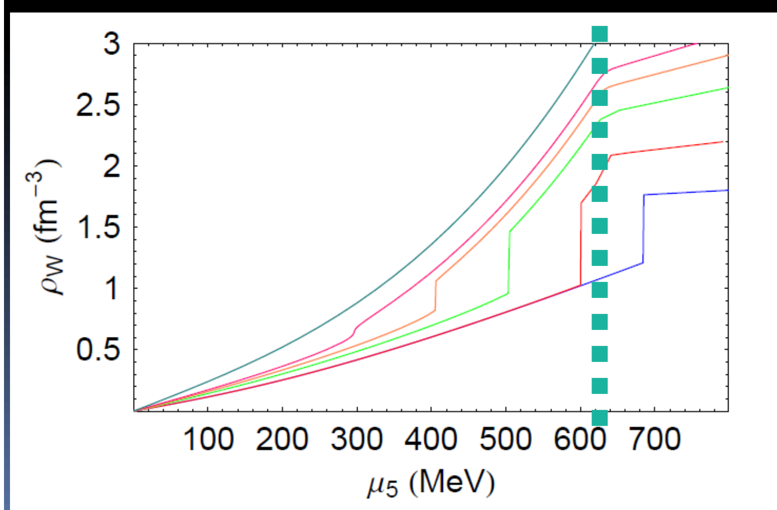
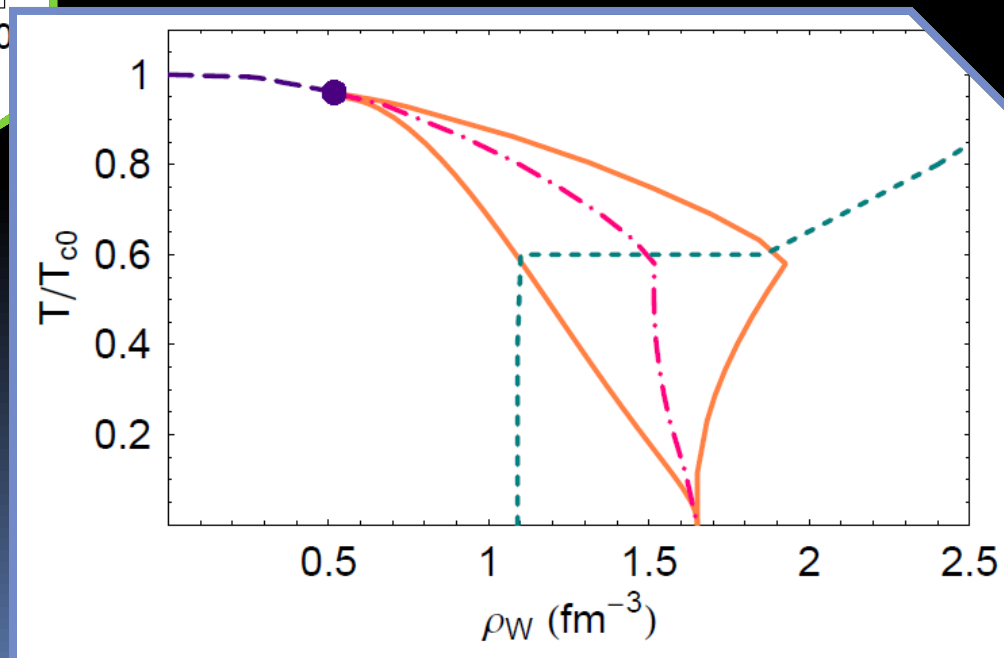
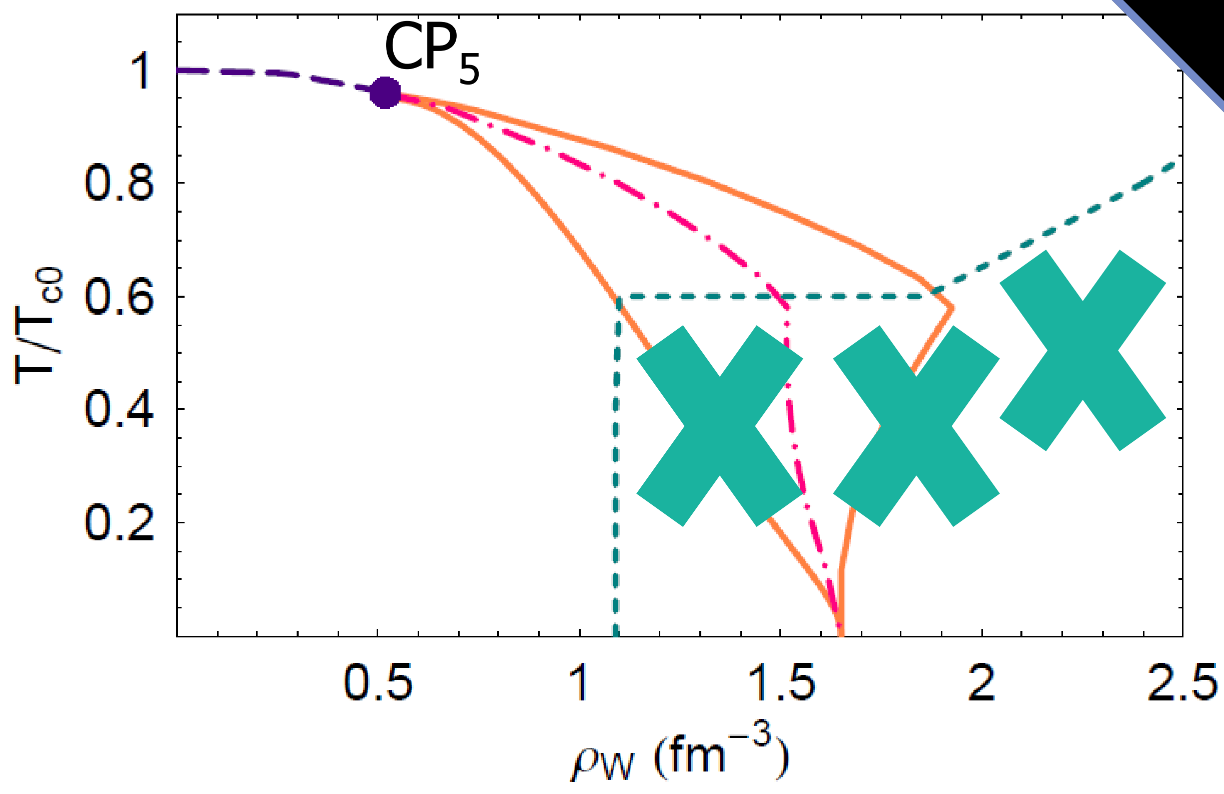




R. Gatto and M.R.: WiP

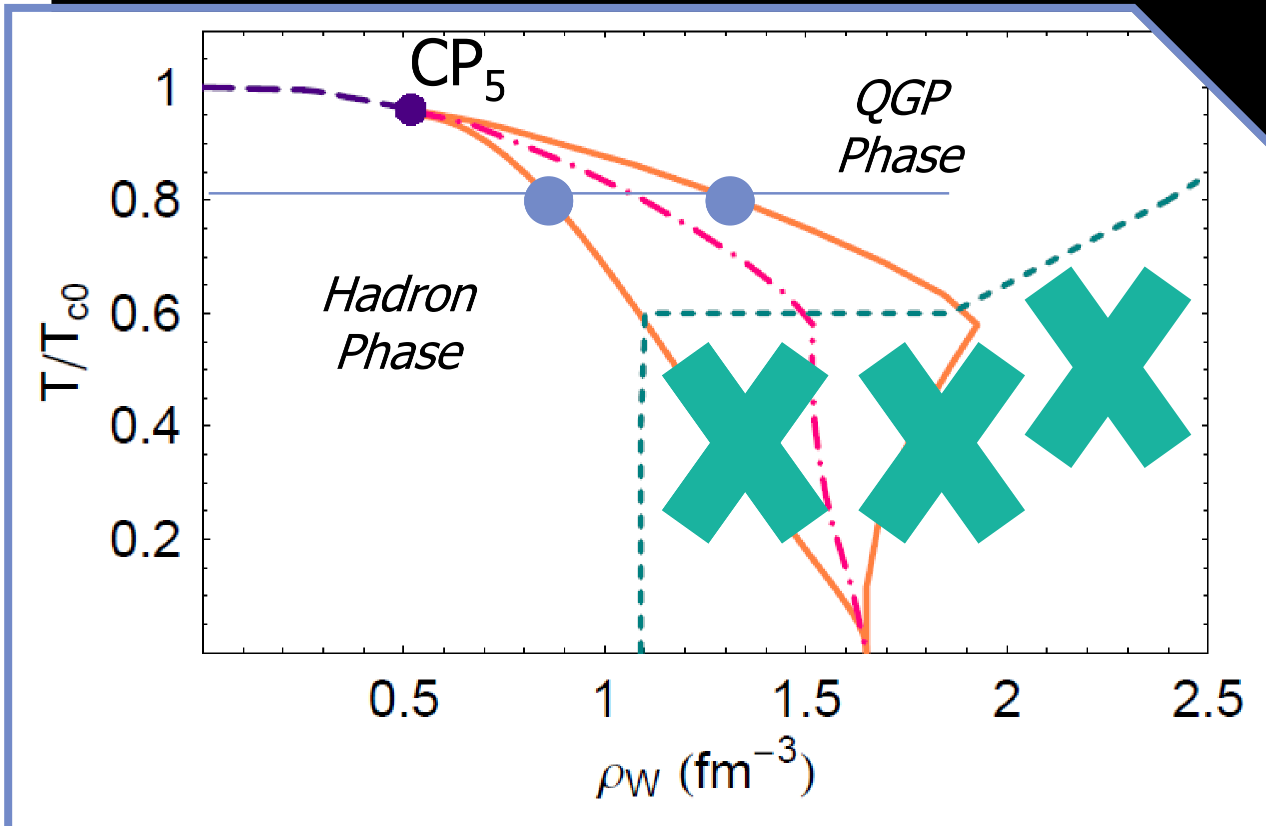
Phase Diagram



$$\rho_{c5} = 0.518\ \mathrm{fm}^{-3}$$
$$T_{c5} = 167\ \mathrm{MeV}$$

R. Gatto and M.R.: WiP

Phase Diagram



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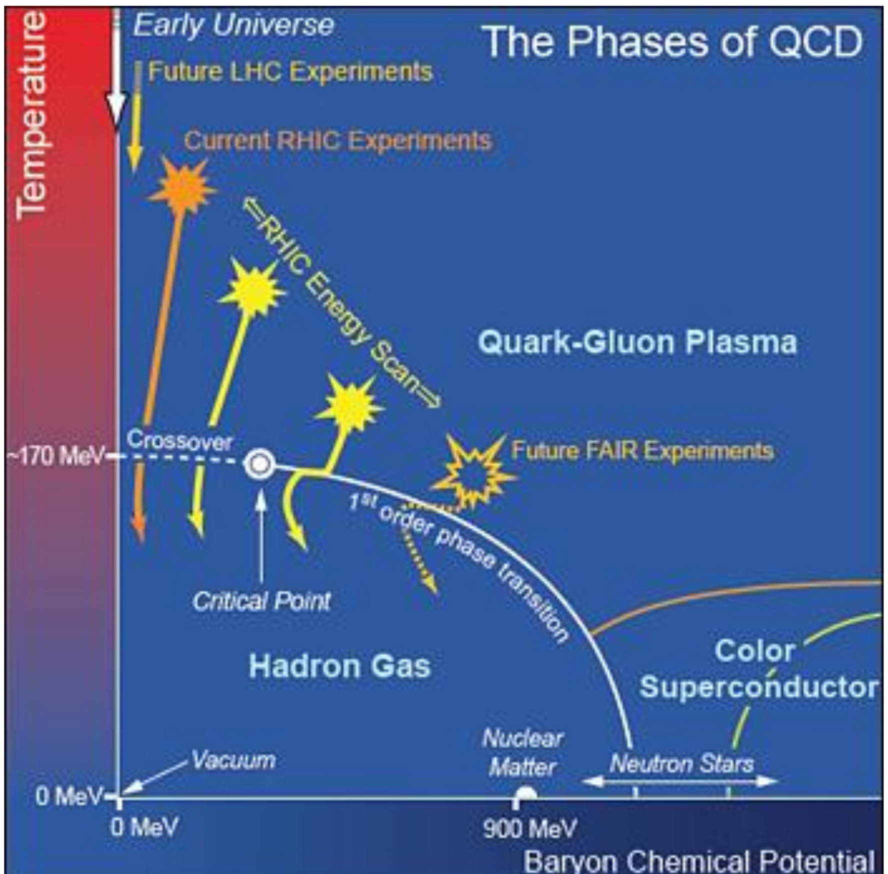
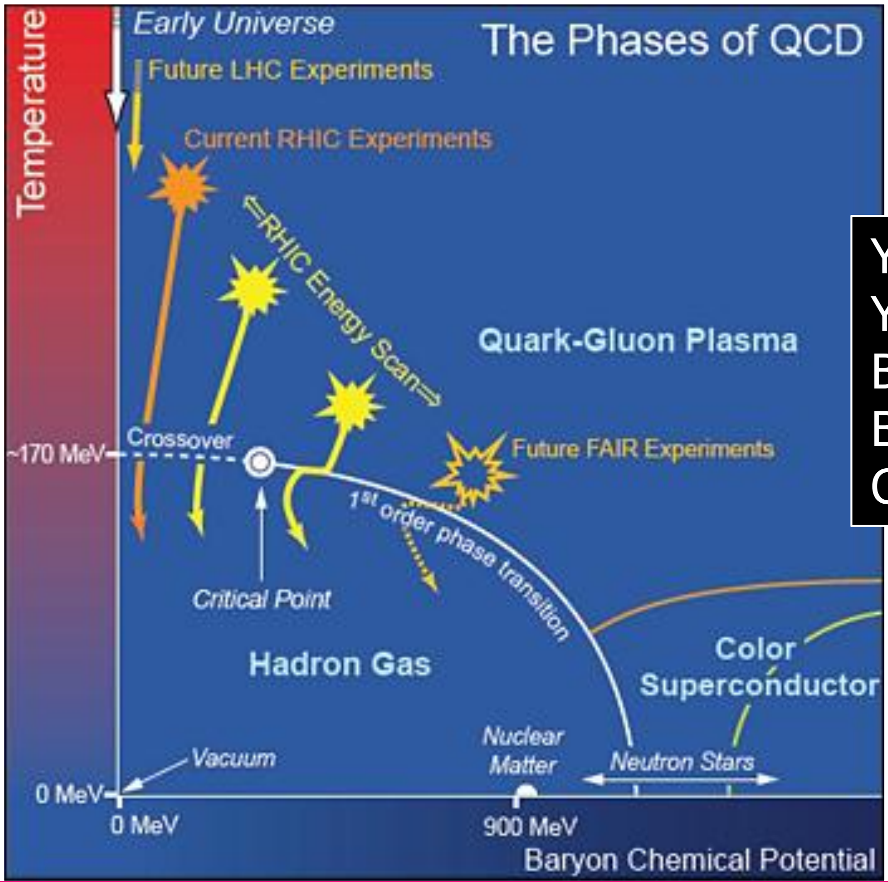
Critical Endpoint of QCD



Sophie Bushwick,
http://www.bnl.gov/today/story.asp?ITEM_NO=1870

Critical Endpoint of QCD

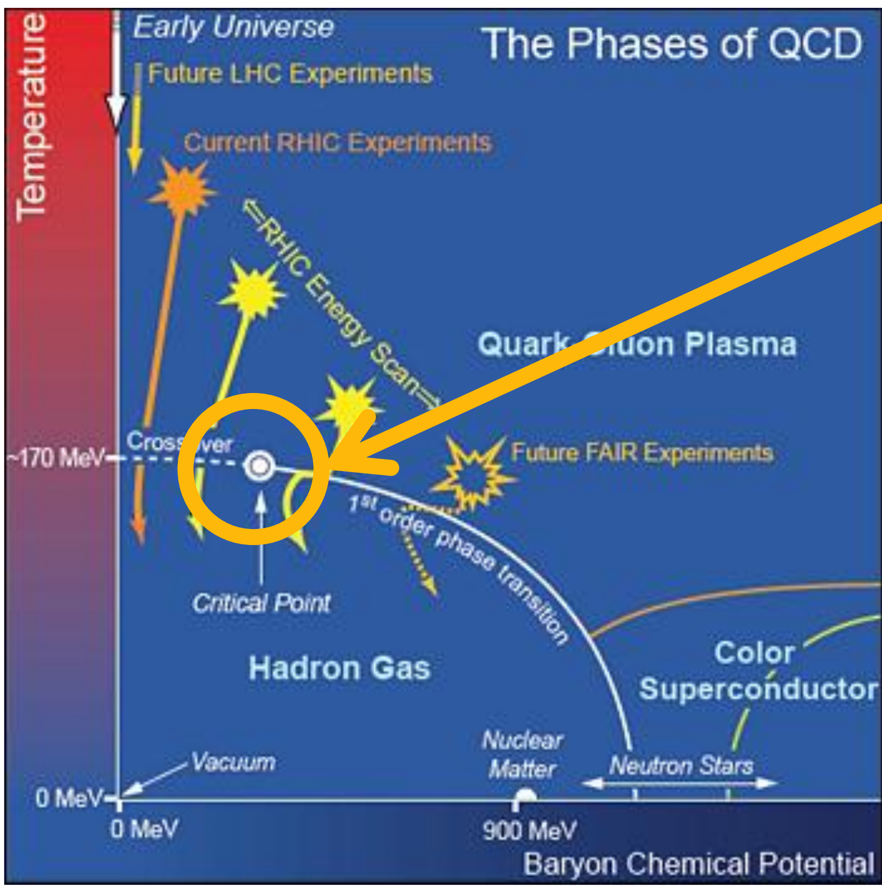


Sophie Bushwick,
http://www.bnl.gov/today/story.asp?ITEM_NO=1870

Evidence of *crossover*
at zero chemical potential:

Y. Aoki et al., Nature 443 (2006) 675
Y. Aoki et al., JHEP06 (2009) 088
Borsanyi et al., JHEP09 (2010) 073
Bazarov et al., Phys. Rev. D80 (2009) 014504
Cheng et al., Phys. Rev. D81 (2010) 054510

Critical Endpoint of QCD



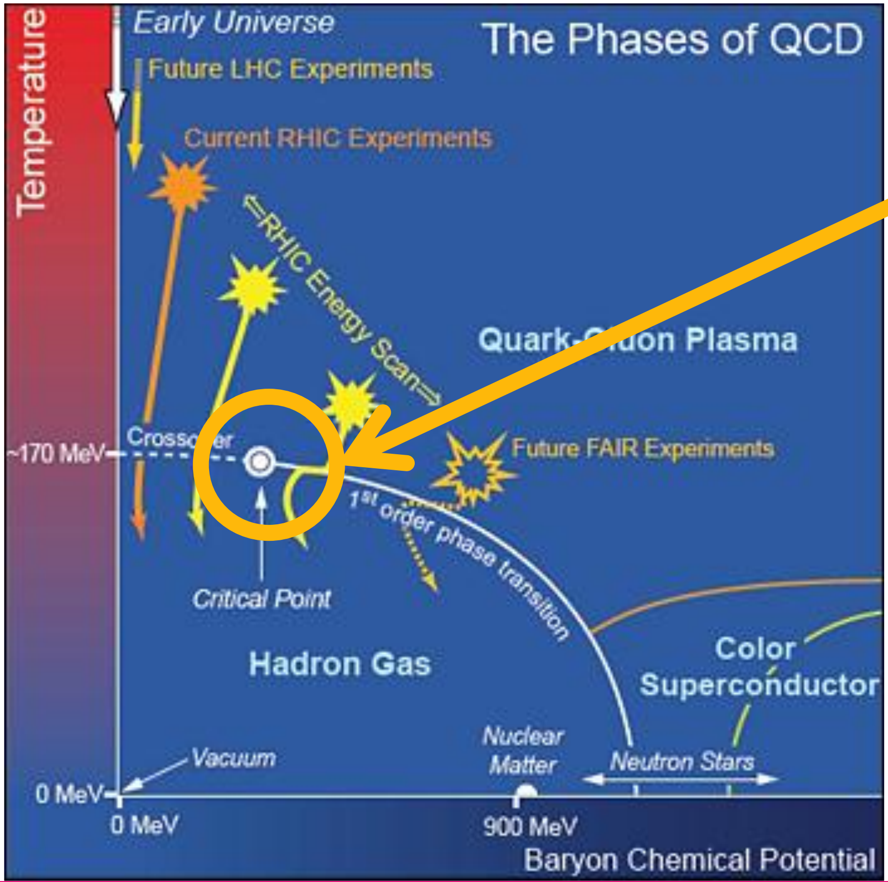
Sophie Bushwick,
http://www.bnl.gov/today/story.asp?ITEM_NO=1870

Critical Endpoint (CP)
First order and crossover lines intersect at CP

Asakawa and Yazaki,
Nucl.Phys. A504 (1989) 668-684

Based on NJL model

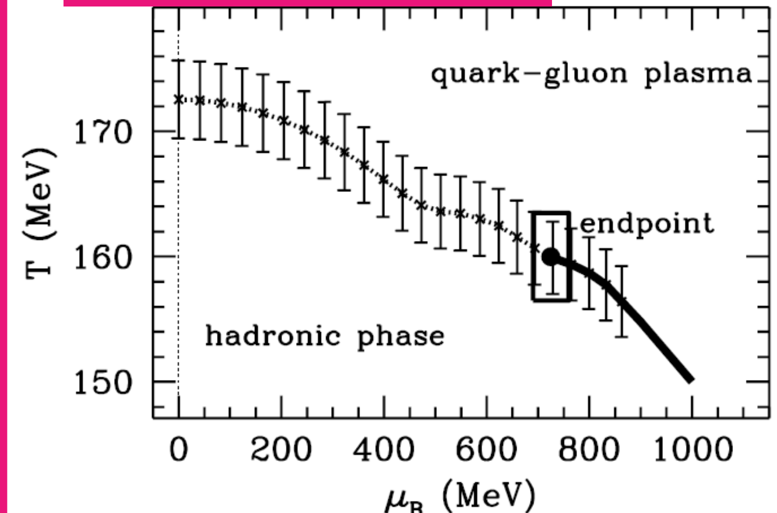
Critical Endpoint of QCD



Sophie Bushwick, http://www.bnl.gov/today/story.asp?ITEM_NO=1870

Critical Endpoint (CP)
First order and crossover lines intersect at CP

It might exist in QCD



Fodor and Katz, JHEP03(2002)014

For a (critical) review see De Forcrand in **PoS LAT2009 (2009) 010**

Critical Endpoint of QCD

Lattice

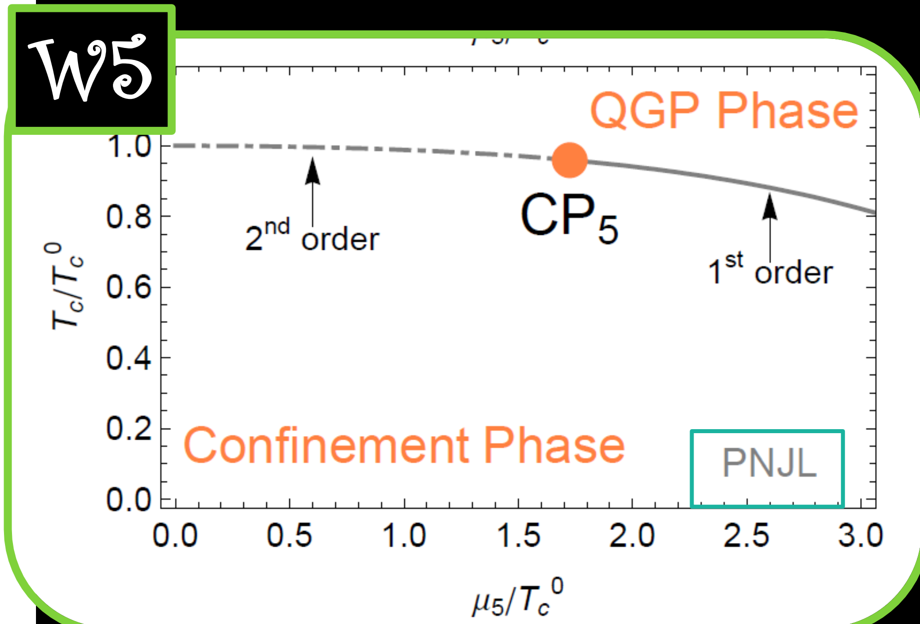
Fodor and Katz, **JHEP03(2002)014**
C. R. Allton *et al.*, **Phys. Rev. D71 (2005) 054508**
Gavai and Gupta, **Phys. Rev. D78 (2008) 114503**
De Forcrand and Philipsen, **Nucl. Phys. B642 (2002) 290**
P. De Forcrand *et al.*, **arXiv:0911.5682**
S. Eijiri, **Phys. Rev. D78 (2008) 074507**
A. Ohnishi *et al.*, **Pos LAT2010 (2010) 202**

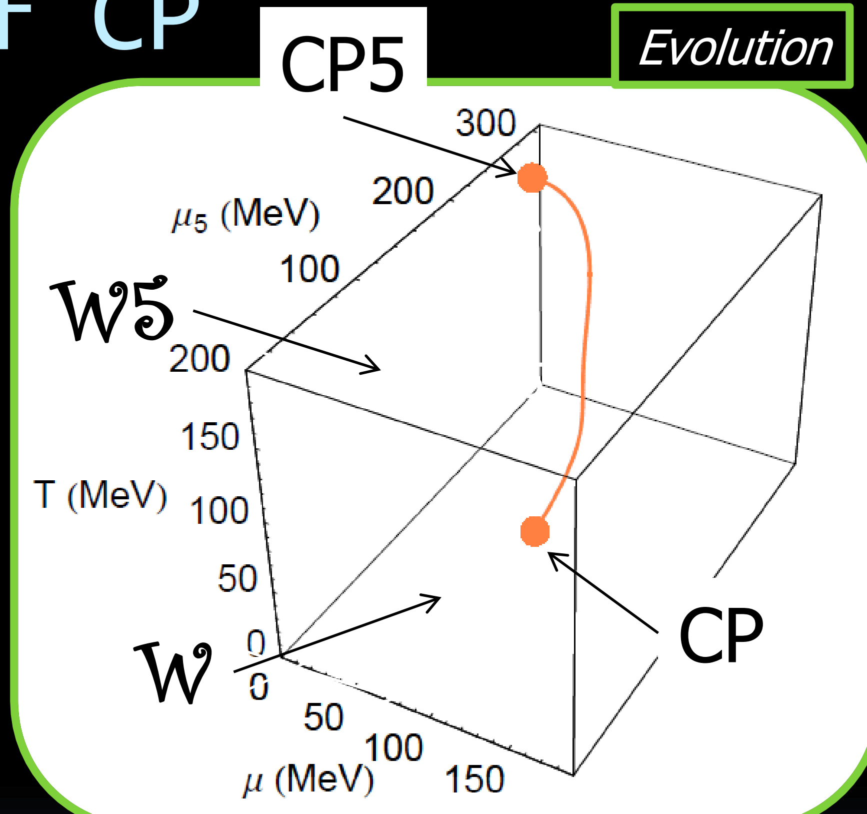
Models

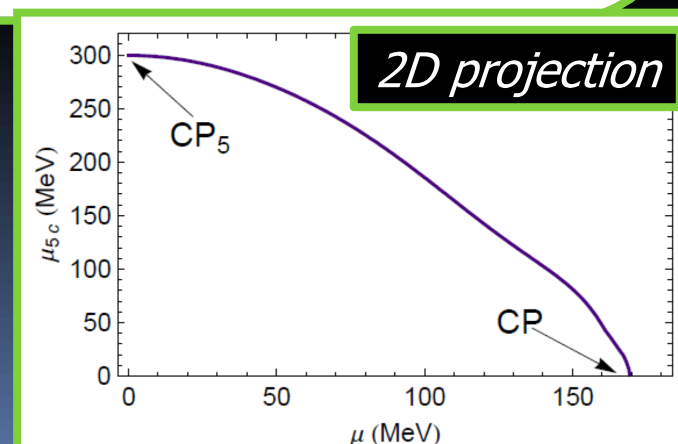
K. Fukushima et al., **Phys. Rev. D80 (2009) 054012**
Bowman and Kapusta, **Phys. Rev. C79 (2009) 015202**
Zhang and Kunihiro, **Phys. Rev. D80 (2009) 290**
A. Ohnishi *et al.*, **arXiv:1102.3753**
M. A. Stephanov, **PoS LAT2006 (2006) 024**
Abuki *et al.*, **Phys. Rev. D81 (2010) 125010**
Basler and Buballa, **Phys. Rev. D82 (2010) 094004**
Hanada and Yamamoto, **arXiv:1103.5480 [hep-ph]**

M.R., **arXiv:1103.6186**

Continuation of CP







CP5 is not an accident, but
CP viewed by hot quark matter in W5.
Its detection (?) can be interpreted as a
theoretical signatureof the real world **CP**.

R. Gatto and M.R.: WiP

Topological Susceptibility

$$\chi = \left.\frac{\partial^2\Omega}{\partial\theta^2}\right|_{\theta=0}$$

Correlator of topological charge density at zero momentum

where:

$$\theta\frac{g^2}{64\pi^2}\varepsilon^{\mu\nu\rho\sigma}F^a_{\mu\nu}F^a_{\rho\sigma}$$

R. Gatto and M.R.: WiP

Topological Susceptibility

$$\chi = \left.\frac{\partial^2 \Omega}{\partial \theta^2}\right|_{\theta=0}$$

Correlator of topological charge density at zero momentum

where: $$\theta \frac{g^2}{64\pi^2}\varepsilon^{\mu\nu\rho\sigma}F^a_{\mu\nu}F^a_{\rho\sigma}$$

After a chiral rotation, the theta-dependence of the grand potential is transmitted to the quark sector; within the model, the quark mass term becomes:

$$\sqrt{\left[m_0 \cos\left(\frac{\theta}{2}\right) - 2\sigma\right]^2 + m_0^2 \sin^2\left(\frac{\theta}{2}\right)}$$

Sakai et al., 2010
Sasaki et al., 2011
Boer and Boomsma, 2008

We can use the effective model to compute the topological susceptibility, at finite temperature and in presence of a background of topological charge density.

Topological Susceptibility: References

Lattice (and related to Lattice)

S. Aoki et al., 2007 (Finite volume QCD at fixed topological charge)
S. Aoki et al., 2008 (Lattice computation of ♍)
Brower et al, 2003 (Several aspects of QCD at fixed topological charge)
Chiu et al., 2008 (♍ with domain wall fermions)
D’Elia et al., 2000 (♍ at zero and finite temperature)
Gattringer et al., 2002 (♍ at zero and finite temperature)
DeGrand et al., 2002 (Veneziano-Witten relation on the Lattice)
Pica et al., 2003 (♍ on the Lattice)
Del Debbio et al., 2005 (♍ on the Lattice)

ChPT, Large Nc, Models

Veneziano, 1979
Di Vecchia and Veneziano, 1980
Leutwyler and Smilga, 1992
Fukushima et al., 2001
Mao and Chiu, 2009

R. Gatto and M.R.: WiP

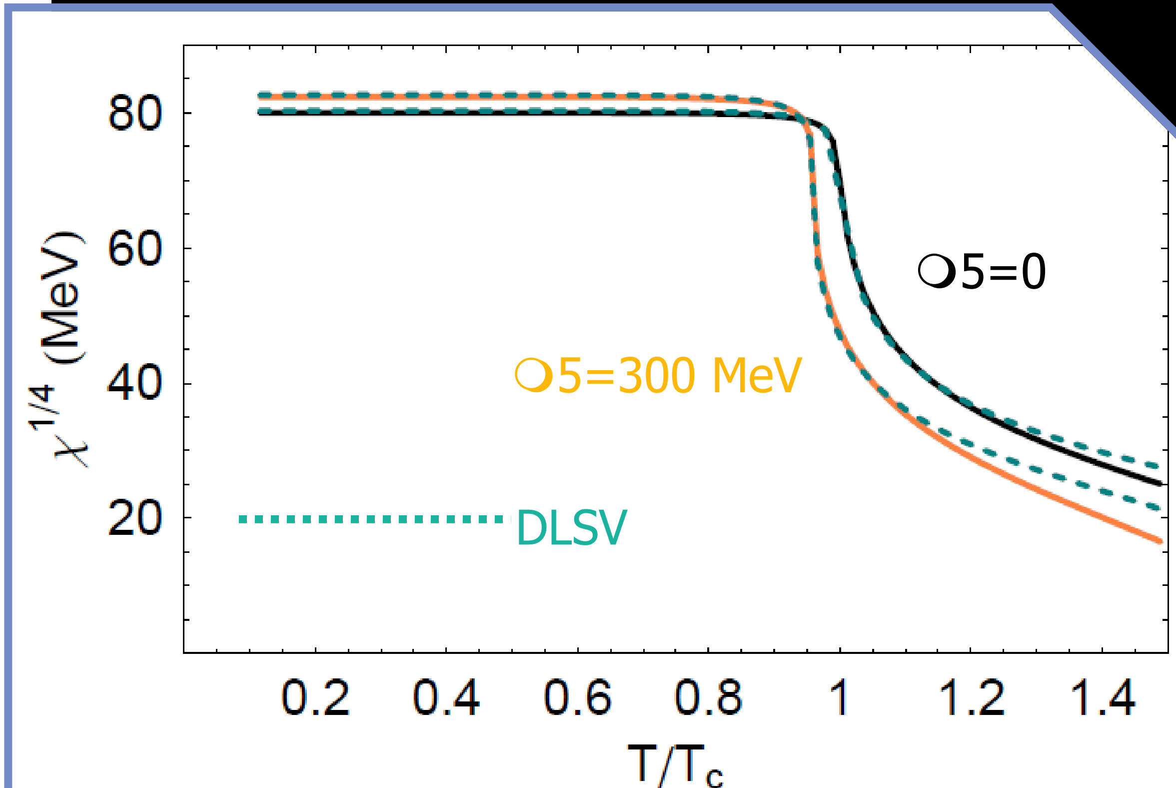
Topological Susceptibility

$$\chi = \left.\frac{\partial^2 \Omega}{\partial \theta^2}\right|_{\theta=0}$$

Correlator of topological charge density at zero momentum

where:

$$\theta \frac{g^2}{64\pi^2} \varepsilon^{\mu\nu\rho\sigma} F^a_{\mu\nu} F^a_{\rho\sigma}$$



Zero temperature:

$$\chi = (79.97\ \mathrm{MeV})^4$$

DLSV

$$\chi = |\langle \bar{q} q \rangle| \left(\sum_f \frac{1}{m_f} \right)^{-1}$$

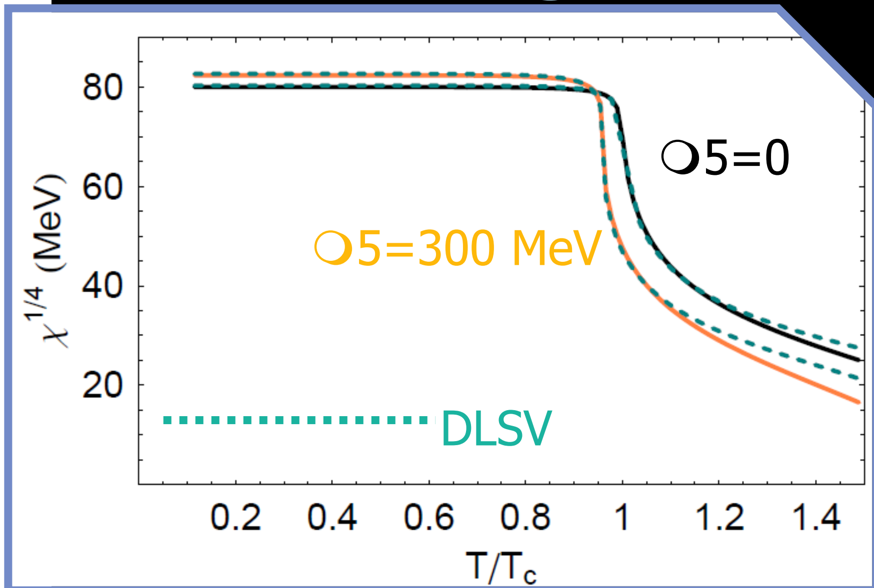
Veneziano, 1979

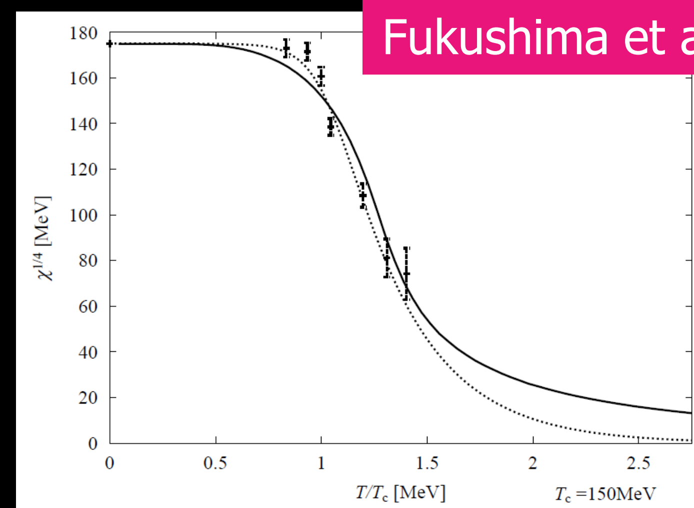
Di Vecchia and Veneziano, 1980

Leutwyler and Smilga, 1992

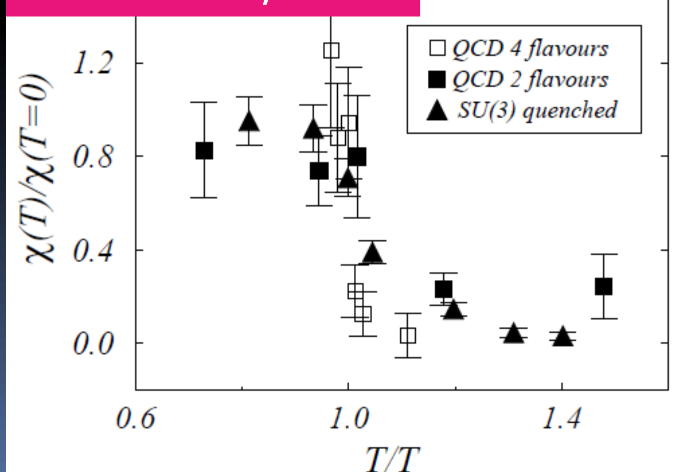
R. Gatto and M.R.: WiP

Topological Susceptibility

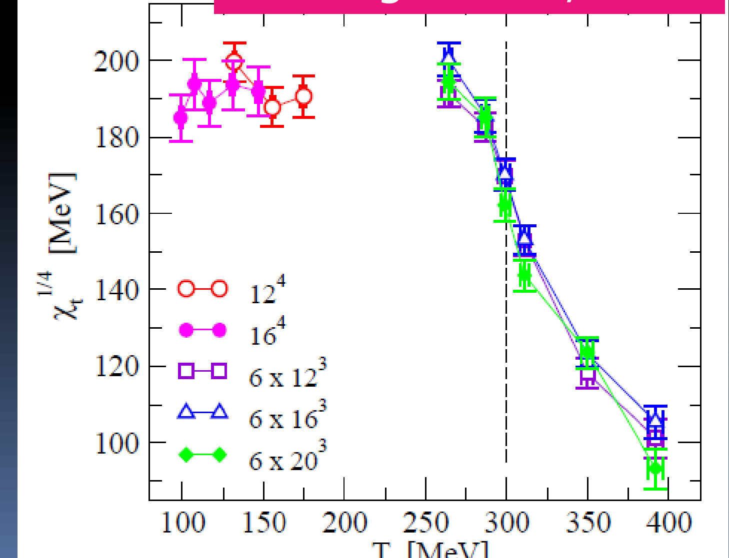




Fukushima et al., 2001



D'Elia et al., 2000



Gattringer et al., 2002

Phase Diagram of the Model

A closer look to the Critical Endpoint

Conclusions and Outlook

Conclusions

- Chiral chemical potential is introduced to mimic chirality-changing processes in hot QCD medium
- QM with ❍□ can be simulated on Lattice (no sign problem)
- Critical Endpoint (CP) of QCD is continued to a new Critical Endpoint, CP5
- Phase Structure of Quark Matter (QM) with ❍□ similar to that of QM of our Universe

Outlook

Interesting comparison with results from SS model,
C. A. Ballon Bayona et al., **arXiv:1104.2291[hep-th]**

- Study of inhomogeneous phases around the critical endpoint at finite ❍5
- Compute quantitative dependence of the critical endpoint on the quark masses
- Study the Nc dependence of the mapping coordinates
- Mapping the critical endpoint within the Ginzburg-Landau effective potential approach
- Studies at fixed topological charge density
- Studies in a finite box
- From 2 to 2+1 flavors

I acknowledge:
K. Fukushima and **R. Gatto** for collaboration on some of the topics discussed here.
Moreover, I acknowledge:
H. Abuki, **M. Chernodub**, **P. De Forcrand**, **M. D'Elia**, **M. Frasca**, **T. Hatsuda**, **A. Ohnishi**, **M. Tachibana**, **A. Yamamoto** and **N. Yamamoto** for interesting discussions about the topics discussed in this talk.

Thanks for your attention.

Non pentirti di ciò che hai fatto, se quando l'hai fatto eri felice
(Do not regret the things you did, if when you did them you were happy)

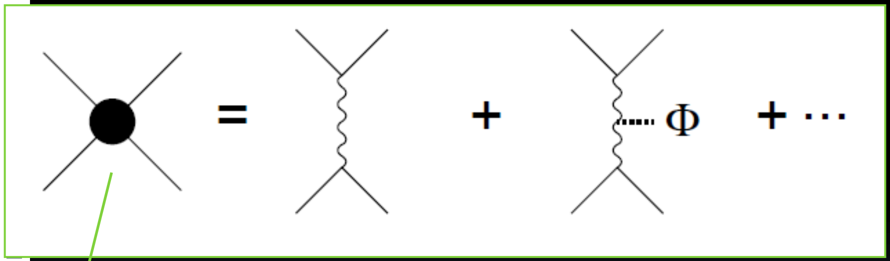
Appendix

M.R., **arXiv:1103.6186**

The L-dependent coupling

NJL Model with the Polyakov Loop

$$\mathcal{L} = \bar{q}\left(i\gamma^\mu D_\mu - m\right)q + G\left[\left(\bar{q}q\right)^2 + \left(i\bar{q}\gamma_5\boldsymbol{\tau}q\right)^2\right]$$



Interaction among background field and gluons leads to a tree-level coupling among G and L

$$G = g\left[1 - \alpha_1 L L^\dagger - \alpha_2 (L^3 + (L^\dagger)^3)\right]$$

M. Yahiro *et al.*, **Phys.Rev. D82 (2010) 076003**
K. Kondo, **Phys.Rev. D82 (2010) 065024**

M.R., **arXiv:1103.6186**

The 1-loop TP

$$\mathcal{L} = \bar{q}\left(i\gamma^\mu D_\mu - m\right)q + G\left[(\bar{q}q)^2 + (i\bar{q}\gamma_5\boldsymbol{\tau}q)^2\right] + \mu_5\bar{q}\gamma^0\gamma^5 q$$

One-loop Thermodynamic Potential

$$V = \mathcal{U}(L, L^\dagger, T) + \frac{\sigma^2}{G} - N_c N_f \sum_{s=\pm1}\int\frac{d^3\boldsymbol{p}}{(2\pi)^3}\omega_s - \frac{N_c N_f}{\beta}\sum_{s=\pm1}\int\frac{d^3\boldsymbol{p}}{(2\pi)^3}\log\left(F_+F_-\right)$$

Minimization of V leads to physical values of
.) • (chiral condensate)
.) L

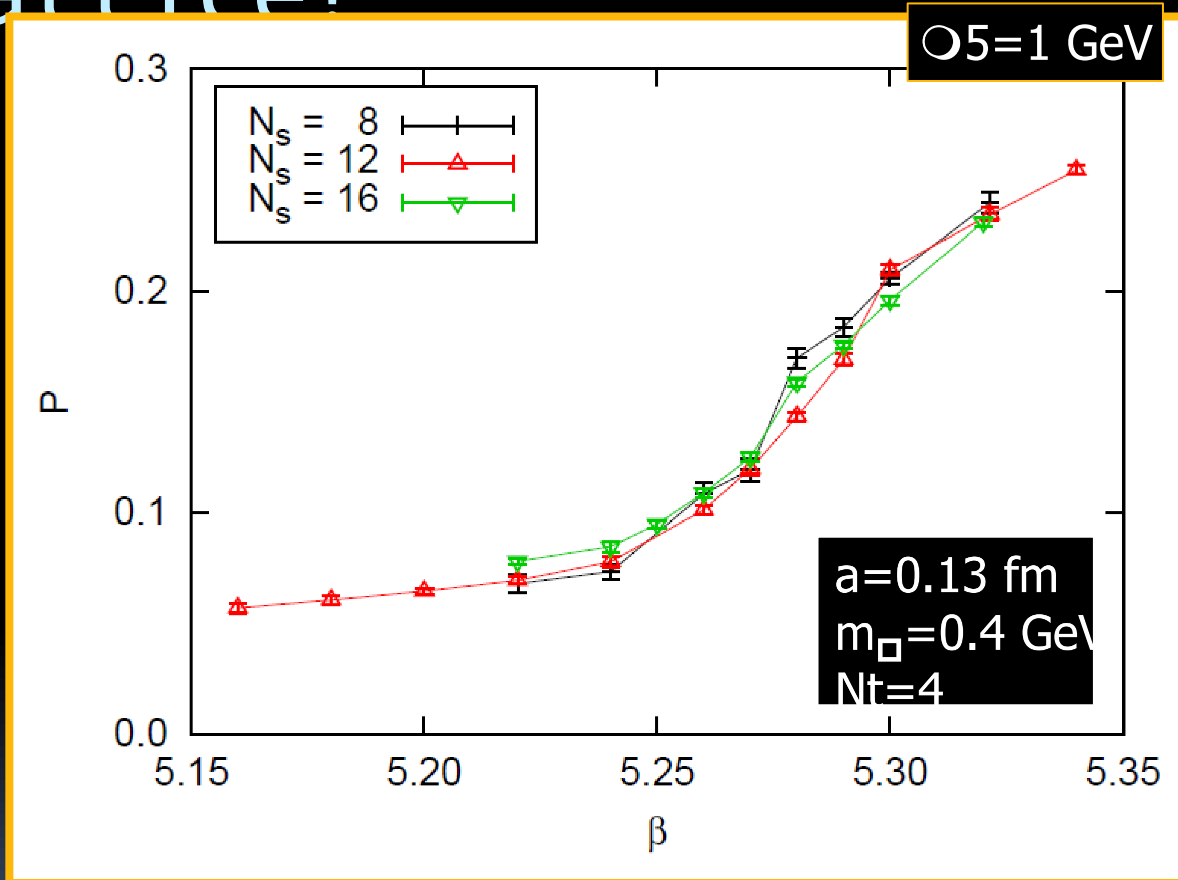
$$F_- = 1 + 3Le^{-\beta(\omega_s-\mu)} + 3L^\dagger e^{-2\beta(\omega_s-\mu)} + e^{-3\beta(\omega_s-\mu)}$$

$$F_+ = 1 + 3L^\dagger e^{-\beta(\omega_s+\mu)} + 3Le^{-2\beta(\omega_s+\mu)} + e^{-3\beta(\omega_s+\mu)}$$

Statistically confining distribution functions

A. Yamamoto, **arXiv:1105.0385 [hep-lat]**

What do we know from Lattice?



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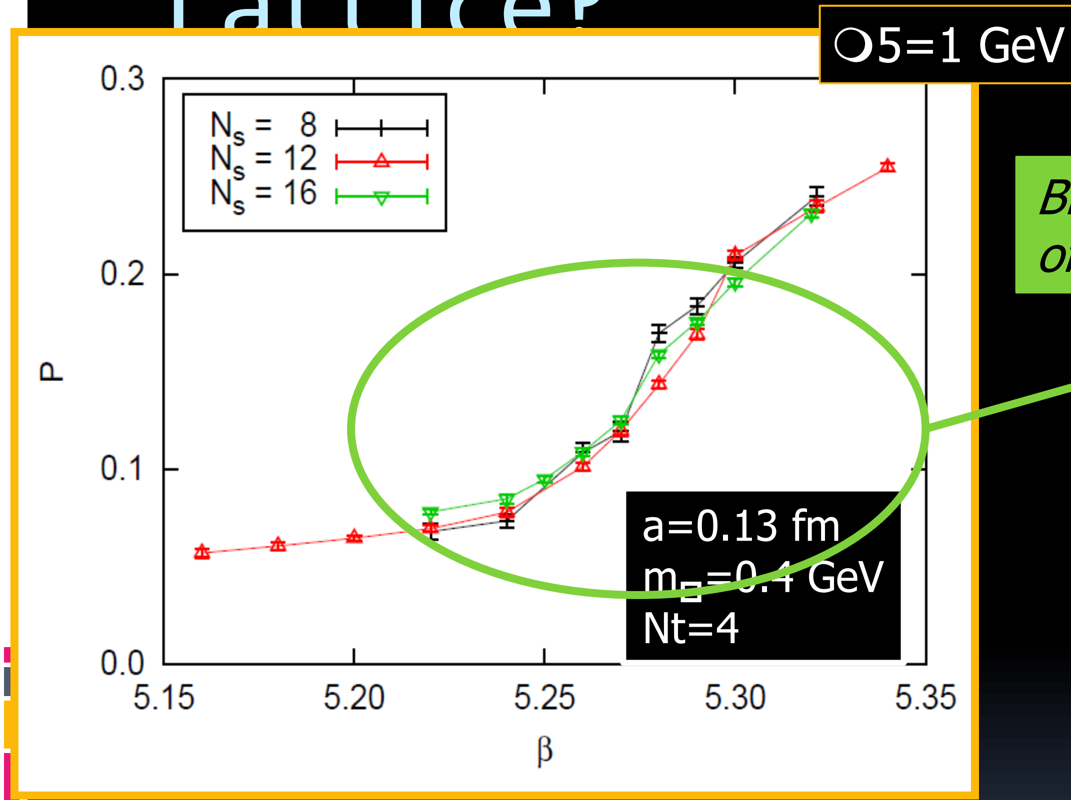


Broad crossover instead of the expected 1st order transition

Speculation

If the result is confirmed with finer lattices and with the physical pion, the crossover could be interpreted as the smoothed phase transition due to inhomogeneous phases.

Phase Diagram: Model Calculation

The effective potential for the Polyakov loop:

$$V = \mathcal{U}(L, L^\dagger, T) + \frac{\sigma^2}{G} - N_c N_f \sum_{s=\pm 1} \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} \omega_s - \frac{N_c N_f}{\beta} \sum_{s=\pm 1} \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} \log\left(F_+ F_-\right)$$

Thermodynamic Potential

$$\mathcal{U}[\Phi, \bar{\Phi}, T] = T^4 \left\{ -\frac{a(T)}{2} \bar{\Phi}\Phi + b(T) \ln\left[1 - 6\bar{\Phi}\Phi + 4(\bar{\Phi}^3 + \Phi^3) - 3(\bar{\Phi}\Phi)^2\right] \right\}$$

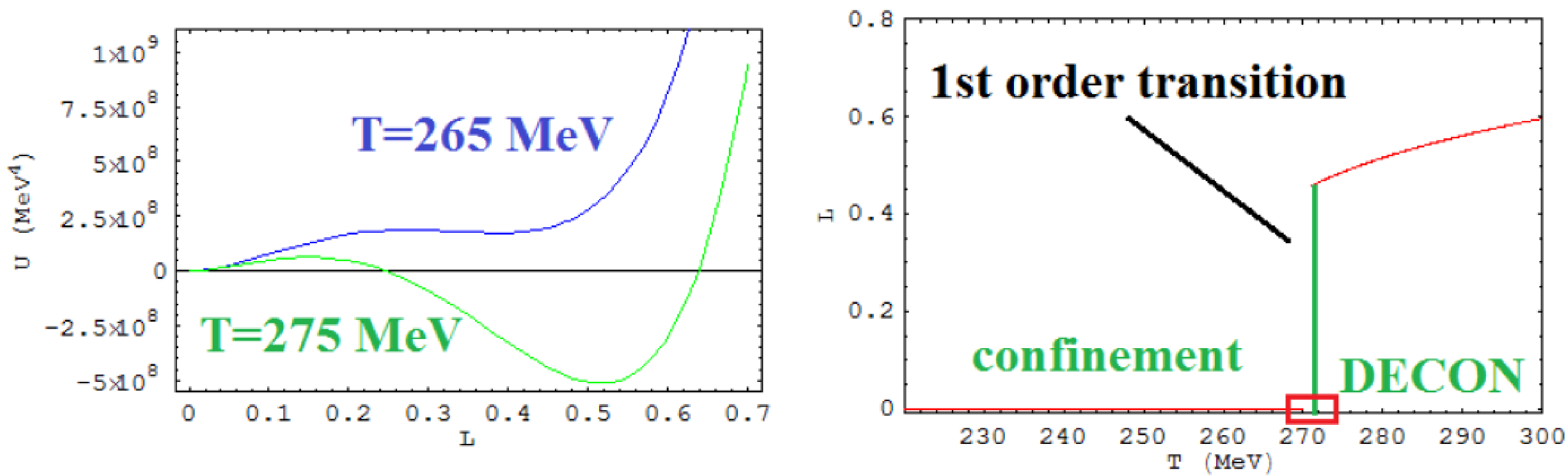
W. Weise *et al*, **Phys.Rev.D75:034007,2007**

W. Weise *et al*, **Phys.Rev.D75:034007,2007**

Polyakov loop effective potential

$$\mathcal{U}[\Phi,\bar{\Phi},T] = T^4\left\{-\frac{a(T)}{2}\bar{\Phi}\Phi + b(T)\ln\left[1-6\bar{\Phi}\Phi+4(\bar{\Phi}^3+\Phi^3)-3(\bar{\Phi}\Phi)^2\right]\right\}$$

Expectation value of L: identified with the global minima of the effective potential.



Generating chirality in QCD

Gluon configurations with winding number

Ward identity in QCD:

$$(N_L - N_R)_{+\infty} - (N_L - N_R)_{-\infty} = 2Q_W$$

with $Q_W \equiv$ winding number of a background gluon configuration:

$$Q_W = \frac{g^2}{32\pi^2} \int d^4x F \cdot \tilde{F}$$

If in a region of space there is a gluon configuration with $Q_W \neq 0$, this will cause the chirality of quarks to change.

- Perturbative QCD: only $Q_W = 0 \rightarrow$ absence of chirality change
- Non-perturbative QCD: classical gluon configurations with $Q_W \neq 0$ can give contribution to physical quantities

Generating chirality in QCD

Connecting winding number to Chern-Simon number

- Pure gauge $SU(3)$ theory: energy minimized by pure gauge configurations
- In the gauge $A_0 = 0$: $A_i(\mathbf{x}) = ig^{-1}U(\mathbf{x})\partial_i U^\dagger(\mathbf{x})$, with $U(\mathbf{x}) \in SU(3)$
- Each vacuum configuration can be labelled by an integer number:

$$N_{CS} = \frac{1}{24\pi^2}\int d^3x\ \epsilon^{ijk}\,\mathrm{Tr}\left[(U^\dagger\partial_i U)(U^\dagger\partial_j U)(U^\dagger\partial_k U)\right]$$

- The different vacua are separated by energy barrier of order Λ_{QCD}
- Gauge field configuration with $Q_W \neq 0$ interpolates between two vacua:

$$Q_W = N_{CS}(t = +\infty) - N_{CS}(t = -\infty)$$

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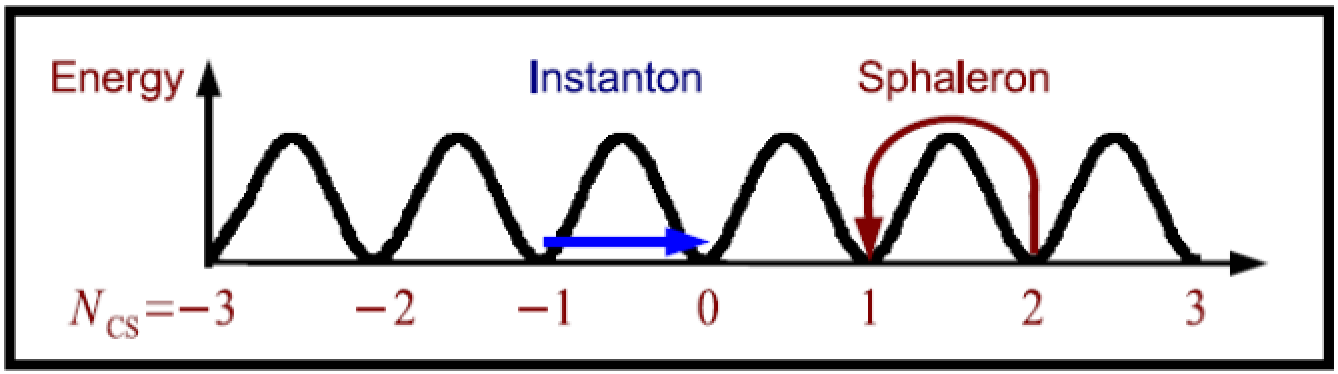
Generating chirality in QCD

Energy Landscape, Instantons and Sphalerons

$$Q_W = N_{CS}(t=+\infty) - N_{CS}(t=-\infty)$$



- Instantons: tunneling between two different vacua.
- Sphalerons: hopping over the barrier.

Transition rate via sphaleron: from Lattice (Moore, 2000):

$$\Gamma = \frac{dN}{d^3x dt} \propto \alpha_S^5 T^4$$

See also Moore and Tassler, **JHEP 1102 (2011) 105**

M.R., **Phys. Rev. D84 (2011)**

Statistical Confinement

$$F_- = 1 + 3Le^{-\beta(\omega_s-\mu)} + 3L^\dagger e^{-2\beta(\omega_s-\mu)} + e^{-3\beta(\omega_s-\mu)}$$

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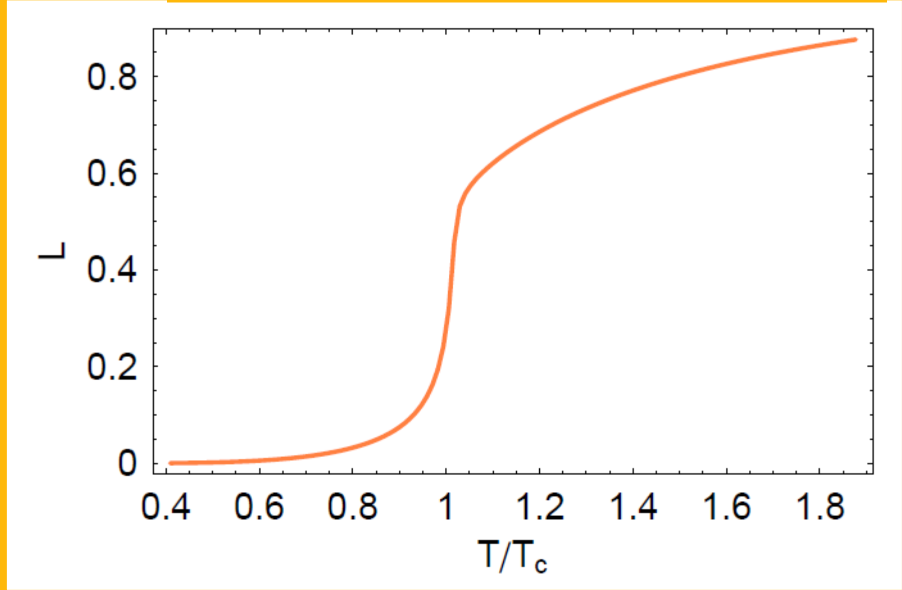
1-quark

2-quark

3-quark

Polyakov loop expectation value



M.R., **Phys. Rev. D84 (2011)**

Statistical Confinement

$$F_- = 1 + 3 \ldots \beta(\omega_s-\mu) + 3 \ldots 2\beta(\omega_s-\mu) + e^{-3\beta(\omega_s-\mu)}$$

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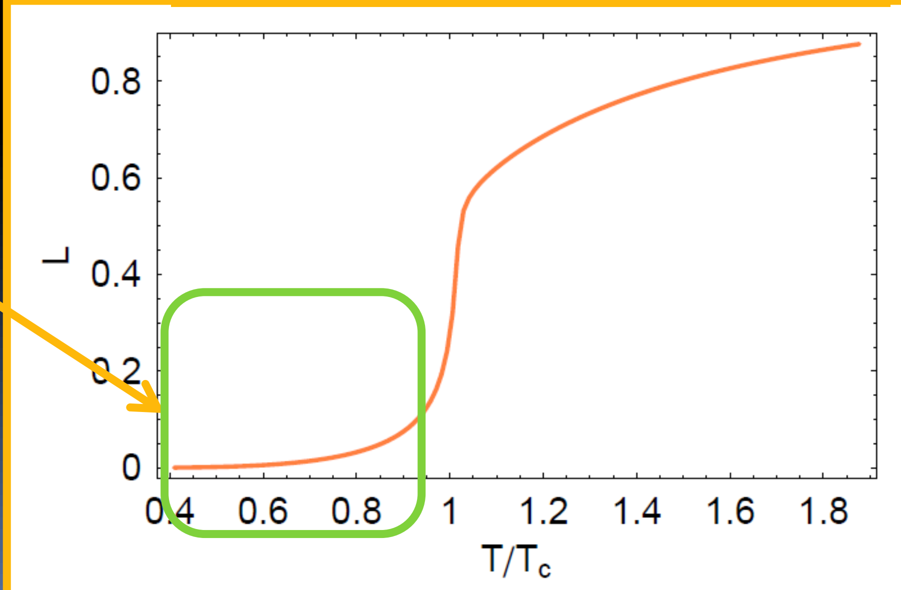
1-quark

2-quark

3-quark

Confinement phase: L=0 (approximately)

Polyakov loop expectation value

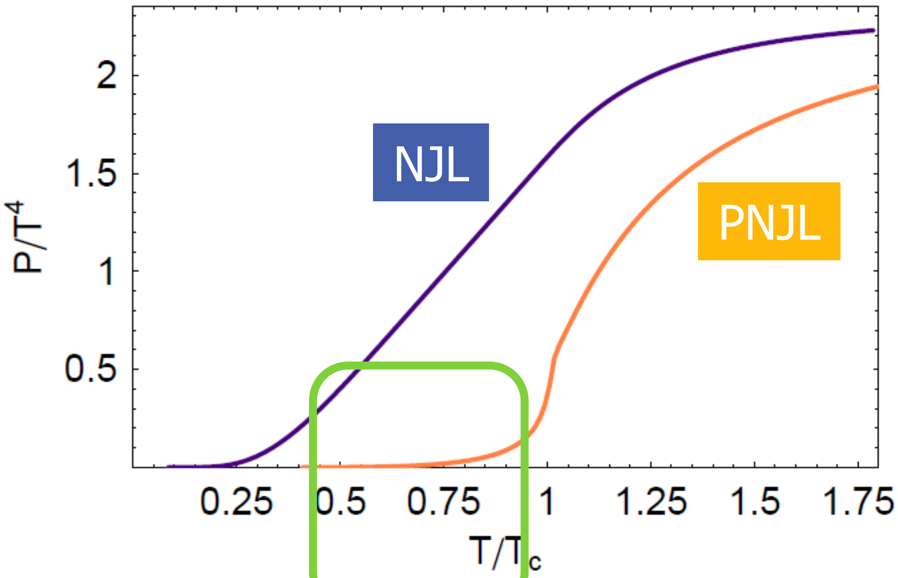


The colorless 3-quark states give the main contribution to the thermodynamic potential in the confinement phase.

M.R., **Phys. Rev. D84 (2011)**

Statistical Confinement

Pressure

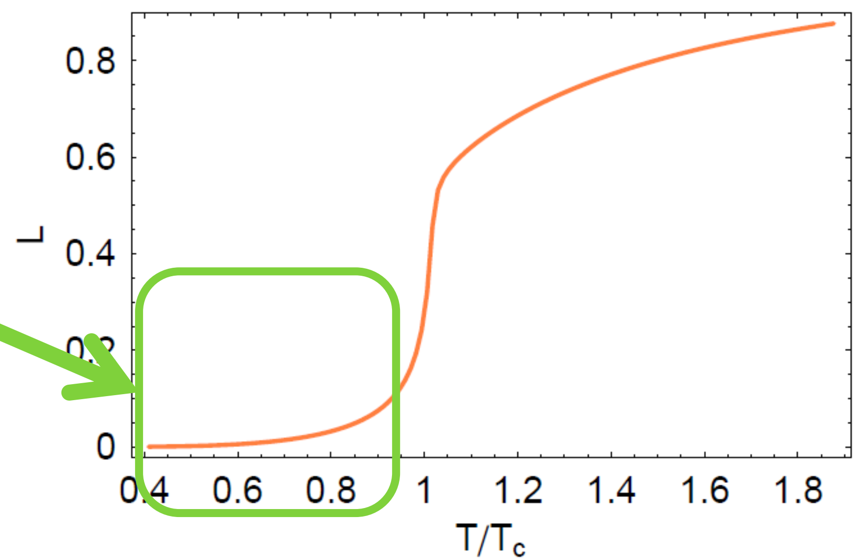


Suppression of colored states

↓

Thermal suppression of pressure

Polyakov loop expectation value



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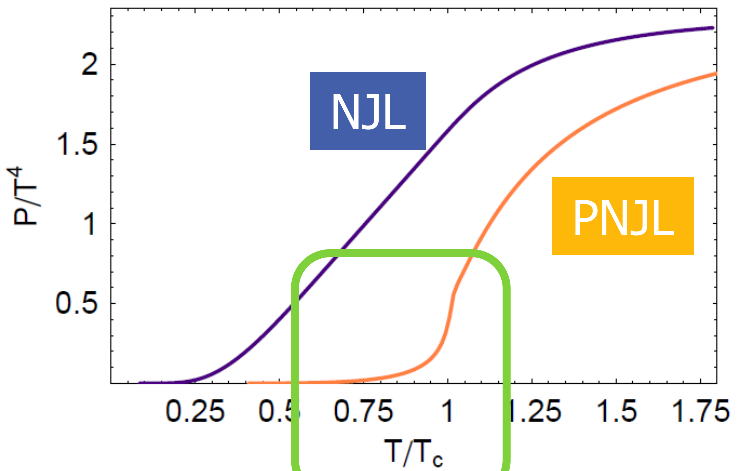

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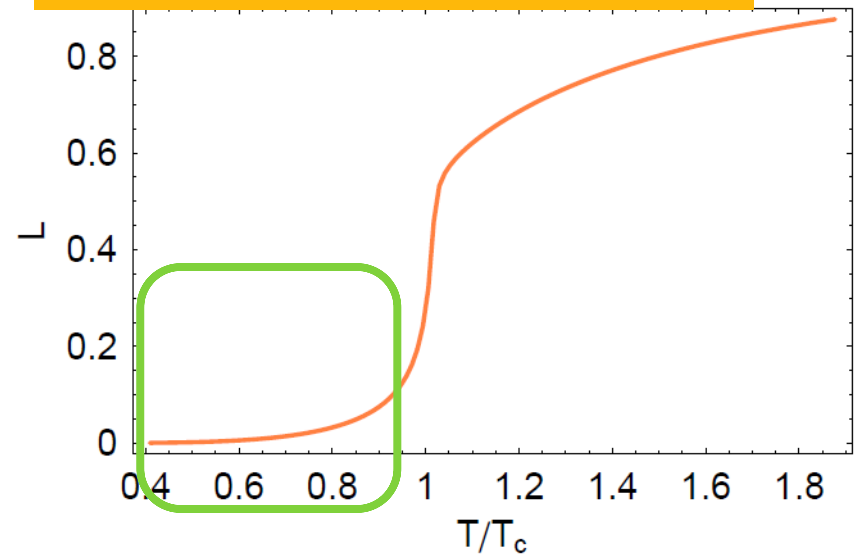
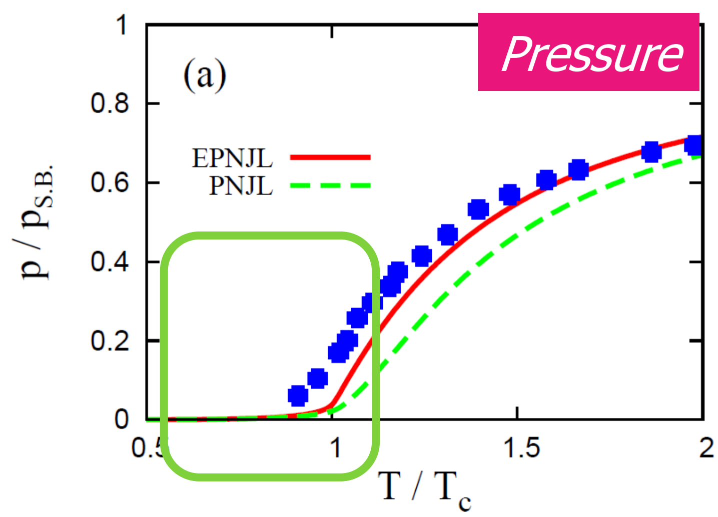


Pressure



Qualitative agreement with Lattice data

Picture from:
M. Yahiro *et al.*, **arXiv:1104.2394 [hep-ph]**
Lattice data from:
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PNJL offers a better description of finite temperature QCD than NJL

M.R., **Phys. Rev. D84 (2011)**

Statistical *De*-confinement

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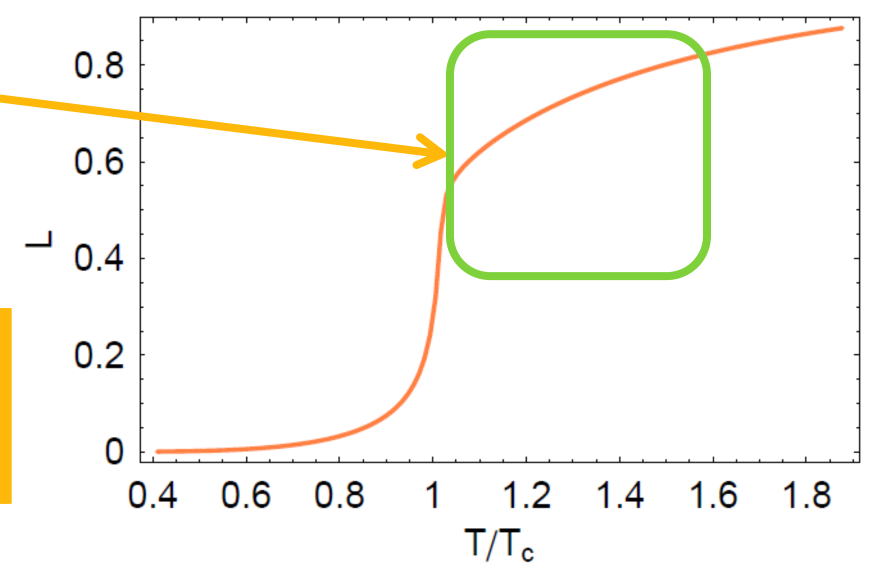
1-quark 2-quark 3-quark

Deconfinement phase: L>0

Polyakov loop expectation value



1- and 2-quark states are liberated

The colored 1-quark and 2-quark states give a finite contribution to the thermodynamic potential in the deconfinement phase.

M.R., **Phys. Rev. D84 (2011)**

Statistical *De*-confinement

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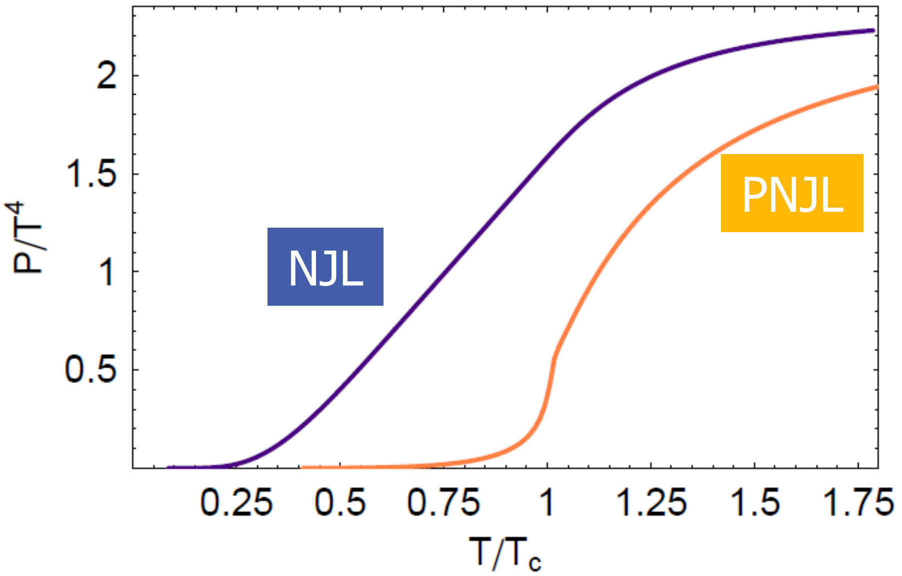

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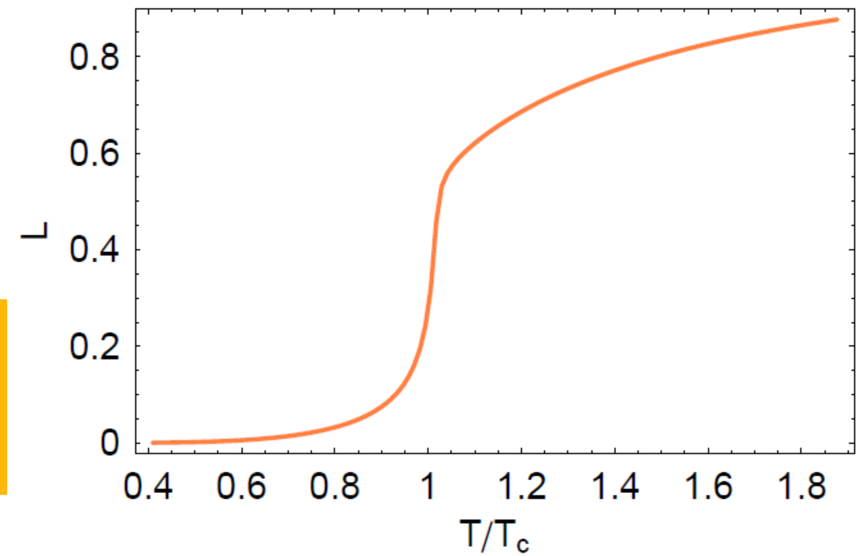

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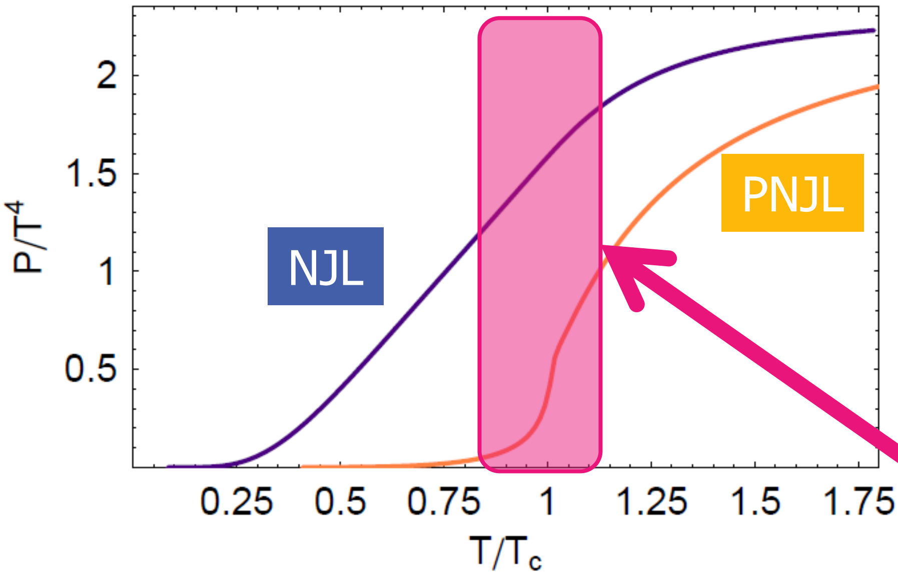

Thermal growht of pressure in correspondence of the crossover

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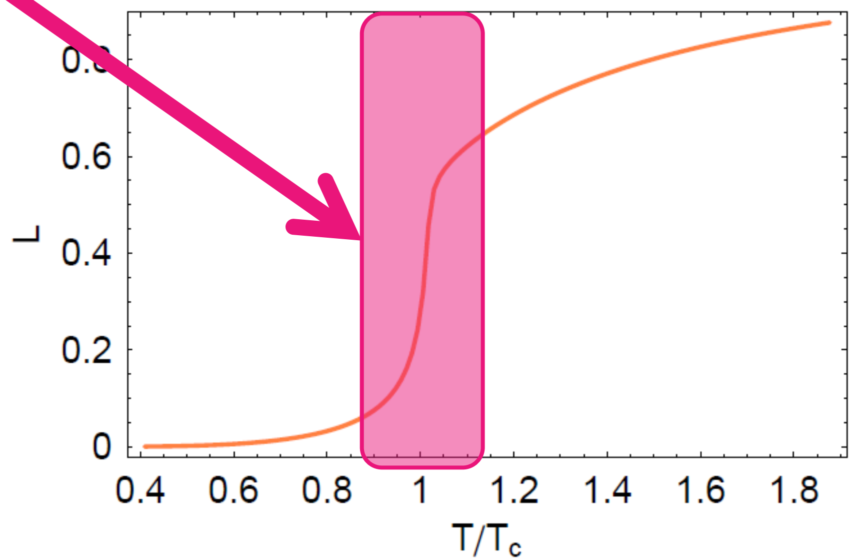

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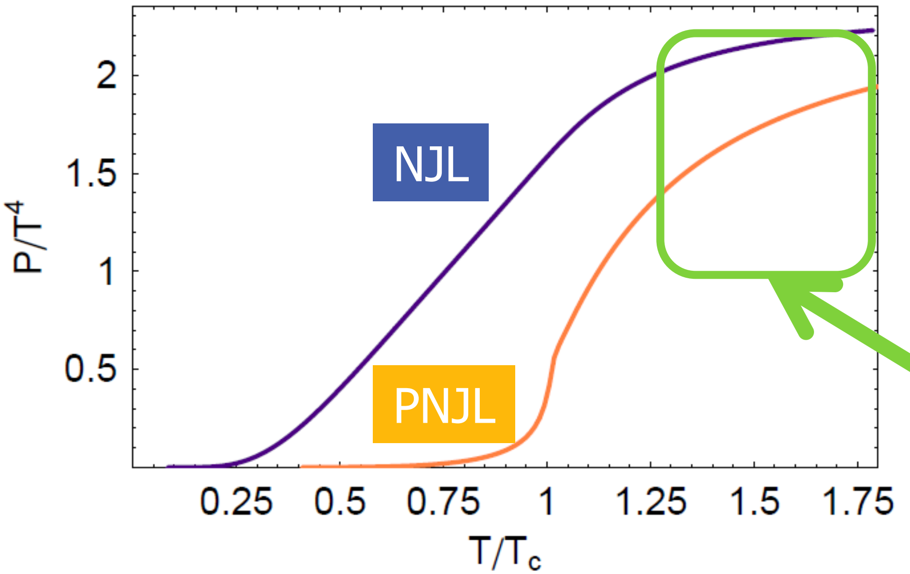

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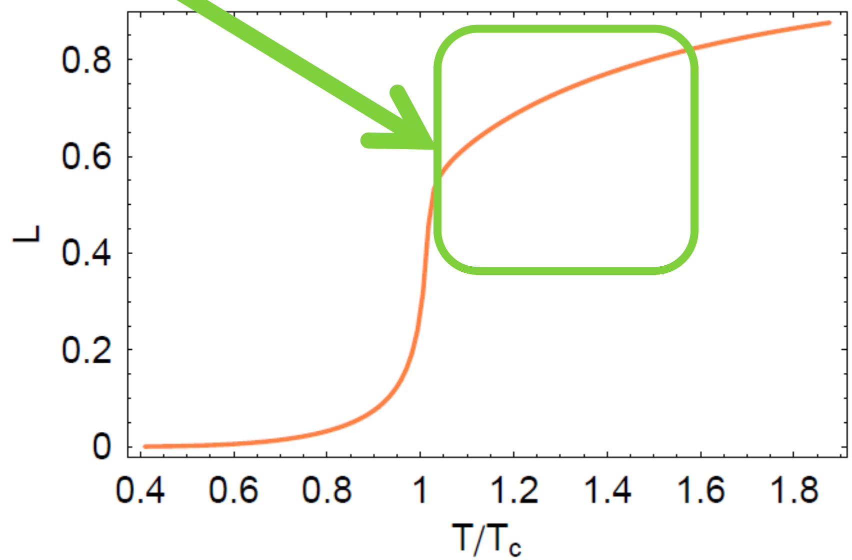

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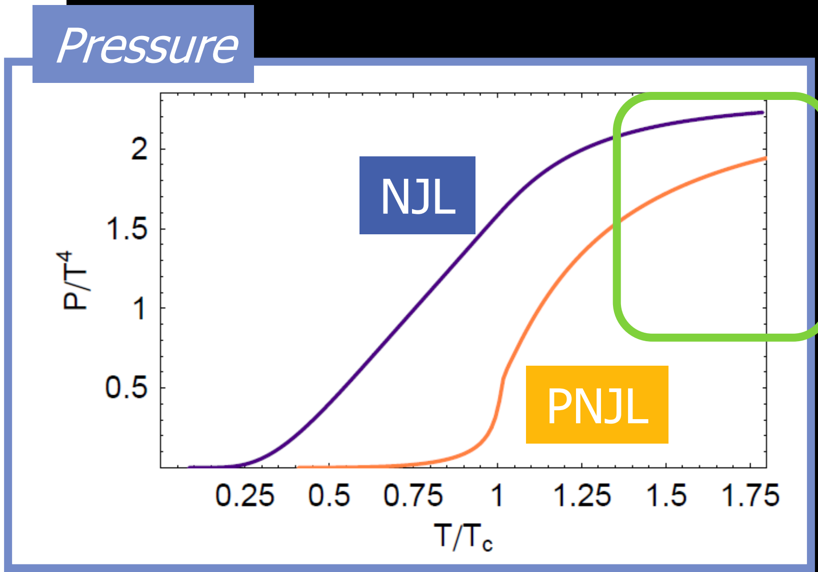

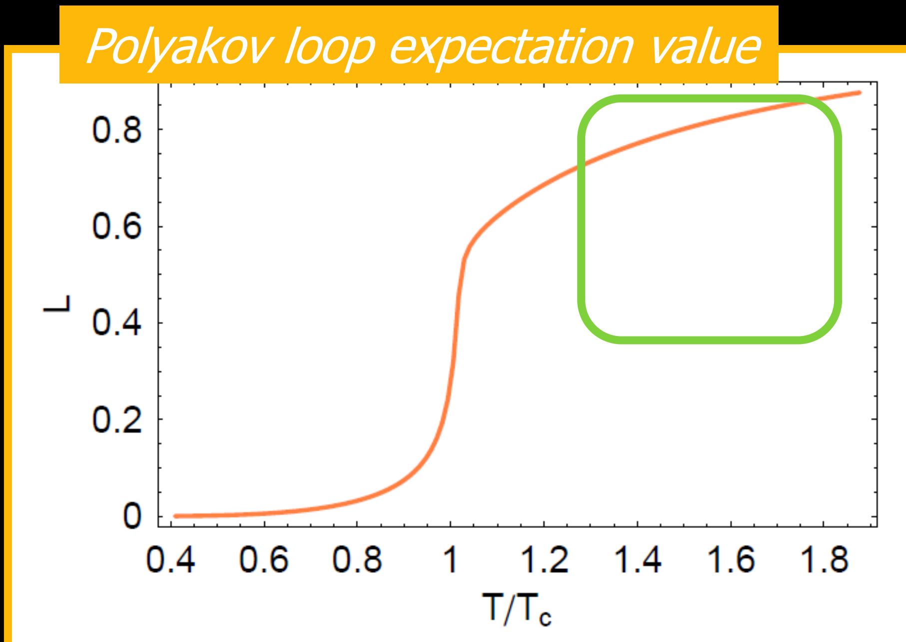

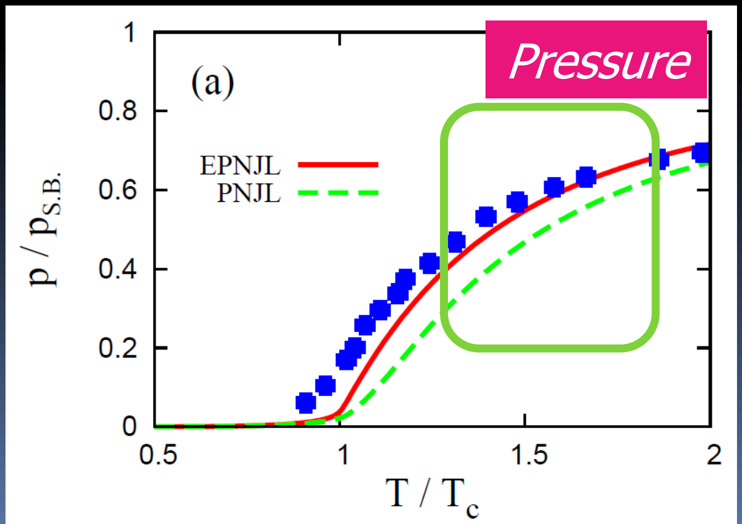

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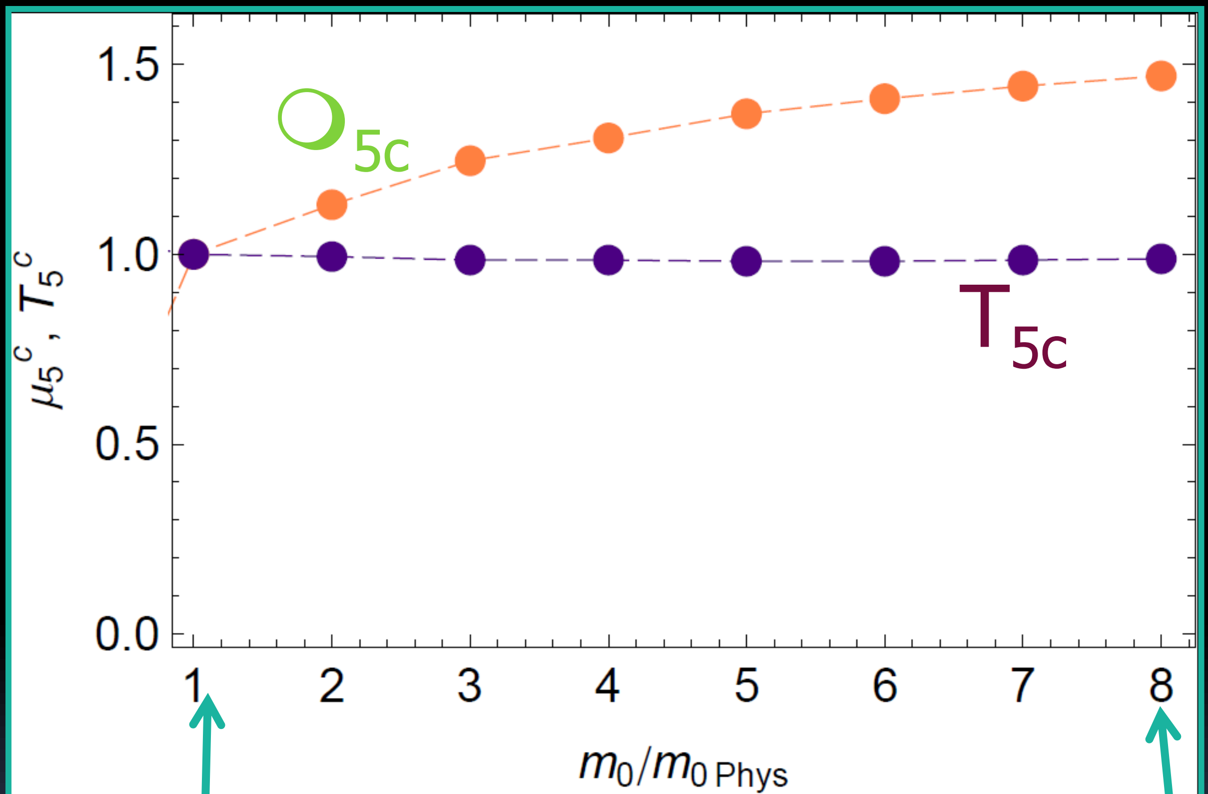
M.R.: WiP

Quark Mass Dependence of CP5



m□=139 MeV

m□=400 MeV