Yang-Mills Superfluidity as On-going Story

General idea:

from 4d confinement at $T < T_c$

to 3d superfluidity at $T > T_c$

Based on papers with Henri Verschelde

Outline of the talk

- Phenomenological motivation

- Looking for implemetation of the standard theory of superfluidity

- Looking for modifications of the standard theory of superfluidity

Why Superefluidity at all?

Favored by phenomenology

three basic features:

- Equation of state close to ideal gas

- Low viscosity-to-entropy ratio η/s
  close to ideal liquid
  (and opposite to the ideal gas)

- $\eta/s \;\approx\; the\ uncertainty-principle\ limit$

Quantum liquid, similar to Helium II

Avoiding contradictions

through Two-components

Weight factors $c_1 + c_2 = 1$

- To explain EoS: $c_1 \approx 5c_2$

- To explain viscosity

$$\eta_2 \ll \eta_1, \quad \frac{1}{\eta_{tot}} = \frac{c_1}{\eta_1} + \frac{c_2}{\eta_2}$$

- Quantum nature is explained by superfluidity

Get phenomenology rather naturally

Standard Two-Component Model

$$T^{\nu\sigma} = (\epsilon + p)u^\nu u^\sigma + \frac{\rho_s}{\mu}\partial^\nu\phi\partial^\sigma\phi$$

$$J^\nu = \rho_n + \frac{\rho_s}{\mu}\partial^\nu\phi$$

$$u^\nu\partial_\nu\phi + \mu = 0$$

where $\rho_n + \rho_s = \rho_{tot}$
μ is chemical potential,
u^ν is 4-velocity,
ϕ is gapless field

Matching to non-relativistic
is not smooth actually

In nonrelativistic case, number of particles conserved (no annihilation graphs)

In relativistic case need 3d spontaneous breaking of a new symmetry

In more detail:

$$< \phi_{3d} > \;\neq\; 0 \;. \quad \phi^*_{3d} \neq \phi_{3d}$$

$$\partial_i^2 \theta \;=\; 0 \quad \phi_{3d} \;=\; |\phi_{3d}| e^{i\theta}$$

$$\partial_t \theta \;=\; \mu$$

No such symmetry in YM?

Stringy quantum numbers

Amusingly enough, stringy models of YM do provide examples of such ϕ_{3d}

The first example: Thermal scalar

New quantum number: wrapping around Euclidean (compact) time direction
Thermal scalar becomes massless
at the Hagedorn phase transtion

Modern holography (cigar-shaped geometry): massless 3d field at $T = T_c$

Digression on holographic models

Holographic model, in the same universality class as YM is known in the infrared,

$$d \gg \Lambda_{QCD}^{-1}$$

Strictly speaking, applies only
in the hydrodynamic limit

Reservations:

- Fails in the UV: has an extra compact dimension

- May apply only to " non-perturbative" physics, whatever it means

Static strings

Known since long: near the phase transion
one long static string dominates
the partition function in Euclidean

This is true
in modern holographic models as well

Non-perturbative defects
(monopoles, vorteces) on the lattice
become time-oriented, i. e. static

A 3d slice of a static string becomes
percolating 3d trajectory, or 3d condensate

Strong lattice evidence in favor of
a 3d scalar field condensed

Summary to part II

- Lattice and stringy models agree on static strings

- Evidence for a 3d condensate crucial also for superfluidity

- Is there massless 3d Goldstone?

- Should we break with the standard scheme since $\mu = 0$?

Massless 3d scalar

Where to look for the scalar?

$$G_R^{ij} \;\equiv\; i \int d^4x e^{-ikx} \theta(t) < |T^{0i}(x), T^{0j}(0)| >$$

In the limit $\omega \equiv 0$, $\mathbf{k} \to 0$

there is a pole sensitive exclusively
to the superfluid component:

$$\lim_{\mathbf{k}\to 0} G_R^{ij} \;=\; \frac{k^i k^j}{\mathbf{k}^2} \mu \rho_s$$

Well suited for the lattice but

Apparently no pole is seen (H. Meyer)

Exotic liquid with vanishing density?

Even if we find a topological quantum number (wrapping, e.g.)

QGP would be neutral with respect to it

Thus: $\mu = 0$

But then: $\rho_{superfluid} = 0$

We come to the idea that

there are alternative descriptions for relativistic superfluidity

Holographic liquid, $\rho = 0$, (1103.3022)

Holographic liquid dual to Rindler vacuum

Scalar field with the action

$$S \; = \; T \int d^4x \sqrt{-\gamma} \sqrt{-(\partial\phi)^2}$$

where $\gamma_{ab} dx^a dx^b \; = \; - r_c d\tau^2 + dx_i dx^i$

In this approximation

$$T^{\mu\nu} \; = \; diag(0, \; p, \; p, \; p)$$

Dissipative corrections introduce $\eta/s = 1/4\pi$

An alternative supefluidity?

Features of the $\rho = 0$ liquid

- Similar to the liquid seen on the surface of black hole (streched horizon)

- In the approxamation considered entropy $s = 0$

- The ratio $\eta/s = 1/4\pi$ is a kind of minimal

- In the limit $r_c = 0$ metric becomes 3d. Minkowskian analogy to the Euclidean staticity

- Pole in the $< T^{0i}, T^{0j} >$ correlator persists

- Clearly, corresponds to static 3d branes

No less superfluid than any other holographic liquid

'New' superfluid vs phenomenology

On positive side:

Contribution of the vorteces to EoS was measured separately on the lattice

$$(\epsilon - 3p)_{non-pert} < 0 \qquad \textit{and big numerically}$$

in agreement with the picture above

On the negative side:

No pole in the $< T^{0i}, T^{0j} >$

seems to exist contrary to the picture above

Summary to part III

- Alternative descriptions of relativistic superfluid might exist

- $\epsilon = 0$ liquid as analytical continuation of $3d$ condensation from Euclidean space

- Phenomenology rather controvesial

Massless states from hiolography

Branes static classically start to fluctuate in extra dimensions quantum-mechanically. Results in massless states on the branes (Luscher term is the simplest example)

In our case, these fields unphysical:

- in 4d case, $T < T_c$ seems to be Kogut-Susskind ghost
- in 3d case, $T > T_c$, this is a superluminal ecitation

Both probably are removed by pert. th. ignored so far

Conclusions

- To answer the question, whether QGP could be supefluid, need to settle first some theoretical issues

- So far, phenomenologically holography looks rather attractive than not

- latest development: non-pert phjysics is not unitary by itself (as knew already from the Kogut-Susskind ghost)

- Further understanding is hopefully imminent